Efficiency vs. Flexibility in Public-Private Partnerships*

Thomas W. Ross** and Jing Yan***

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** UPS Foundation Professor of Regulation and Competition Policy, Sauder School of Business, University of British Columbia

*** Ph.D. Candidate, Sauder School of Business, University of British Columbia
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Abstract

Public-private partnerships (PPP) have become an increasingly popular way for governments to procure for their citizens certain public services (e.g. roads, hospitals, schools, prisons etc.). This paper models a basic trade-off associated with a government’s decision whether to use a PPP model or more traditional public procurement approaches to procure public services: while PPPs marshal the power of competitive markets and stronger incentives to lower the costs of producing those services, they also involve long-term contracts that may prove relatively inflexible. While, *ex ante*, governments can expect competition to control private bidders’ prices, after the contract has been signed any changes will have to be negotiated in bilateral bargaining which will almost certainly raise the costs to government. The paper shows that the optimal choice between PPP and public procurement depends on a number of factors including the likelihood that changes will be necessary, the productivity of non-contractible effort exerted by private sector partners, the costs of switching, the difference between first best and second-best projects, and the bargaining power of governments vis-à-vis private parties. It also shows that the optimal choice may depend on whether the government’s objective is to maximize “value for money” (i.e. get the right project delivered for the lowest cost to taxpayers) or to maximize total social surplus.

Keywords: Public-Private Partnerships; Contracting-Out; Providing Public Services
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I. Introduction

Continuing a movement that had earlier involved the privatization of many state-owned firms, the deregulation of major industries and the more widespread use of contracting-out,¹ in the early 1990s a number of governments began to experiment with another innovation involving expanded use of the private sector in the delivery of public services: public-private partnerships (PPP).² A large and diverse set of definitions of PPPs exist today, but one that conveys the main idea simply defines a PPP as “a cooperative venture between the public and private sectors, built on the expertise of each partner, that best meets clearly defined public needs through the appropriate allocation of resources, risks and rewards.”³

The increased private involvement in the provision of public sector infrastructure and related services has become a global phenomenon. Yescombe (2007) describes major PPP programs and projects in several countries in Europe, North America, Asia, Australia and Africa. Even middle and lower income countries have come to use various forms of the PPP model for the provision of public services.⁴ Data from the Infrastructure Journal suggests that major PPP projects with a combined value of US$394 Billion have been undertaken world-wide since 2005.⁵ In both higher and lower income countries, common industries for this kind of private

¹ On privatization and deregulation generally see, for example, Viscusi et al. (2005). With respect to privatization in particular, see, e.g., the work by Jones et al. (1990) and a survey of empirical work by Netter and Megginson (2001). For an introduction to the literature on contracting out, see Domberger and Rimmer (1994), Domberger and Jensen (1997) and Hodge (2000).

² An early version of PPPs was the Private Finance Initiative (PFI) pioneered in the U.K. from the early 1990s.

³ Canadian Council for Public Private Partnerships: http://www.pppcouncil.ca/resources/about-ppp/definitions.html

⁴ The World Bank and the Public-Private Infrastructure Advisory Facility (a multi-donor technical assistance facility financed by a number of national and multi-national development agencies) have jointly created the Private Participation in Infrastructure Database (at http://ppi.worldbank.org) which contains data on more than 4,600 infrastructure projects in 137 low and middle-income countries.

participation included those related to power, transportation, water and telecommunications. Various kinds of social infrastructure such as hospitals, prisons and schools are common as well.

In a typical PPP, the private (generally for-profit) partner will assume responsibility for some parts of the delivery of a public service, for example, the design, construction and maintenance of a new hospital. Responsibility for the remaining parts, for example, the provision of medical services in that hospital, will stay with the public sector partner. Very long-term, and frequently complex, agreements govern the relationship between the private and public partners.

Advocates of the PPP model argue that such arrangements, by giving a larger set of tasks to the private sector and subjecting their provision to competitive bidding, will unleash the superior power of competitive, for-profit enterprises to minimize costs and to find innovative approaches to addressing social needs.

Critics of the PPP model have pointed to a number of potential problems with this model of public sector procurement. One of these – our focus here – is the loss of flexibility that comes with the long-lived contractual obligations governments must respect when changing circumstances may require significant changes in the way the public service is provided.

This suggests a trade-off faced by governments choosing whether to proceed with a project as a PPP or by traditional public procurement: while PPPs might have the potential to generate substantial productive efficiencies, they may limit a government’s ability to react to changing demands for the public service. This trade-off is the subject of this paper. Our model allows us to illustrate the trade-off very simply and to explore various factors that will influence that trade-off. We can show that the optimal choice of procurement mode will depend on the exact nature of the government’s objective function. In this regard we study two commonly proffered objectives: (i) minimizing the cost to taxpayers (or users) of the service and (ii) maximizing total social surplus. We also demonstrate that the relative advantages of the PPP mode will depend on the nature of the PPP contract considered – that is whether the private partner is compensated based on simply having completed the project or based on the actual usage of the facilities.

II. Some Background on PPPs and Related Literature

In a PPP, the private and public partners must come to an agreement as to what elements of service delivery each will contribute, how costs will be covered and how the private partner(s) is
to be compensated for its contributions. Most of the larger, higher profile, partnerships involve the building of new public infrastructure, and the delivery of services using that infrastructure.

Essentially, a PPP involves contracting out at a scale and complexity well beyond what is normal for governments. That said, most governments have had experience with contracting out to some extent, so we might ask what really distinguishes modern PPPs from contracting out. De Bettignies and Ross [2004 and 2011] suggest that three key differences distinguish modern PPPs from standard contracting out: (i) compared to standard contracting out, PPP contracts assign a larger number of tasks on a single project to the private sector. For instance, in the famous ‘FDBOOM’ model, the private sector partner(s) finances, designs, builds, owns, operates and maintains the facility; (ii) PPPs typically bundle multiple tasks in one large contractual agreement between the government and a single contractor typically a special-purpose corporation created by consortium members to develop and operate the project. In contrast, standard contracting out would allocate one task (or part of a task) per contract, and the various contractors would be independent of each other; and (iii) PPPs often involve the privatization of the finance function and some operations function -- tasks that were historically the exclusive preserve of the public sector.

As suggested, the alleged benefits of PPPs derive from their potential to apply the forces of competition and higher-powered incentives to generate higher levels of efficiency and innovation. While the experiences of governments with PPP are varied, the evidence seems to suggest that the PPP model can be successful in the right circumstances.6

However, critics of PPPs point out that these efficiencies, if they exist, come at some cost. First, given the long lives of many PPP agreements (which can last 50 years or longer) the bidding and contracting process is very complicated and expensive, and the long projects require on-going monitoring of the private partner by the public, all suggesting the potential for significantly higher transactions costs with this form of procurement.7

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6 There has not been enough work done to properly evaluate the success of PPP projects ex post, in part because these are generally very long-lived – and on-going – agreements; and a full accounting cannot really be done until the agreements have expired. The UK was an early adopter of the model and some work by the National Audit Office there suggests some success with the private finance initiative (PFI) version of the PPP model – there are many NAO reports at [http://www.nao.org.uk/](http://www.nao.org.uk/). Australia has also been a leader in the use of the PPP model, and there is evidence of success there as well, e.g. see Infrastructure Partnerships Australia (2009).

7 On this see, e.g. Boardman and Vining (2004) and Daniels and Trebilcock (1996).
argued that some key elements of service quality may be very difficult to enforce by contract if measurable, verifiable metrics are elusive.8

Third, many have argued that upon entering into long-term partnership arrangements governed necessarily by complicated and detailed contracts, governments lose control of key aspects of service delivery that they may wish to adjust in the future. To make changes requires negotiation and not only will the renegotiation process involve a new set of transactions costs, it will involve bargaining without competition on the selling side. This paper is about this third challenge and how the government might trade off flexibility for efficiency.

