A Heterogeneous Agents Equilibrium Model for the Term Structure of Bond Market Liquidity[*]

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Abstract

We analyze the impact of market frictions on trading volume and liquidity premia of finite maturity assets when investors differ in their trading needs. In equilibrium, investors who trade frequently only invest in short-term assets and illiquidity spills over from short-term to long-term maturities. Our model predicts i) a hump-shaped relation between trading volume and maturity, ii) lower trading volumes of older compared to younger assets, iii) an increasing liquidity term structure when considering ask prices, and iv) a liquidity term structure from bid prices that is decreasing or U-shaped. Empirical tests for U.S. corporate bonds support our theoretical predictions.

JEL classification: G11, G12
Keywords: bond liquidity, term structure of liquidity premia, heterogeneous agents, aging effect, trading volume, equilibrium

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1 Introduction

The risk of being unable to sell an asset at its fair value is one of the main risks associated with securities investment. Such liquidity risk is of particular interest for bonds, since they offer investors the opportunity to wait for a bond's maturity and thereby avoid transaction costs. This option creates a relation between the time until a bond's maturity and the liquidity premium investors require to invest in the bond. As this relation affects trading strategies, optimal portfolio allocations, price discounts, and capital costs, it is important to all investors and issuers active in global bond markets.

Although there are numerous papers empirically investigating the relation between liquidity premia and maturity, there is little consensus even on the most fundamental question: What is the shape of the term structure of liquidity premia? Empirically, the term structure is found to be decreasing (Ericsson and Renault, 2006), increasing (Dick-Nielsen, Feldhütter, and Lando, 2012), or U-shaped (Longstaff, 2004). Moreover, the literature offers no explanation for our puzzling empirical observation that bonds with very short or long maturities are rarely traded, while there is an active secondary market for bonds with intermediate maturities.

We suggest a parsimonious equilibrium model that explains these seemingly conflicting empirical results on the shape of the term structure of liquidity premia. Our unified framework also explains the empirically observed hump-shaped term structure of trading volume and the well-known aging effect (see, e.g., Warga, 1992; Edwards, Harris, and Piwowar, 2007): other things equal, old bonds trade less frequently than newly issued bonds.

In our model, agents with heterogeneous investment horizons trade bonds with a continuum of different maturities in a market with two simple frictions: transaction costs and shocks to investors' time preference parameter. If a preference shock occurs, the investor faces the trade-off between the cost (in terms of utility) of awaiting the asset's maturity,

which is higher for long-term bonds, and the bid-ask spread charged by an exogenous market maker or dealer. Prior to the preference shock, the investor determines her optimal portfolio allocation by comparing the higher return earned when holding a long-term bond until the maturity date to the higher expected costs of selling this asset in case of a preference shock. Due to these investor-specific endogenous decisions on the portfolio composition and bonds' decreasing time to maturity, we obtain spill-overs from the short to the long end of the term structure. This agrees with empirical evidence on liquidity transmission between different maturity segments by Goyenko, Subrahmanyam, and Ukhov (2011).

Our model offers four key testable predictions. First, assets with very short maturities are traded less frequently, as are assets with long maturities. The first effect arises because investors have lower disutility from waiting than from paying the bid-ask spread when maturity is short. As only investors who experience comparatively few preference shocks hold assets with long maturities, these assets are rarely traded as well. Second, since these low preference shock investors still hold a proportion of aged (formerly long-term, but now short-term) bonds, our model endogenously explains the well-documented aging effect. We believe that ours is the first equilibrium model to explain the impact of aging on trading volume via a simple transaction cost friction. Third, liquidity premia in bond yields computed from ask prices are negligible for short maturities, and increase for longer maturities. The increasing term structure arises, even for constant bid-ask spreads, because the disutility from waiting increases with maturity. For longer maturities, the term structure flattens out as investors with low probabilities of preference shocks dominate. Fourth, liquidity premia from bid yields depend on the term structure of bid-ask spreads. If transaction costs do not depend on the bond's maturity, short-term liquidity premia are large, then decrease and flatten out at longer maturities. If transaction costs are increasing in maturity, the term structure takes on a U-shape.

We verify these key model predictions empirically using transaction data for highly rated

U.S. corporate bonds from the TRACE database. The results of multiple regressions confirm that transaction volume is hump-shaped and bonds are traded less frequently as they age. To calculate the liquidity component in bond yields, we employ two completely different approaches. First, we follow Longstaff, Mithal, and Neis (2005) and compute liquidity premia as the difference of bond yields from trade prices and theoretical prices that are computed from a bootstrapped credit risky curve using Treasury yields and CDS premia. Second, we implement the methodology of Dick-Nielsen, Feldhütter, and Lando (2012) and identify the liquidity component using an indirect, regression-based approach. All analyses as well as multiple robustness checks show that liquidity premia computed from ask prices are monotonically increasing with a decreasing slope. Liquidity premia computed from bid prices are U-shaped with significant liquidity premia for very short maturities

Our paper adds to several strands of literature. Ericsson and Renault (2006) model the liquidity shock for assets with different maturities as the jump of a Poisson process that forces investors to sell their entire portfolio to the market maker, who charges a proportional spread. Liquidity premia are downward-sloping because only current illiquidity affects asset prices, and because investors have the option to sell assets early to the market maker at favorable conditions. Kempf, Korn, and Uhrig-Homburg (2012) extend this analysis by modeling the intensity of the Poisson process as a mean-reverting process. In this setting, liquidity premia depend on the difference between the average and the current probability of a liquidity shock, and can exhibit a number of different shapes. In contrast to these papers, we allow investors to trade-off the transaction costs when selling immediately versus the disutility from awaiting the bond's maturity. By endogenizing investors' trading decisions in bonds of different maturities, our model provides an equilibrium-based explanation for spill-overs of liquidity shocks between different ends of the maturity range.

Feldhütter (2012) is most closely related to our study, since he considers an investor's optimal decision to a holding cost shock. Search costs allow market makers to charge a spread,

which results in a difference between the asset's fundamental value and its bid price. However, Feldhütter (2012) abstracts from aging because in his model, bonds mature randomly with a rate of $\frac{1}{T}$. Additionally, his model cannot accommodate any spill-over effects between maturities because bonds of different maturities T are not considered *simultaneously*.

Besides supporting the equilibrium model predictions, our results provide an explanation for the variation in the term structures found in previous empirical studies. Studies that document a decreasing term structure (Amihud and Mendelson, 1991; Ericsson and Renault, 2006) or a U-shaped term structure (Longstaff, 2004) use mid quotes or ask quotes net of a spread component such as brokerage costs. In contrast, Dick-Nielsen, Feldhütter, and Lando (2012) find an increasing term structure for the U.S. corporate bond market computed from average quarter-end trade prices. However, trade prices in this market are dominated by buy transactions, as the numbers of observations in Panel A of our Table II document. Hump-shaped (Koziol and Sauerbier, 2007) or variable term structures (Kempf, Korn, and Uhrig-Homburg, 2012) arise from a varying mixture of bid and ask prices. Hence, consistent with our theoretical predictions, the shape of the liquidity term structure is crucially driven by whether most transactions occur at the dealer's bid or ask price.

Last, our paper contributes to the growing literature on asset pricing in heterogeneous agents models. Like Beber, Driessen, and Tuijp (2012), we study optimal portfolio choice of heterogeneous investors faced with exogenous transaction costs in a stationary equilibrium setting. Duffie, Gârleanu, and Pedersen (2005) and Vayanos and Wang (2007) endogenize transaction costs through search costs and bargaining power. None of these studies, however, can address the relation between maturity and liquidity as they do not simultaneously consider assets with different finite maturities. We show that even when transaction costs are identical for all maturities, liquidity shocks are transmitted from short-term to long-term bonds via heterogeneous investors.

2 Model Setup

This section presents an extension of the Amihud and Mendelson (1986) model adapted to bond markets. We present the model setting in Section 2.1, describe the equilibrium in Section 2.2, and provide a discussion of the differences between our model and the one of Amihud and Mendelson (1986) in Section 2.3. Our objective is to derive equilibrium relations between a bond's time to maturity, trading volume, and liquidity premia. We derive these relations in Section 3 and test them in Section 5.

2.1 Setting

In our continuous-time economy with cash as the numeraire, there are two types of assets: the money market account in infinite supply paying a constant non-negative return r and a continuum of illiquid zero-coupon bonds with maturity between 0 and $T_{\max}$ at which they pay one unit of cash. Bonds are perfectly divisible and, for each initial maturity T_{init} between 0 and $T_{\max}$, are issued at a constant rate a.[1] Hence, in steady state, bonds of the same initial maturity are equally distributed with respect to their remaining time-to-maturity.

We consider two types of agents: high-risk investors (type H) and low-risk investors (type L). Each investor is infinitesimally small, but all type-i investors together have aggregate wealth W_i. Investors are risk-neutral and have utility from consumption of cash (received from the money market account or from sold/matured bonds) $U_i(c)$, $i \in \{H, L\}$. In addition, we assume that (unmodeled) dealers act as intermediaries: they provide liquidity via bid and ask quotes at which they stand ready to trade. They are compensated only for providing immediacy by an exogenous bid-ask spread $s(T)$, depending on a bond's (remaining) maturity T, i.e., they quote an ask price $P^{\text{ask}}(T) = P(T)$ and a bid price

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[1]Note that with the assumption of a given issuance rate a, we take maturity dispersion as given. This assumption is supported, for example, by firms managing rollover or funding liquidity risk by spreading out the maturity of their debt (Choi, Hackbarth, and Zechner, 2015; Norden, Roosenboom, and Wang, 2013).

$$P^{\text{bid}}(T) = (1 - s(T)) \cdot P(T).^2$$

Investors choose their portfolio allocation across available assets taking into account that each investor experiences a single preference shock with Poisson rate λ_i, $i \in \{H, L\}$, $\lambda_L < \lambda_H$, that increases her time preference rate from r to $r + b > r$. We can economically interpret this event as a funding shock that leads to an incentive for the investor to reduce her security holdings (see Brunnermeier and Pedersen, 2009).[3] As a reaction to the shock, the investor decides for each bond whether to sell it at the bid price and consume the proceeds, or to hold the bond despite the increased time preference rate. It is intuitive that the disutility from waiting approaches zero for bonds with very short maturities. Therefore, investors avoid paying the bid-ask spread for short-term bonds and never sell them prematurely. We denote the maturity for which an investor is indifferent between selling the bond and holding it until maturity by τ (which is identical for both investors). Below, we only consider steady-state equilibria, in which neither prices nor aggregate wealth changes over time. An investor who does not experience a preference shock then has no incentive to change her (initially optimal) portfolio allocation. It is therefore sufficient to consider the investor's decision problem at time $t = 0$, where each investor maximizes her expected utility from consumption by choosing the amount of money invested into the money market account and into bonds with different maturities. We formally derive this decision problem and calculate first order conditions in Appendix A, we compute equilibrium prices in Appendix B, and show that markets clear, given these prices, in Appendix C.

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[2]In contrast to our approach, some recent papers model bid-ask spreads endogenously in a search-based framework. Applying a search-model introduces limiting restrictions on the number of different assets traded at the same time (e.g., in Feldhütter, 2012, all bonds have the same (expected) maturity T and mature randomly). A further advantage of our approach is the use of easily observable bid-ask spreads as an input. In contrast, search intensities and search costs, which are needed in search-based models, are hard to quantify empirically.

[3]Duffie, Gârleanu, and Pedersen (2005), Feldhütter (2012), and He and Milbradt (2014) obtain a similar effect through an increased holding cost for the bond. Our model is thus operationally equivalent to those search-based models in this aspect.

2.2 Clientele Effect

In equilibrium, if the wealth of low-risk investors alone is not sufficient to buy all bonds, there arises a clientele effect related to the ones in Amihud and Mendelson (1986) and Beber, Driessen, and Tuijp (2012). In our case, low-risk investors buy only bonds with maturity above T_{lim}, and high-risk investors buy only bonds with maturity below T_{lim}. Note that dealers (in aggregate) do not absorb any inventory. Hence, low-risk investors absorb the supply of long-term bonds and high-risk investors absorb the supply of short-term bonds. Proposition 1 summarizes the results on equilibrium prices and the clientele effect, which we prove in Appendix B. For ease of exposition, we set $r = 0$.

