Watering a Lemon Tree: Heterogeneous Risk Taking and Monetary Policy Transmission

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Abstract

We build a general equilibrium model with financial frictions that impede monetary policy transmission. Agents with heterogeneous productivity can increase investment by levering up, which increases liquidity risk. In equilibrium, productive agents choose higher leverage, which limits their responsiveness to interest rate changes. A reduction in the interest rate then leads to a deterioration in aggregate investment quality, which decreases liquidation values. This, in turn, reduces loan demand, decreasing the interest rate further and generating a negative spiral. Overall, the allocation of credit is distorted and monetary stimulus can become ineffective even with significant interest rate drops.

JEL Classification: E52, E58, G20

Keywords: monetary policy transmission; financial frictions; heterogeneous agents; financial intermediation

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1 Introduction

The run-up to the recent financial crisis as well as its aftermath have led to concerns about monetary policy being "too loose for too long" and leading to "excessive risk taking." However, monetary policy is loose for a reason—to stimulate investment and output—and is supposed to encourage risk taking—by lowering the hurdle rate so more projects receive funding. This apparent paradox raises questions about the effectiveness of monetary stimulus and the nature of risk taking involved.

In this paper, we try to give answers by taking inspiration from policy makers such as Stein (2014) and Tarullo (2014) who point to the stimulus of low-quality investment financed with risky maturity transformation. We build a general equilibrium model with heterogeneous agents facing financial frictions and show how heterogeneous responses and risk taking by the wrong agents can significantly impair the transmission of monetary stimulus. Aggregate output can become unresponsive to monetary stimulus even with significantly lowered interest rates due to a feedback between investment quality deterioration in response to the lower interest rate and decreased aggregate loan demand further lowering the interest rate, leading to inefficient credit reallocation in the economy. Our mechanism is therefore different from the conventional liquidity trap in which output becomes unresponsive to monetary stimulus because the interest rate becomes unresponsive, as well as from the conventional credit channel where monetary policy has an amplification effect.

The model features heterogeneous entrepreneurs that differ in their constant-returns-to-scale productivity and have to borrow to invest. In the first-best case without any financial frictions only the most productive entrepreneur would invest—absorbing all the loanable funds—which would maximize aggregate output in this economy. However, we impose frictions that disrupt the efficiency of credit allocation. As a first friction, we assume that borrowing comes with interim liquidity risk because of financial intermediation with maturity transformation. Investment projects are long-term but borrowing is short-term such that borrowers are subject to liquidity shocks at an interim date. When hit by the shock, a borrower has to liquidate her assets in the secondary market at a discount.

The probability of a liquidity shock is higher with more leverage, thus ex ante liquidity risk increases as borrowers lever up. This implies that the marginal liquidity cost of additional borrowing becomes higher as a borrower's leverage increases, pushing the allocation away from the first-best. In equilibrium, each borrower equates the marginal excess return of her project to the marginal cost of liquidity risk. Since more productive entrepreneurs have higher excess returns, they can afford to take on more liquidity risk

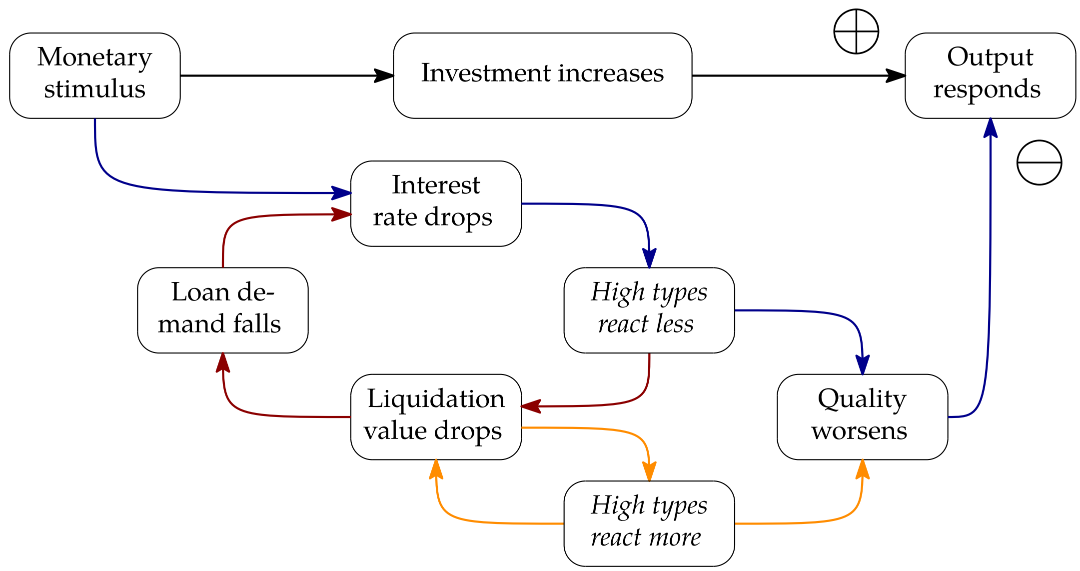

Figure 1: Negative feedback spirals dampening the effect of monetary policy

and, as a result, they borrow more and invest more in equilibrium.

Our novel effects arise from the fact that, for given equilibrium values of interest rate and secondary market liquidation value, each borrower is at a type-specific interior optimum. When monetary policy leads to changes in these equilibrium values, different borrower types respond heterogeneously in adjusting their leverage, which leads to a change in the distribution of investment across types and therefore affects the aggregate response to the policy.

As all agents in our model are risk neutral and have rational expectations, every borrower pays the risk free interest rate in expectation and a change in the rate has the same effect on every borrower type's first order condition. However, high types endogenously face higher marginal liquidity risk and therefore adjust their borrowing *less* to changes in the interest rate than low types. In contrast, due to high types' greater exposure to liquidity risk, a change in the liquidation value has a bigger effect on high types' first order condition. This can lead high types to adjust their borrowing *more* to changes in the liquidation value than low types—the opposite from the response to the interest rate.

Figure 1 illustrates the general equilibrium mechanisms of the model. When the central bank provides monetary stimulus, the market clearing interest rate drops, leading to an increase in investment which has a standard positive effect on output (black arrows). In our model, however, the quality of investment changes since agents with different investment productivity respond heterogeneously to the interest rate drop. Since high types are already more exposed to liquidity risk, they are more reluctant to lever up further. Overall, the direct effect of a decrease in the interest rate is therefore a shift in the distribution of investment towards low types so the average quality of investment in the economy worsens which has a negative effect on output (blue arrows).

In addition, the shift in investment towards low types opens the door for indirect feedback effects when we introduce our second financial friction: a lemons problem in the secondary market for liquidated assets such that the liquidation value depends on the overall quality of assets sold. With this lemons pricing, e.g. due to opaqueness or complexity of the underlying assets, the heterogeneous response to monetary stimulus leads to a drop in the equilibrium liquidation value. This raises the cost of being hit by a liquidity shock and reduces all borrowers' demand for funds, causing a feedback loop of further downward pressure on the interest rate (red arrows). In addition, the drop in the liquidation value tightens the trade-off between investment return and liquidity risk *more* for *high* types. Thus, high types can react more to the liquidation value than low types, so that quality deteriorates further (orange arrows) and a lemons spiral arises.

In this feedback process, the overall quality of investment deteriorates significantly as funds are reallocated from high types to low types. Overall, monetary stimulus can therefore lead to a large drop in the interest rate but only a small increase (or potentially even a decrease) in aggregate output due to the composition of investment changing from agents with high productivity to agents with low productivity. Our model therefore helps understand the apparent paradox of weak monetary policy transmission concurrent with undesirable risk taking as stimulus leads to low-quality investment with increased liquidity risks.

In an extension of the model, we show that the lemons pricing is not essential for our results. In particular, we analyze the case where the buyers can distinguish the individual assets in the secondary market but the cash in the secondary market is limited e.g. due to limited participation (Allen and Gale, 1994, 1998). This, in turn, leads to cash-in-the-market pricing and we show that even in this setup monetary stimulus gets dampened due to liquidity risk and the heterogeneous response of agents.

We discuss several policy implications of our model. First, the importance of liquidation values suggests a role for unconventional monetary policy intervening in secondary markets ex post to complement the effectiveness of conventional monetary policy ex ante.[1] We further show that the dampening effect in our model is stronger as the liquidity risk becomes more severe. This suggests that liquidity regulation such as the Liquidity Cov-

[1]See Choi, Eisenbach, and Yorulmazer (2016) for a more detailed discussion on the effectiveness of different forms of interventions that support asset prices such as asset purchases and the discount window.

erage Ratio and Net Stable Funding Ratio of Basel III which reduce liquidity risks can be complementary to the transmission of monetary policy. In terms of business cycles, our dampening effect of monetary stimulus does not imply a symmetric amplification effect of monetary tightening which is usually implemented during booms when liquidity risks are lower. Finally, we argue that other costs that are increasing in balance sheet size can generate similar heterogeneous responses. For example, in a setting with compliance costs for regulation, our mechanism predicts a relative growth of unregulated "shadow" banking in response to prolonged monetary loosening.

Related literature: This paper is related to an emerging literature that focuses on the role of heterogeneous agents in the transmission of monetary policy. Di Maggio et al. (2015) and Keys et al. (2014) analyze the consumption responses of heterogeneously indebted households, while Ippolito et al. (2015) focus on firms with different levels of bank debt. Auclert (2015) provides a theoretical model with agents heterogeneously exposed to interest rate risk and study the monetary policy transmission in general equilibrium. Sufi (2015) provides a literature review on recent findings, emphasizing the importance of "redistribution channels of monetary policy."

While these papers focus on the transmission through households or firms, our paper focuses on the endogenous allocation of credit and analyzes how introducing heterogeneity changes the efficiency implications of monetary policy.[2] In that regard, it is related to the literature on the credit channel of monetary policy. Our mechanism is different from the standard balance sheet channel (e.g. Bernanke and Gertler 1989, 1995) in which an external finance premium resulting from agency problems is the main driver. In that case, monetary policy has an *amplifying* effect since it relaxes the financial constraints of borrowers, whereas in our case there are no agency problems and a *dampening* effect arises. Thus, agents in a standard setup face binding financial constraints and the shadow costs of capital are different across agents in equilibrium. In our setup, agents are making an unconstrained decision and thus the marginal costs of capital are equalized in equilibrium. In addition, since we assume constant returns to scale for each entrepreneur there is no lack of good projects.