While there have been a large number of case studies, consulting reports and government agency reports on PPP, there has been relatively less work done by economists, particularly economic theorists.9 Some early research considered the question of whether or not PPPs can dominate public procurement methods if quality is not completely verifiable and contractible.10 A larger number of papers have focused on the optimality of “bundling” of the various tasks so that one private partner tackles multiple tasks (e.g. building and operating) rather than having each task performed by a single independent private partner.11 While, to some researchers, it is the bundling of tasks that defines a PPP, our focus is on the “privatization” aspect of PPPs – that they involve the outsourcing of tasks that might otherwise have been performed by public sector agencies and their employees.12

We investigate a different type of contractual incompleteness arising as a result of uncertainty regarding future demands for the public goods or services. Given the very long lives of some of these contracts, it is impossible to anticipate and optimally prepare for every potential contingency. PPP contracts may therefore need to be revisited and renegotiated – a potentially costly process. In this sense, we say PPP agreements lack flexibility. The flexibility question strikes us as particularly important in dynamic industries such as healthcare, where the kind of

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8 This was a concern, for example, of Hart et al (1997) and Hart (2003) with respect to prison projects.
9 For a recent review of theoretical work by economists on PPPs see de Bettignies and Ross (2011).
10 See, for example, Hart et al. (1997).
11 For example, see Benz et al. (2002), Hart (2003), Bennett and Iossa (2006), and Iossa and Martimort (2008).
12 Not surprisingly, it is this aspect of PPPs that have led to their being strongly opposed by public sector unions. For example: “The labour movement is opposed to privatization, including public-private partnerships, because it undermines both the values and ethos of the public sector, and the goal of enhancing the public good.” Canadian Labour Congress, 2011, Document No. 8, 24th Constitutional Convention. Or see “The case against PFI” from the website of the largest public service union in the UK (UNISON): www.unison.org.uk/pfi/caseagainst.asp.
hospital services appropriate today may not closely resemble the services we will be demanding in twenty or fifty years, given advances in medical knowledge and technologies. On the other hand, flexibility may be less important for other kinds of projects, such as roads and bridges.

A great deal of experience suggests that flexibility challenges are very real in PPP delivery. A review of changes made in PPP type projects in the United Kingdom by the National Audit Office (2008) concluded that changes undertaken in UK projects in 2006 involved extra payments to contractors of approximately £180 million. While much of this spending provided additional value to taxpayers and users, the NAO noted that “several components of the cost of changes were problematic” (p.5). The report cites some examples where changes were not smoothly and cost-efficiently managed, including the Blackburn Hospital Project.

While many writers have mentioned the potential costs to the public sector of lost flexibility in lengthy PPP contracts, there has been very little formal modelling to our knowledge. Two papers consider flexibility questions that are different from the one examined here. In “Case 3” of Iossa and Martimort (2008), the authors are principally focused on the

13 In the UK, PPP (or PFI) are not recommended for the provision of information technology services, in part because of the high probabilities and costs of changes. See Yescombe (2007, p. 27). From H.M. Treasury (2006, p. 32): “...the PFI procurement structure is unlikely to deliver value for money....where authorities require a significant degree of short-term flexibility due to fast-changing service requirements. It is for this reason and from the evidence of past projects that PFI is not used by the Government for information technology projects...”

14 Making the point that the surrendering of decision-making authority to the private sector in PPPs makes it more difficult for the public sector to adapt to changing demands for public services, see, e.g. OECD (2008, pp 65-69), Yescombe (2007, section 2.12) and PricewaterhouseCoopers (2005, Chapter 2). In recent proceedings about Private Finance Initiatives, the UK Commons Treasury Committee has heard arguments from Members of Parliament about “the ‘inflexible’ nature of PFI contracts, arguing it is locking the public sector into long-term contracts that do not allow changes in terms, if and when it becomes clear that the terms of the contract are inappropriate.” (Partnerships Bulletin, June 14, 2011 at www.partnershipsbulletin.com.)

15 From page 8 of the NAO report: “Under PFI, almost any requested change, even as small as a new electrical socket, has to be processed through the SPV as it manages the asset during the contractual period and bears the risk of failing to meet service obligations. Often lacking the option of going to a different supplier, even for major changes, there is a risk that the public sector will have reduced leverage in negotiation and that the SPV or FM provider may not be incentivised to keep down the cost of changes or to process them quickly.” This report goes on to list the kinds of changes that come up frequently in long-term PPP contracts.

16 A particularly famous example of inflexibility played out in the British press over attempt by famous chef Jamie Oliver to get government funded schools in Britain to offer better, healthier food to students. Some of these schools were operated as PPPs and, given that the original contracts did not contemplate the provision of healthier and more expensive meals, those contracts had to be renegotiated in what turned out to be a somewhat challenging process. “Private deals block Jamie’s school dinners”, by Felicity Lawrence and Katharine Quarmby, The Guardian, Monday April 25, 2005.
efficiencies of bundling two tasks: building and operating a new facility. They consider a government choosing between sequentially contracting separately with first a builder then an operator (non-PPP approach) versus contracting to one firm (consortium) that will perform both tasks. Information from the building stage might allow a better contract to be drawn with the operator in a non-PPP approach. However, in separating the two tasks, some externalities between them may not be internalized with an associated loss of efficiency.

Athias and Saussier (2010) take the PPP form as the given choice, and ask how much flexibility to introduce into the long-term contract between the public sector and the operator in order to allow adaptation in pricing. Long term contracts that are too flexible risk opportunistic renegotiations trigged by the private sector, while contracts that are too rigid may not allow for efficient adaptation to changing circumstances.17

The paper that is most closely related to the questions addressed here is, in fact, not about public-sector procurement. Bajari and Tadelis (2001), hereafter BT, study private sector procurement, emphasizing an example from the construction industry. As here, they emphasize the trade-off between contracts that provide maximum flexibility to incorporate changes and those that provide the strongest incentives for cost minimization. While there are many parallels between the analysis of BT and that provided here, there are several important points of differentiation. First, our focus on the special problems associated with public-sector procurement provide for a number of new insights. For example: (i) we consider the role played by the actual objective pursued by the public sector – does it simply seek the lowest quality-adjusted costs (as a private buyer would) or does it care about social welfare more broadly defined; and (ii) we study the two most standard types of actual PPP contracts – those that pay the supplier a fixed sum and those in which the supplier is paid via tolls paid by users – to compare them on the extent to which each supports the attainment of the public objectives. Second, our modelling approach is different in a number of ways. For example, we use a less “reduced form” model that, while less general in some respects, provides for a larger set of specific and intuitive comparative statics results on the importance of key parameters. Third, the

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17 These authors go on to examine the amount of pricing flexibility actually incorporated in a multinational set of 71 toll road projects.
source of the inefficiencies that can arise in PPP or fixed-price contracts differs in our two models. Fourth, we model the re-negotiation process differently.\textsuperscript{18}

In what follows, Section III lays out the general model while Section IV studies the two special cases, PPP and public provision, that form the core of our analysis. Section V extends the basic model in three directions. The last section concludes.

III. General Model

We begin by taking a fairly general approach to the problem of a government, “G” (as principal), procuring certain public services (the “project”) from a firm, “F” (the agent). Special cases of this model will correspond to provision via public-private partnerships (denoted PPP) and more traditionally by public agencies (PUB).\textsuperscript{19} The services that G wishes to procure could involve the construction of an important piece of infrastructure plus its operation for many years.

Basic Setup

The (net present value of the) gross social benefit of the project is given by $b_0 > 0$ which is not verifiable or contractible. The government can contract with F to deliver these services. There are in general many identical firms willing to compete to provide these services.

The firm’s cost to deliver the project depends, in part, on the non-contractible level of effort or innovation, $e$, chosen by the firm, and the productivity of that effort\textsuperscript{20}:

$$C = K - \delta e$$

where $K$ is the observable innate cost of the project (including cost of materials, labour, etc) and $\delta > 0$ captures the marginal productivity of the firm’s effort. We assume that $\delta$ is private information of the firm, hence that it is not known by the government, $\textit{ex ante}$. The monetary

\textsuperscript{18}We will highlight the differences between this paper and the analysis of BT at various points below.

\textsuperscript{19}In BT’s private sector procurement model, they consider fixed price contracts which have properties similar to our PPP contract, and cost-plus contracts which have incentive properties similar to our PUB arrangements.

\textsuperscript{20}It is possible that effort is observable. Here we only assume that effort is not contractible, i.e. that its value cannot be demonstrated for a court or any contract enforcement mechanism.
costs of this effort for the firm is $\emptyset(e)$. For simplicity, we assume that these costs can be represented by a quadratic disutility function:

$$\emptyset(e) = \frac{e^2}{2}$$

The non-verifiability of effort will generate familiar moral hazard problems.

Firms wishing to secure the contract to deliver this project will bid competitively. The firm winning the project will be paid a lump sum $\alpha$ by the government and then be responsible for project costs. The firm’s profits are taxed by the government at the rate $t \in [0,1]$. We will explore the consequences of different rates of taxation but assume the level is set by forces outside the model. A firm winning the contract to deliver the project will have after-tax profits given by:

$$\pi = (1 - t)(\alpha - K + \delta e) - \emptyset(e)$$

The original specification of the project comes from government and the government can be right or wrong about this specification. If the government specifies the project correctly, the social benefits will be $b_0 > 0$ as described. However, there is a chance -- with probability $\mu$ -- that the government will have incorrectly specified the project, or that circumstances will change such that a different design is appropriate. We will refer to this as an unexpected change in the nature or level of demand for the services. To suggest a specific example, the project might involve construction of a bridge over a river, and it might become apparent at some point into the project that one lane should be devoted to bicycles (and maybe pedestrians) and not cars.