Proposition 1. (Equilibrium prices and clientele effect)

For constant or monotonically increasing bid-ask spreads $s(T)$ with $0 < s(T) < 1$, prices of illiquid bonds $P(T)$ are given in closed form

$$P(T) = \begin{cases} \frac{b \cdot e^{-\lambda_H \cdot T} - \lambda_H \cdot e^{-b \cdot T}}{b - \lambda_H}, & \text{if } T \le \min(\tau, T_{\text{lim}}) \\ e^{-\int_\tau^T \lambda_H \cdot s(x)\, dx} \cdot P(\tau), & \text{if } \tau < T \le T_{\text{lim}} \\ e^{-\int_{T_{\text{lim}}}^T \frac{(\Delta_L(T_{\text{lim}}) + s(x)) \cdot \lambda_L}{1 + \Delta_L(T_{\text{lim}})}\, dx} \cdot P(T_{\text{lim}}), & \text{if } \tau < T_{\text{lim}} < T \\ e^{-T \cdot \lambda_L} \cdot \left(1 - \frac{\lambda_L \cdot \left(1 - e^{T \cdot (\lambda_L - b)}\right)}{(1 + \Delta_L(T_{\text{lim}})) \cdot (\lambda_L - b)}\right), & \text{if } T_{\text{lim}} < T \le \tau \\ e^{-\int_\tau^T \frac{(\Delta_L(T_{\text{lim}}) + s(x)) \cdot \lambda_L}{1 + \Delta_L(T_{\text{lim}})}\, dx} \cdot P(\tau), & \text{if } T_{\text{lim}} \le \tau < T \end{cases} \quad , \qquad (1)$$

where $\Delta_L(T_{\text{lim}})$ denotes marginal utility of low-risk investors when investing in bonds with maturity T_{lim} and τ is the maturity for which investors are indifferent between selling the bond when experiencing a preference shock and holding it until maturity. In equilibrium, there arises a clientele effect that leads to low-risk investors investing only in long-term bonds with $T > T_{\text{lim}}$ and high-risk investors investing in short-term bonds with $T \le T_{\text{lim}}$.

The terms within the integrals in Equation (1) can be interpreted as the instantaneous returns of bonds with maturity x. Subtracting the expected transaction costs $\lambda_i \cdot s(x)$ gives

the expected excess returns net of transaction costs, which are 0 for high-risk investors and $\tau < T \leq T_{\lim}$, and positive for low-risk investors for all maturities.

2.3   Comparison to the Amihud and Mendelson (1986) Model

Before proceeding with the predictions of our model for trading volume and liquidity premia, it is instructive to compare our model to that of Amihud and Mendelson (1986). With respect to the optimization problem, we endogenize the investor's decision to sell assets as a reaction to the preference shock. With respect to assets, we consider a continuum of assets with (almost) arbitrary bid-ask spreads. The clientele effect in Amihud and Mendelson (1986) results from *different* bid-ask spreads of assets with identical (infinite) maturity. Hence, investors with low trading needs invest in assets with large bid-ask spreads in the Amihud and Mendelson (1986) model. In contrast, our clientele effect also applies if bid-ask spreads are identical for all bonds. The reason for this difference is the investor's endogenous decision to sell, which allows her to trade-off one type of illiquidity (the disutility of higher transaction costs) against another type of illiquidity (the disutility from awaiting the bond's maturity). Hence, even if short-term bonds are not more "liquid" with respect to transaction costs, they are more "liquid" due to the lower disutility from awaiting their (closer) maturity. In contrast, if investors are forced to sell immediately after a liquidity shock (like, e.g., in Amihud and Mendelson, 1986; Ericsson and Renault, 2006), this second source of liquidity is irrelevant since a bond's maturity and a liquidity shock never coincide in a continuous-time setting. For that reason, there would be no advantage from investing in short-term bonds.

3 Hypotheses on Trading Volume and Liquidity Term Structure

3.1 Trading Volume

We first present the model-implied relations between trading volume, maturity, and age. These relations are intuitive: First, bonds of short maturities are not sold prematurely, since the disutility from awaiting maturity is low. Second, the clientele effect (high-risk investors with strong trading needs only hold short-term bonds) translates into lower trading volumes for bonds with longer maturities. The first and second effect lead to a hump-shaped relation between maturity and trading volume. Third, an aged bond (formerly long-term but now short-term) is still partially locked up in the portfolios of investors with low trading needs. This leads to a lower trading volume of this bond compared to a young short-term bond. We are not aware of any other model that is both able to endogenously derive relations between maturity, age, and trading volume and predict term structures of liquidity premia.

The predictions regarding trading volume are summarized in the following proposition, which we prove in Appendix E. We exclude trading volume from issuing activities in the primary market, which are exogenous in our setting, and focus on secondary-market trading volume. Since in aggregate, dealers do not hold any inventory, trading volume equals twice the volume sold by investors to dealers.[4]

Proposition 2. (Trading volume)
Consider the case that $\tau < T_{\lim}$.

1. *Secondary-market turnover is hump-shaped in the time to maturity T, more specifically, it is zero for $T < \tau$ and equals $2 \cdot \lambda_L$ for $T > T_{\lim}$. For T with $\tau < T < T_{\lim}$, turnover exceeds $2 \cdot \lambda_L$.*

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[4]We look at turnover, i.e., trading volume in percent of the outstanding volume for each maturity since the outstanding volume of short-term bonds exceeds that of long-term bonds due to the latter's aging.

2. *For two bonds 1 and 2 that both have a remaining maturity T with $\tau < T < T_{\lim}$, but a different initial maturity $T_{\text{init},1} < T_{\lim}$ and $T_{\text{init},2} > T_{\lim}$, secondary-market turnover is higher for the younger bond 1 than for the older bond 2.*

In the (less interesting) case that $T_{\lim} \leq \tau$, high-risk investors never sell bonds prematurely, and turnover is determined by low-risk investors only. Hence, turnover is zero for $T < \tau$, and equals $2 \cdot \lambda_L$ for $T > \tau$. Then, no aging effect arises.

We illustrate the relation between maturity and trading volume with the help of a baseline parameter specification in Figure 1. In this specification, bid-ask spreads are 0.3% for all maturities T. High-risk investors experience preference shocks with a rate of $\lambda_H = 0.5$, i.e., they experience on average one preference shock every 2 years. Low-risk investors experience half as many shocks ($\lambda_L = 0.25$).[5] b equals 2%, i.e., if a shock arises, investors' time preference rate increases by 2% which can be thought of as the additional borrowing cost in excess to the risk-free rate.

Insert Figure 1 about here.

The dependence of trading volume on the distribution of bonds over the portfolios of low- and high-risk investors leads to the aging effect (second part of Proposition 2). Bonds with initial maturity $T_{\text{init}} < T_{\lim}$ (dotted line in Figure 1) are only held by high-risk investors. These investors sell the bonds when experiencing a preference shock if the remaining maturity T is larger than τ. This leads to a turnover of these bonds which equals $2 \cdot \lambda_H$ for $T > \tau$ and drops to zero for $T < \tau$.

The same intuition applies for low-risk investors and bonds with initial maturity $T_{\text{init}} > T_{\lim}$ (dashed line in Figure 1) and remaining maturity $T > T_{\lim}$. If they reach a remaining maturity T below $T_{\lim}$, only high-risk investors purchase the bonds. Hence, these bonds

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[5]Our parameter values are comparable to Feldhütter (2012), who estimates for the U.S. corporate bond market that investors experience a preference shock once every three years.

gradually move into the portfolios of high-risk investors, who suffer preference shocks with a higher rate. Therefore, turnover increases for decreasing maturity (until it drops to zero at τ). As a direct consequence, a bond with remaining maturity $T < T_{\text{lim}}$ has a lower turnover if its initial maturity was larger than T_{lim} (the bond is older), compared to a younger bond with initial maturity $T_{\text{init}} < T_{\text{lim}}$.

The solid line in Figure 1 shows turnover for all bonds. It corresponds to the weighted average of the other two lines, with weights equal the proportion of bonds of remaining maturity T. Our model predictions are consistent with the aging effect discussed in Warga (1992) and empirically documented, e.g., in Fontaine and Garcia (2012) for U.S. Treasuries and Hotchkiss and Jostova (2007) for corporate bonds. Note, however, that our aging effect is of a cross-sectional nature, i.e., it compares two bonds with the same remaining maturity but different age. It therefore differs from the on-the-run/off-the run effect, which describes the decreasing trading volume over a single bond's life.[6] In the empirical analysis, we isolate the impact of aging not due to the pure on-the-run/off-the-run effect.

3.2   The Term Structure of Liquidity Premia

To demonstrate the effect of illiquidity on the term structure of interest rates, we separately compute liquidity premia from ask prices $P^{\text{ask}}(T) = P(T)$ and from bid prices $P^{\text{bid}}(T) = (1 - s(T)) \cdot P(T)$. Liquidity premia are defined as the bond yield minus the risk free rate r, i.e.,

$$\text{Illiq}^{\text{ask}}(T) = -\frac{\log\left(P^{\text{ask}}(T)\right)}{T} - r = -\frac{\log\left(P(T)\right)}{T} - r,$$
$$\text{Illiq}^{\text{bid}}(T) = -\frac{\log\left(P^{\text{bid}}(T)\right)}{T} - r = -\frac{\log\left(1 - s(T)\right)}{T} - \frac{\log\left(P(T)\right)}{T} - r. \tag{2}$$

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[6]Vayanos and Wang (2007) provide an explanation for this effect based on coordination. In their model, it is more attractive for speculators to trade bonds that are expected to be more actively traded in the future. For that reason, liquidity concentrates in newly issued on-the-run bonds.

The formulas for liquidity premia can be interpreted as distributing the "liquidity discount" over the time to maturity T. Bid premia are increased in addition by bid-ask spreads $s(T)$, which are distributed over T, as $s(T) \approx -\log(1 - s(T))$ for small $s(T)$. We summarize our model predictions regarding the term structure of liquidity premia in Proposition 3, which we prove in Appendix E.

Proposition 3. (Term structure of liquidity premia)

1. *The term structure of liquidity premia from ask prices* $\mathrm{Illiq}^{\mathrm{ask}}(T)$ *is monotonically increasing in time to maturity* T *for all* T *and goes to zero for* $T \to 0$. *The term structure flattens at* T_{lim}, *i.e.,*

$$\lim_{T \uparrow T_{\mathrm{lim}}} \left(\mathrm{Illiq}^{\mathrm{ask}}(T)\right)' > \lim_{T \downarrow T_{\mathrm{lim}}} \left(\mathrm{Illiq}^{\mathrm{ask}}(T)\right)'. \tag{3}$$

2. *The term structure of liquidity premia from bid prices* $\mathrm{Illiq}^{\mathrm{bid}}(T)$ *is decreasing in* T *at the short end.*

The predictions in Proposition 3 are illustrated in Figure 2 for constant and in Figure 3 for the empirically relevant case of increasing bid-ask spreads. The parametric form of bid-ask-spreads in Figure 3 is calibrated to observed bid-ask spreads (for details, see Section 5.1). Both figures show that ask premia $\mathrm{Illiq}^{\mathrm{ask}}(T)$ (solid lines) always go to zero for $T \to 0$ as the disutility from awaiting the bond's maturity vanishes. In Figure 2, the ask term structure $\mathrm{Illiq}^{\mathrm{ask}}(T)$ flattens out quickly. Since the slope is already close to zero for $T \uparrow T_{\mathrm{lim}}$, the small kink at T_{lim} is hard to detect as $\mathrm{Illiq}^{\mathrm{ask}}(T)$ cannot decrease in maturity. Otherwise, low-risk investors would invest in bonds with shorter maturities. In Figure 3, ask liquidity premia increase more strongly for longer maturities as expected trading costs increase due to the increasing term structure of bid-ask spreads $s(T)$. Hence, the kink at T_{lim} becomes more apparent. Bid-premia $\mathrm{Illiq}^{\mathrm{bid}}(T)$ (dashed lines) always exhibit an inverse shape.

<center>Insert Figures 2 and 3 about here.</center>

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Apart from the predictions of Proposition 3, Figure 2 and Figure 3 allow us to make two additional observations. First, for constant bid-ask spreads, bid liquidity premia also flatten out for longer maturities because the fixed bid-ask spread is distributed over a longer time period.[7] Second, for the (empirically relevant) case of increasing bid-ask spreads maturity, bid liquidity premia slightly increase at the long end. The increasing shape of the bid-ask spread curve therefore results in increasing term structures of liquidity for ask and flat or U-shaped ones for bid liquidity premia.

Finally, Figure 2 also illustrates a spill-over effect of high-risk investors' liquidity demand on long-term premia. For the thin lines, all parameters are identical as before, except λ_H which is twice as large as in the baseline case. Although only high-risk investors', who hold bonds with maturities smaller than $T_{\lim}$, are affected by this change, liquidity premia of all maturities increase. The reason for this spill-over is the same as above: low-risk investors would prefer short-term bonds over long-term bonds if long-term ask liquidity premia were lower than short-term premia.

Taking into account that empirically observed bond yield spreads are typically computed from mid prices and incorporate a liquidity component, our model can shed light on the credit spread puzzle. This puzzle refers to the observation that empirically observed bond yield spreads are too high, especially at the short end, compared to what structural models à la Merton (1974) can explain (see, e.g., Huang and Huang, 2012). If we average ask and bid prices to compute mid liquidity premia in our framework, we get an inverse shape with large premia for very short maturities: in our baseline specification, we obtain about 185 bps for one month time to maturity.

In summary, the model predicts four main testable hypotheses. First, turnover is hump shaped. Second, for bonds that have identical maturity but a different age, the older bond

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[7]A recent working paper of Huang et al. (2014) confirms our model predictions empirically. The authors find that investors with low liquidity needs on average hold more illiquid bonds (with higher liquidity premia), but demand less compensation for less liquid bonds than investors with higher trading needs would. These empirical results correspond to our clientele effect and the flattening term structure of liquidity premia.

has lower or equal turnover compared to the younger bond. Third, liquidity premia computed from ask prices are monotonically increasing in maturity at the short end, and, depending on the shape of the bid-ask spreads $s(T)$, either flatten out or keep increasing for longer maturities. Fourth, liquidity premia computed from bid prices are monotonically decreasing at the short end and, again depending on the shape of $s(T)$, flatten out or start increasing for longer maturities.