Our model also presents a novel distortion of monetary transmission within a bank lending channel (Bernanke and Blinder, 1992; Kashyap and Stein, 2000), driven by het-

[2] In a recent paper, Agarwal et al. (2015) find that bank-mediated stimulus was less effective during the Great Recession due to inefficient pass-through leading to liquidity misallocation. They argue that facing a reduction of funding costs, banks extended additional credit to the agents with lower marginal propensity to borrow, due to concern about asymmetric information problems.

erogeneous agents' endogenously chosen risk exposures that consequently limit debt capacity. Benmelech and Bergman (2012) also study how the real economy becomes unresponsive to monetary stimulus due to financial frictions in credit intermediation. Our distinction between "quality" and "quantity" of lending is related to the emerging literature on the risk taking channel of monetary policy (for an overview, see Borio and Zhu, 2012; De Nicolò et al., 2010; Adrian and Shin, 2010), although our focus is on productivity rather than credit risks. Empirical evidence relating monetary loosening and quality deterioration is documented by Ioannidou et al. (2015), Altunbas and Marques-Ibanez (2014), Peydró and Maddaloni (2011), Paligorova and Santos (2012), Dell'Ariccia et al. (2016), and Jiménez et al. (2014). Dell'Ariccia et al. (2014) provide a theoretical argument.

In our paper, intermediation features a maturity mismatch. Hence, our paper is related to the literature on maturity structure of debt and the associated fragility that arises (e.g. Diamond and Dybvig, 1983; Flannery, 1986; Diamond, 1991; Calomiris and Kahn, 1991; Diamond and Rajan, 2001; Brunnermeier and Yogo, 2009; Brunnermeier and Oehmke, 2013).

Our paper is related to the literature on fire sales and costly liquidation of assets. The idea that fire sales can occur when potential buyers are financially constrained and assets are not easily deployable was shown by Williamson (1988) and Shleifer and Vishny (1992). Holmström and Tirole (1998) study an ex-ante investment decision facing this interim risk, and Allen and Gale (1994, 1998) feature models where the price of assets is determined by the level of liquidity in the market, resulting in cash-in-the-market pricing. There is strong empirical support for this idea in the corporate-finance literature, documented by Pulvino (1998), Acharya et al. (2006), Berger et al. (1996) and Stromberg (2000). The evidence of such effects specifically for financial intermediaries is studied by James (1991), Shin (2009) and Gorton and Metrick (2010, 2012). Rosenthal and Wang (1993) use a model in which sellers may not be able to extract the fundamental value due to the informational rents earned by the privately informed bidders. However, in contrast to other banking models with costly liquidation, we assume that interim liquidations do *not* result in any direct welfare losses but only amount to transfers between agents. Hence, our inefficiency is entirely driven by disruption in the allocation of credit across heterogeneous agents, i.e. the redistribution of investment from high to low productivity agents.

Finally, our paper contributes to the broad literature on incorporating financial frictions into macroeconomic analysis. In particular, we analyze how frictions in the secondary market generate macro effects. Kiyotaki and Moore (1997) study the effect of resalability of financial assets in secondary markets on aggregate investment, and Kurlat (2013) builds a model in which this friction comes from a lemons problem in the secondary market. Bolton et al. (2011) and Malherbe (2014) also study an economy in which incomplete information in the secondary market affects investment decisions. For a general review, see e.g. Brunnermeier et al. (2013).

The paper is organized as follows. Section 2 discusses the model setup. Section 3 analyzes the leverage and investment decisions of individual agents, as well as the effects of interest rates and asset prices on such choices. Section 4 analyzes the mechanism of monetary transmission impairment in a general equilibrium setup. Section 5 illustrates the model with a numerical example. Section 6 discusses policy implications and the model's assumptions and Section 7 concludes. All proofs are in Appendix B.

2 Model setup

Primitives: Consider a model with three dates $t = 0, 1, 2$. There are two groups of agents: lenders, e.g. households, and borrowers, e.g. entrepreneurs. All agents are risk neutral and have discount factors of 1. At $t = 0$, borrowers have an investment opportunity but no endowment while lenders, with measure 1, have a perishable endowment of E that can be invested. Among borrowers, we have two types, high and low, denoted by $i = h, \ell$, each with measure 1.[3] Borrowers have access to a type-specific investment technology with constant returns to scale which pays off a random return at $t = 2$: one unit of investment implemented by an agent of type i yields $R\theta_i$ with probability p and 0 with probability $1 - p$. We assume $\theta_h > \theta_\ell$ so that types reflect heterogeneous productivity across agents whereas p and R capture aggregate productivity variables, such as TFP. Assuming constant returns to scale is both for simplicity and to ensure that our dampening effects are not driven by exogenous technology but by endogenous agent behavior. Note that there is no heterogeneity in the exogenous riskiness of agents' projects at $t = 2$ as p is the same for all types. To simplify the notation, we denote the expected return from the investment as $R_i \equiv pR\theta_i$ and assume that both types' investment is productive, $R_h > R_\ell > 1$.[4]

Borrowing/lending: In order to focus on liquidity risk originating in financial intermediation with maturity transformation, we only consider debt financing. At $t = 0$, the borrowers invest by borrowing from the lenders in the loanable funds market. Let D_i denote the amount type i borrows at $t = 0$ so that the total investment by borrowers of type i is $I_i = D_i$. Lenders are competitive so the interest rate r_i promised by a borrower of type i

[3] This setup is for simplicity and we could generalize the number of types as well as the distribution of the endowment.

[4] Only the product $p \times R$ matters in our analysis with risk neutral agents, while the separation of p and R becomes useful when we provide a micro-foundation of the liquidity risk in Appendix A.

guarantees that all lenders receive the risk-free rate r in expectation. Because debt is fairly priced, our results are not driven by distortions such as deposit insurance or agency problems, which are common in other models with financial intermediation. The risk-free rate r, in turn, is determined endogenously by market clearing in the market for loanable funds at $t = 0$. We assume $R_\ell > 1 + r$ so that even the low type has a high enough expected return from the investment to cover her expected funding cost.

Total investment in this economy at $t = 0$, denoted by I, can be written as

$$I = D_h + D_\ell.$$

Taking the heterogeneous productivity into account, the average quality of investment as measured by its productivity is given by

$$q = \frac{R_h D_h + R_\ell D_\ell}{D_h + D_\ell}, \tag{1}$$

which depends on the distribution of D_i across the two types.

Note that type h has a higher expected return from the risky investment. Hence, the first-best allocation would require that all the funds in the economy are invested by the high types.

Liquidity risk: The key friction in our setup is that borrowers face liquidity risk in the interim period $t = 1$ and that this risk is a function of leverage. We assume that all debt is short-term and needs to be rolled over at $t = 1$, which creates the potential for liquidity problems due to creditor runs. This is a typical maturity-mismatch problem that financial institutions face, and thus we effectively assume that financial intermediation is involved in channeling funds from lenders to borrowers. As is customary, we collapse the financial intermediary and the borrower into a single economic agent for simplicity (see, e.g. Brunnermeier and Sannikov, 2014). When experiencing a run, a borrower is forced to liquidate the long-term assets in a secondary market at a discount, which is costly for the agent. Thus debt becomes endogenously more costly to the borrower as she increases leverage, although its expected rate of return to the lender stays constant at r.

In Appendix A, we micro-found how the probability of a creditor run increases with leverage, using a global game setup. In the main text, we use a reduced-form setup, denoting by $\alpha(D)$ the ex-ante, as of $t = 0$, probability that an agent with debt level D experiences a run at $t = 1$.[5] We assume the following properties for the liquidity risk $\alpha(D)$: it is in-

[5] Kashyap et al. (2014) adopt a similar reduced-form setup where interim liquidity risk is captured by

creasing and weakly convex in leverage, $\alpha'(D) > 0$ and $\alpha''(D) \geq 0$, with $\alpha(0) = 0$, and effects are not driven by higher order terms, $\alpha'''(D) \approx 0$.

In contrast to the failure risk $1 - p$ at $t = 2$, which is exogenous and the same across agents, the liquidity risk $\alpha(D_i)$ at $t = 1$ is endogenously determined by each agent's borrowing decision D_i and can therefore differ across agents.[6] Morris and Shin (2010) propose a decomposition of the unconditional 'credit risk' into an 'insolvency component' conditional on no run and an 'illiquidity component' accounting for the difference. Using this language, we can decompose total 'credit risk' of type i as follows:

$$\begin{aligned}\text{'credit risk'} &= \big(1 - \alpha(D_i)\big)\,(1-p) + \alpha(D_i) \\ &= \underbrace{1-p}_{\text{'insolvency risk'}} + \underbrace{\alpha(D_i)\,p}_{\text{'illiquidity risk'}}\end{aligned} \tag{2}$$

Any heterogeneity in total credit risk across agents therefore originates from the endogenous heterogeneity in illiquidity risk.

Liquidation values: Our second financial friction is in the secondary market where borrowers facing a run at $t = 1$ have to liquidate their assets. In contrast to borrowing at $t = 0$, where lenders have time to assess each individual borrower, we assume that liquidation following a run at $t = 1$ happens quickly and disorderly such that potential buyers cannot distinguish the individual quality of the liquidated assets.[7] Instead, potential buyers only know the average quality of the assets being liquidated in the secondary market given by

$$Q = \frac{\alpha(D_h)\,R_h D_h + \alpha(D_\ell)\,R_\ell D_\ell}{\alpha(D_h)\,D_h + \alpha(D_\ell)\,D_\ell}, \tag{3}$$

where the denominator characterizes the amount of liquidated assets, and the numerator characterizes long-term output from the liquidated assets.[8] Formally, we assume lemons pricing due to incomplete information such that $P = Q - \delta$ where δ is a discount relative

ex-ante probability of a run, which is a function of balance-sheet variables. With constant borrower equity, changes in leverage are entirely due to changes in debt. Since our borrowers' equity is constant and normalized to 0, we make liquidity risk a function of debt.

[6] Note that the function α itself does not vary across agents which follows from the micro-foundation in Appendix A.

[7] In Section 6.5, we also consider cash-in-the-market pricing (Allen and Gale, 1994, 1998), where the buyers *can* distinguish individual asset quality but there is limited liquidity in the secondary market. We show that our main results go through with cash-in-the-market pricing so that asymmetric information in the secondary market is not necessary but gives us additional effects.

[8] For evidence on asset opacity, especially in case of financial intermediation, see, e.g. Morgan (2002), Hirtle (2006) and Flannery et al. (2013).

to the assets' expected fundamental value, and we only focus on the case with $P < 1 + r$ such that liquidation is costly for the borrower.[9]

Importantly, we assume that no output is lost through the secondary market liquidation process. The difference δ between expected fundamental value and liquidation value is simply a transfer, e.g. to bankruptcy lawyers. This assumption implies that the dampening effect we show is *not* due to resources lost in inefficient liquidation; the effect is due purely to changes in the equilibrium distribution of borrowing levels $\{D_i\}$ across types. This also implies that a social planner can only improve efficiency by changing the distribution of investment across types.