While the change in demand is public information, we assume that because demand could have changed in so many different ways, contracts cannot practically be made contingent on changes in demand. Adapting to the new design will restore the $b_0$ benefits. However if the project is not changed, a lower social benefit $b_1$ will be realized, where $0 \leq b_1 < b_0$.

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21 In fact, the most interesting cases involve changes in the nature of demand – i.e. the kinds of services to be provided. Uncertainty about simply the level of demand for a project can often be dealt with in carefully-drafted contracts.

22 We are deliberately suggesting an example in which the design change will not obviously greatly change the scope or cost of the project as this fits other assumptions in our model. It would be trivial to extend the model to cases in which the design change significantly expands the scope and cost of the project – and there are many examples of these kinds of changes – but the additional implications derived would be quite obvious given the results we present here.
There are many reasons why projects may need to be re-designed or adjusted. In some cases, demand may have been badly estimated or technological changes may alter the way services should be delivered (common in healthcare, for example). While we will model this adjustment, if it is needed, as occurring before any services have been delivered, this is a modelling device meant only to capture the need for change sometime during the long life of the project.

If demand changes, efficiency dictates that the project adapt (e.g. the bike lane be added). Since this change represents a change in the contract between G and F, it must be negotiated. We model the negotiations using the familiar Nash bargaining model: the parties will bargain over the surplus created by efficiently adapting to the demand changes. We assume that there are real economic costs associated with switching to a new project design (this could include the renegotiation costs) and each of the two parties will bear this cost by paying an amount $s \geq 0$. After the contracts are settled and, if necessary, any design changes negotiated, the firm picks its level of effort, costs are incurred, contracts honoured and payoffs received.

**Timing**

Reviewing the timing of the game then, we have:

1. Government announces it has a project for which it wishes to receive bids.
2. Firms bid for the project, and it is awarded to the firm offering to provide it at the lowest fixed fee, $\alpha_0$.
3. Nature may move to change demand – if no change, proceed to 5.
4. If demand changes, $\alpha_0$ is renegotiated (to $\alpha_1$) via Nash bargaining and the design is changed, both parties incur switching costs $s$.
5. F chooses level of effort, $e$.
6. Benefits are realized and the government honours its contract.

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23 In contrast, BT use a “take-it-or-leave-it” renegotiation model.

24 If this is solely a renegotiation cost, it may seem likely that $s$ would be smaller (or even zero) when the changes can simply be ordered by the government under a traditional public procurement process. We will consider this possibility below.
The objective of the firm in this model is straightforward. It will make decisions to maximize its profits subject to honouring its contracts. The government’s objectives are more interesting. In much of the literature on public-private partnerships, it is argued – and often just assumed – that a PPP will dominate the public procurement method in a particular case if the PPP form provides “value for money” (VFM) relative to traditional methods of procurement. Value for money in this context is typically taken to mean that services of the quality desired are provided at a lower cost to the ultimate payers – taxpayers or (in the case of tolled services) end users. This will be our starting point here: we will look for conditions under which a PPP can provide services at a lower cost to the government.

Value for money as an objective can be contrasted with the objective of maximizing total social surplus (TSS) that is often applied in cost-benefit analysis. The key differences arise due to transfers that will not matter under a TSS standard but that will affect VFM. For example, firm profits will hurt VFM by pushing up the price the government pays for the project, but as simple transfers from taxpayers to firms will not affect TSS. A later section of this paper will consider how the choice between PPP and traditional methods (PUB) can depend on the government’s objective function.

**Solving the Model**

After solving this model for the more general case, we will focus on two specific special cases that will allow us to highlight the differences between public-private partnership and traditional public procurement. Traditional public procurement, PUB, will correspond to the case in which $t = 1$ (so that all profits revert to the government) and all bargaining power remains with the government (as it can order its employees to take actions without the need for renegotiation required with an outside contractor). We will contrast this with something more “private”, here by considering as our PPP example, the case in which $t = 0$ and the bargaining weights are more equal.

We solve the model backwards, beginning with the firm’s choice of effort.

**Firm's Optimal Effort ($e^*$):** Given $\alpha$, $F$ chooses a level of $e$ to maximize its profit. This gives us the incentive compatibility constraint:
\[
\begin{align*}
\max_{e} \pi &= \max_{e} (1 - t)(\alpha - K + \delta e) - \varnothing(e) \\
\text{subject to } e &\geq 0
\end{align*}
\]

The optimal level of effort is then given by:

\[
(1 - t)\delta = \varnothing'(e^*) = e^* \tag{1}
\]

Clearly, higher rates of taxation discourage effort. At the extreme, when \( t = 1 \), \( e^* = 0 \). To be clear, under our assumptions \( e^* = (1 - t)\delta \) is the profit-maximizing choice of effort for either the original or a revised design, as the costs of delivering to either design are described by the same cost function and the effort decision is made after any required renegotiations.

**Renegotiation (if needed):** As indicated, with probability \( \mu \) there is a shift in demand that requires that changes be made to project design to preserve the maximum social benefits (\( b_0 \)). In such a case, if switching costs are not too large – which is assumed for now \(^{25}\) – G and F will renegotiate and sign a new contract with payment \( \alpha_1 \) going from G to F. Under Nash bargaining, G's threat point is: \( b_1 + t(\alpha_0 - K + \delta e^*) - \alpha_0 \). If renegotiation fails, G gets only \( b_1 \) of social benefit from the project but still pays \( \alpha_0 \) to F as provided in the initial contract. It also collects taxes on F’s profits to reduce its net cost of the project. Correspondingly, F still receives \( \alpha_0 \) and its optimal cost reducing effort remains the same. Thus F’s threat point will be given by:

\[
(1-t)(\alpha_0 - K + \delta e^*) \cdot \left( \frac{\alpha_0^2}{2} \right) = 0.
\]

We allow for the possibility that the government and the firm have unequal bargaining power, represented by differing bargaining weights in the Nash Product. It may be, for example, that governments will have larger weights given their powers to enact and revise laws and regulations and even to rewrite contacts while shielding themselves from damage actions. In other cases, however, sophisticated and highly motivated private partners may be able to retain higher quality advisory services to assist in their renegotiations, in the process capturing a larger share of the surplus generated.\(^{26}\) To explore the implications of unequal bargaining power, suppose G has a bargaining weight \( \lambda \), and F has the weight \( 1 - \lambda \), where \( 0 \leq \lambda \leq 1 \).

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\(^{25}\) This requires that the total benefits of making the change (\( b_0 - b_1 \)) exceed the costs of making the change (\( 2s \)).

\(^{26}\) A common concern raised about PPPs, particularly when the public partner is an under-resourced government department of a developing country, is that the private partners will have more legal and technical “firepower” at the table than will the government.
The Nash Product (NP) will then be:

$$NP = [(b_0 + t(\alpha_1 - K + \delta e^* - s) - \alpha_1 - s) - (b_1 + t(\alpha_0 - K + \delta e^*) - \alpha_0)]^\lambda$$

$$* \left[ \left( (1 - t)(\alpha_1 - K + \delta e^* - s) - \frac{(e^*)^2}{2} \right) - \left( (1 - t)(\alpha_0 - K + \delta e^*) - \frac{(e^*)^2}{2} \right) \right]^{1-\lambda}$$

Maximizing this with respect to $\alpha_1$ yields:

$$\alpha_1 = \frac{1 - \lambda}{1 - t} (b_0 - b_1) + \frac{2\lambda - 1 - t}{1 - t} s + \alpha_0 \quad (2)$$

**Initial Payment ($\alpha_0$):** Assuming perfectly competitive bidding and absent any uncertainty about which design is optimal, we would expect potential private partners to bid down to the level of costs (which would be perfectly known to the firm and revealed through bidding), leaving the firm with zero profits. When we introduce uncertainty in the efficient design and the opportunity for renegotiation in the absence of competition, however, the picture is more complicated. Renegotiation under Nash bargaining, if needed, will necessarily involve moving to the new, superior design and to a readjustment upward of the fixed fee (to $\alpha_1$) to share the gains of the renegotiation with the private partner. If the original contract was going to provide zero profits to the private partner (because it had been competitively bid), the renegotiated contract will necessarily generate positive profits.