4   Data

We use bond transaction data from Enhanced TRACE (Trade Reporting and Compliance Engine) to test the predictions of our model. TRACE contains information concerning secondary-market transactions of U.S. corporate bonds, e.g., actual trade prices, yields resulting from these prices, and trade sizes. In contrast to standard TRACE, Enhanced TRACE additionally includes information on the side of a trade for the full sample, and trading volumes are not capped at 5 million USD. Since Enhanced TRACE contains information that has previously not been disseminated to the public, it is only available with a lag of 18 months. Therefore, our observation period ends in September 30, 2012. As the beginning of our observation period, we select the full implementation of TRACE in October 1, 2004. We then collect the transaction yield, price, volume, and the information whether the trade is an interdealer trade or a customer buy or sell trade as well as the reporting date and time.

We start with filtering out erroneous trades as described in Dick-Nielsen (2009, 2014). For the remaining bonds, we collect information on the bond's maturity, coupon, and other features from Reuters and Bloomberg using the bond's CUSIP. We drop all bonds which are not plain vanilla fixed rate bonds without any extra rights. We also collect the rating history from Reuters and drop all observations for bonds on days on which fewer than two rating agencies (S&P's, Moody's, Fitch) report an investment-grade rating. We exclude private placements, bonds with more than 30 years remaining maturity, and all bonds that are not

classified as senior unsecured in the Markit database.

For the sample used in the analysis of liquidity premia, we follow Dick-Nielsen (2009) and additionally drop all transactions with non-standard trade or settlement conditions. Moreover, we exclude all trades for which the yield calculated from the reported price does not exactly match the reported yield (less than 1% of the trades). For the turnover analysis and as a control variable, we collect the history of outstanding notional amounts for each bond from Reuters. We use Treasury yields as the risk-free interest rate curve and employ swap rates instead as a robustness check in Section 6.1.[8] Table I summarizes the data selection procedure and the number of observations for our final sample and the subsamples used in our robustness checks in Section 6.

Insert Table I about here.

Since our theoretical predictions are for zero coupon bonds, but traded bonds are mainly coupon bonds, we use duration instead of time to maturity in our empirical tests. We obtain similar results when using the time to maturity as an explanatory variables. Turnover is computed from outstanding amounts and notional volumes. Determining the liquidity component in bond yields, on the other hand, is less straightforward. We apply two different methodologies:

First, we compute the liquidity premium as the simple difference between the observed bond yield and the yield of a theoretical bond with identical promised cash flows, but which is only subject to credit risk. This approach is in line with, e.g., Longstaff, Mithal, and Neis (2005), and does not depend on a specific proxy for bond illiquidity. In the first step, we determine a zero credit-risky curve with which we discount the promised payments of

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[8]For the par Treasury yield curve, we use updated data from Gürkaynak, Sack, and Wright (2007) published on the Federal Reserve's website. We use USD swap curves available via Bloomberg for maturities larger than three months and extend the curve at the short end by linearly interpolating the six-months rate with USD LIBOR rates for a maturity of one and three months. We also account for the different day count conventions in swap and LIBOR markets.

the bond. We collect a time series of daily CDS mid quotes of all available maturities for each bond issuer from Markit, and derive a full term structure by interpolating between the available maturities. Since the shortest available maturity for CDS quotes is six months, we extrapolate the term structure of CDS premia at the very short end. We then bootstrap a zero credit-risky curve using Treasury par yields, accounting for different day count conventions and payment frequencies of CDS and Treasury markets. In the second step, we calculate the liquidity premium as the difference between the observed bond yield (again differentiating between customer buys and customer sells) and the hypothetical yield of the bond that is only subject to credit risk. We denote the resulting liquidity premium by $\text{Illiq}_{\text{diff}}^{\text{ask/bid}}(T)$, which we calculate for each trade in our sample and which we winsorize at the 1% and 99% level. We also use the derived theoretical risk-free bond price, instead of the reported transaction price, to calculate the bond's duration since we do not want duration (our right hand side variable) to be affected by construction by the liquidity premium (our left hand side variable).

Second, we follow Dick-Nielsen, Feldhütter, and Lando (2012) and identify the liquidity component in bond yields by regressing monthly bond yield spreads on a liquidity measure. We calculate the Amihud (2002) liquidity measure, imputed roundtrip costs as in Feldhütter (2012), and their intra-month standard deviations. We winsorize all four measures at the 1% and 99% quantile, transform them to a standard deviation of one, and take the equally weighted average lm_{it} as our aggregated measure of illiquidity for bond i in month t (for details, we refer to the appendix of Dick-Nielsen, Feldhütter, and Lando, 2012).[9] In the second step, we compute the bond's yield spread ys_{it} as the difference between the observed yield and the yield of a risk-free bond with identical promised cash flows (where all payments are discounted at the Treasury curve) and again winsorize yield spreads at the 1% and 99% level. If the observed yield belongs to a transaction marked as a customer buy in TRACE,

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[9]In contrast to Dick-Nielsen, Feldhütter, and Lando (2012) who demean each measure, the individual measures are strictly positive in our analysis. The reason is that a perfectly liquid bond should have a liquidity measure of 0, not a large negative value, for our subsequent regressions to be meaningful.

we denote it by $\mathrm{ys}_{it}^{\mathrm{ask}}(T)$, and by $\mathrm{ys}_{it}^{\mathrm{bid}}(T)$ if it belongs to a transaction marked as a customer sell. We then compute the average over all observed trades for this bond at the bid or ask side on the last day of the month (we only use the last day to reduce endogeneity of the liquidity proxy). To identify the liquidity component in bond yield spreads separately for bid and ask yields and for different maturities, we use dummy variables for the side of a trade ($\mathbb{1}_{\mathrm{ask}}$ and $\mathbb{1}_{\mathrm{bid}}$) and for monthly duration buckets, i.e., $\mathbb{1}_{\{T_m \leq T < T_m + \frac{1}{12}\}}$. We then run the following regression model, pooled across all bonds i and months t:

$$
\begin{aligned}
\mathrm{ys}_{it}^{\mathrm{ask/bid}}(T) = \ &\alpha \\
&+ \sum_{T_m \in \{\frac{1}{12}, \frac{2}{12}, \dots, 30\}} \beta_{T_m}^a \cdot \mathbb{1}_{\mathrm{ask}} \cdot \mathbb{1}_{\{T_m \leq T < T_m + \frac{1}{12}\}} \cdot lm_{it} \\
&+ \sum_{T_m \in \{\frac{1}{12}, \frac{2}{12}, \dots, 30\}} \beta_{T_m}^b \cdot \mathbb{1}_{\mathrm{bid}} \cdot \mathbb{1}_{\{T_m \leq T < T_m + \frac{1}{12}\}} \cdot lm_{it} \\
&+ \sum_{T_m \in \{\frac{1}{12}, \frac{2}{12}, \dots, 30\}} \gamma_{T_m} \cdot \mathbb{1}_{\{T_m \leq T < T_m + \frac{1}{12}\}} \cdot \mathrm{CDS}_{it} \\
&+ \delta \cdot \mathrm{Controls} + \varepsilon_{it}.
\end{aligned}
\tag{4}
$$

where T is the bond's duration in years, lm_{it} is as described above, CDS_{it} is the month-end five-year CDS Markit mid quote for issuer of bond i for month t, and Controls include the month-end numerical rating of the bond (where AAA (D) corresponds to a rating of 1 (22)), bond age in years, and the logarithm of the ouststanding amount of the bond. The impact of the control variables is as expected: CDS premia affect bond yield spreads positively and significantly for each duration bracket, with higher coefficient estimates for bonds with higher duration. Rating and age also have a positive (0.09 and 0.03, respectively) and significant impact, outstanding amount has a negative (-0.05) and significant impact. Finally, we can compute an average liquidity component for each duration bucket $[T_m, T_m + \frac{1}{12})$ in the bond

yield as

$$\text{Illiq}_{\text{reg}}^{\text{ask/bid}}(T_m) = \widehat{\beta_{T_m}^{a/b}} \cdot lm_{\text{Mean}}(T_m), \tag{5}$$

where $\widehat{\beta_{T_m}^{a/b}}$ is the estimate from Equation (4) and $lm_{\text{Mean}}(T_m)$ is the mean across all observations lm_{it} that fall in the corresponding duration bucket.

5 Empirical Analysis

5.1 Bid-ask spreads

Our model predictions for the long end of the term structure of liquidity premia depend on the shape of the term structure of bid-ask spreads. Therefore, we calibrate a parametric form for $s(T)$ to our data set. Using non-linear least squares, we minimize the sum of squared errors ϵ_i in the following equation:[10]

$$s(T) = a^{\text{bid-ask}} + b^{\text{bid-ask}} \cdot \left(1 - e^{-c^{\text{bid-ask}} \cdot T}\right) + \epsilon, \tag{6}$$

where bid-ask spreads $s(T)$ are calculated for each bond with duration T on days with trades on both sides as the difference between the average bid and ask transaction price. We winsorize bid-ask spreads at the 1% and 99% quantile. Figure 4 presents the calibrated function $s(T)$ together with average bid-ask spreads for monthly duration buckets.

Insert Figure 4 about here.

Figure 4 shows two important properties of bid-ask spreads. First, bid-ask spreads are

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[10]Since the bond-specific spread should be limited between 0 and 1, a range of simple functions such as a linear form $s(T) = a^{\text{bid-ask}} + b^{\text{bid-ask}} \cdot T$ or its exponential counterpart $s(T) = a^{\text{bid-ask}} + b^{\text{bid-ask}} \cdot \left(e^{c^{\text{bid-ask}} \cdot T}\right)$ for $c^{\text{bid-ask}} > 0$ are not suitable for all possible T_{max}.

small but distinctly positive even for securities with very short maturities, which corresponds to a fixed component of transaction cost. Second, bid-ask spreads increase in maturity.

5.2   Hypotheses

Our model forecasts non-linear relations between maturity T and bid and ask liquidity premia. More formally, it predicts that the sensitivity of liquidity premia to maturity T is different for short- and long-term bonds. To test these relations, we employ piecewise linear regressions that explicitly allow for such a different sensitivity for maturities below and above a breakpoint y:

$$\text{Illiq}^{\text{ask}}(T) = \alpha^a + \beta_1^a \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta_2^a \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \gamma^a \cdot \text{Controls} + \varepsilon,$$

$$\text{Illiq}^{\text{bid}}(T) = \alpha^b + \beta_1^b \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta_2^b \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \gamma^b \cdot \text{Controls} + \varepsilon, \quad (7)$$

where $\text{Illiq}^{\text{ask}}(T)$ ($\text{Illiq}^{\text{bid}}(T)$) is the liquidity premium computed from ask (bid) prices, T is the duration of the bond, and ε is an error term. We explore a wide range of possible breakpoints y between three months and three years and do not endogenously derive an optimal breakpoint to avoid overfitting.[11]

If our hypotheses regarding the liquidity term structure are confirmed, we expect the following behavior. For ask premia, we should find positive and significant estimates for β_1^a and β_2^a as the slope of the ask liquidity premium term structure is positive for all maturities. Because our model predicts a flattening term structure, we expect β_1^a to be larger than β_2^a. For bid premia, we should find significant negative estimates for β_1^b. The shape of the bid term structure at the long end depends on the shape of bid-ask spreads $s(T)$. Figure 3 shows for the empirically calibrated bid-ask spread curve that bid premia are relatively flat but slightly increasing at the long end. Therefore, we expect β_2^b to be either not significantly

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[11]Naturally, our model does not imply that our predictions hold for every possible duration breakpoint.

different from zero or slightly positive.

A similar intuition holds for trading volume. There, we use a regression of the form

$$\text{Turnover}(T) = \alpha + \beta_1 \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta_2 \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \beta_3 \cdot \text{Age} + \gamma \cdot \text{Controls} + \varepsilon,$$
(8)

and expect positive and significant estimates for β_1 and negative and significant estimates of β_2. We expect a negative estimate for β_3, since our model predicts a lower trading volume for an older but otherwise equal bond compared to a younger one.

5.3   Liquidity Premia Analysis

We first present an overview of the average term structure of ask and bid liquidity premia together with the respective model predictions for both approaches to measure liquidity premia in Figure 5. Visual inspection of Panels A and B suggests that our main hypotheses regarding liquidity premia hold for the full sample. Ask liquidity premia are mostly increasing in maturity, while bid liquidity premia exhibit an inverse shape. At the long end, both bid and ask premia slightly increase with maturity.

Insert Figure 5 about here.