Monetary policy: In addition to the lenders' initial endowment E, the central bank provides liquidity L to the market for loanable funds at $t = 0$. The equilibrium risk-free rate r then equates aggregate loan supply, consisting of the public supply L and the private supply E from lenders, with aggregate loan demand consisting of agents' borrowing, that is, $E + L = D_h + D_\ell$. We identify monetary policy as changes in the central bank's supply of loanable funds L. In this setup, the central bank can effectively create loanable funds at $t = 0$ which are then invested by borrowers and produce output at $t = 2$.

We discuss in Section 6.4 how our modeling of monetary policy can be interpreted, e.g. as the reduced form of a New-Keynesian setup or of a setup with reserve requirements. Furthermore, examining changes in L is equivalent to examining changes in the central bank target rate r since there is a one-to-one correspondence between L and r in equilibrium.[10] In our model, an advantage of analyzing changes in L is that we can directly compare the equilibrium allocation to the first-best allocation where all resources are invested by the high type.

Similar to Allen et al. (2014) and Keister (2016), we assume that monetary stimulus at $t = 0$ has costs at $t = 2$ given by a function $c(L)$ which is increasing in L to ensure that monetary policy is not a "free lunch." Although not explicitly modeled in this paper, these costs can be interpreted as, e.g. welfare losses from nominal price distortions, additional taxes necessary to meet the government's consolidated budget constraint, or less public goods provision.

Definition of equilibrium: The equilibrium of our economy is characterized by private decision variables (D_h, D_ℓ) and price variables (r, P) satisfying the following conditions:

[9] The assumption that all liquidation is at a loss, $P < 1 + r$, rules out strategic borrowing, where agents choose to borrow since P is higher than the borrowing cost and then always liquidate at $t = 1$.

[10] See Svensson (2003) for a discussion on the interchangeability between interest rates and money in New-Keynesian models.

1. Borrowers $i = h, \ell$ choose optimal debt levels $D_i(r, P)$ taking prices r and P as given.

2. The risk-free rate r clears the market for loanable funds:

$$E + L = D_h + D_\ell$$

3. The secondary market price P satisfies the pricing rule given the private decision variables (D_h, D_ℓ) such that $P = Q - \delta$ where Q is defined by (3).

3 Individual agent behavior

We first analyze the optimizing behavior of individual agents taking prices r and P as given. Our analysis specifically focuses on how different types change their $t = 0$ leverage—and therefore investment levels—differently in response to changes in these prices. We first show that high types react less elastically to changes in the interest rate r. We then show that high types can react more elastically to changes in the liquidation value P.

Since the lenders don't have access to the investment technology, they will lend their entire endowment. Borrowers choose how much to borrow taking the prices P and r as given. Since all agents are risk neutral and the loanable funds market is competitive with no agency problem, the equilibrium market clearing rate r is the expected rate of return for lending and the expected cost of borrowing, common across all agents in this economy. Intuitively, a borrower's expected payoff is therefore the total expected payoff from the investment minus the expected funding cost. Formally, we have the following result. All proofs are relegated to Appendix B.

Lemma 1. *When every borrower promises to pay a type-specific interest rate r_i such that all lenders receive the risk free rate r in expectation, we can write a borrower's payoff as*

$$\Pi_i(D; r, P) = \big(1 - \alpha(D)\big) R_i D + \alpha(D) PD - (1 + r) D. \tag{4}$$

When an agent i borrows D, she ex-ante anticipates that a run occurs at $t = 1$ with probability $\alpha(D)$ leaving only P per unit of investment whereas she expects to collect R_i when she does not experience a run. Since $R_\ell > 1 + r$, both types borrow and invest in their projects. Note that we can also write the expected payoff (4) as follows:

$$\Pi_i(D; r, P) = \underbrace{R_i D}_{\text{gross payoff}} - \underbrace{(1 + r) D}_{\text{funding cost}} - \underbrace{\alpha(D) (R_i - P) D}_{\text{liquidity cost}} \tag{5}$$

This illustrates that the liquidity risk effectively imposes an implicit 'liquidity cost' which is deducted from the gross investment return just like the funding cost. Differentiating (5) with respect to D, we get the first order condition characterizing borrower i's optimal loan demand D_i:[11]

$$\underbrace{R_i - (1+r)}_{\text{marginal excess return}} = \underbrace{\left(\alpha'(D_i)\, D_i + \alpha(D_i)\right)(R_i - P)}_{\text{marginal liquidity cost}} \tag{6}$$

Without the liquidity risk, an agent should keep on increasing her investment as long as the marginal excess return—the wedge between the marginal product of investment R_i and the marginal funding cost $1+r$—is positive. However, liquidity risk increases as leverage goes up making additional borrowing more costly. At the optimal level of borrowing, each type's wedge is filled with the type-specific liquidity risk premium. The wedge is larger for the high types, and thus they can take more liquidity risk by building up higher leverage.[12]

Proposition 1. *For given r and P, high types borrow more than low types, i.e. $D_h > D_\ell$.*

As a result of optimal leverage increasing in type, total credit risk defined by (2) is also increasing in type—higher types are riskier borrowers since high types choose to take on more liquidity risk. This may seem counterintuitive if high types are thought of as "good borrowers" who should be "safe borrowers." However, in our model type corresponds to investment productivity, which induces more productive types to *endogenously* take on higher liquidity risk.

3.1 Response to interest rate

We now analyze how borrowers respond to changes in the interest rate. The wedge between the marginal product of investment R_i and the marginal funding cost $1+r$ becomes larger when the interest rate is lower, so that agents have more "room" to take additional liquidity risk when the funding cost is lower.

Proposition 2. *For a reduction in r, all borrowers increase their debt, i.e. $\partial D_i/\partial r < 0$ for $i = h, \ell$. High types respond less than low types, i.e. $|\partial D_h/\partial r| < |\partial D_\ell/\partial r|$.*

[11] The second order condition is satisfied with weakly convex and non-decreasing α:

$$-\left(\alpha''(D_i)D_i + 2\alpha'(D_i)\right)(R_i - P) < 0$$

[12] Note that the marginal funding cost is equal to $1+r$ for all agents with the binding first order condition, and thus there is no external finance premium that could be different across types, unlike in the conventional credit channel models.

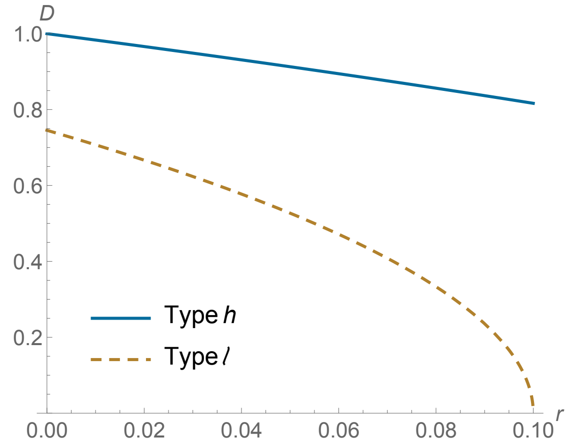

Figure 2: Optimal borrowing D_i as a function of the interest rate r for the two types h and ℓ. The functional forms and parameter values used are the same as in Section 5 with $\alpha(D) = 0.2D^2$ and $P = 0.8$.

Figure 2 illustrates the optimal borrowing D_i as a function of type i for different levels of r. The intuition for the heterogeneous response can be seen from the first-order condition (6) where a drop in r leads to an identical increase in the marginal excess return on the LHS for both types which has to be balanced by an increase in the marginal liquidity cost on the RHS. To achieve this, high types require a smaller increase in borrowing than low types for two reasons:

1. High types are more levered than low types so both their exposure $\alpha(D_h)$ per dollar of additional debt and their additional exposure $\alpha'(D_h)D_h$ on their existing debt are higher than those for low types.

2. High types suffer a bigger discount $R_h - P$ per dollar of investment when forced into liquidation.

These two reasons both imply that the marginal liquidity cost is more sensitive to changes in leverage for high types. Figure 3 illustrates the different sensitivities by plotting marginal excess return and marginal liquidity cost for the two types. Since the marginal liquidity cost is steeper for high types, the same parallel shift in the marginal excess return leads to a smaller response in high types' borrowing.[13]

[13]Contrary to the second-order effect that a change in the choice variable has on the maximized objective function (envelope theorem), we are dealing with the *first-order* effect that a change in a price variable has on the choice variable.

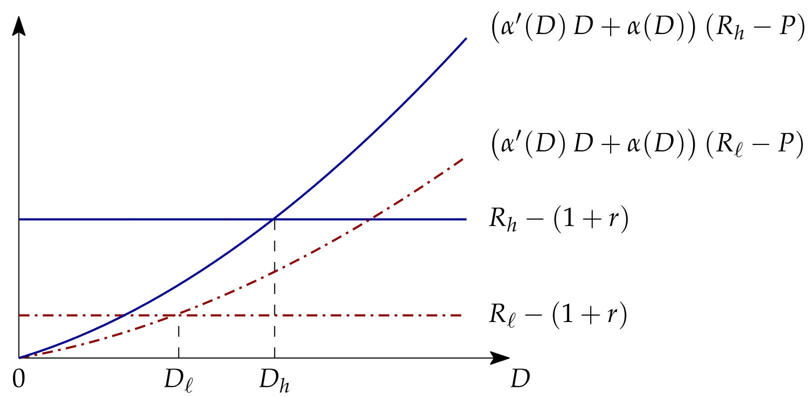

Figure 3: Marginal excess return and liquidity cost for both types.

3.2 Response to secondary market price

We next analyze how borrowers respond to the changes in the secondary market price P. An increase in P makes liquidation less costly and therefore reduces marginal liquidity cost on the RHS of the first order condition (6). Similar to a drop in the interest rate r, this leads both types to borrow more. However, the relative response for high and low types to P is different than for r.

In contrast to r, which enters the first-order condition (6) of both types with a factor of -1, the liquidation value P enters with a factor of $\left(\alpha'(D_i)D_i + \alpha(D_i)\right)$, which is larger for high types. While a drop in r generates the same slack in the first-order condition for all types, an increase in P therefore generates *more* slack for high types than for low types. This effect on its own would imply that high types respond *more* to changes in P than low types. However, it is combined with the effect discussed in Section 3.1 that high types need smaller increases in borrowing to achieve the same level of tightening of their first-order condition.

With these competing effects, we have the following result.