Clearly, the potential for this renegotiation -- and the new profits it will provide the private partners -- could affect the original bidding. In a world in which private, risk neutral, bidders could commit to honoring their promises to complete projects regardless of the subsequent profitability of those projects -- perhaps by posting bonds -- these prospective profits in renegotiation would translate into lower prices bid in the initial round. The result would be that ex ante expected profits of bidders would still be zero -- and if the design did not need to be changed, the private partner would in fact suffer losses ex post. However, it is not clear that such commitment -- particularly on large projects -- is possible. Private partners will often be able to walk away from projects once the profits going-forward have turned negative, or to at least make credible threats to walk away if subsidies are not forthcoming. This behavior is facilitated by the “special purpose vehicle” (SPV) structure that is adopted for many large PPP projects in which
the SPV can simply declare bankruptcy and shut without there being any remaining claims on the SPV’s joint-venture parents.

Therefore, we invoke a limited-liability constraint, familiar in the contracting literature, and assume here that the government will not accept as a winning bid any offer that would not allow the private partner to at least break even absent any renegotiations. Governments, in their scrutiny of bids, will need to be persuaded that the contract can be honored absent any renegotiations. Therefore, if we assume that private firm has limited liability and no wealth it can commit -- that is, it cannot be pushed to negative profits in any state (or it would exit) -- F will again bid up to the point where it will break even without renegotiation. The limited liability/wealth approach to the agency problem has been applied by many others as an alternative to introducing risk aversion on the part of one or both players.

We can then write the limited liability constraint as:

\[(1 - t)(\alpha_0 - K + \delta e^*) - \frac{(e^*)^2}{2} = 0\]

And the winning bid is given by:

\[\alpha_0 = K - \delta e^* + \frac{(e^*)^2}{2(1-t)}\]

Given that \(e^* = (1 - t)\delta\), we can see that

\[\alpha_0^* = K - \frac{\delta^2}{2}(1 - t) \quad (3)\]

This result suggests that the winning bid will be lower when tax rates are lower, because of the higher levels of effort that follow lower taxes.

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27 This is not to deny that there can be a problem of firms adopting strategies in which they win the contract with an apparently attractive bid, only to threaten later that the deal must be renegotiated or they will not continue. However, sophisticated governments will be alert to this possibility, and will want to scrutinize bids carefully to assure themselves that they are feasible. Also, in a world in which most of these private sector PPP players wish to continue to win future bids with the same or other governments, such gaming could be damaging to their reputations as trustworthy partners.

28 The classic reference is Sappington (1983). See also the text by Laffont and Martimort (2002). It can be shown, as well, that the key trade-off between efficiency and flexibility remains even if firms are permitted to bid down to zero ex ante expected profits (and can commit to their bids) as long as there are costs associated with renegotiation.

29 If the limited liability constraint is satisfied it is very easy to see that a participation or individual rationality constraint (requiring firms earn non-negative profits in expectation ex ante) will never be binding. For this reason we do not include participation constraints here.

30 This does require that the winning bidder can credibly convince the government that its bid will allow it to break even in the absence of any renegotiations.
**Value for Money (VFM):** Given the costs of the contract to the government with and without renegotiation, we can easily determine the expected value for money. If there were no change in demand, the value for money would be simply given by the difference between the benefits of the project and the government’s costs to procure it ($\alpha_0$ minus any tax collected on profits):

\[
VFM = b_0 + t(\alpha_0^* - K + \delta e^*) - \alpha_0^* = b_0 - K + \frac{\delta^2}{2}(1 - t^2)
\]

If the project does change, the new contract price, $\alpha_1^*$, will be given by, substituting (3) into (2):

\[
\alpha_1^* = \frac{1 - \lambda}{1 - t}(b_0 - b_1) + \frac{2\lambda - 1 - t}{1 - t}s + K - \frac{\delta^2}{2}(1 - t)
\]

(4)

And the VFM after renegotiation will then be given by:

\[
VFM = b_0 + t(\alpha_1^* - K + \delta e^* - s) - \alpha_1^* - s = \lambda b_0 + (1 - \lambda)b_1 - K + \frac{\delta^2}{2}(1 - t^2) - 2\lambda s
\]

Combining the VFM results with and without a change in project design, we see that the *ex ante* expected VFM is then

\[
E(VFM) = \mu\left(\lambda b_0 + (1 - \lambda)b_1 - K + \frac{\delta^2}{2}(1 - t^2) - 2\lambda s\right) + (1 - \mu)\left(b_0 - K + \frac{\delta^2}{2}(1 - t^2)\right)
\]

\[
= (1 - \mu(1 - \lambda))b_0 + \mu(1 - \lambda)b_1 - K + \frac{\delta^2}{2}(1 - t^2) - 2\mu\lambda s
\]

(5)

A set of intuitive results regarding the determinants of VFM of a project follow directly.

**Result 1:** The value for money of a project will be greater: the lower is the cost of the project ($K$); the greater is the gross benefit of first and second-best projects ($b_0$ and $b_1$); the greater is the cost reducing effect of effort ($\delta$); the smaller is the probability ($\mu$) the project design will need to change; the smaller is the switching cost ($s$); and the lower the tax rate ($t$). Assuming that the net benefit of renegotiation ($b_0 - b_1 - 2s$) is always positive -- that is, renegotiation is efficient and therefore always occurs when there are changes in demand -- VFM is higher when the government is in a stronger bargaining position (i.e. when $\lambda$ is greater).

**Proof:** the results are demonstrated by the following simple comparative statics, assuming $b_0 - b_1 - 2s > 0$:

\[
\frac{d(E(VFM))}{dK} = -1 < 0 ;
\]

\[
\frac{d(E(VFM))}{db_0} = 1 - \mu(1 - \lambda) > 0 ; \frac{d(E(VFM))}{db_1} = \mu(1 - \lambda) > 0 ;
\]
\[
\frac{d(E(VFM))}{d\delta} = \delta(1 - t^2) > 0;
\]
\[
\frac{d(E(VFM))}{du} = -(1 - \lambda)(b_0 - b_1) - 2\lambda s < 0;
\]
\[
\frac{d(E(VFM))}{ds} = -2\lambda \mu < 0
\]
\[
\frac{d(E(VFM))}{dt} = -\delta^2 t < 0
\]
\[
\frac{d(E(VFM))}{d\lambda} = \mu(b_0 - b_1 - 2s) > 0.
\]

Most of these results are easily understood and relate to the simple point that the greater the benefits and/or the lower are the costs of the project, the greater will be its VFM. The least obvious results here are those related to the tax rate, the bargaining weights and the likelihood of the need for renegotiation. By discouraging effort, higher tax rates actually hurt the achievement of VFM, even though the taxes received reduce government expenditures. By limiting the ability of the private partner to drive up prices in renegotiation, a greater government bargaining weight increases expected VFM. Finally, the more likely it is that the project will need to be modified, the less attractive it becomes \textit{ex ante} from a VFM perspective, even though the final gross benefits and costs of production will be in the same in any case.

**IV. Comparing PPPs and Traditional Public Procurement**

We turn now to the key results of the paper: considering two special cases of the model above will allow us to compare the relative costs and efficiency of a PPP and more traditional methods of public procurement (PUB).

By taxing back all the profits \((t \text{ approaching } 1)\) and granting the government full power in renegotiation \((\lambda = 1)\) we essentially transform the firm into an arm of the government and create the traditional public procurement scenario, PUB.\textsuperscript{31} By contrast when taxes are lower and bargaining weights more balanced, we have more private interests and control in the project.\textsuperscript{32}

---

\textsuperscript{31} If we allow \(t\) to exactly equal 1 we lose uniqueness to our solutions – since the government taxes back every dollar of profit, it does not care what price it pays. Therefore when we speak of \(t = 1\) here, we more precisely mean \(t = 1-\varepsilon\), where \(\varepsilon\) can be an arbitrarily small but positive quantity.