We now formally explore the effect of maturity on bond liquidity premia and estimate Equation (7). In the analysis of $\text{Illiq}_{\text{diff}}^{\text{ask/bid}}$, we use the numerical rating, bond age, and outstanding amount as control variables.[12]   When the illiquidity component is calculated with the second approach, i.e., for $\text{Illiq}_{\text{reg}}^{\text{ask/bid}}$, these effects are already controlled for in the first-step regression (4). For $\text{Illiq}_{\text{diff}}^{\text{ask/bid}}$, for which we have an observation for each trade in our sample period, we also include month and firm fixed effects to adjust for unobservable

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[12]Edwards, Harris, and Piwowar (2007) report a dependence of transaction costs on age and outstanding volume which is not directly captured by our model.

20

variation in our bond sample over time or across firms, and cluster standard errors by firm as suggested by Petersen (2009). The results of the regression are given in Table II.

Insert Table II about here.

Panel A and B of Table II confirm our hypotheses regarding liquidity premia. Irrespective of the way we measure liquidity premia, we find that the estimates for the slope at the short end, β_1^a, are always positive and significant for ask liquidity premia for 11 out of 12 breakpoint specifications. The estimates for the slope at the long end, β_2^a, are always positive and again significant in 11 out of 12 cases, and consistently smaller than the estimates for β_1^a by around a factor of 10 to 20. This relation indicates a much higher slope at the short end. When we formally test this relation, we obtain an always negative difference between the long and the short end which is significant in 11 out of 12 cases. Overall, the results strongly support our model prediction that ask liquidity premia increase more strongly for shorter durations, and flatten out for longer durations.

For bid liquidity premia, we obtain negative and significant estimates for the slope at the short end for 10 out of 12 specifications. Consistent with the shape of the predicted bid curve in Figure 5, 11 out of 12 estimates for the slope at the long end are positive but only one of them is significant. This implies a relatively flat term structure at the long end. When we again test formally for differences between the long and the short end, the differences are positive in 11 and significant in 10 out of 12 cases. Overall, bid liquidity premia exhibit an inverse shape with a strongly negative slope for short durations which flatten out for longer durations.

For $\text{Illiq}_{\text{diff}}^{\text{ask/bid}}$, the impact of the control variables is also as expected. Age has a positive impact on liquidity premia, the numerical rating has a positive impact whenever significant, and outstanding volume has a negative impact whenever significant.

Overall, the results of the regression analysis confirm our model predictions. Ask liquidity

21

premia are monotonically increasing with a decreasing slope, while bid liquidity premia are decreasing for short maturities and flatten out at the long end.

5.4 Turnover Analysis

To formally explore the hypotheses regarding secondary-market trading volume, we consider two subsamples. First, we use all transactions available in TRACE, standardized with the outstanding amount of the bond under consideration. Second, we exclude bonds immediately around changes in their outstanding volume (through new issues, reopenings, and bond repurchases) since we do not consider these events in our model. When bonds are newly issued, they are often first held by dealers, who distribute them to clients and other dealers. Hence, the time interval around new issues of bonds might consist of multiple inter-dealer trades. We therefore exclude transactions two months prior to a new issue and six months following the issue, and denote this sample by Excl[-2,+6].[13] Since newly issued bonds are excluded, this sample should also be less affected by the on-the-run/off-the-run effect.

We now apply our piecewise regression approach with age as an additional explanatory variable according to Equation (8). The control variables we use are again the outstanding amount and credit risk. Since turnover cannot be calculated on a trade-by-trade basis, we aggregate traded volume for each bond and calendar month and compute average daily turnover to account for a different number of business days per month. As for liquidity premia, we winsorize turnover at the 1% and 99% quantile. The regression results are displayed in Table III.

<center>Insert Table III about here.</center>

Table III confirms our model predictions regarding the hump-shaped turnover. For both subsamples, trading volume first increases strongly: the factor loadings for the slope at the

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[13]We exclude the time two months before changes in the amount outstanding mainly because of trades taking place in connection with bond repurchases that are typically announced about one month in advance.

<center>22</center>

short end, β_1, are positive and significant whenever we consider breakpoints below two years (in eight out of twelve cases). Following the breakpoint, trading volume decreases slowly, and the effect is significant in nine out of twelve cases. The negative loading for age in all specifications is consistent with our prediction of a lower trading volume for older but otherwise equal bonds compared to younger ones. The results for the outstanding amount are also as expected: bonds with a higher outstanding volume are more liquid, and thus display a higher trading volume.

6 Robustness

In the previous section, we compute ask and bid liquidity premia under two important assumptions. First, we use Treasury rates as the risk-free curve. Second, we use CDS premia and ratings as a proxy for the credit risk premium. In this section, we show that these assumptions do not affect the relation between liquidity premia and maturity by repeating our analysis for different subsamples. In Section 6.1, we repeat the regression analysis using swap rates instead of Treasury yields as a proxy for the risk-free interest rates. In Section 6.2, we restrict liquidity premia to AAA bonds where the impact of credit risk on yield spreads is minimized and we do not correct for credit risk.[14]

6.1 Swap Rates as Risk-Free Interest Rates

As mentioned in Section 4, instead of using Treasury rates, we also interpolate swap rates to obtain an alternative risk-free yield curve. Table IV shows the results when we re-estimate Equation (7) using Swap rates as the risk-free reference curve to calculate liquidity premia.

Insert Table IV about here.

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[14]In additional robustness checks, we restrict our dataset to transactions with a volume of \$100,000 or more, or only consider the time before the onset of the subprime crisis. All results confirm our hypotheses and are available upon request.

Table IV shows that our estimation results are mostly unaffected by the use of swap rates as risk-free rates. For ask liquidity premia, the estimates for the slope at both the short and the long end are positive in all and significant in 7 out of 12 cases. The estimates for the long end consistently are below those for the long end, and the difference is significant in 5 out of 12 cases. For bid liquidity premia, the slope at the short end is always negative and significant in 10 out of 12 cases. The slope at the long end is now positive and significant in all cases, but, compared to the short end, quantitatively small. The difference between long- and short-term premia is always positive and significant in 11 out of 12 cases.

Our main conclusions remain unaffected: ask liquidity premia increase more strongly for shorter maturities, bid liquidity premia exhibit an inverse shape at the short end and are flat or increase slightly for longer maturities.

6.2   Analysis of AAA Bonds

In our second robustness check, we analyze whether our results are sensitive to how we adjust the observed yield spreads for credit risk. To do so, we concentrate on those bonds which are least likely to be affected by credit risk: AAA rated bonds. We therefore drop all transactions where the traded bond does not exhibit a AAA rating by at least two rating agencies on the transaction date. We also drop all transactions which occurred after March 31, 2007 since a AAA rating might not be indicative of negligible credit risk during the financial crisis. General Electric bonds, e.g., exhibited increasing yields long before the downgrade from AAA to AA+ by Standard&Poor's on March 12, 2009. For the calculation of $\text{Illiq}_{\text{diff}}^{\text{ask/bid}}$, we interpret the difference between the bond's yield minus a theoretical yield calculated by discounting the bond's cash flows with the Treasury curve as a pure liquidity premium. Since all bonds exhibit a AAA rating, we exclude rating as an explanatory variable. For $\text{Illiq}_{\text{reg}}^{\text{ask/bid}}$, we exclude CDS quotes and ratings in the first-step regression in Equation (4).[15] We explore

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[15]In an alternative robustness check, we use agency bonds instead of AAA rated bonds. The results are virtually the same.

the relation between liquidity premia and maturity for AAA rated bonds in Table V.

Insert Table V about here.

Table V shows that our results are, if anything, stronger for the AAA sample than for the entire sample. For ask liquidity premia, the estimates for β_1^a are always positive and significant in 11 out of 12 cases. The slope at the long end is always positive and significantly flatter than at the short end in all specifications. Bid liquidity premia exhibit always negative (always positive) estimates for the slope at the short (long) end which are significant in 10 (12) out of 12 cases. As for ask liquidity premia, the difference between the long and the short end is always significant.

7 Summary and Conclusion

In this paper, we develop a parsimonious equilibrium model that generates a hump-shaped term structure of trading volume and, depending on whether we consider bid or ask prices, different shapes for the term structure of liquidity premia. Investors sell bonds of intermediate and long maturities because they experience a preference shock. Liquidity is supplied by exogenous market makers who charge a positive spread. We then analyze liquidity premia of corporate bonds with a wide range of maturities, and show that the observed trading behavior and liquidity premia from ask and bid yields are consistent with our model predictions. The main conclusion from our analysis is that the different term structures can arise because of two frictions which are prevalent in bond markets. First, traders who provide liquidity charge a non-zero spread for bonds of all maturities. Second, investors differ with respect to their probability of experiencing a liquidity shock. Such a difference is obvious if, for example, we consider insurance companies who are unlikely to experience frequent liquidity shocks, and bond market funds which frequently experience cash outflows.

Our model also yields two central implications for market microstructure and financial stability. First, our model allows us to quantify the well-established price impact of any given bid-ask spread term structure for assets of different maturities. This is important because of two effects. First, artificially increasing transaction costs, especially at the short end such as through a fixed financial transaction tax, lead to uniformly higher required yields, and thus lower prices, for all bonds. Conversely, a decrease of transaction costs, e.g., via a subsidized dealer system, uniformly decreases yields and increases prices. Second, an increase (decrease) of transaction costs shifts the maturity limit below which investors do not sell bonds in spite of a preference shock to higher (lower) values. Therefore, our model predicts that higher bid-ask spreads can dry out the market for short-term securities.

The second important implication of our results concerns the interplay of liquidity and credit risk. As He and Xiong (2012) show, liquidity premia for corporate bonds can have a strong impact on the issuer's optimal default boundary. Hence, higher bid-ask spreads, which lead to higher liquidity premia, increase individual and aggregate credit risk. To the best of our knowledge, we are the first to show that a higher probability of a liquidity shock for investors with high trading needs (who hold only short-term securities) also affects liquidity premia for long-term securities. Hence, firms that issue long-term bonds might be affected by shocks to institutional investors who hold short-term debt to a similar extent as firms with short-term debt. This mechanism implies that liquidity risk management of investors with short investment horizons (e.g., liquidity buffers for banks under Basel III, or for mutual funds under the Investment Company Act) might increase financial stability for the entire economy.

Appendix - Formal Derivation of the Equilibrium

We first discuss the model setting and the investor's individual optimization problem in Appendix A. Our model can be viewed as a continuous modification of a linear exchange model (see Gale, 1960) for which unique solutions exist. The equilibrium mechanism is similar to the ones in Amihud and Mendelson (1986) and Beber, Driessen, and Tuijp (2012). For a given model parameter set $(\lambda_L, \lambda_H, W_L, W_H, a, b, T_{\max})$ and bid-ask spread function $s(T)$, equilibrium prices $P(T)$, which we calculate in Appendix B, depend on critical maturities τ (below which it is not optimal to sell bonds after a preference shock) and $T_{\lim}$ (below which it is optimal for low-risk investors not to invest, and above which it is optimal for high-risk investors not to invest). Reversely, critical maturities depend on equilibrium prices. We use a market clearing argument in Appendix C to calculate $T_{\lim}$ and iterate until convergence over the calculation of equilibrium prices $P(T)$, τ (see Appendix A), and $T_{\lim}$. We finally demonstrate in Appendix D that the assumptions used in formulating the investor's optimization problem hold, and prove Propositions 2 and 3 in Appendix E.

Appendix A – Model Setting and Optimization Problem

We consider a continuum of illiquid zero-coupon bonds with maturity between 0 and $T_{\max}$. Each bond is characterized by its initial maturity at issuance $T_{\text{init}} \leq T_{\max}$, and bonds of each initial maturity are issued with rate a. In steady state, for each T_{init}, there are $a \cdot T_{\text{init}}$ bonds outstanding, and equally distributed with respect to their remaining time to maturity T in $(0, T_{\text{init}}]$. Hence, total outstanding volume of all bonds is $\int_0^{T_{\max}} a \cdot T_{\text{init}} \, dT_{\text{init}} = \frac{1}{2} \cdot a \cdot (T_{\max})^2$.

Investors experience a single preference shock with Poisson rate λ_i, $i \in \{H, L\}$, depending on their type i. Conditional on a liquidity shock at time $\tilde{T}_i$, total utility from consumption for an (infinitesimally small) investor of group i is given by $U_i(c) = \int\limits_0^{\tilde{T}_i} e^{-r \cdot t} c_t \, dt + \int\limits_{\tilde{T}_i}^{\infty} e^{-r \cdot \tilde{T}_i - (r+b) \cdot (t - \tilde{T}_i)} c_t \, dt$. The maturity for which an investor experiencing a preference shock

27

is indifferent between selling a bond and holding it until maturity, τ, satisfies

$$P(\tau) \cdot (1 - s(\tau)) = e^{-(r+b) \cdot \tau} \tag{9}$$

and is identical for both investor types.

Investors are risk-neutral implying an additive structure of the expected utility function. Therefore, they want to invest either nothing or the maximum amount possible in a particular maturity (see, e.g., Feldhütter, 2012). An investor who initially invests in a bond of some maturity T thus re-invests in a bond with this maturity if her old bond matures. The investor's decision to invest in a particular maturity hence neither depends on her wealth nor on her holdings in other maturities. In summary, each investor chooses an initially optimal allocation strategy when she first enters the market and has no incentive to change her portfolio prior to a preference shock.