Proposition 3. *For an increase in P, all borrowers increase their debt, i.e. $\partial D_i/\partial P > 0$ for $i = h, \ell$. High types respond more than low types to a change in P, i.e. $\partial D_h/\partial P > \partial D_\ell/\partial P$, if and only if*

$$\frac{\alpha''(D_h)D_h + 2\alpha'(D_h)}{\alpha''(D_\ell)D_\ell + 2\alpha'(D_\ell)} < \frac{\left(R_h - (1+r)\right)/(R_h - P)^2}{\left(R_\ell - (1+r)\right)/(R_\ell - P)^2}. \tag{7}$$

Condition (7) captures the two competing effects of P on the first order condition (6) and can hold locally or globally, depending on the parameters chosen. For example, Figure 4 illustrates the optimal borrowing D_i as a function of P for quadratic liquidity risk $\alpha(D) = aD^2$ and shows high types responding more than low types at every level of P. In case of

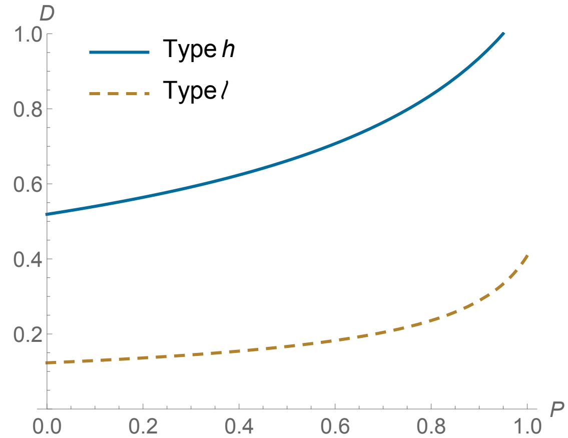

Figure 4: Optimal borrowing D_i as a function of the liquidation value P for the two types h and ℓ. The functional forms and parameter values used are the same as in Section 5 with $\alpha(D) = 0.2D^2$ and $r = 0.09$.

linear liquidity risk $\alpha(D) = aD$, we have a simple sufficient condition for (7).

Corollary 1. *For linear liquidity risk, a sufficient condition for high types to respond more to P than low types is* $2(1+r) > R_h + P$.

4 Monetary policy with heterogeneous risk taking

We are interested in the effect of monetary policy in the initial period $t = 0$ on aggregate output in the final period $t = 2$. Since agents in our model are heterogeneous in their investment productivity, changes in aggregate output also depend on how the distribution of initial investment across different types changes. Therefore we have two channels of monetary policy transmission: Monetary policy—a change in L—affects aggregate output (i) through its effect on the quantity of aggregate investment—a change in I—and (ii) through its effect on the average quality of investment—a change in q.

Recall that we assume no output is lost through the secondary market liquidation process in the interim period $t = 1$. Aggregate output in the final period $t = 2$ can therefore be written as the average quality of investment times the aggregate amount invested:

$$\begin{aligned} Y &= R_h D_h + R_\ell D_\ell \\ &= q \times I, \end{aligned}$$

where q is the average productivity of investment defined in (1). Denoting output net of the costs of monetary policy by $\bar{Y} = Y - c(L)$, the effect of monetary policy in the form of changes in central bank liquidity L can then be decomposed into three parts:

$$\frac{d\bar{Y}}{dL} = \underbrace{q \times \frac{dI}{dL}}_{\text{new investment}} + \underbrace{\frac{dq}{dL} \times I}_{\text{change in quality}} - \underbrace{c'(L)}_{\text{marginal cost}}$$

The first and third part are straightforward and standard. In our model, total investment equals total loanable funds, $I = L + E$, so investment changes one-for-one with monetary policy, $dI/dL = 1$.[14] Our focus is therefore on the second part, how monetary policy affects the average quality of investment. While the effect on aggregate investment is always positive, the effect on average quality can be negative, dampening the effectiveness of monetary policy. If quality deteriorates sufficiently, it may even reverse the effect of monetary stimulus on output such that $d\bar{Y}/dL < 0$.

We can decompose the effect of L on quality as follows:

$$\frac{dq}{dL} = \underbrace{\frac{dq}{dr}}_{\text{'quality elasticity'}} \times \underbrace{\frac{dr}{dL}}_{\text{'stimulus pass-through'}} \tag{8}$$

Monetary policy affects the average quality of investment through its effect on the equilibrium risk-free rate which, in turn, affects average quality. If the first factor in the decomposition (8), which we refer to as 'quality elasticity', is positive and the second factor, which we refer to as 'stimulus pass-through', is negative, monetary stimulus decreases the interest rate but at the same time lowers the quality of investment. Digging deeper into these two parts highlights the effects of our model and the mechanism of negative feedback between the two factors, (i) a deterioration in investment quality in response to a lower interest rate, and (ii) a decrease in aggregate loan demand in response to the quality deterioration, leading to a further decrease in the interest rate.

First, consider the quality elasticity, i.e. the effect of the risk-free rate r on the average quality of investment q. Recall that average quality q is determined by the distribution of borrowing D_h and D_ℓ. The optimal borrowing, in turn, depends on the risk-free rate r as well as the secondary-market price P. When the secondary market price is an endogenous

[14] We don't have any hoarding of liquidity which would reduce investment, e.g. as in Diamond and Rajan (2011) or Gale and Yorulmazer (2013). See Choi et al. (2016) for an analysis that allows for hoarding, such that an increase in L at $t = 0$ doesn't necessarily lead to the same increase in I.

variable, we can further decompose the quality elasticity into a direct and an indirect effect:

$$\frac{dq}{dr} = \underbrace{\frac{\partial q}{\partial r}}_{\text{direct effect}} + \underbrace{\frac{\partial q}{\partial P} \times \frac{dP}{dr}}_{\text{indirect effect}} \tag{9}$$

Next, consider the stimulus pass-through, i.e. the effect of a liquidity injection L on the interest rate r. Note that the market clearing condition equating supply and demand of loanable funds is given by:

$$L + E = D_h + D_\ell$$

Implicit differentiation yields the equilibrium stimulus pass-through as the inverse of the effect of r on the aggregate demand for loanable funds:

$$\frac{dr}{dL} = \left(\frac{d}{dr} \left(D_h + D_\ell \right) \right)^{-1} \tag{10}$$

When additional funds are injected, the market clearing interest rate drops more if aggregate loan demand is less elastic. Given the dependence of optimal borrowing D_i on the risk-free rate r and the price P, the change in leverage also goes through two channels:

$$\frac{dD_i}{dr} = \underbrace{\frac{\partial D_i}{\partial r}}_{\text{direct effect}} + \underbrace{\frac{\partial D_i}{\partial P} \times \frac{dP}{dr}}_{\text{indirect effect}} \tag{11}$$

4.1 Direct effects of monetary stimulus

First, we analyze the direct effect of a change in liquidity L, assuming, for now, that the price P in the secondary market is fixed so that $dP/dr = 0$. We show that even in the absence of any price effects, our model generates a dampening effect on monetary stimulus because of the heterogeneous response of different types to changes in the interest rate.

Consider first the stimulus pass-through in equations (10) and (11). Without a change in P, the shift in the supply of loanable funds leads to a move along the demand for funds which is decreasing in the interest rate, $\partial D_i/\partial r < 0$ (Proposition 2). The market clearing rate therefore drops in response to an injection of loanable funds:

$$\frac{dr}{dL} = \left(\frac{\partial}{\partial r} \left(D_h + D_\ell \right) \right)^{-1} < 0 \quad \text{for} \quad \frac{dP}{dr} = 0$$

Consider next the quality elasticity in equation (9). Without a change in P, we now have:

$$\frac{dq}{dr} = \frac{\partial q}{\partial r} \quad \text{for} \quad \frac{dP}{dr} = 0 \tag{12}$$

Using the definition of q, we can write this as follows:

$$\frac{\partial q}{\partial r} = -\frac{\sum_i \big((q - R_i) \times \partial D_i/\partial r\big)}{\sum_i D_i} \tag{13}$$

Intuitively, for a lower interest rate, average quality should decrease (increase) if D_i increases more for the low (high) type. Formally, note the two factors in the summation in the numerator of (13): The first factor, $q - R_i$, is positive for the low type and negative for the high type and, since q is biased upward with $D_h > D_\ell$, summation only over $q - R_i$ would yield a positive result. The second factor, $\partial D_i/\partial r$, the direct effect of the risk-free rate r on the borrowing D_i of type i is negative; this factor plays the role of a weighting of different types, determining whether the positive or the negative part of $q - R_i$ dominates. The weighting and ultimately the sign of $\partial q/\partial r$ therefore depends on differences in sensitivity across types. Since Proposition 2 shows that $|\partial D_h/\partial r| < |\partial D_\ell/\partial r|$, i.e. high types are less sensitive to interest rate changes, we have that $\partial q/\partial r$ is positive. Therefore, overall investment quality deteriorates when the interest rate decreases.

Corollary 2. *Without changes in P, monetary stimulus leads to a decline in the interest rate, i.e. $dr/dL < 0$, which leads to a deterioration in investment quality, i.e. $dq/dr > 0$. The overall effect is a dampening of monetary policy transmission:*

$$\frac{dq}{dL} = \frac{dq}{dr} \times \frac{dr}{dL} < 0 \quad \textit{for} \quad \frac{dP}{dr} = 0$$

Hence, while monetary loosening leads to an increase in investment, it also leads to a deterioration of the quality of investments. This, in turn, dampens the effect of monetary stimulus. The effect is illustrated in Figure 1 through the blue arrows.

Note that we have a constant returns to scale investment technology as opposed to a decreasing returns to scale, which is common in the literature. Hence, there is no lack of good investment opportunities in our model, that is, the dampening effect of stimulus comes from the heterogeneous responses of agents and the change in the composition of investment.

4.2 Feedback through liquidation values

We now account for the endogeneity of the liquidation value P and examine how changes in the equilibrium value of P can strengthen the impairment of monetary transmission. Recall that we include the indirect effects through the secondary market price P in the quality elasticity (9) as well as in the stimulus pass-through (11). The direction of the indirect effects is determined by three derivatives:

1. dP/dr: the equilibrium comovement between the liquidation value P and the interest rate r
2. $\partial D_i/\partial P$: the effect of the liquidation value on the borrowing of type i
3. $\partial q/\partial P$: the direct effect of the liquidation value on the average quality of investment

We are interested in determining when the indirect effects further dampen the transmission of monetary policy. In particular, when a drop in the equilibrium interest rate r coincides with a drop in the equilibrium liquidation value P, that is, $dP/dr > 0$. This appears in both the quality elasticity and the stimulus pass-through and is necessary for the feedback effects. Recall that we assume buyers in the secondary market in $t = 1$ cannot observe individual quality but know the average quality Q of assets sold, and the secondary market price therefore reflects this average quality such that $P = Q - \delta$. The average quality Q of assets being sold in the secondary market (defined in equation (3)) is a function of each type's optimal debt level D_i, and thus depends on the risk-free rate r as well as the liquidation value P. The equilibrium liquidation value is therefore implicitly defined by the fixed-point condition

$$P = Q(r, P) - \delta. \tag{14}$$

Given this implicit definition of P in (14), the equilibrium effect of r on P is given by

$$\frac{dP}{dr} = \frac{\partial Q/\partial r}{1 - \partial Q/\partial P}. \tag{15}$$

Sufficient conditions for $dP/dr > 0$ are therefore $\partial Q/\partial r > 0$, that is, the average quality of liquidated assets has to decrease after a drop in the interest rate, and $\partial Q/\partial P < 1$ to guarantee a stable fixed point.[15]

[15] There is an important difference between average quality of all assets q and average quality of liquidated assets Q. Since high types borrow more, they are more likely to face liquidation, $\alpha(D_h) > \alpha(D_\ell)$, so their assets are over-represented in the secondary market, $Q > q$. While average quality of all assets always declines in response to a drop in the interest rate, $\partial q/\partial r > 0$ (Corollary (2)), we need an additional condition to guarantee $\partial Q/\partial r > 0$.