\textsuperscript{32} An alternative modeling technique could simply involve assuming that under PUB, the government does not need to renegotiate changes and can just order them. This approach, taken in an earlier version of this paper, produces essentially identical results.
To fix ideas here, we take the case in which $t = 0$ and $0 \leq \lambda \leq 1$ to represent the PPP model. Recall that for a given level of $\lambda$, lower taxes raise VFM, so indeed in a PPP scenario in which the government is ceding some control rights (i.e. $\lambda < 1$), the government will maximize VFM by setting the tax rate to zero.\(^\text{33}\)

Setting $t = 0$ we can solve for the chosen level of effort, from (1), the original contract price, from (3), and the renegotiated price, from (4):

$$e^{PPP} = \delta,$$

$$\alpha_0^{PPP} = K - \frac{\delta^2}{2},$$

$$\alpha_1^{PPP} = (1 - \lambda)(b_0 - b_1) + K - \frac{\delta^2}{2} + (2\lambda - 1)s$$

This will provide an expected VFM in the PPP case given by:

$$E(VFM^{PPP}) = \left(1 - \mu(1 - \lambda)\right)b_0 + \mu(1 - \lambda)b_1 - K + \frac{\delta^2}{2} - 2\mu\lambda s \quad (6)$$

Then, by setting $t = 1$ and $\lambda = 1$, we can construct the PUB alternative. This will yield:

$$e^{PUB} = 0$$

$$\alpha_0^{PUB} = K$$

$$\alpha_1^{PUB} = K + s$$

As a result of the lack of profit incentives for $F$, no effort or innovation will be forthcoming. The fact that it holds all the bargaining power, however, does mean that $G$ will have to pay no more than the costs actually incurred by $F$. The expected VFM under PUB will be given by:

$$E(VFM^{PUB}) = \mu(b_0 - K - 2s) + (1 - \mu)(b_0 - K) = b_0 - K - 2\mu s \quad (7)$$

Our second set of results highlights the intuitive trade-offs between efficiency and flexibility in the choice of procurement mode.

**Result 2:** Under the objective of maximizing VFM, a PPP procurement will dominate public procurement (PUB) when:

\(^{33}\) To be clear, we do not see $\lambda$ as a choice variable except to the extent that a government can choose to procure using traditional public methods, thereby effectively setting $\lambda$ to one. Any form of private provision will involve, for the purposes of our model, a bargaining weight that is exogenous to the government.
\[ E(\text{VFM}^{\text{PPP}}) - E(\text{VFM}^{\text{PUB}}) = \frac{\delta^2}{2} - \mu(1 - \lambda)(b_0 - b_1) + 2(1 - \lambda)\mu s > 0 \quad (8) \]

Therefore, procurement using a PPP is more likely to dominate procurement under PUB: the greater is the cost reducing effect of effort (\( \delta \)); the smaller is the probability the project design will need to change (\( \mu \)); the greater is the switching cost (\( s \)); and, the smaller is the difference between the social values of the best project and the other project (\( b_0 - b_1 \)). Assuming that the net benefit of renegotiation (\( b_0 - b_1 - 2s \)) is always positive, then the VFM of a PPP is relatively higher when government is in a stronger bargaining position (\( \lambda \)).

**Proof:** Follows directly from simple comparative statics as above.\(^\text{34} \)

The basic trade-off between the efficiency of private providers and the flexibility of public providers comes through very clearly here. When flexibility does not matter because demand is extremely unlikely to change (\( \mu = 0 \)), \( E(\text{VFM}^{\text{PPP}}) > E(\text{VFM}^{\text{PUB}}) \), implying that PPP procurement must dominate PUB because of its efficiency benefits. On the other hand, when the need to provide extra incentive for efficiency is less important, perhaps because there is little potential for innovation in the project (\( \delta = 0 \)), procurement via PUB will dominate, that is, \( E(\text{VFM}^{\text{PPP}}) < E(\text{VFM}^{\text{PUB}}) \).

Other intuitive results follow from (8) quite simply. When the difference between the “right” project and the “wrong” one (\( b_0 - b_1 \)) is large it is more important for G to renegotiate – improving the terms that F can extract and reducing the VFM benefits of the PPP. Greater bargaining power on the part of the government expands the range of parameter values under which PPP procurement is preferred under the VFM standard as greater bargaining power on the part of the government limits the ability of the private partner to raise the government’s costs in renegotiation. Finally, higher levels of switching costs (\( s \)), other things equal, tend to favour the

\(^{34}\text{The basic comparative statics are: (assuming } b_0 - b_1 - 2s > 0)\)

\[
\frac{d(E(\text{VFM}^{\text{PPP}}) - E(\text{VFM}^{\text{PUB}}))}{d\delta} = \delta > 0, \quad \frac{d(E(\text{VFM}^{\text{PPP}}) - E(\text{VFM}^{\text{PUB}}))}{d\mu} = -(1 - \lambda)(b_0 - b_1 - 2s) < 0 \\
\frac{d(E(\text{VFM}^{\text{PPP}}) - E(\text{VFM}^{\text{PUB}}))}{ds} = 2\mu(1 - \lambda) > 0, \quad \frac{d(E(\text{VFM}^{\text{PPP}}) - E(\text{VFM}^{\text{PUB}}))}{d(b_0 - b_1)} = -\mu(1 - \lambda) < 0 \\
\frac{d(E(\text{VFM}^{\text{PPP}}) - E(\text{VFM}^{\text{PUB}}))}{d\lambda} = \mu(b_0 - b_1 - 2s) > 0
\]
PPP mode. Under PUB, the public provider bears the full switching costs of both partners. However, under the PPP structure -- and given Nash bargaining in renegotiation -- the private partners will end up sharing the loss of surplus associated with higher switching costs. Less than 100% of this cost is then passed on to the public sector.

We can illustrate the efficiency-flexibility trade-off with a simple graph based on values of $\delta$ (capturing the importance of effort for efficiency) and $\mu$ (capturing the need for flexibility) that lead the PPP and PUB procurement modes to deliver identical levels of VFM (i.e. that set condition (8) equal to zero). These levels will be given by $\delta = \left(2\mu(1 - \lambda)(b_0 - b_1 - 2s)\right)^{\frac{1}{2}}$, which is illustrated in Figure 1, where the area under the curve (shaded) is the range within which PUB dominates PPP while the area above represents values such that the PPP mode dominates. Other aspects of Result 2 can be illustrated with this graph. For instance, increases in $b_0 - b_1$ or decreases in $s$ rotate the curve upward (around the origin), which expands the area in which PUB dominates.

![Figure 1: The Efficiency-Flexibility Trade-Off](image)

The interpretation of Result 2 is therefore quite straightforward. Under a PPP contract, F has incentives to exert cost-reducing effort because it captures the gains of its efforts. Those incentives are absent under PUB. The disadvantages of the PPP, however, derive from the incompleteness of the contract. For the government, this involves new costs –the extra surplus that must be transferred to the private partner under Nash bargaining, which lowers the government’s VFM.
Therefore, PPPs will be more attractive when there is little chance for a need to redesign the project – road or bridge projects may come to mind as examples. On the other hand, when it is more likely that the kind of services needed in the future could be very different from those anticipated ex ante – as might be the case for sophisticated health care projects, for example -- the flexibility of public procurement contracts may make them superior.

V. Extensions

V.1. A Total Social Surplus (TSS) Objective

In the baseline case, the government pursues a value for money objective, a measure widely used in the PPP literature and by PPP practitioners. However, this is not the most commonly assumed objective for governments in normative policy work by economists. More typically, for example in standard cost-benefit analysis, government objectives would be modelled as the maximization of some measure of social welfare or total social surplus. This is a different measure than value for money to be sure – principally differing as a result of transfers from one party to another. Two examples of transfers that make these objectives different are particularly relevant. First, it has been argued – for example, by public sector labour unions -- that PPPs facilitate the substitution of poorly paid private sector labour for more highly paid (and unionized) public sector labour. To the extent that this happens, these transfers do improve value for money, but do not represent increases in total social surplus. Second, surpluses moved from taxpayers to private firms in the form of profits will hurt the achievement of value for money but will not necessarily affect total social surplus. In this section, we explore the implications of using a total social surplus (TSS) criterion for selecting between PPP and PUB. Under a TSS objective, the government does not care about the distribution of surpluses, only about the total surplus generated.

To do this however, we need to add a new element to the model. Paying higher prices to private sector partners will not be seen as a cost to a government that maximizes TSS (since inflated prices are merely a transfer) unless raising the required tax revenues generates deadweight losses to the economy. Therefore, without some shadow price of public funds that
incorporates this cost of raising revenue, there will not be a unique solution to the question of what price the government will pay for the services provided.

Therefore, we add a deadweight cost of government financing (from tax revenues), $\gamma > 0$, which represents the additional cost to the economy when a government extracts $1$ to pay for the project. Hence, if the government pays the firm $\Psi$, the cost to the government is $(1 + \gamma)\Psi$.

To be clear, we could have introduced this cost of financing parameter into the VFM analysis above, but it would have made no difference. That analysis identified which procurement mode would provide the lowest cost to government; adding this additional cost would not have changed any of those comparisons, though it would lower the overall desirability of doing the project at all.