We make two assumptions in deriving the investor's optimization problem. First, we assume that in the case of a preference shock, it is optimal to either sell the bond immediately or hold it until maturity. Second, we assume that it is never optimal to sell bonds when no preference shock has occurred. In Appendix D, we derive general conditions under which these assumptions hold.

Since we consider a steady-state equilibrium, investors who have experienced a preference shock are replaced by new investors such that aggregate wealth W_i from each investor group i remains constant. To ensure this, the wealth of any investor group cannot grow at a higher rate than that at which members of the respective investor group leave the market. In equilibrium, $r + b$ is an absolute upper bound for the growth rate of wealth such that we assume $r + b < \lambda_L < \lambda_H$.[16]

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[16]As investors are infinitesimally small, the distribution of the investors' age remains constant over time in steady state. An investor's age determines her individual wealth gains because it determines how long she was able to collect liquidity premia and risk-free returns. Hence, as all newly-arriving investors have an identical capital endowment, the constant distribution of investors' age directly leads to constant aggregate wealth.

As neither aggregate wealth nor bond supply change over time, prices of bonds for a given maturity are constant over time. We therefore only consider the decision problem at time $t = 0$, where each type-i investor maximizes her expected utility $E[U_i(c)]$ from consumption by choosing the amount of money X_i invested into the money market account ($X_i(0)$) and into bonds with maturity T between 0 and $T_{\max}$ ($X_i(T)$). Short sales are not allowed, so $X_i(T) \geq 0, \ \forall T \in [0, T_{\max}]$. Hence, a type-$i$ investor solves the following optimization problem:

$$
\max_{X_i} E \left\{ \int_0^{T_{\max}} X_i(T) \cdot \sum_{j=1}^{\infty} \frac{1}{P(T)^j} \cdot (1 - s(T \cdot j - \tilde{T}_i)) \cdot P(T \cdot j - \tilde{T}_i) \cdot e^{-r \cdot \tilde{T}_i} \right.
$$

$$
\cdot \mathbb{1}_{\{T \cdot (j-1) < \tilde{T}_i < T \cdot j - \min(\tau, T)\}} \, dT
$$

$$
+ \int_0^{T_{\max}} X_i(T) \cdot \sum_{j=1}^{\infty} \frac{1}{P(T)^j} \cdot e^{-r \cdot \tilde{T}_i - (r+b) \cdot (T \cdot j - \tilde{T}_i)} \cdot \mathbb{1}_{\{T \cdot j - \min(\tau, T) \leq \tilde{T}_i \leq T \cdot j\}} \, dT
$$

$$
\left. + X_i(0) \right\}. \tag{10}
$$

The first summand in Expression (10) denotes utility of consumption from bonds which the investor sells to the dealer at the bid price $(1 - s(T \cdot j - \tilde{T}_i)) \cdot P(T \cdot j - \tilde{T}_i)$ immediately after a preference shock. The amount invested in bonds $X_i(T) \cdot \frac{1}{P(T)^j}$ grows for as many investment rounds j as the investor (re-)invests in the bond until the preference shock and thereby in each round collects the price difference between the notional value of the bond and the price of the bond $P(T)$. The second summand gives the utility of consumption from bonds which the investor holds after the preference shock until their maturity date. The third summand measures the utility from cash invested in the money market account.

The investor's budget constraint is $W_i = \int_0^{T_{\max}} X_i(T) \, dT + X_i(0)$. Simplifying Expression (10), taking expectations, and replacing $X_i(0)$ via the budget constraint yields the

29

following optimization problem:

$$
\max_{X_i} \left\{ \int_0^{T_{\max}} X_i(T) \cdot \frac{\lambda_i \cdot e^{\lambda_i \cdot T}}{P(T) \cdot e^{r \cdot T} \cdot e^{\lambda_i \cdot T} - 1} \cdot \int_{\min(\tau,T)}^{T} P(x) \cdot e^{r \cdot x} \cdot (1 - s(x)) \cdot e^{-\lambda_i \cdot (T-x)} \, dx \, dT \right.
$$
$$
+ \int_0^{T_{\max}} X_i(T) \cdot \frac{\lambda_i \cdot (1 - e^{(\lambda_i - b) \cdot \min(\tau,T)})}{(1 - P(T) \cdot e^{r \cdot T} \cdot e^{\lambda_i \cdot T}) \cdot (\lambda_i - b)} \, dT
$$
$$
\left. + W_i - \int_0^{T_{\max}} X_i(T) \, dT \right\}. \tag{11}
$$

Taking partial derivatives with respect to each $X_i(T)$ yields the marginal utility of holding bonds with maturity T for a type-i investor:

$$
\frac{\partial E[U_i(c)]}{\partial X_i(T)} = \frac{\lambda_i \cdot e^{\lambda_i \cdot T}}{P(T) \cdot e^{r \cdot T} \cdot e^{\lambda_i \cdot T} - 1} \cdot \int_{\min(\tau,T)}^{T} P(x) \cdot e^{r \cdot x} \cdot (1 - s(x)) \cdot e^{-\lambda_i \cdot (T-x)} \, dx
$$
$$
+ \frac{\lambda_i \cdot (1 - e^{(\lambda_i - b) \cdot \min(\tau,T)})}{(1 - P(T) \cdot e^{r \cdot T} \cdot e^{\lambda_i \cdot T}) \cdot (\lambda_i - b)} - 1 =: \Delta_i(T). \tag{12}
$$

The fact that marginal utility does not depend on X_i simplifies the equilibrium: As investors are indifferent between all bonds they invest in, the marginal utility of these bonds must be equal. As marginal utility does not depend on X_i, it is sufficient to consider whether an investor buys a bond at all. Given that the investor buys the bond, she is indifferent on how she distributes her wealth across all bonds she invests in.

Equation (12) also shows that the time preference rate r which applies prior to the liquidity shock does not affect the investor's optimization problem. To see why, we rewrite bond prices as $P(T) = e^{-r \cdot T} \cdot Q(T)$. Here, $Q(T)$ is the discount of an illiquid bond compared to the price of a perfectly liquid bond $e^{-r \cdot T}$. Substituting $Q(T) = e^{r \cdot T} \cdot P(T)$ into Equations (9) and (12) would lead to an identical optimization problem independent of r. To simplify notation, we therefore set $r = 0$ in the following analysis.

Appendix B – Equilibrium Prices and Clientele Effect

Marginal utility for holding bonds is larger for low-risk investors than for high-risk investors. Marginal utility of the money market account is equal for both. Hence, the allocation that high-risk investors buy bonds and at the same time, low-risk investors invest in the money market account cannot be an equilibrium. We therefore focus on the most general remaining allocation: both high- and low-risk investors hold bonds, and high-risk investors additionally invest in the money market account.

Below, we show that given the calculated equilibrium prices, there exists a limiting maturity $T_{\lim}$ such that low-risk investors buy only bonds with maturity between $T_{\lim}$ and $T_{\max}$, and high-risk investors buy only bonds with maturity between 0 and $T_{\lim}$ (clientele effect). The equilibrium conditions are then given by

$$\Delta_H(T) = 0 \qquad \text{for all } T \in (0, T_{\lim}], \tag{13}$$

$$\Delta_L(T) = \Delta_L(T_{\lim}) \qquad \text{for all } T \in (T_{\lim}, T_{\max}], \tag{14}$$

where $\Delta_i(T)$ is defined as in Equation (12). High-risk investors are indifferent between holding bonds with a maturity up until $T_{\lim}$ and the money market account, low-risk investors are indifferent between buying bonds with maturities between $T_{\lim}$ and $T_{\max}$.

Calculation of Equilibrium Prices

For given limiting maturities τ and $T_{\lim}$, the conditions in Equations (13) and (14) lead to closed-form solutions for $P(T)$. We must consider five different ranges for maturity T: $T \leq \min(\tau, T_{\lim})$, $\tau < T \leq T_{\lim}$, $\tau < T_{\lim} < T$, $T_{\lim} < T \leq \tau$, and $T_{\lim} < \tau < T$.

(i) For $T \leq \min(\tau, T_{\lim})$, the integral term of Equation (12) is zero. Using the first order

condition (13), we get

$$\Delta_H(T) = \frac{\lambda_H \cdot (1 - e^{(\lambda_H - b) \cdot T})}{(1 - P(T) \cdot e^{\lambda_H \cdot T}) \cdot (\lambda_H - b)} - 1 \overset{!}{=} 0. \tag{15}$$

Solving Condition (15) for $P(T)$ directly yields

$$P(T) = \frac{b \cdot e^{-\lambda_H \cdot T} - \lambda_H \cdot e^{-b \cdot T}}{b - \lambda_H} \qquad \text{for } T \leq \min(\tau, T_{\text{lim}}). \tag{16}$$

(ii) For $\tau < T \leq T_{\text{lim}}$, again using Equation (12), the first order condition (13) evaluates to

$$\Delta_H(T) = \frac{\lambda_H \cdot e^{\lambda_H \cdot T}}{P(T) \cdot e^{\lambda_H \cdot T} - 1} \cdot \int_\tau^T P(x) \cdot (1 - s(x)) \cdot e^{-\lambda_H \cdot (T-x)} \, dx$$
$$+ \frac{\lambda_H \cdot (1 - e^{(\lambda_H - b) \cdot \tau})}{(1 - P(T) \cdot e^{\lambda_H \cdot T}) \cdot (\lambda_H - b)} - 1 \overset{!}{=} 0. \tag{17}$$

The solution of this integral equation is given as

$$P(T) = e^{-\int_\tau^T \lambda_H \cdot s(x) \, dx} \cdot P(\tau) \qquad \text{for } \tau < T \leq T_{\text{lim}}, \tag{18}$$

which can be verified by plugging in (18) into (17). It is instructive to note that (18) corresponds to the market value of a defaultable bond with a default intensity λ_H and a "recovery-rate" of $(1 - s(T))$ when using the "recovery of market value assumption" in Duffie and Singleton (1999).

(iii) For $\tau < T_{\text{lim}} < T$, we insert Equation (12) into the first order condition for the low-risk investors (14) and get

$$\Delta_L(T) = \frac{\lambda_L \cdot e^{\lambda_L \cdot T}}{P(T) \cdot e^{\lambda_L \cdot T} - 1} \cdot \int_\tau^T P(x) \cdot (1 - s(x)) \cdot e^{-\lambda_L \cdot (T-x)} \, dx$$
$$+ \frac{\lambda_L \cdot (1 - e^{(\lambda_L - b) \cdot \tau})}{(1 - P(T) \cdot e^{\lambda_L \cdot T}) \cdot (\lambda_L - b)} - 1 \overset{!}{=} \Delta_L(T_{\text{lim}}). \tag{19}$$

32

By plugging in

$$P(T) = e^{-\int_{T_{\lim}}^{T} \frac{(\Delta_L(T_{\lim})+s(x)) \cdot \lambda_L}{1+\Delta_L(T_{\lim})} \, dx} \cdot P(T_{\lim}) \qquad \text{for } \tau < T_{\lim} < T, \tag{20}$$

we show that (20) solves the integral equation (19).

(iv) For $T_{\lim} < T \leq \tau$, we can ignore the first term of Equation (12) and then again employ the first order condition for the low-risk investors (14) to get

$$\Delta_L(T) = \frac{\lambda_L \cdot (1 - e^{(\lambda_L - b) \cdot T})}{(1 - P(T) \cdot e^{\lambda_L \cdot T}) \cdot (\lambda_L - b)} - 1 \overset{!}{=} \Delta_L(T_{\lim}). \tag{21}$$

Rearranging terms directly yields

$$P(T) = e^{-T \cdot \lambda_L} \cdot \left(1 - \frac{\lambda_L \cdot \left(1 - e^{T \cdot (\lambda_L - b)}\right)}{(1 + \Delta_L(T_{\lim})) \cdot (\lambda_L - b)}\right) \qquad \text{for } T_{\lim} < T \leq \tau. \tag{22}$$

(v) For $T_{\lim} \leq \tau < T$, as in (iii), we obtain (19). Since $T_{\lim} \leq \tau < T$, we get the solution

$$P(T) = e^{-\int_{\tau}^{T} \frac{(\Delta_L(T_{\lim})+s(x)) \cdot \lambda_L}{1+\Delta_L(T_{\lim})} \, dx} \cdot P(\tau) \qquad \text{for } T_{\lim} \leq \tau < T, \tag{23}$$

which we again verify by plugging it into (19), but now use $\Delta_L(T_{\lim})$ from (21).

This verifies the first part of Proposition 1.

Clientele Effect

We prove that for the derived equilibrium prices and constant or monotonically increasing bid-ask spreads $s(T)$ with $0 < s(T) < 1$, there is a maturity $T_{\lim}$ such that high-risk investors have no incentive to invest in bonds with longer maturity:

$$\Delta_H(T) < 0 \qquad \text{for all } T \in (T_{\lim}, T_{\max}], \tag{24}$$

and low-risk investors have no incentive to invest in bonds with shorter maturity:

$$\Delta_L(T) < \Delta_L(T_{\lim}) \qquad \text{for all } T \in (0, T_{\lim}), \tag{25}$$

nor in the money market account, since they hold only bonds, i.e., $\Delta_L(T) > 0$ for at least one $T \in (0, T_{\max}]$.