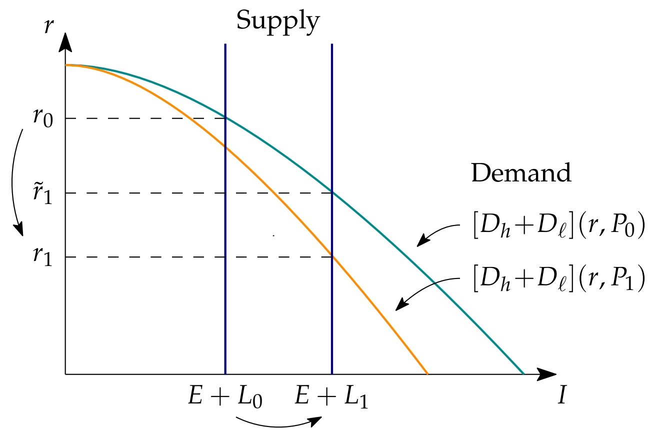

Figure 5: Stimulus pass-through for an increase in liquidity from L_0 to L_1. The direct effect is along the original demand curve from r_0 to $\tilde{r}_1$; the indirect effect is from $\tilde{r}_1$ to r_1 due to a shift in the demand curve as P drops from P_0 to P_1.

For stimulus pass-through, the indirect effect works by changing the responsiveness of borrowing demand D_i to the interest rate r and is illustrated in Figure 5. If the indirect effect in (11) is positive, it makes borrowing demand less responsive to r, which means stronger stimulus pass-through—a larger drop in r following an increase in L. Since the liquidation value P captures (inversely) how costly a liquidity shock is, agents borrow less for a lower liquidation value, $\partial D_i/\partial P > 0$, as shown in Proposition 3. With $dP/dr > 0$, the indirect effect through P offsets the direct effect and strengthens the stimulus pass-through, i.e. dr/dL becomes more negative.

For quality elasticity, heterogeneous response to the change in P can strengthen the effect as illustrated in Figure 6. If the indirect effect is positive, it means that quality of investment deteriorates further due to the heterogenous response of different types to changes in P (orange arrow in Figure 1). For $dP/dr > 0$, the sign of the indirect effect depends on $\partial q/\partial P$. As in the case of the direct effect of the risk-free rate on quality, $\partial q/\partial r$ in (13), the difference in sensitivity across types is key: average quality decreases if high types reduce their borrowing more than low types in response to a lower liquidation price.

Note that these heterogeneous responses to P can also impair the stimulus effect by directly depressing the liquidation value itself. Average quality of the liquidated assets decreases if high types reduce their borrowing more than low types in response to a lower liquidation price, i.e. $\partial Q/\partial P > 0$, which leads to greater dP/dr as in (15). This affects both stimulus pass-through and quality elasticity, amplifying the feedback.

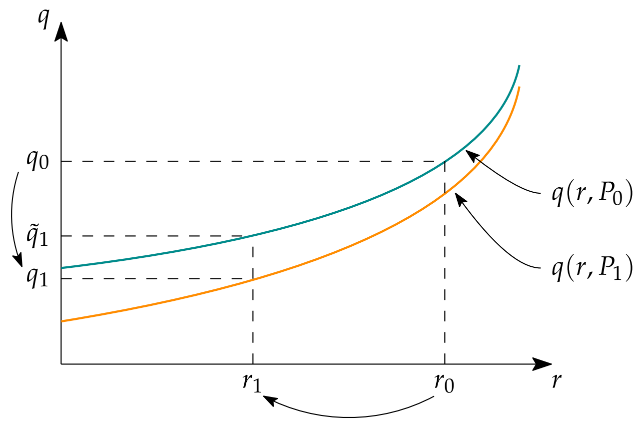

Figure 6: Quality elasticity for a drop in the interest rate from r_0 to r_1. The direct effect is along the original quality curve from q_0 to $\tilde{q}_1$; the indirect effect is from $\tilde{q}_1$ to q_1 due to a shift in the quality curve as P drops from P_0 to P_1.

Corollary 3. *The conditions for amplifying indirect effects are:*

$$\partial Q/\partial r > 0 \tag{16}$$

$$\partial Q/\partial P < 1 \tag{17}$$

$$\partial q/\partial P > 0 \tag{18}$$

$$\partial Q/\partial P > 0 \tag{19}$$

We have the following:

1. *Conditions* (16) *and* (17) *are sufficient for feedback in stimulus pass-through.*
2. *Conditions* (16), (17) *and* (18) *are sufficient for feedback in quality elasticity.*
3. *Under condition* (19), *there is a feedback in P itself, strengthening the feedbacks through both stimulus pass-through and quality elasticity.*
4. *The four conditions are not mutually exclusive.*

In sum, combining the effects of r and P on average quality q we see that our model can generate a strong spiral illustrated by Figure 7. Injections of liquidity increase the supply of loanable funds which puts downward pressure on the interest rate. Any reduction in the interest rate leads worse borrowers to lever up relatively more than higher quality borrowers, directly leading to a deterioration in the average quality of investment. In addition, under mild conditions, the expected quality of assets sold in the secondary market

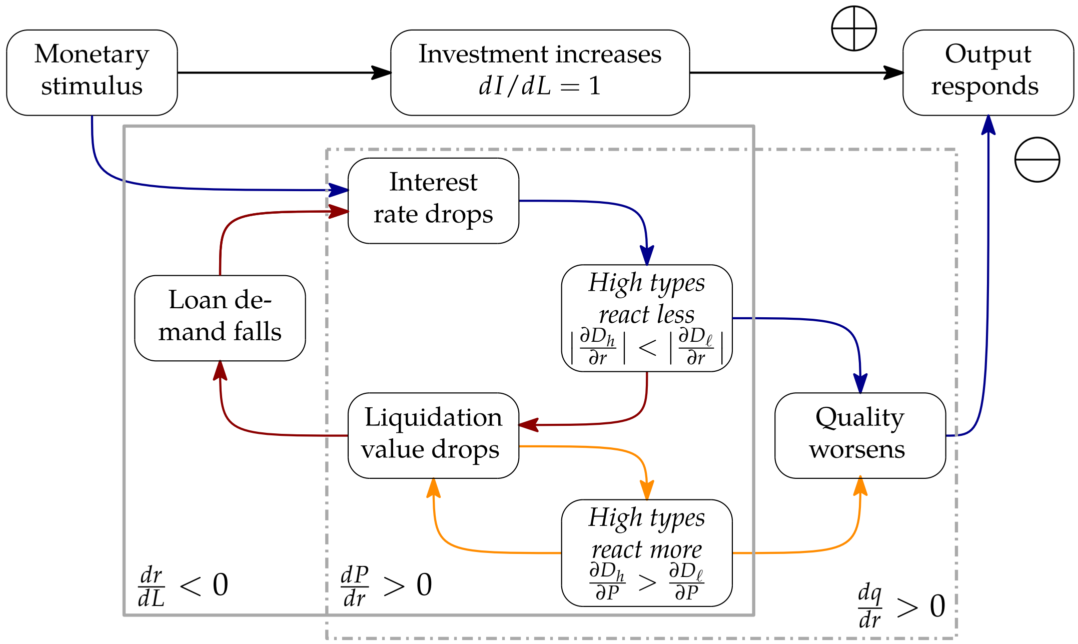

Figure 7: Summary of negative feedback spirals dampening the effect of monetary policy

at $t = 1$ also deteriorates. This, in turn, leads to a decrease in the liquidation value which reduces borrowers' demand for funds, causing further downward pressure on the interest rate. Finally, if better borrowers react more to the decrease in the liquidation value, (i) average quality deteriorates also indirectly (through P), and (ii) the liquidation value itself becomes more depressed, which strengthens the spiral further. The overall effect of the liquidity injection is then a large drop in the interest rate but only a small increase or potentially even a decrease in total output since total borrowing shifts from agents with high productivity to agents with low productivity.

5 Numerical example

In this section we present a numerical example to illustrate the impaired transmission of monetary policy in our framework. We choose quadratic functions for the run probability at $t = 1$ and the cost of monetary policy at $t = 2$:

$$\alpha(D) = aD^2, \quad c(L) = b_1 L + b_2 L^2$$

For the parameters of the model we use the values in Table 1.

Figure 8 compares the effect of monetary policy in the first-best economy ($a = 0$) and

Table 1: Parameters of numerical example

Parameter	Description
$E = 1$	Lender endowment
$(R_h, R_\ell) = (1.3, 1.1)$	Borrower expected returns
$a \in \{0, 0.2, 0.25\}$	Liquidity risk parameter
$(b_1, b_2) = (1, 0.5)$	Cost parameters
$\delta = 0.5$	Liquidation discount