To facilitate comparison with VFM analysis, we continue to focus on the two special cases: (i) PUB with $\lambda = 1, t = 1$, and (ii) PPP with $0 \leq \lambda \leq 1, t = 0$. As in the case of VFM, under the PUB contract, G pays $\alpha_0^{PUB} = K$ to cover the (non-effort) cost and F exerts no cost reducing effort, that is we have again, $\epsilon^{PUB} = 0$. Changes will be made, if necessary, and G will only have to cover F’s actual costs of the change ($s$). The expected total social surplus under PUB, including the cost of government financing, is now

$$E(TSS^{PUB}) = \mu(b_0 - (1 + \gamma)K - 2(1 + \gamma)s) + (1 - \mu)(b_0 - (1 + \gamma)K)$$

$$= b_0 - (1 + \gamma)K - 2(1 + \gamma)\mu s$$

Under the PPP contract, G initially pays a lump sum $\alpha_0^{PPP}$ and F again chooses an optimal level of effort, $\epsilon^{PPP} = \delta$, to maximize its profit: $\alpha_0^{PPP} - C - \frac{(\epsilon^{PPP})^2}{2} = \alpha_0^{PPP} - K + \frac{\delta^2}{2}$. The limited liability constraint still holds so we have $\alpha_0^{PPP} = K - \frac{\delta^2}{2}$.

If changes in demand necessitate changes in project design, G and F will again enter into Nash bargaining. As we now assume that there is a deadweight cost of financing the project, the benefit of the original project is now $b_1 - \gamma \alpha_0^{PPP}$. Since the government’s current object is to maximize the total social surplus of the project, G has a threat point that depends on the TSS without renegotiation: $b_1 - \gamma \alpha_0^{PPP} - C - \frac{(\epsilon^{PPP})^2}{2} = b_1 - \gamma \alpha_0^{PPP} - K + \frac{\delta^2}{2}$, which is the benefit of the “wrong” project minus the cost of providing it. F has the same objective function as before, so its threat point remains: $\alpha_0^{PPP} - K + \frac{\delta^2}{2} = 0$. The Nash product in this case is
\[ NP = \left[ b_0 - \gamma \alpha_1^{PP} - K + \frac{\delta^2}{2} - (s + (1 + \gamma)s) - \left( b_1 - \gamma \alpha_0^{PP} - K + \frac{\delta^2}{2} \right) \right]^\lambda \]
\[ \times \left[ (\alpha_1^{PP} - K + \frac{\delta^2}{2} - s) - (\alpha_0^{PP} - K + \frac{\delta^2}{2}) \right]^{1-\lambda} \]

Solving yields the new payment, \( \alpha_1^{PP} \):
\[ \alpha_1^{PP} = \frac{(1 - \lambda)(b_0 - b_1) + (2\gamma\lambda + 2\lambda - \gamma - 2)s}{\gamma} + K - \frac{\delta^2}{2} \]

The expected TSS under a PPP contract is then
\[ E(TSS^{PPP}) = \mu \left( b_0 - \gamma \alpha_1^{PP} - K + \frac{\delta^2}{2} - (2 + \gamma)s \right) + (1 - \mu) \left( b_0 - \gamma \alpha_0^{PP} - K + \frac{\delta^2}{2} \right) \]
\[ = \left( 1 - \mu(1 - \lambda) \right) b_0 + \mu(1 - \lambda) b_1 - (1 + \gamma)(K - \frac{\delta^2}{2}) - 2(1 + \gamma)\lambda \mu s \]

This leads to our third set of results.

**Result 3:** Under the standard of maximizing TSS, a PPP procurement will dominate public procurement (PUB) when:
\[ E(TSS^{PPP}) - E(TSS^{PUB}) = (1 + \gamma)\frac{\delta^2}{2} - \mu(1 - \lambda)(b_0 - b_1) + 2(1 + \gamma)(1 - \lambda)\mu s > 0 \quad (9) \]

As before, a PPP approach is more likely to dominate PUB, the greater is the cost reducing effect of effort (\( \delta \)), the smaller is the probability the project design will need to change (\( \mu \)), the larger is the switching cost (\( s \)), and, the smaller is the difference between the social values of the best project and the other project (\( b_0 - b_1 \)). When the deadweight loss from taxation (\( \gamma \)) is close to 0 (and, as a result, the renegotiated payment \( \alpha_1^{PP} \) becomes very large) the conditions for PPP to dominate under a TSS standard are the same as those under a VFM standard. The effect of increases in the deadweight loss of government finance makes the PPP more attractive. Assuming that the net benefit of renegotiation \( b_0 - b_1 - 2(1 + \gamma)s \) is always positive, so renegotiation always occurs when there are changes in demand, the TSS of a PPP is higher when government is in a stronger bargaining position.
Proof: Follows from straightforward comparative statics.\(^{35}\)

While these results parallel those from Result 2, there is one new parameter here (\(\gamma\)). Increases in the marginal deadweight loss of government funding (\(\gamma\)) favour the PPP alternative. This is because increases in the deadweight loss of government funding will favour the procurement mode that involves the lowest cost to government. The PPP mode lowers the cost of production because of the higher levels of effort, and it also shares the switching costs with the private partner whose financing does not create deadweight loss. The intuition with respect to changes in the other parameters is the same as provided with respect to Result 2.

**Comparing TSS and VFM Results**

Would a government pursuing a VFM objective adopt the PPP mode when the PUB approach would yield greater total surplus under the TSS objective? To compare conditions under which either mode is preferred under VFM vs. TSS standards, we need to make the models more directly comparable by adding cost of government financing to our VFM calculations. The expected VFM of PPP in the presence of financing cost is: \(^{36}\)

\[
E(VFM_{PPP}) = (1 - \mu(1 - \lambda))b_0 + \mu(1 - \lambda)b_1 - (1 + \gamma)\left(K - \frac{\delta^2}{2}\right) - (\gamma + 2\lambda)\mu s 
\] (10)

The expected VFM of PUB is the same as the expected TSS of PUB:

\[
E(VFM_{PUB}) = b_0 - (1 + \gamma)K - 2(1 + \gamma)\mu s
\]

So the difference between VFM of the two types of contracts is then:

\[
E(VFM_{PPP}) - E(VFM_{PUB}) = (1 + \gamma)\frac{\delta^2}{2} - \mu(1 - \lambda)(b_0 - b_1) + (\gamma + 2(1 - \lambda))\mu s 
\] (11)

\(^{35}\) The basic comparative statics are very similar to those in the VFM case, with the inclusion of the deadweight loss parameter: assuming \(b_0 - b_1 - 2(1 + \gamma)s > 0\)

\[
\frac{d[E(TSS_{PPP})-E(TSS_{PUB})]}{ds} = (1 + \gamma)\delta > 0, \quad \frac{d[E(TSS_{PPP})-E(TSS_{PUB})]}{d\mu} = -(1 - \lambda)(b_0 - b_1 - 2(1 + \gamma)s) < 0
\]

\[
\frac{d[E(TSS_{PPP})-E(TSS_{PUB})]}{d\gamma} = 2(1 + \gamma)(1 - \lambda)\mu > 0, \quad \frac{d[E(TSS_{PPP})-E(TSS_{PUB})]}{d(b_0-b_1)} = -\mu(1 - \lambda) < 0
\]

\[
\frac{d[E(TSS_{PPP})-E(TSS_{PUB})]}{d\delta} = \frac{\delta^2}{2} + 2(1 - \lambda)\mu s > 0
\]

\[
\frac{d[E(TSS_{PPP})-E(TSS_{PUB})]}{d\lambda} = \mu (b_0 - b_1 - 2(1 + \gamma)s) > 0
\]

\(^{36}\) See the appendix for a detailed derivation.
Comparing the TSS and VFM results provides our fourth set of results which demonstrate that the different objectives can indeed lead to different choices.

**Result 4:** When the public and private partners have different bargaining weights the following cases become possible:

(i) When the government has the greater bargaining weight (i.e. \( \lambda > \frac{1}{2} \)) it is possible for a PPP to maximize VFM while PUB maximizes TSS;

(ii) When the government has the lesser bargaining weight (i.e. \( \lambda < \frac{1}{2} \)) it is possible for a PPP to maximize TSS while PUB maximizes VFM.

(iii) When the government and firm have equal bargaining weight (i.e. \( \lambda = \frac{1}{2} \)), comparisons under VFM are the same as those under TSS.