Low-risk investors have higher marginal utility for all bonds than high-risk investors, who have a marginal utility of 0 for bonds with maturity $T_{\lim}$, therefore $\Delta_L(T_{\lim}) > \Delta_H(T_{\lim}) = 0$. Hence, the last condition $\Delta_L(T) > 0$ trivially holds for $T = T_{\lim}$.

Proof of Equation (25): We verify that $\Delta_L(T)$ is strictly monotonically increasing in T for $T \leq T_{\lim}$ and arbitrary $T_{\lim}$, i.e., $\Delta'_L(T) > 0$: For the case $T \leq \tau$, $\Delta_L(T)$ is given as

$$\Delta_L(T) = \frac{\lambda_L \cdot (1 - e^{(\lambda_L - b) \cdot T})}{(1 - e^{\lambda_L \cdot T} \cdot P(T)) \cdot (\lambda_L - b)} - 1. \tag{26}$$

By employing Equation (16) for $P(T)$, using $0 < b < \lambda_L < \lambda_H$ (see Appendix A), and substituting $b = \lambda_L - c1$ and $\lambda_H = \lambda_L + c2$ with $c1, c2 > 0$ and $c1 < \lambda_L$, the condition $\Delta'_L(T) > 0$ simplifies to

$$e^{(c1+c2) \cdot T} \cdot c1 + c2 > e^{c1 \cdot T} \cdot (c1 + c2). \tag{27}$$

(27) holds for all $T > 0$ since for $T = 0$, both sides are equal $(c1 + c2)$, and the first derivative with respect to T of the left-hand side of (27) is larger than that of the right-hand side, i.e.,

$$(c1 + c2) \cdot c1 \cdot e^{(c1+c2) \cdot T} > (c1 + c2) \cdot c1 \cdot e^{c1 \cdot T}, \tag{28}$$

which is always true since $c1, c2 > 0$.

For the second case with $T > \tau$, rearranging terms and again using $0 < b < \lambda_L < \lambda_H$,

the condition $\Delta'_L(T) > 0$ simplifies to

$$(1 - s(T)) \cdot \left(e^{T \cdot \lambda_L} \cdot P(T) - 1\right) - \left(\frac{\left(1 - e^{(\lambda_L - b) \cdot \tau}\right) \cdot \left(-\lambda_L - \frac{P'(T)}{P(T)}\right)}{(\lambda_L - b)}\right.$$

$$+ \left(\int_\tau^T e^{-(T-x) \cdot \lambda_L} \cdot (1 - s(x)) \cdot P(x) \, dx\right) \cdot \left.\left(e^{T \cdot \lambda_L} \cdot \lambda_L + e^{T \cdot \lambda_L} \cdot \frac{P'(T)}{P(T)}\right)\right) > 0. \qquad (29)$$

We prove that (29) holds in two steps: In step (a), we show that (29) holds for $T \downarrow \tau$, i.e., we look at the right-side limit of (29). In step (b), we show that the first derivative with respect to T of the left-hand side of (29) is positive. For (a), rearranging Equation (9) yields[17]

$$s(\tau) = \frac{b \cdot \left(e^{(b - \lambda_H) \cdot \tau} - 1\right)}{b \cdot e^{(b - \lambda_H) \cdot \tau} - \lambda_H}. \qquad (30)$$

Using again our substitutions $b = \lambda_L - c1$ and $\lambda_H = \lambda_L + c2$ with $c1, c2 > 0$ and $c1 < \lambda_L$, plugging in (30) as well as (18) for $P(T)$, we can simplify (29) to

$$e^{c2 \cdot \tau} \cdot c1 + e^{-c1 \cdot \tau} \cdot c2 - (c1 + c2) > 0. \qquad (31)$$

Again, it is easy to show that (31) holds for all $\tau > 0$ by verifying that its left-hand side equals 0 for $\tau \to 0$ and its first derivative with respect to τ is larger than 0.

For (b), we rearrange (29) by employing (18) for $P(T)$ and substituting $g(T) = T \cdot \lambda_L - \int_\tau^T \lambda_H \cdot s(x) \, dx$ and $g'(T) = \lambda_L - \lambda_H \cdot s(T)$ to finally get

$$\frac{\left(e^{g(T)} \cdot P(\tau) - 1\right) \cdot (\lambda_H - \lambda_L)}{\lambda_H} + \left(\frac{e^{\lambda_L \cdot \tau} P(\tau) - 1}{\lambda_H} + \frac{e^{(\lambda_L - b) \cdot \tau} - 1}{b - \lambda_L}\right.$$

$$\left. - \int_\tau^T \frac{e^{g(x)} \cdot P(\tau) \cdot (\lambda_H - \lambda_L)}{\lambda_H} \, dx\right) \cdot g'(T) > 0 \qquad (32)$$

and it remains to show that the first derivative with respect to T of the left-hand side of

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[17]Note that for $T > \tau$, we implicitly assume that τ exists. If τ does not exist because bid-ask spreads are too large, the already-discussed case for $T \leq \tau$ applies for all T.

(32) is positive:

$$\left(\frac{e^{\lambda_L \cdot \tau} \cdot P(\tau) - 1}{\lambda_H} + \frac{e^{(\lambda_L - b) \cdot \tau} - 1}{b - \lambda_L} - \int_\tau^T \frac{e^{g(x)} \cdot P(\tau) \cdot (\lambda_H - \lambda_L)}{\lambda_H} \, dx \right) \cdot g''(T) > 0. \tag{33}$$

As $g''(T) = -\lambda_H \cdot s'(T) \leq 0$ for monotonically increasing $s(T)$ and $-\int_\tau^T \frac{e^{g(x)} \cdot P(\tau) \cdot (\lambda_H - \lambda_L)}{\lambda_H} \, dx < 0$ (since all factors in the numerator of the integrand are positive), a sufficient condition for (33) to hold is that

$$\frac{e^{\lambda_L \cdot \tau} \cdot P(\tau) - 1}{\lambda_H} + \frac{e^{(\lambda_L - b) \cdot \tau} - 1}{b - \lambda_L} < 0. \tag{34}$$

Using once more our substitutions $b = \lambda_L - c1$ and $\lambda_H = \lambda_L + c2$ with $c1, c2 > 0$ and $c1 < \lambda_L$ and utilizing (16) for $P(\tau)$, (34) simplifies to

$$c1 \cdot (\lambda_L - c1) + e^{c2 \cdot \tau} \cdot \left(c1^2 - c1 \cdot \lambda_L + \left(e^{c1 \cdot \tau} - 1 \right) \cdot c2 \cdot (c2 + \lambda_L) \right) > 0. \tag{35}$$

As before, it is easy to show that (35) holds for all $\tau > 0$ by verifying that its left-hand side equals 0 for $\tau \to 0$ and its first derivative with respect to τ is larger than 0. $\qquad \square$

Proof of Equation (24): Inequality (24) directly follows from $\Delta_L'(T) > 0$ for $T \leq T_{\lim}$. To see this, assume that for some parameter set $(\lambda_H, \ \lambda_L, \ a, \ b, \ T_{\max})$ and given bid-ask spread function $s(T)$, the wealth of high-risk investors is sufficient to buy all bonds and the wealth of low-risk investors goes to zero ($W_L^* \to 0$), so that $T_{\lim}^* \to T_{\max}$. Suppose now, that for the same parametrization $(\lambda_H, \ \lambda_L, \ a, \ b, \ T_{\max})$ and bid-ask spread function $s(T)$, the wealth of low-risk investors $W_L^+ >> 0$, so that $T_{\lim}^+ << T_{\max}$. Then it follows with the low-risk investors' first order condition (14) that

$$\Delta_L^+(T) = \Delta_L^+(T_{\lim}^+) \quad \text{for all } T \in (T_{\lim}^+, T_{\max}], \tag{36}$$

where we use $(+)$ to indicate for which case of W^+/W^* $\Delta_L(T)$ applies. Moreover, it follows

that

$$\Delta_L^+(T_{\text{lim}}^+) = \Delta_L^*(T_{\text{lim}}^+) \tag{37}$$

as $P(T_{\text{lim}}^+)$ is not affected by the choice of $T_{\text{lim}} \geq T_{\text{lim}}^+$ (dependent on τ, but independent of T_{lim}, either Equation (16) or (18) applies for $P(T)$). From the fact that $\Delta_L'(T) > 0$ for $T \leq T_{\text{lim}}$, we directly get

$$\Delta_L^*(T_{\text{lim}}^+) < \Delta_L^*(T) \qquad \text{for all } T \in (T_{\text{lim}}^+, T_{\text{lim}}^* = T_{\text{max}}]. \tag{38}$$

Putting together (36)-(38), we get

$$\Delta_L^+(T) < \Delta_L^*(T) \qquad \text{for all } T \in (T_{\text{lim}}^+, T_{\text{max}}]. \tag{39}$$

From the last Inequality (39), it directly follows that

$$P^+(T) > P^*(T) \qquad \text{for all } T \in (T_{\text{lim}}^+, T_{\text{max}}] \tag{40}$$

since lower prices $P(T)$ directly result in higher marginal utilities due to higher wealth gains. Turning this argument around, we get

$$\Delta_H^+(T) < \Delta_H^*(T) \qquad \text{for all } T \in (T_{\text{lim}}^+, T_{\text{max}}]. \tag{41}$$

Employing the high-risk investors' first order condition (13)

$$\Delta_H^*(T) = 0 \qquad \text{for all } T \in (0, T_{\text{lim}}^* = T_{\text{max}}], \tag{42}$$

it directly follows from (41) that

$$\Delta_H^+(T) < 0 \qquad \text{for all } T \in (T_{\text{lim}}^+, T_{\text{max}}], \tag{43}$$

which equals Inequality (24) for $T_{\text{lim}} = T_{\text{lim}}^+$. $\qquad\square$

This verifies the second part of Proposition 1.

Appendix C – Market Clearing

In this section, we verify that markets clear for given equilibrium prices. In doing so, we perform the final step of our iterative approach: we compute a new value of T_{lim} for given equilibrium prices.

As outlined in Appendix B, we focus on the allocation where both high- and low-risk investors hold bonds, and high-risk investors additionally invest in the money market account. In this allocation, markets clear if aggregate wealth of both investor types exceeds total bond supply (left inequality), but on the other hand, total wealth of low-risk investors alone does not suffice to buy all bonds (right inequality):

$$W_H + W_L > \int_0^{T_{\text{max}}} P(T) \cdot \int_T^{T_{\text{max}}} a \, dT_{\text{init}} \, dT > W_L. \tag{44}$$

The right inequality of (44) is automatically satisfied if the condition we derive below to determine T_{lim} yields a $T_{\text{lim}} \in (0, T_{\text{max}})$. By inserting the closed form solutions for $P(T)$ from Proposition 1 for a given parameter set, it is easy to verify the left inequality of (44).[18]

[18]If low-risk investors' wealth alone is sufficient to buy all bonds, prices are as in Proposition 1 with $T_{\text{lim}} = 0$. Markets then clear if $W_L > \int_0^{T_{\text{max}}} P(T) \cdot \int_T^{T_{\text{max}}} a \, dT_{\text{init}} \, dT$. However, this case is less interesting as high-risk investors do not play a role. We do not consider the degenerate allocation where $W_H + W_L$ is equal to $\int_0^{T_{\text{max}}} P(T) \cdot \int_T^{T_{\text{max}}} a \, dT_{\text{init}} \, dT$. In this case, bond prices would primarily reflect the economy's wealth constraint and strongly depend on total wealth, which is hard to quantify empirically.

To compute a $T_{\lim}$ consistent with equilibrium prices, we exploit the market clearing condition for bonds with maturities $T_{\text{init}} \in (T_{\lim}, T_{\max}]$ that are held by low-risk investors, i.e., we solve

$$W_L = \int_0^{T_{\max}} P(T) \cdot \int_T^{T_{\max}} a \cdot Y_L\left(T, T_{\text{init}}, T_{\lim}\right) \, dT_{\text{init}} \, dT \tag{45}$$

for $T_{\lim}$. Here, $Y_L(T, T_{\text{init}}, T_{\lim})$ denotes the fraction of bonds with remaining maturity T and initial maturity T_{init} for a given $T_{\lim}$ held by low-risk investors, i.e.,

$$Y_L\left(T, T_{\text{init}}, T_{\lim}\right) = \begin{cases} 0, & \text{if } T, T_{\text{init}} \leq T_{\lim} \\ e^{-\lambda_L \cdot (T_{\lim} - T)}, & \text{if } T \leq T_{\lim} \text{ and } T_{\text{init}} > T_{\lim} \\ 1, & \text{if } T > T_{\lim}. \end{cases} \tag{46}$$

For bonds with initial maturity $T_{\text{init}} > T_{\lim}$ and current maturity $T \leq T_{\lim}$, a fraction of $e^{-\lambda_L \cdot (T_{\lim} - T)}$ is held by old low-risk investors. Bonds with initial and current maturity smaller than $T_{\lim}$ are not held by low-risk investors, bonds with current and initial maturity larger than $T_{\lim}$ are only held by low-risk investors. To illustrate this, consider the extreme case of $W_L \to 0$. For Equation (45) to hold, $Y_L(T, T_{\text{init}}, T_{\lim})$ has to be 0 for all T and T_{init}. Hence, $T_{\lim} \to T_{\max}$.