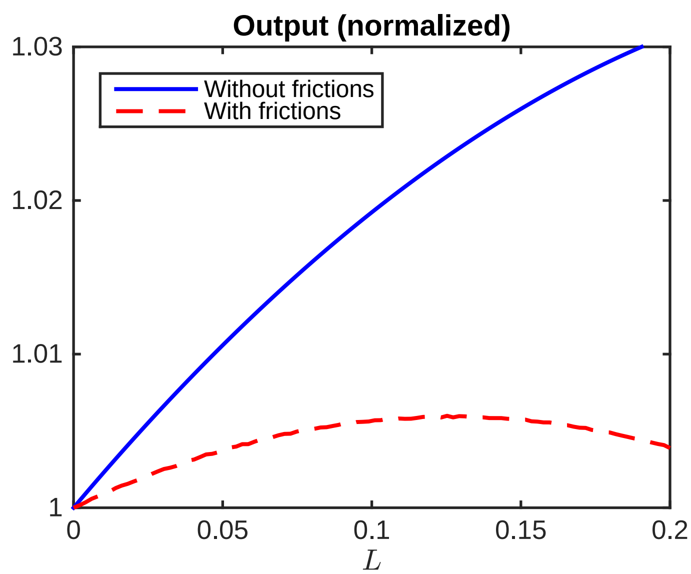

Figure 8: Effect of monetary policy on output with and without frictions

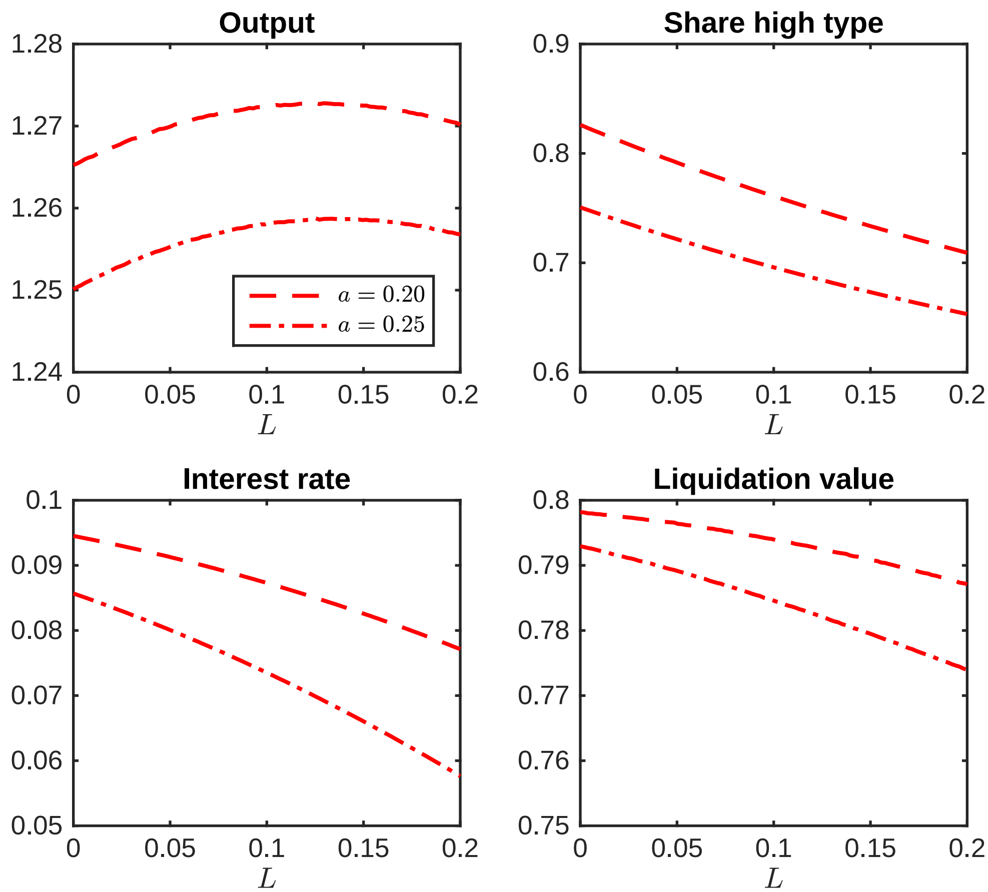

Figure 9: Effect of monetary policy under different severities of liquidity risk

in a second-best economy characterized by our frictions ($a = 0.2$). In the first-best economy without frictions, only the high type h invests and any liquidity injected is allocated only to the high type. Starting from $L = 0$, monetary stimulus at $t = 0$ increases output at $t = 2$ at a rate equal to the highest type's expected return, $R_h = 1.3$. Since we assume that monetary policy at $t = 0$ has costs $c(L)$ at $t = 2$, the stimulus effect is concave even in the first-best economy (blue solid line in Figure 8). In contrast, in the second-best economy with agents facing liquidity risk and lemons pricing, the effect of monetary policy is considerably impaired (red dashed line in Figure 8). Since it is no longer the case that only the high type invests, any stimulus is split across the high and low type, resulting in a flatter path starting at $L = 0$. As stimulus increases, the quality deterioration effect kicks in and final output is strongly concave and eventually decreasing in L.

Figure 9 shows the effect of monetary policy under two scenarios that differ in the severity of the liquidity risk, $a \in \{0.20, 0.25\}$. The first row shows final output $\bar{Y}$ and the share of the high type's investment $D_h/(D_h + D_\ell)$, while the second row shows the equilibrium interest rate r and the lemons price P. The first thing to note is that more severe liquidity risk reduces the level of output overall. This is significant, since aggregate investment is always $I = E + L$ so the difference in output for a given level of L is due *only* to the endogenous distribution of borrowing across types, as is clear from the top right panel. This is also reflected in the different levels of the interest rate r and the liquidation value P. With higher liquidity risk a, overall borrowing demand is lower so the equilibrium interest rate is lower. This, however, leads worse types to invest which is reflected in the lower liquidation value (recall that $P = Q - \delta$). Furthermore, we see that the effect of monetary policy is weaker in the scenario with more severe liquidity risk: output responds less and flattens earlier while the interest rate and liquidation value drop faster as stimulus increases.

6 Discussion

In this section, we discuss policy implications of our model as well as its critical assumptions. See Choi et al. (2016) for complementary discussion in a simplified framework.

6.1 Secondary market intervention

Our policy analysis so far has focused on monetary loosening at $t = 0$, which can be strongly impaired by the feedback between liquidation values at $t = 1$ and loan demand at $t = 0$. Naturally, this feedback effect could be alleviated through an intervention in secondary markets to support liquidation values. If such a program for $t = 1$ were announced or anticipated at $t = 0$, it could counteract the credit misallocation at $t = 0$.

One such program would be to announce a floor for the secondary market price, which would result in an exogenously fixed P. Such a policy would eliminate the indirect effects through changes in P, both for the quality elasticity as well as for the stimulus pass-through. However, this could be costly since the policy maker has to credibly commit to purchasing any amount of assets at that price. An alternative program would be to support private buyers with subsidies or loss-sharing arrangements. In our setup, this would correspond to a reduction in the wedge δ between average quality and liquidation value.

Comparing interest rate policy and secondary market intervention, we have the following result on their relative cross-sectional effects.

Corollary 4. *Comparing the heterogeneous responses of the two policies, we have:*

$$\left|\frac{\partial D_h/\partial r}{\partial D_\ell/\partial r}\right| < \left|\frac{\partial D_h/\partial P}{\partial D_\ell/\partial P}\right|$$

Thus, the response in leverage of high types relative to low types for a change in P is larger than that for a change in r. Suppose that the central bank wishes to induce further investment by productive high types without affecting less productive low types to minimize efficiency losses. Corollary 4 implies that this goal can be achieved more effectively by raising P than by lowering r as liquidity provision in the secondary market directly affects liquidity risk, our primary source of financial frictions. See Choi et al. (2016) for more discussion on various policy measures as well as their timing.

Another benefit of this intervention is that it could generate a positive spiral that partially offsets the negative spiral discussed in the paper; higher P increases aggregate loan demand and raises the interest rate, which leads to an improvement in overall investment quality through the heterogeneous responses and thus a further increase in P. If the policy maker implements monetary stimulus and simultaneously announces these programs, the impairment effect could be alleviated.

6.2 Liquidity regulation and monetary transmission

In response to the prominence of liquidity problems in the recent crisis, the Basel Committee has introduced new liquidity regulations. The Liquidity Coverage Ratio (LCR) requires financial institutions to hold sufficient high quality liquid assets to cover their cash needs over a 30-day stress scenario, whereas the Net Stable Funding Ratio (NSFR) requires financial institutions to maintain stable funding sources that would cover funding requirements for a period of at least one year.

In a recent paper, Bech and Keister (2013) argue that these liquidity requirements are likely to impact monetary policy implementation. The requirements affect banks' trade-off between short- and long-term funding and therefore the demand for overnight interbank loans which is crucial for the implementation of monetary policy.

Our paper provides an additional perspective on the interaction of liquidity requirements and monetary policy implementation. In our model, liquidity risk impedes the transmission of monetary policy. The new liquidity requirements aim at lowering the maturity and liquidity mismatch on the balance sheets of financial intermediaries. By reducing liquidity risk and strengthening the transmission mechanism, the regulations are therefore complementary to the implementation of monetary policy.

6.3 Shadow banking

The heterogeneous response underlying the dampening mechanism in our model relies on liquidity risk that is increasing in leverage. However, any other cost that is similarly increasing in balance sheet size can generate the heterogeneous response. For example, such costs could be from regulatory burden or the cost of deviating from a target leverage ratio.

When the interest rate decreases, traditional banks facing regulatory costs respond less compared to shadow banks with less regulatory burden. Hence, our mechanism predicts a relative growth of the shadow banking sector in response to prolonged monetary loosening. More resources are allocated to "opaque" shadow banks and as a result secondary markets can become more illiquid, generating a negative feedback analogous to the one described in our paper.

6.4 Interpretation of monetary policy

In our stylized setup, we assume that the central bank can change L to implement monetary policy. In this section, we discuss several interpretations of this setup.

We could interpret an increase in L as an injection of more central bank "money" in a New Keynesian model with perfect price stickiness (thus there is no distinction between real price and nominal price). In this case, the amount of available money limits total investment in the economy, and the central bank is able to increase aggregate investment by increasing the money supply. We can apply a similar argument when interpreting changes in L as changes in central bank reserves, which affect aggregate lending.[16] Although our focus is on monetary policy that affects the supply of money or loanable funds, any inflows of liquidity into the economy can generate the same effect, e.g. international capital flows.[17] However, our policy intervention is clearly different from fiscal policy where stimulus has a crowding-out effect that increases the interest rate rather than decreasing it.

Finally, our model doesn't specifically distinguish monetary stimulus and tightening. However, the quality effect in the transmission mechanism (characterized by dq/dL) should not be thought of as symmetric for stimulus and tightening since the macroeconomic contexts—captured by the exogenous parameters in our model—for the two scenarios are different. Note that our mechanism critically depends on financial frictions, in particular

[16]See Bernanke and Blinder (1992), Kashyap and Stein (2000), and Bianchi and Bigio (2014) for general equilibrium models in which the central bank uses reserves to implement monetary policy. Our simplified setup would be a case with 100% reserve requirements in which total reserves equal total lending.

[17]See Bruno and Shin (2015) on the role of the international banking system in global capital flows, and Justiniano et al. (2015) on the foreign capital inflows and the housing boom.

endogenously increasing cost of leverage and secondary market frictions. Both of these frictions should not be assumed constant over the business cycle but rather more severe in downturns (in particular during a crisis) than in upturns. In our specific setup, the liquidity risk underlying the heterogeneous responses is more relevant—implying higher and faster increasing α—during downturns with low aggregate productivity p.[18]

6.5 Cash-in-the-market pricing

We now analyze an alternative case to illustrate how the dampening mechanism due to heterogeneous responses can arise even without the lemons pricing assumption. Similar to the framework in Choi et al. (2016), we drop the assumption of incomplete information in the secondary market and examine instead a case where buyers can distinguish seller types but secondary market liquidity is limited, leading to cash-in-the-market pricing. In this case, the increase in aggregate investment due to monetary stimulus leads to an increase in the discount in the secondary market, also resulting in a dampening effect on output.