**Proof:** To see this, we write \( E(TSS_{PPP}) - E(TSS_{PUB}) \) in terms of \( E(VFM_{PPP}) - E(VFM_{PUB}) \).

\[
E(TSS_{PPP}) - E(TSS_{PUB}) = (1 + \gamma) \left( \frac{s^2}{2} - \mu(1 - \lambda)(b_0 - b_1) + 2(1 + \gamma)(1 - \lambda)\mu s \right) = E(VFM_{PPP}) - E(VFM_{PUB}) + (1 - 2\lambda)\gamma \mu s
\]

The last term on the right-hand side of this expression will be positive when \( \lambda < \frac{1}{2} \) and negative when \( \lambda > \frac{1}{2} \). As a result, when the government has the lesser bargaining weight (i.e. \( \lambda < \frac{1}{2} \)), it is possible for a PPP to maximize TSS (\( E(TSS_{PPP}) - E(TSS_{PUB}) > 0 \)) while PUB maximizes VFM (\( E(VFM_{PPP}) - E(VFM_{PUB}) < 0 \)). And when the government has the greater bargaining weight (i.e. \( \lambda > \frac{1}{2} \)), it is possible that a PPP contract is the optimal choice in terms of VFM while PUB is the optimal choice in terms of TSS.\(^{37} \) When the government and firm have equal bargaining weight (i.e. \( \lambda = \frac{1}{2} \)), if delivery of a project using a PPP dominates its delivery using public procurement under a VFM objective, it also dominates under a TSS objective (and vice versa). In other words, in this case, whether the objective involves maximizing VFM or total social surplus does not alter the optimal choice of procurement method.

\(^{37}\)An example: When \( \lambda < 0.5 \), \( \lambda = 0.1 \), \( \gamma = 0.5 \), \( \mu = 0.5 \), \( s = 15 \), \( \delta = 8 \), \( b_0 = 200 \), \( b_1 = 50 \). \( E(VFM_{PPP}) - E(VFM_{PUB}) = -2.25 < 0 \) and \( E(TSS_{PPP}) - E(TSS_{PUB}) = 0.75 > 0 \). Therefore PUB dominates under a VFM standard but PPP dominates under a TSS standard. When \( \lambda > 0.5 \), \( \lambda = 0.6 \), \( \gamma = 0.5 \), \( \mu = 0.5 \), \( s = 15 \), \( \delta = 5 \), \( b_0 = 200 \), \( b_1 = 60 \). \( E(VFM_{PPP}) - E(VFM_{PUB}) = 0.5 > 0 \) and \( E(TSS_{PPP}) - E(TSS_{PUB}) = -0.25 < 0 \). Here PPP dominates under a VFM standard while PUB dominates under a TSS standard.
VI.2 Toll Revenue PPPs

In the PPP model we have used to this point, the private partner is paid a set sum to deliver the project. This could have been in the form of a lump sum or a stream of “availability payments” (i.e., payments dependant only on the facility operating but not on the actual demand for that facility or the extent of its usage). In many PPP arrangements, however, private parties are paid according to the use of the services. Road projects funded by tolls are a common example in which users rather than the government pay the private partners. In other cases, private partners are paid according to demand or use of the facilities, but are paid by the government.38

In cases in which the private partner is paid based on the project’s success in terms of meeting demand, we can imagine that this player will be much more willing—even eager—to amend the project if demand changes make design changes optimal. Similarly, users (or the government if it pays) are not as disadvantaged by a failure to adapt to change since they will not have to pay as much. Put another way, the players’ threat points are different in such a situation.

To see the implications of this alternative design, consider a PPP model in which the government and firm share the benefits generated by the project. In this case, the firm’s threat point deteriorates if the project must change and renegotiated terms will improve for government.

We will continue to assume that \( t = 0 \) in the PPP model with tolls. Suppose that, under the PPP contract, the fraction of the benefit that F gets (via usage fees) is \( \tau_0 \). The government’s VFM objective here then will be to secure the best project by surrendering the smallest fraction of the benefits to the private partner.

Given \( \tau_0 \), F chooses an optimal level of effort to maximize its profit. Once again F’s incentive compatibility constraint is:

\[
Max \pi = Max \tau_0 b_0 - (K - \delta e) - \frac{e^2}{2}
\]

38 Shadow tolls (where use is measured but the tolls are paid by the government) on road and bridge projects would be an example.
and the optimal effort level is again $e_{PPP}^* = \delta$. \(^{39}\)

Potential private partners will bid according to what level of $\tau_0$ they are willing to accept, with the contract going to the lowest bidder. We maintain the assumption that the winning firm, F, will have bid to the point where it will break even (its limited liability constraint) absent renegotiation. This implies

$$\pi = \tau_0 b_0 - (K - \delta e_{PPP}) - \frac{(e_{PPP})^2}{2} = \tau_0 b_0 - K + \frac{\delta^2}{2} = 0$$

so $\tau_0^* = \frac{1}{b_0} \left( K - \frac{\delta^2}{2} \right)$

If changes in demand materialize, G and F renegotiate and sign a new contract. G’s threat point is $(1 - \tau_0^*)b_1$. If we think of $\tau_0^* b_1$ as G’s payment to F, then G actually “pays” less now as $\tau_0^* b_1 < \tau_0^* b_0$. In this sense, we say that G has a stronger bargaining position than it would in an availability case. If the firm does not agree to change the contract, its payoff is $\tau_0^* b_1 - K + \frac{\delta^2}{2}$, which is negative as $b_1 < b_0$. Consistent with our earlier assumption, we assume that F can walk away. So in this case F’s threat point still generates zero profits (in this case through exit). Assume that the target of G is to maximize VFM. The Nash Product is then

$$NP = [(1 - \tau_1)b_0 - s - (1 - \tau_0^*)b_1]^{\lambda} [\tau_1 b_0 - K + \frac{\delta^2}{2} - s - 0]^{1-\lambda}$$

The new toll rate, $\tau_1^*$, will then be given by:

$$\tau_1^* = \frac{1}{b_0} \left( (1 - \lambda)(b_0 - b_1) + (\lambda b_0 + (1 - \lambda)b_1) \frac{1}{b_0} \left( K - \frac{\delta^2}{2} \right) - (1 - 2\lambda)s \right)$$

Therefore, the expected VFM of the toll contract is

$$E(VFM_{toll}) = \mu((1 - \tau_1^*)b_0 - s) + (1 - \mu)(1 - \tau_0^*)b_0$$

$$= \left( (1 - \mu(1 - \lambda))b_0 + \mu(1 - \lambda)b_1 \right) (1 - \tau_0^*) - 2\lambda \mu s$$

$$= \left( (1 - \mu(1 - \lambda))b_0 + \mu(1 - \lambda)b_1 \right) \left( 1 - \frac{1}{b_0} \left( K - \frac{\delta^2}{2} \right) \right) - 2\lambda \mu s$$

\(^{39}\) The optimal effort is the same here in part because effort does not affect demand in this model. It would be interesting to explore the implications of having effort influence demand as well as costs.
We can now ask whether a toll contract or an availability contract generate greater VFM for the government. Recall that expected VFM of the availability contract, for clarity here now labeled \( E(VFM_{\text{avail}}) \), is given by (6).

This leads to our fifth set of results.

**Result 5:** When the objective of the government is to maximize VFM, the toll contract dominates the availability contract.

**Proof:** The difference between the VFM of the two contracts is

\[
E(VFM_{\text{toll}}^{\text{PPP}}) - E(VFM_{\text{avail}}^{\text{PPP}}) = \left( (1 - \mu(1 - \lambda))b_0 + \mu(1 - \lambda)b_1 \right)(1 - \tau_0^*) - 2\lambda\mu s - \left( (1 - \mu(1 - \lambda))b_0 + \mu(1 - \lambda)b_1 - \tau_0^*b_0 - 2\lambda\mu s \right)
\]

Thus, the toll contract dominates the availability contract in terms of VFM. Additionally, we see that the advantage of the toll contract over the availability contract is greater: the more likely it is that the project will need to change (higher \( \mu \)); the greater the cost of the project (bigger \( K \)); the smaller the cost reducing effect of effort (\( \delta \)); and the greater the percentage difference between the "right" and "wrong" project (larger \( (b_0 - b_1)/b_0 \)). This is because under the toll contract, the government essentially pays less if renegotiation fails and hence the government is in a stronger bargaining position. Finally, as \( \lambda \) gets larger, renegotiation is less costly to \( G \) with the result that the VFM advantages of the toll contract are greater when \( G \) has less bargaining power (i.e. when \( \lambda \) is small).

Not surprisingly then, the use of tolling expands the conditions under which the PPP approach can dominate public procurement methods. The condition for a toll-based PPP to dominate the PUB alternative is given in **Result 6**.