Appendix D – Verification of Assumptions On Investor Behavior

In this section we check the assumptions from Appendix A: First, for $T > \tau$, it is always optimal to immediately sell the bond if an investor experiences a preference shock. Second, no investor has an incentive to sell bonds prematurely without having experienced a preference shock.

Bonds are sold immediately after a preference shock occurs if $T > \tau$: We define the utility of an investor she receives from selling a T-year bond d time periods after she

experienced a preference shock:

$$f(d) = (1 - s(T - d)) \cdot P(T - d) \cdot e^{-b \cdot d}. \tag{47}$$

Bonds are always sold immediately, iff $f'(d) < 0$. For $\tau < T \leq T_{\text{lim}}$, by plugging in prices $P(T)$ from (18), it can be shown that this condition holds iff

$$s'(T - d) < (1 - s(T - d)) \cdot (b - \lambda_H \cdot s(T - d)), \tag{48}$$

i.e., if bid-ask spreads do not grow with maturity "too strongly". For constant bid-ask spreads $s(T) = s$, (48) always holds since $s'(T) = 0$, $s < 1$, and $b - \lambda_H \cdot s > 0$. The latter condition holds as inserting $b - \lambda_H \cdot s \leq 0$ into Equation (9) leads to a contradiction (i.e., τ would not exist). Condition (48) also ensures that Equation (9) cannot have more than one solution for τ.

For the other two relevant cases $T_{\text{lim}} \leq \tau < T$ and $\tau < T_{\text{lim}} < T$, $f'(d) < 0$ also holds when (48) applies. This follows directly from the clientele effect since $P(T)$ decreases more slowly for increasing T when $T > T_{\text{lim}}$ than when $T \leq T_{\text{lim}}$ (low-risk investors demand lower compensation for holding longer term bonds compared to high-risk investors). Thus, the incentive to wait in the case of a preference shock is reduced, compared to $T \leq T_{\text{lim}}$ (since gains from increasing prices when the maturity decreases are smaller).

It is never optimal to sell bonds without preference shock: High-risk investors are indifferent between all bonds with maturities between 0 and T_{lim}. Hence, selling one bond with $T \in (0, T_{\text{lim}}]$, paying the bid-ask spread $s(T)$, and buying another bond with $T_{\text{new}} \in (0, T_{\text{lim}}]$ cannot be optimal. Using the same argument, low-risk investors can never have an incentive to sell bonds with maturity $T \geq T_{\text{lim}}$. For them, selling bonds with $T < T_{\text{lim}}$ without a preference shock can only be optimal if the marginal utility through the early reinvestment in a bond with maturity $T_{\text{new}} \in (T_{\text{lim}}, T_{\text{max}}]$ plus the proceeds from

selling the bond with maturity $T \in (0, T_{\lim})$ is higher than the marginal utility from the later reinvestment (at maturity of the respective bond) plus the proceeds from the maturing bond if no preference shock occurs, or the proceeds from the optimal decision given that a preference shock occurs:

$$(\Delta_L(T_{\lim}) + 1) \cdot P(T) \cdot (1 - s(T)) > Pr(\tilde{T}_L > T) \cdot (\Delta_L(T_{\lim}) + 1)$$

$$+ \int_0^{T-\min(T,\tau)} \underbrace{\lambda_L \cdot e^{-\lambda_L \cdot y}}_{\text{density function of the preference shock time}} \cdot (1 - s(T - y)) \cdot P(T - y) \, dy$$

$$+ \int_{T-\min(T,\tau)}^{T} \lambda_L \cdot e^{-\lambda_L \cdot y} \cdot e^{-b \cdot (T-y)} \, dy. \tag{49}$$

Note that in deriving (49), we exploit the fact that marginal utility does not depend on the invested amount (see Equation(12)), i.e., the optimal investment of an amount z for a low-risk investor leads to an expected utility of $(1 + \Delta_L(T_{\lim})) \cdot z$. Rearranging Equation (49) shows that low-risk investors have no incentive to sell bonds without having experienced a preference shock iff

$$(1 + \Delta_L(T_{\lim})) \cdot e^{-T\lambda_L} + \frac{e^{-T \cdot \lambda_L} \left(-1 + e^{(-b+\lambda_L) \cdot \min(T,\tau)}\right) \cdot \lambda_L}{-b + \lambda_L}$$

$$+ \int_{\min(T,\tau)}^{T} e^{(-T+x) \cdot \lambda_L} \cdot \lambda_L \cdot P(x) \cdot (1 - s(x)) \, dx - (1 + \Delta_L(T_{\lim})) \cdot P(T) \cdot (1 - s(T)) > 0. \tag{50}$$

Condition (50) holds for $T \leq \tau$ since for $T < \tau$, a sell is not optimal even in the case of a preference shock. As b is an upper bound for the ask liquidity premium of an arbitrary maturity (and thus the maximum return a selling investor could gain from her new bonds), the incentive to sell is lower when no preference shock occurs. For constant bid-ask spreads $s(T) = s$, it can also never be optimal to sell prematurely for $\tau \leq T < T_{\lim}$, as the relative wealth gain $-\frac{P'(T)}{P(T)}$ is higher than for $T > T_{\lim}$. Since we have already shown that it is never optimal to sell prematurely for $T \leq \tau$ and $T \geq T_{\lim}$, it can also not be optimal to sell

41

during the time of highest wealth gains. In the most general case with increasing bid-ask spreads $s(T)$ and for $T \in (\tau, T_{\text{lim}})$, (50) has to be verified by plugging in prices $P(T)$ from Proposition 1.

Appendix E – Proof of Propositions 2 and 3

Proposition 2

The fact that seller-initiated turnover is 0 for $T < \tau$ with $\tau > 0$ follows directly from Equation (9) as $P(\tau) \cdot (1 - s(\tau)) < 1$. The fact that secondary-market turnover is larger for $T < T_{\text{lim}}$ than for $T > T_{\text{lim}}$ if $\tau < T_{\text{lim}}$ is a direct consequence of the clientele effect.

Since dealers in aggregate do not hold any inventory, secondary-market trading volume can be calculated as twice the trading volume initiated by customers who sell their bond position prematurely. To calculate turnover, we divide by the total outstanding volume of all bonds with the respective maturity:

$$\text{Turnover}(T) = \frac{2 \cdot \mathbb{1}_{\{T > \tau\}} \cdot \int_{T}^{T_{\text{max}}} a \cdot \sum_{i=S,L} Y_i(T, T_{\text{init}}, T_{\text{lim}}) \cdot \lambda_i \, dT_{\text{init}}}{\int_{T}^{T_{\text{max}}} a \, dT_{\text{init}}}, \qquad (51)$$

where in the numerator and the denominator, we integrate over all bonds with initial maturity T_{init} and remaining maturity T that are held by both investor types. $Y_i(T, T_{\text{init}}, T_{\text{lim}})$ denotes the fraction of bonds with remaining maturity T and initial maturity T_{init} held in the portfolios of type-i investors (where $Y_S(T, T_{\text{init}}, T_{\text{lim}}) = 1 - Y_L(T, T_{\text{init}}, T_{\text{lim}})$ and $Y_L(T, T_{\text{init}}, T_{\text{lim}})$ is formally defined in Equation (46) in Appendix C). This fraction is multiplied with the rate at which preference shocks arrive. The denominator gives the total volume of all bonds with remaining maturity T and initial maturity T_{init} between T and T_{max}. The entire fraction is multiplied by $\mathbb{1}_{\{T > \tau\}}$, since investors who experience a preference shock only sell bonds with maturity $T > \tau$.

As elaborated in the main text, the second part of Proposition 2 also directly follows from the clientele effect. □

Proposition 3

Illiq$^{\text{ask}}(T)$ is monotonically increasing in T: To formalize this requirement, we calculate the first derivative with respect to T of Illiq$^{\text{ask}}(T)$ and show that it is greater than or equal to zero, i.e.,

$$\left(\text{Illiq}^{\text{ask}}(T)\right)' = \frac{\log(P(T))}{T^2} - \frac{P'(T)}{T \cdot P(T)} \geq 0. \tag{52}$$

(i) For $T \leq \min(\tau, T_{\text{lim}})$, plugging in prices $P(T)$ from Equation (16) into (52) and multiplying with T^2 leads to the condition

$$b \cdot T + \frac{b \cdot e^{b \cdot T} \cdot T \cdot (b - \lambda_H)}{-b \cdot e^{b \cdot T} + e^{T \cdot \lambda_H} \cdot \lambda_H} + \log\left(\frac{b \cdot e^{-T \cdot \lambda_H} - e^{-b \cdot T} \cdot \lambda_H}{b - \lambda_H}\right) \geq 0. \tag{53}$$

(53) trivially holds for $T = 0$. Moreover, for the first derivative with respect to T of the left-hand side of (53) it holds

$$\frac{b \cdot e^{T \cdot (b + \lambda_H)} \cdot T \cdot (b - \lambda_H)^2 \cdot \lambda_H}{\left(b \cdot e^{b \cdot T} - e^{T \cdot \lambda_H} \cdot \lambda_H\right)^2} \geq 0 \tag{54}$$

such that (53) is true for all T.

(ii) For T with $\tau < T \leq T_{\text{lim}}$, multiplying (52) by T and exploiting the relation $\frac{P'(T)}{P(T)} = -s(T) \cdot \lambda_H$ from (18) as well as $-\frac{\log(P(T))}{T} = \text{Illiq}^{\text{ask}}(T)$ yields

$$\text{Illiq}^{\text{ask}}(T) \leq s(T) \cdot \lambda_H. \tag{55}$$

$s \cdot \lambda_H$ is the liquidity premium for the extreme case that bid-ask spreads s remain constant

and investors are forced to sell immediately after a preference shock (see Equation (18)). In the general case, however, investors have the option to wait until maturity and for $s'(T) > 0$ bid-ask spreads even decrease over the bond's lifetime. Hence, $s(T) \cdot \lambda_H$ is an upper bound for $\text{Illiq}^{\text{ask}}(T)$.

(iii) For T with $\tau < T_{\lim} < T$, multiplying again (52) by T and exploiting the relation $\frac{P'(T)}{P(T)} = -\frac{(\Delta_L(T_{\lim})+s(T)) \cdot \lambda_L}{1+\Delta_L(T_{\lim})}$ from (20) yields

$$\text{Illiq}^{\text{ask}}(T) \leq \frac{(\Delta_L(T_{\lim}) + s(T)) \cdot \lambda_L}{1 + \Delta_L(T_{\lim})}. \tag{56}$$

$P^{\text{forced}}(T) = e^{-T \cdot \text{Illiq}^{\text{forced}}}$ with $\text{Illiq}^{\text{forced}} = \frac{(\Delta_L(T_{\lim})+s) \cdot \lambda_L}{1+\Delta_L(T_{\lim})}$ solves the indifference condition

$$\frac{\lambda_L \cdot e^{\lambda_L \cdot T}}{P^{\text{forced}}(T) \cdot e^{\lambda_L \cdot T} - 1} \cdot \int_0^T P^{\text{forced}}(x) \cdot (1-s) \cdot e^{-\lambda_L \cdot (T-x)} \, dx \overset{!}{=} \Delta_L(T_{\lim}). \tag{57}$$

Therefore, $\text{Illiq}^{\text{forced}}$ can be interpreted as the liquidity premium low-risk investors would demand for an artificial bond with the following characteristics: (a) only low-risk investors are allowed to invest in this bond, (b) the bond has constant bid-ask spreads s, (c) investors are forced to sell immediately after a preference shock (see also (19)). As high-risk investors are not excluded, bid-ask spreads $s(T)$ can only decrease when the bond ages, and investors have the option to wait until maturity, $\frac{(\Delta_L(T_{\lim})+s(T)) \cdot \lambda_L}{1+\Delta_L(T_{\lim})}$ is again an upper bound for $\text{Illiq}^{\text{ask}}(T)$.

The same reasoning for (iii) applies for our case (v), i.e., $T_{\lim} \leq \tau < T$.