Suppose that the amount of cash available to purchase assets at $t = 1$ is limited to an amount C, e.g. due to limited participation as in Allen and Gale (1994, 1998). As a result, when sufficiently large amounts of assets are being sold in the secondary market, the asset prices suffer from a discount, which increases in the aggregate amount of assets liquidated.

Let V denote the total value of assets being sold in the secondary market:

$$V = \sum_i \alpha(D_i)\, R_i D_i$$

When the cash available in the market is less than V, assets are sold at a discount to expected fundamental value. Note that the buyers in the secondary market can perfectly identify each asset so that each asset has to offer the same rate of return, i.e. suffer the same proportional discount $1 - \Delta$, where $\Delta = C/V$. Hence, the price in the secondary market for an asset sold by a borrower of type i has to satisfy $P_i = \Delta\, R_i$.

In this case, liquidations lead to a loss of $(1 - \Delta)R_i$, which is the only difference in the first-order condition in equation (6). We can easily show that $\partial D_i / \partial \Delta > 0$ and $\partial q / \partial \Delta > 0$ as in Proposition 3 and Corollary 3, respectively. Hence, for $d\Delta/dr > 0$, we get the additional dampening effect on output due to the indirect effect through the price in the secondary market.

[18]See Appendix A for more discussion of the link between α and aggregate conditions.

Note that the equilibrium liquidation value is implicitly defined by the fixed-point condition:

$$\Delta V(r, \Delta) = C \tag{20}$$

We can show that we still have equilibrium comovement between the liquidation values and the interest rate.

Corollary 5. *With cash-in-the-market pricing, the liquidation values $P_i = \Delta R_i$ and the interest rate r are positively related, i.e. $d\Delta/dr > 0$.*

This implies that insufficient market liquidity leads to a drop in the equilibrium liquidation value if more funds are injected, $dP_i/dL < 0$ for all i. This affects dr/dL through the indirect effect of (11); monetary stimulus increases aggregate investment but at the same time lowers the interim liquidation prices with limited liquidity in the secondary market, leading to a dampening effect on output.

Hence, asymmetric information in the secondary market is not essential to get the dampening effect on output due to the indirect effect through the price and we can get similar effects even when buyers of the assets can perfectly distinguish the quality of the assets being sold. Furthermore, while we assume that the liquidity in the secondary market is fixed at C, this is not necessary for our results. As long as capital is slow-moving to the secondary market, there is cash-in-the-market pricing in the secondary market and our results go through qualitatively (Mitchell et al., 2007; Duffie, 2010; Acharya et al., 2013).

7 Conclusion

We build a general equilibrium model with heterogeneous agents facing financial frictions and show that monetary policy can become less effective than desired in stimulating output. More productive agents choose to invest more by borrowing, but at the same time they become exposed to higher liquidity risk due to high leverage. Agents increase their debt when the interest rate is lowered, but this additional risk taking is greater for less productive agents because high productivity agents are reluctant to lever up due to the existing high liquidity risk. This, in turn, dampens the effect of monetary stimulus on output even in the absence of price effects.

Furthermore, the drop in the overall quality of investment decreases liquidation values of opaque assets, increasing liquidity risks. The elevated liquidity risk then depresses aggregate loan demand, which lowers the interest rate further. This again affects the agents differently and further decreases the investment quality. When the economy is trapped in this negative spiral, aggregate output becomes less sensitive to monetary policy (potentially decreasing) even with a significant reduction in the interest rate. This effect is purely a product of credit reallocation among heterogeneous agents, rather than a direct loss from inefficient liquidation or a pecuniary externality.

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Appendix

A Microfoundation of run likelihood $\alpha(D)$

In this section, we derive an ex-ante likelihood of interim creditor runs on borrowers using global game techniques analogous to the analysis in Eisenbach (2014).[19] Importantly, we show how run risk depends on the borrower type i only indirectly through the leverage level, rationalizing our type independent functional assumption $\alpha(D)$.

We can interpret θ_i as the idiosyncratic productivity of type i and the success probability p as a macroeconomic variable common across all types as of $t = 0$. Now suppose that at $t = 1$ each type receives an idiosyncratic shock to p, updating it to $p_i = p + u_i$ where u_i is i.i.d. across types with mean 0 and cumulative distribution F_u on $[\underline{u}, \overline{u}]$.

Suppose that at $t = 0$ each borrower is endowed with equity of e and additionally borrows D to invest $D + e$ in total. We normalize each borrower's creditors to a continuum of measure 1. Each creditor $k \in [0, 1]$ can choose at $t = 1$ whether to withdraw or roll over until $t = 2$. Denote by $\lambda \in [0, 1]$ the fraction of creditors who choose to withdraw and by r_s the one-period interest rate promised to creditors.[20] The borrower fails at $t = 1$ if total withdrawals are larger than the total liquidation value of the borrower's assets:

$$\lambda > \hat{\lambda} \equiv \frac{P\left(D + e\right)}{D\left(1 + r_s\right)}$$

The threshold $\hat{\lambda}$ is decreasing in D so for higher leverage D/e, a smaller fraction of withdrawals can cause failure. A creditor who withdraws at $t = 1$ receives $\left(1 + r_s\right) D$ if the bank doesn't fail and the liquidation value $P\left(D + e\right)$ if it does fail. A creditor who rolls over to $t = 2$ expects to receive $p_i \left(1 + r_s\right)^2 D$ if the bank survives and 0 otherwise.

We now introduce a global game setup so that a unique failure threshold p_i^* can be derived where a borrower i fails if $p_i \leq p_i^*$ and survives otherwise. Suppose that p_i is not common knowledge but creditor $k \in [0, 1]$ of borrower i receives an i.i.d. noisy signal $s_i^k = p_i + \varepsilon_i^k$ instead, where $\varepsilon_i^k \sim \mathcal{U}[-\varepsilon, \varepsilon]$. Each borrower then chooses whether to roll over or withdraw after observing this private signal. We focus on the threshold strategy equilibrium for $\varepsilon \to 0$ such that a creditor chooses to withdraw if and only if $s_i^k < p_i^*$ for some threshold p_i^*.

[19]See Morris and Shin (2010) for a similar approach. Eisenbach et al. (2014) provide a model where banks can fail due to poor fundamentals and/or a loss of significant short-term funding as well as the interaction between the two.

[20]Note that r_s is endogenous and is set by an ex-ante break-even condition as shown below.

A creditor exactly at the switching point, $s_i^k = p_i^*$, has to be indifferent between the two actions which requires that

$$\Pr\left[\lambda \leq \hat{\lambda} \,|\, s_i^k = p_i^*\right] \times (1 + r_s)\, D + \Pr\left[\lambda > \hat{\lambda} \,|\, s_i^k = p_i^*\right] \times P\,(D + e)$$
$$= \Pr\left[\lambda \leq \hat{\lambda} \,|\, s_i^k = p_i^*\right] \times p_i^*\,(1 + r_s)^2\, D + \Pr\left[\lambda > \hat{\lambda} \,|\, s_i^k = p_i^*\right] \times 0$$

For $\varepsilon \to 0$, the distribution of $\lambda \mid s_i^k = p_i^*$ becomes uniform on $[0, 1]$ (Morris and Shin, 2003; Goldstein and Pauzner, 2005) and the indifference condition simplifies to

$$\hat{\lambda}\,(1 + r_s)\, D + \left(1 - \hat{\lambda}\right) P\,(D + e) = \hat{\lambda} p_i^*\,(1 + r_s)^2\, D$$

Substituting in for $\hat{\lambda}$ and solving for p_i^* we get

$$p_i^* = \frac{2\,(1 + r_s)\, D - P\,(D + e)}{(1 + r_s)^2\, D} \tag{21}$$

For given r_s the run threshold and therefore run risk is increasing in D. Note, however, that r_s is an endogenous variable that depends on p_i^* and D. The interest rate r_s is determined by a $t = 0$ break-even conditions for creditors:

$$F_u(p_i^* - p)\, P\,(D + e) + \int_{p_i^* - p}^{\overline{u}} (p + u_i)\, dF_u(u_i)\,(1 + r_s)^2\, D = (1 + r)\, D \tag{22}$$

The $t = 1$ indifference condition (21) and the $t = 0$ break-even condition (22) implicitly define the interim run threshold p_i^* as a function of the ex-ante leverage D. Lemma 1 of Eisenbach (2014) shows that the mapping $p_i^*(D)$ is one-to-one and satisfies $dp_i^*/dD > 0$.

Note that p_i^* here depends on D but is independent of i. Therefore, the ex-ante run risk $\alpha(D) = \Pr(p_i \leq p_i^* \mid p)$ depends only on D. We can thus write α as a function of D but not i, as we did in main part of the paper. Note also that given D, $\alpha(D)$ becomes larger when the fundamental p is lower. Thus, the run risk is higher when the aggregate productivity is lower.

B Proofs

Proof of Lemma 1. The creditors of borrower type i are promised an interest rate r_i on their loan D. If there is no run in $t = 1$, which happens with probability $1 - \alpha(D)$, and the project is successful, which happens with probability p, they receive $(1 + r_i)\, D$. If there is a run in $t = 1$, which happens with probability $\alpha(D)$, the entire project is liquidated and the creditors receive the lesser of their face value or the liquidation value of the borrower's assets, i.e. $\min\{(1+r_i)\,D, PD\}$. For the creditors of type i to break even, given a risk free rate of r, the promised interest rate r_i therefore has to satisfy:

$$\big(1-\alpha(D)\big)\,p\,(1+r_i)\,D + \alpha(D)\,\min\big\{(1+r_i)\,D, PD\big\} = (1+r)\,D \tag{23}$$

With limited liability, the borrower is the claimant to any positive residual payoff. If there is no run and the project succeeds, the borrower type i receives the project payoff net of debt repayment, i.e. $R\theta_i D - (1+r_i)\,D$ (we verify below that this is nonnegative). If there is a run, the borrower receives whatever is left over after repaying the creditors, i.e. $\max\{PD - (1+r_i)\,D, 0\}$. The expected payoff of borrower type i at $t = 0$ therefore is:

$$\big(1-\alpha(D)\big)\,p\,\big(R\theta_i D - (1+r_i)\,D\big) + \alpha(D)\,\max\big\{PD - (1+r_i)\,D, 0\big\} \tag{24}$$

To eliminate the interest rate r_i from the borrower payoff (24) using the creditor breakeven condition (23), note that we can rewrite the borrower payoff (24) as:

$$\begin{aligned}&\big(1-\alpha(D)\big)\,pR\theta_i D + \alpha(D)\,PD \\ &\quad - \Big[\big(1-\alpha(D)\big)\,p\,(1+r_i)\,D - \alpha(D)\,\max\big\{-(1+r_i)\,D,\; -PD\big\}\Big]\end{aligned}$$