**Result 6:** Under the standard of maximizing VFM, a PPP procurement based on toll payments will dominate public procurement when:

\[
E(VFM_{\text{toll}}^{\text{PPP}}) - E(VFM_{\text{PUB}}^{\text{PPP}}) = \frac{\delta^2}{2} - \mu(1 - \lambda)(b_0 - b_1) \left( 1 - \frac{1}{b_0} \left( K - \frac{\delta^2}{2} \right) \right) + 2(1 - \lambda)\mu s > 0
\]
**Proof:** Recall that the expected VFM under the PUB contract is $E(VFM_{PUB}) = b_0 - K - 2\mu s$. Then the difference between VFM of the two types of contracts is

\[
E(VFM_{toll}) - E(VFM_{PUB}) = ((1 - \mu(1 - \lambda))b_0 + \mu(1 - \lambda)b_1)(1 - \tau_0^*) - 2\lambda \mu s - (b_0 - K - 2\mu s)
\]

\[
= (1 - \mu(1 - \lambda))b_0 + \mu(1 - \lambda)b_1 - (b_0 - \mu(1 - \lambda)b_0 + \mu(1 - \lambda)b_1)\tau_0^* + 2(1 - \lambda)\mu s - b_0 + K
\]

\[
= -\mu(1 - \lambda)(b_0 - b_1) - \tau_0^*b_0 + \mu(1 - \lambda)(b_0 - b_1)\tau_0^* + 2(1 - \lambda)\mu s + K
\]

Since $\tau_0^*b_0 = K - \frac{\delta^2}{2}$, we have

\[
E(VFM_{toll}) - E(VFM_{PUB}) = \frac{\delta^2}{2} - \mu(1 - \lambda)(b_0 - b_1)(1 - \tau_0^*) + 2(1 - \lambda)\mu s
\]

\[
= \frac{\delta^2}{2} - \mu(1 - \lambda)(b_0 - b_1) \left(1 - \frac{1}{b_0} \left(K - \frac{\delta^2}{2}\right)\right) + 2(1 - \lambda)\mu s
\]

Derivatives of this expression reveal comparative statics results similar to those from Result 2.

While in general we might expect a toll-based PPP project to help G secure value for money by both weakening the private partner’s threat-point (because it will lose money without renegotiating) and by strengthening the government’s threat-point (because it will not have to pay as much without renegotiation) only the second effect is operative here. In both cases (tolls and availability payments) the private partner’s alternative to a renegotiated contract is zero profits, in the toll case because it exits to avoid losses, in the availability case because its payments will just cover its costs. As a result, the advantages of the toll version derive here from the strengthening of G’s bargaining position.

**VI.3 Renegotiation Costs**

In the base model presented in Section IV, a change in the project requires – under PPP or PUB modes – that both G and F need to expend resources as switching costs. We indicated that

\[40\] For example: the PPP toll contract is more likely to dominate PUB, the greater is the cost reducing effect of effort ($\delta$), the smaller is the probability the project design will need to change ($\mu$), the larger is the switching cost ($s$), and, for a given $b_0$, the greater is the value of the other project (i.e. $b_1$). There is a slightly different condition necessary to guarantee this effect of $\mu$. We must assume that the benefits of the doing the right project are big enough:

\[b_0 - b_1(1 - \tau_0^*) > 2s\]. If this condition were not satisfied, the government could not benefit from renegotiation.

\[41\] This said, it would not be difficult to expand the model, for example, by introducing an exit cost for the private partner, which would lead it to have an inferior threat point in the PPP with tolls.
some of these switching costs could be the costs of negotiating a new agreement, but in such an interpretation we might expect that these costs would be lower under the PUB mode since G can more directly order changes. We can model the implications of these kinds of negotiation costs very easily by comparing the VFM generated under the PPP mode with these negotiation costs with the VFM generated under the PUB mode in which these costs are set to zero. To avoid confusion, we will denote these renegotiation costs borne under PPPs with the parameter \( m \) (rather than \( s \)) and assume that there are no renegotiation costs for either party under PUB.

The expected VFM\(^{PPP} \) will then be simply (6) again, with \( m \) replacing \( s \). The expected VFM\(^{PUB} \) will be given by (7) with \( s \) set to zero. We can then set out a very intuitive result.

**Result 7:** The presence of renegotiation costs under a PPP that do not arise under the PUB mode will lower the expected VFM of a PPP approach to the project relative to the PUB approach.

**Proof:** The PPP mode will dominate the PUB mode for project delivery under a VFM standard if:

\[
E(\text{VFM}^{PPP}) - E(\text{VFM}^{PUB}) = \frac{\delta^2}{2} - \mu(1 - \lambda)(b_0 - b_1) - 2\lambda\mu m > 0
\]

It is clear that higher renegotiation costs favour the PUB mode which avoids those costs as long as there is some positive probability of change (\( \mu > 0 \)) and the government has some bargaining power under a PPP (\( \lambda > 0 \)).

**VII. Conclusions and Directions for Future Research**

Public-private partnerships are being used to deliver public services in many sectors, such as transportation, water, health care, education and prisons. In this paper, we have examined an important trade-off associated with the choice between PPP and more traditional public procurement methods (PUB). While the PPP model provides the private contractor with greater incentives for cost reducing effort and innovation, it locks the government into a long-term contract that may be costly to renegotiate if changing circumstances make a project redesign optimal. We show that the PPP model will be superior when possible efficiencies are large, the probability there will be a need to change the project is small, the gains to project redesign are small, the government’s bargaining power in renegotiation is greater and when renegotiation costs are low. This result holds no matter whether government's objective is to maximize value of money (VFM) or total social surplus (TSS), though the different objectives can imply
different choices between the PPP and PUB approaches. Our analysis has also demonstrated that PPP contracts based on usage sensitive payments (e.g. tolls) can generate higher VFM for the government.

To our knowledge, this paper is the first to offer a formal model that focuses on this trade-off, though the idea that efficiency comes at a price of flexibility is very intuitive and appears in other contexts (e.g. in the choice of flexible vs inflexible production technologies). Our results may shed some light on why PPPs have become popular in some sectors such as roads and water – where it could be argued that the need for large changes to designs is likely to be relatively smaller – while they remain less common (and more controversial) in areas that might seem more dynamic, such as health care and information technology.

We suggest a few directions in which future work could advance our understanding of these trade-offs. First, the basic model here could be generalized in a number of ways, for example incorporating differential switching costs to change project designs, by allowing contracts to be contingent on observed signals of unobservable variables, or by considering alternative functional forms to test the robustness of our results. Second, as efficient risk shifting is a big part of successful PPP projects, we could consider the implications for our model of leaving risk averse private firms subject to some risk that they cannot control. Finally, we could consider some of the strategies that public and private sector partners employ to deal with the deficiencies of the PPP and PUB models presented above. For example, in PPP projects the parties may anticipate the need to renegotiate in the future and may then put into place mechanisms (e.g. third-party arbitration) to limit the ability of the private partner to take advantage of its strong position to extract much higher payments. PUB modes can also be improved by creating incentives for innovation and effort on the part of public sector providers. Indeed, a great deal of effort has been put into making governments more efficient providers of services generally, through, for example, giving managers more authority for the way their units operate but making them also more accountable for the quantity and quality of the services provided.\[42\]

\[42\] Some of these would be viewed as aspects of the “New Public Management” approach to public administration. See, e.g. Osborne and Gaebler (1992).
Appendix

Derivation of VFM in the general case taking into consideration the government’s cost of raising revenue

The Nash product is

\[ [b_0 - (1 + y)a_1^{PPP} - (1 + y)s - (b_1 - (1 + y)a_0^{PPP})]^1 \left[ (a_1^{PPP} - K + \frac{\delta^2}{2} - s) - (a_0^{PPP} - K + \frac{\delta^2}{2}) \right]^{1-\lambda} \]

The optimal solution is

\[ a_1^{PPP} = \frac{(1 - \lambda)(b_0 - b_1)}{1 + y} - (1 - 2\lambda)s + K - \frac{\delta^2}{2} \]

So the VFM of PPP after renegotiation is

\[ b_0 - (1 + y)a_1^{PPP} - (1 + y)s = \lambda b_0 + (1 - \lambda)b_1 - (1 + y) \left( K - \frac{\delta^2}{2} \right) - (2\lambda + y)s \]

Without any changes in demand, the contract remains the same with VFM is

\[ b_0 - (1 + y) \left( K - \frac{\delta^2}{2} \right) \]

The expected VFM under the PPP contract is then

\[ VFM^{PPP} = \mu \left( \lambda b_0 + (1 - \lambda)b_1 - (1 + y) \left( K - \frac{\delta^2}{2} \right) - (2\lambda + y)s \right) + (1 - \mu) \left( b_0 - (1 + y) \left( K - \frac{\delta^2}{2} \right) \right) \]

\[ = \left( 1 - \mu(1 - \lambda) \right) b_0 + \mu(1 - \lambda)b_1 - (1 + y) \left( K - \frac{\delta^2}{2} \right) - (2\lambda + y)\mu s \]

This is expression (10) in the main text.
References


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