Using the fact that $-\max\{-a, -b\} = \min\{a, b\}$, we can rewrite this expression as:

$$\begin{aligned}&\big(1-\alpha(D)\big)\,pR\theta_i D + \alpha(D)\,PD \\ &\quad - \Big[\big(1-\alpha(D)\big)\,p\,(1+r_i)\,D + \alpha(D)\,\min\big\{(1+r_i)\,D, PD\big\}\Big]\end{aligned}$$

Note that the term in square brackets on the second line of this expression is the same as the left-hand side of the breakeven condition (23). Combining the two, we therefore arrive at the expected payoff as stated in the Lemma where $R_i \equiv pR\theta_i$:

$$\Pi_i(D; r, P) = \big(1-\alpha(D)\big)\,R_i D + \alpha(D)\,PD - (1+r)\,D$$

It remains to verify that $R\theta_i D - (1+r_i)\,D \geq 0$ as assumed above. Note that it is sufficient to show this for $(1+r_i)\,D \geq PD$, in which case we can rewrite condition (23) as:

$$(1+r_i)\,D = \frac{(1+r)\,D - \alpha(D)\,PD}{\big(1-\alpha(D)\big)\,p}$$

Using this expression, we have:

$$\begin{aligned}
&R\theta_i D - (1+r_i)\,D \geq 0 \\
\Leftrightarrow \quad & R\theta_i D - \frac{(1+r)\,D - \alpha(D)\,PD}{\big(1-\alpha(D)\big)\,p} \geq 0 \\
\Leftrightarrow \quad & \Pi_i(D; r, P) \geq 0
\end{aligned}$$

This is clearly satisfied for any borrower optimally choosing $D \geq 0$ since any agent can guarantee herself at least $\Pi_i(0; r, P) = 0$. □

Proof of Proposition 1. For simplicity, we suppress the subscript i in R_i and D_i. From the implicit function theorem, the first order condition (6) implies:

$$\frac{\partial D}{\partial R} = \frac{\big(1 - \alpha'(D)D - \alpha(D)\big)}{\big(\alpha''(D)D + 2\alpha'(D)\big)\,(R-P)}$$

Note that the denominator is positive with weakly convex and non-decreasing α. The numerator is also positive since the first order condition implies

$$\begin{aligned}
\alpha'(D)D + \alpha(D) &= \frac{R-(1+r)}{R-P} \\
&< 1 \quad \text{for} \quad P < 1+r
\end{aligned}$$

Therefore $\partial D/\partial R$ is positive and thus $D_h > D_\ell$. □

Proof of Proposition 2. Note that by the implicit function theorem, we have:

$$\frac{\partial D_i}{\partial r} = \frac{-1}{\big(\alpha''(D_i)D_i + 2\alpha'(D_i)\big)\,(R_i - P)} < 0$$

The denominator is larger for the high types since $D_h > D_\ell$ and $\alpha'' \geq 0$, and the third derivative of α is very small. □

Proof of Proposition 3. From the implicit function theorem, the first order condition implies

$$\frac{\partial D_i}{\partial P} = \frac{\alpha'(D_i)D_i + \alpha(D_i)}{\big(\alpha''(D_i)D_i + 2\alpha'(D_i)\big)\,(R_i - P)}$$

Note that the denominator is positive as before. The numerator is also positive since all terms are positive. Thus $\partial D_i/\partial P$ is positive.

From the first order condition we have

$$\alpha'(D_i)D_i + \alpha(D_i) = \frac{R_i - (1+r)}{R_i - P}$$

so that $\partial D_h/\partial P > \partial D_\ell/\partial P$ if and only if

$$\frac{\alpha''(D_h)D_h + 2\alpha'(D_h)}{\alpha''(D_\ell)D_\ell + 2\alpha'(D_\ell)} < \frac{R_h - (1+r)}{R_\ell - (1+r)} \left(\frac{R_\ell - P}{R_h - P}\right)^2$$

as stated in the proposition. □

Proof of Corollary 1. For linear liquidity risk $\alpha(D) = aD$, using the first order condition (6) we obtain

$$D = \frac{R - (1+r)}{2a\,(R-P)},$$

so that

$$\frac{\partial^2 D}{\partial P \partial R} = \frac{2a\,(R-P)^2 - (R-(1+r))\,4a\,(R-P)}{\left(2a\,(R-P)^2\right)^2}.$$

Hence,

$$\frac{\partial^2 D}{\partial P \partial R} > 0 \quad \Leftrightarrow \quad 2\,(1+r) > R + P,$$

which constitutes a sufficient condition for $\partial D_h/\partial P > \partial D_\ell/\partial P$. □

Proof of Corollary 2. Since $R_\ell < R_h$, we have $q - R_\ell > 0 > q - R_h$. Further, since $D_h > D_\ell$, q is biased towards R_h so $\sum_i (q - R_i) > 0$. From Proposition 2, we have $-\partial D_\ell/\partial r > -\partial D_h/\partial r$ so that

$$-\sum_i \left((q - R_i)\,\frac{\partial D_i}{\partial r}\right) > -\sum_i (q - R_i)\,\frac{\partial D_h}{\partial r} > 0$$

and therefore $\partial q/\partial r > 0$. □

Proof of Corollary 3. First, note that $\partial Q/\partial r > 0$ and $\partial Q/\partial P < 1$ are sufficient for $dP/dr > 0$ from (15). This is also sufficient for a positive indirect effect in (11) since $\partial D_i/\partial P > 0$ from Proposition 3.

Second, $\partial Q/\partial r > 0$ and $\partial Q/\partial P < 1$ are sufficient for $dP/dr > 0$, and this with $\partial q/\partial P > 0$ implies the indirect effect in (9) is positive, which amplifies the quality elasticity.

Third, using the definition of Q, we have that $\partial Q/\partial r > 0$ holds if

$$-\sum_i \big((Q - R_i)\,\partial(\alpha(D_i)D_i)/\partial r\big) > 0. \tag{25}$$

Using

$$\frac{\partial(\alpha(D_i)D_i)}{\partial r} = \big(\alpha'(D_i)D_i + \alpha(D_i)\big)\,\frac{\partial D_i}{\partial r} = -\frac{\partial D_i}{\partial P},$$

(25) can be written as

$$\frac{\partial D_\ell}{\partial P} > \frac{R_h - Q}{Q - R_\ell}\,\frac{\partial D_h}{\partial P} \equiv \beta_1\,\frac{\partial D_h}{\partial P}; \tag{26}$$

and $\partial Q/\partial P < 1$ holds if

$$-\,(Q - R_\ell)\,\frac{\partial\,(\alpha(D_\ell)\,D_\ell)}{\partial P} - (Q - R_h)\,\frac{\partial\,(\alpha(D_h)\,D_h)}{\partial P} < \alpha(D_h)\,D_h + \alpha(D_\ell)\,D_\ell,$$

which can be written as

$$\begin{aligned}\frac{\partial D_\ell}{\partial P} &> \frac{(R_h - Q)\,\big(\alpha'(D_h)\,D_h + \alpha(D_h)\big)}{(Q - R_\ell)\,\big(\alpha'(D_\ell)\,D_\ell + \alpha(D_\ell)\big)}\,\frac{\partial D_h}{\partial P} - \frac{\alpha(D_h)\,D_h + \alpha(D_\ell)\,D_\ell}{(Q - R_\ell)\,\big(\alpha'(D_\ell)\,D_\ell + \alpha(D_\ell)\big)}\\ &\equiv \beta_3\,\frac{\partial D_h}{\partial P} - \gamma;\end{aligned} \tag{27}$$

and $\partial q/\partial P > 0$ holds if

$$(q - R_\ell)\frac{\partial D_\ell}{\partial P} < (R_h - q)\frac{\partial D_h}{\partial P},$$

which can be written as

$$\frac{\partial D_\ell}{\partial P} < \frac{R_h - q}{q - R_\ell}\,\frac{\partial D_h}{\partial P} \equiv \beta_2\,\frac{\partial D_h}{\partial P}. \tag{28}$$

Similarly, $\partial Q/\partial P > 0$ holds if

$$-\sum_i \big((Q - R_i)\,\partial(\alpha(D_i)D_i)/\partial P\big) > 0,$$

and thus

$$(Q - R_\ell)\,\big(\alpha'(D_\ell)\,D_\ell + \alpha(D_\ell)\big)\,\frac{\partial D_\ell}{\partial P} < (R_h - Q)\,\big(\alpha'(D_h)\,D_h + \alpha(D_h)\big)\,\frac{\partial D_h}{\partial P},$$

which can be written as

$$\frac{\partial D_\ell}{\partial P} < \frac{(R_h - Q)\left(\alpha'(D_h)\,D_h + \alpha(D_h)\right)}{(Q - R_\ell)\left(\alpha'(D_\ell)\,D_\ell + \alpha(D_\ell)\right)}\,\frac{\partial D_h}{\partial P} = \beta_3\,\frac{\partial D_h}{\partial P}. \tag{29}$$

Now, note that $\beta_2 > \beta_1 > 0$ with $Q > q$, $\beta_3 > \beta_1 > 0$ with $D_h > D_l$, while $\gamma > 0$. Therefore, (26), (27), (28), and (29) are not mutually exclusive. □

Proof of Corollary 4. Note that

$$\frac{\left|\frac{\partial D_h/\partial P}{\partial D_\ell/\partial P}\right|}{\left|\frac{\partial D_h/\partial r}{\partial D_\ell/\partial r}\right|} = \frac{\alpha'(D_h)D_h + \alpha(D_h)}{\alpha'(D_\ell)D_\ell + \alpha(D_\ell)}$$

This is greater than 1 since $\alpha(D)$ is increasing and weakly convex in D, and $D_h > D_\ell$. □

Proof of Corollary 5. Given this implicit definition of Δ in (20), the equilibrium effect of r on Δ is given by

$$\frac{d\Delta}{dr} = -\frac{\Delta\partial V/\partial r}{V + \Delta\partial V/\partial\Delta}$$

and sufficient conditions for $d\Delta/dr > 0$ are $\partial V/\partial r < 0$ and $\partial V/\partial\Delta > 0$. We obtain

$$\frac{\partial V}{\partial r} = \sum_i \left(\alpha'(D_i)\,D_i + \alpha(D_i)\right)\frac{\partial D_i}{\partial r}R_i < 0$$
$$\frac{\partial V}{\partial \Delta} = \sum_i \left(\alpha'(D_i)\,D_i + \alpha(D_i)\right)\frac{\partial D_i}{\partial P_i}R_i^2 > 0$$

since $\partial D_i/\partial r < 0$ from Proposition 2 and $\partial D_i/\partial P_i > 0$ from Proposition 3.[21] Hence, we obtain

$$\frac{dP_i}{dr} = \frac{d\Delta}{dr}R_i > 0,$$

as desired. □

[21] Making the liquidation value type-dependent does not affect the comparative statics of individual borrowing with respect to r or P_i.