(iv) For the last case of T with $T_{\lim} < T \leq \tau$, we exploit that $P(T)$ is continuously differentiable at $T = \tau$ (which can be shown using (22) and (23) for $P(T)$ as well as (9) solved for $s(\tau)$). If $P(T)$ is continuously differentiable at τ, $\left(\text{Illiq}^{\text{ask}}(T)\right)'$ is continuous at τ (see (52)). Since we have already shown that $\left(\text{Illiq}^{\text{ask}}(T)\right)'$ is larger than or equal to zero for T with $T_{\lim} \leq \tau < T$ (case (v)), $\left(\text{Illiq}^{\text{ask}}(T)\right)' \geq 0$ then also holds for $T = \tau$. To show that $\left(\text{Illiq}^{\text{ask}}(T)\right)' \geq 0$ for any T with $T_{\lim} < T \leq \tau$, we introduce an artificial bid-ask spread function $\hat{s}(T) \leq s(T)$ such that the corresponding $\hat{\tau}$ that solves Equation (9) equals

T. Now, we can again exploit case (v) with the artificial bid-ask spread function $\hat{s}(T)$, i.e., $\left(\widehat{\text{Illiq}}^{\text{ask}}(T)\right)' \geq 0$ for T with $T_{\lim} \leq \hat{\tau} < T$. As prices do not depend on bid-ask spreads when investors wait when experiencing a preference shock (see also Equation (1)), it holds that $P(T) = \hat{P}(T)$ for $T \leq \hat{\tau} < \tau$. Applying the same continuity argument as above for $\left(\widehat{\text{Illiq}}^{\text{ask}}(T)\right)'$ then proves the assertion for all $T(= \hat{\tau})$ with $T_{\lim} < T \leq \tau$. $\qquad\square$

Illiq$^{\text{ask}}(T)$ goes to zero for $T \to 0$: Applying l'Hopital's rule and using (16) for $P(T)$ directly leads to $\lim_{T \to 0} \text{Illiq}^{\text{ask}}(T) = \lim_{T \to 0} \frac{-\log(P(T))}{T} = 0$. $\qquad\square$

Illiq$^{\text{ask}}(T)$ flattens at $T_{\lim}$: We prove Condition (3) separately for $T_{\lim} < \tau$, $T_{\lim} = \tau$, and $T_{\lim} > \tau$. For $T_{\lim} < \tau$, using (16) and (22) for $P(T)$, (3) transforms to the condition

$$\frac{b \cdot e^{b \cdot T_{\lim}} \cdot \left(b \cdot \left(e^{T_{\lim} \cdot \lambda_L} - e^{T_{\lim} \cdot \lambda_H}\right) - e^{T_{\lim} \cdot \lambda_L} \cdot \lambda_H + e^{b \cdot T_{\lim}} \cdot (\lambda_H - \lambda_L) + e^{T_{\lim} \cdot \lambda_H} \cdot \lambda_L\right)}{\left(e^{b \cdot T_{\lim}} - e^{T_{\lim} \cdot \lambda_L}\right) \cdot T_{\lim} \cdot \left(b \cdot e^{b \cdot T_{\lim}} - e^{T_{\lim} \cdot \lambda_H} \cdot \lambda_H\right)} > 0.$$

$$(58)$$

Exploiting that the denominator of (58) is positive and using our earlier substitutions $b = \lambda_L - c1$ and $\lambda_H = \lambda_L + c2$ with $c1, c2 > 0$ and $c1 < \lambda_L$, (58) simplifies to

$$e^{T_{\lim} \cdot c2} \cdot c1 + e^{-T_{\lim} \cdot c1} \cdot c2 - c1 - c2 > 0. \qquad (59)$$

We show that this condition holds by again verifying that the left-hand side of (59) equals 0 for $T_{\lim} \to 0$, and its first derivative is strictly positive for $T_{\lim} > 0$.

For $T_{\lim} = \tau$, exactly the same line of arguments as for $T_{\lim} < \tau$, but using (23) instead of (22), proves the assertion.

For $T_{\lim} > \tau$, using (16) and (18), condition (3) evaluates to

$$s(T_{\lim}) \cdot \lambda_H > \frac{\lambda_L \cdot (\Delta_L(T_{\lim}) + s(T_{\lim}))}{1 + \Delta_L(T_{\lim})}, \qquad (60)$$

which always holds due to the clientele effect. To see why, note that due to the clientele

effect (see Equation (24)), high-risk investors are not willing to invest in long-term bonds. Thus, for a fixed T, the price $P(T)$ is lower if $T_{\lim}$ is below T than when $T_{\lim}$ is above T. From that, it directly follows that the integrand in Equation (1) for $\tau < T \leq T_{\lim}$ is larger than the integrand for $\tau < T_{\lim} < T$, which directly implies (60). $\qquad\square$

Illiq$^{\text{bid}}(T)$ is decreasing in T at the short end: We use (2) and (16) to calculate

$$\left(\text{Illiq}^{\text{bid}}(T)\right)' = \frac{T \cdot \left(b + \frac{b \cdot e^{b \cdot T} \cdot (b - \lambda_H)}{e^{T \cdot \lambda_H} \cdot \lambda_H - b \cdot e^{b \cdot T}} + \frac{s'(T)}{1 - s(T)}\right) + \log\left(\frac{(1 - s(T)) \cdot \left(b \cdot e^{-T \cdot \lambda_H} - e^{-b \cdot T} \cdot \lambda_H\right)}{b - \lambda_H}\right)}{T^2}. \quad (61)$$

Plugging in $T = 0$, the numerator of (61) evaluates to $\log(1 - s(0))$, which is strictly negative for $s(0) > 0$. Hence, $\lim_{T \to 0} \left(\text{Illiq}^{\text{bid}}(T)\right)' = -\infty$. $\qquad\square$

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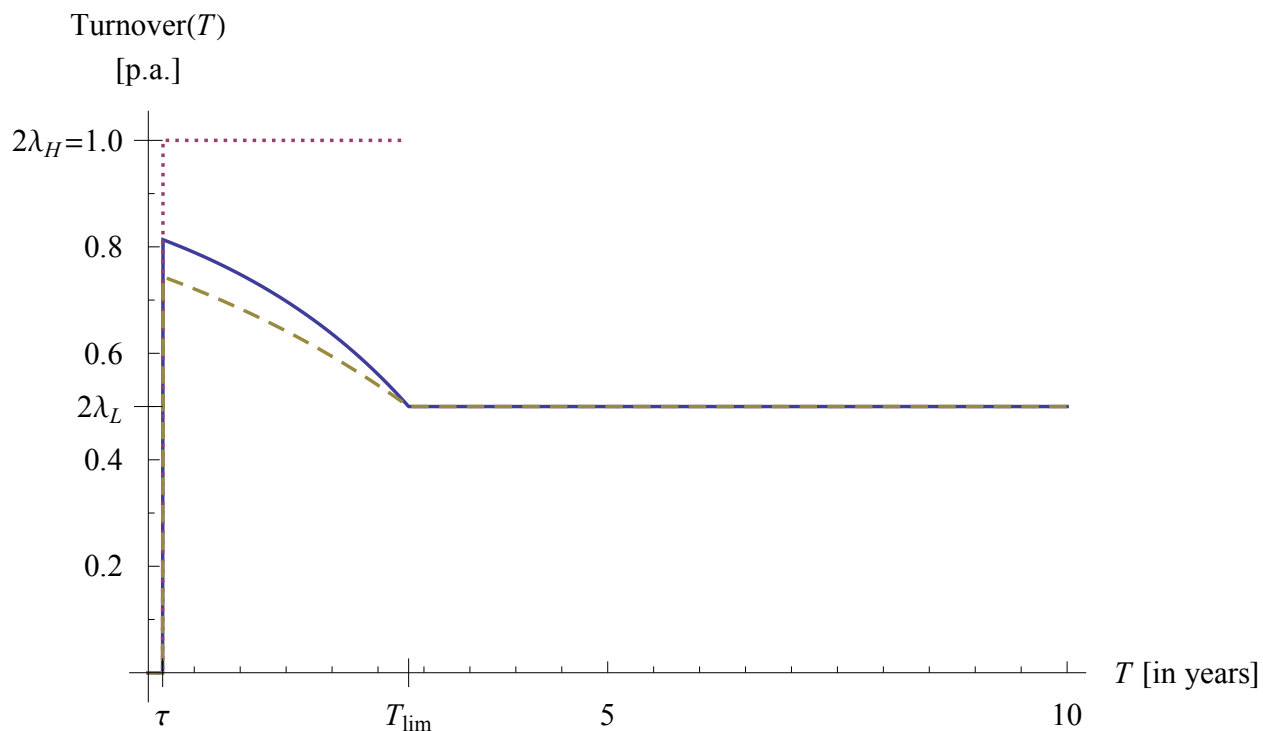
Figure 1: **Turnover – Hump-shaped trading volume and aging effect**
The figure presents secondary-market turnover for the baseline case where the rate λ at which preference shocks occur equals 0.5 for high-risk investors and 0.25 for low-risk investors, the time preference rate increases from 0 to $b = 0.02$ if a preference shock occurs, bid-ask spreads s equal 0.3%, the maximum bond maturity $T_{\max}$ equals 10 years, both investor types have the same aggregate wealth W of 1, and for each initial maturity, bonds are issued with a rate of $a = 0.025$. In the resulting equilibrium allocation, high-risk investors invest in bonds with maturities up to $T_{\lim} = 2.833$ years, and only bonds with a maturity higher than $\tau = 0.156$ years are sold if a preference shock occurs. The dotted line presents turnover of bonds with initial maturity $T_{\text{init}} < T_{\lim}$, the dashed line depicts turnover of bonds with initial maturity $T_{\text{init}} > T_{\lim}$, and the solid line aggregates turnover over all bonds.
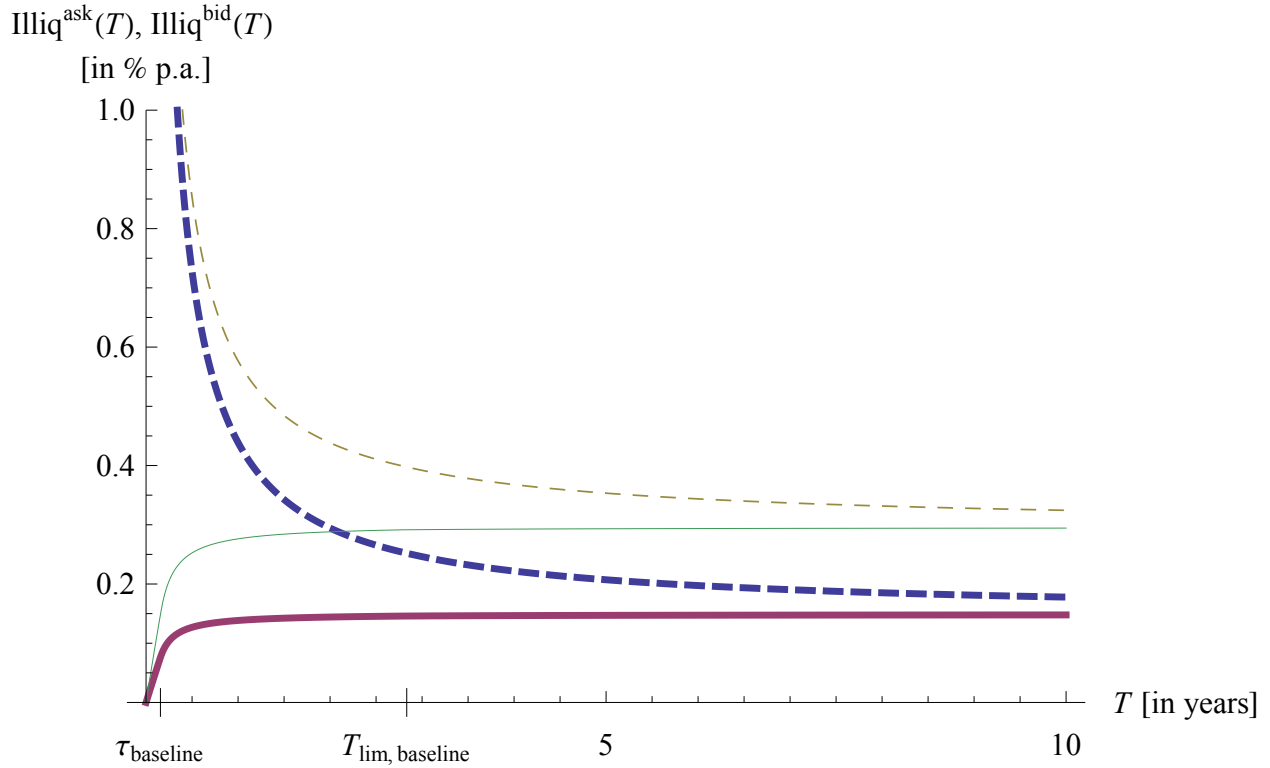
Figure 2: **Liquidity premia – Baseline case and spill-over effects**
The figure presents liquidity premia for the baseline case (thick lines) where the rate λ at which preference shocks occur equals 0.5 for high-risk investors and 0.25 for low-risk investors, the time preference rate increases from 0 to $b = 0.02$ if a preference shock occurs, bid-ask spreads s equal 0.3% for all maturities, the maximum bond maturity $T_{\max}$ equals 10 years, both investor types have the same aggregate wealth W of 1, and for each initial maturity, bonds are issued with a rate of $a = 0.025$. In the resulting equilibrium allocation, high-risk investors invest in bonds with maturities up to $T_{\text{lim, baseline}} = 2.833$ years, and only bonds with a maturity higher than $\tau_{\text{baseline}} = 0.156$ years are sold if a preference shock occurs. Thin lines present liquidity premia for the case of higher liquidity demand for high-risk investors ($\lambda_H = 1.0$). All other parameters are identical to the baseline case. For this specification, critical maturities $\tau_{\lambda_H=1.0} = 0.163$ and $T_{\text{lim, } \lambda_H=1.0} = 2.793$ only change marginally compared to the baseline specification. Solid lines depict $\text{Illiq}^{\text{ask}}(T)$, dashed lines $\text{Illiq}^{\text{bid}}(T)$.
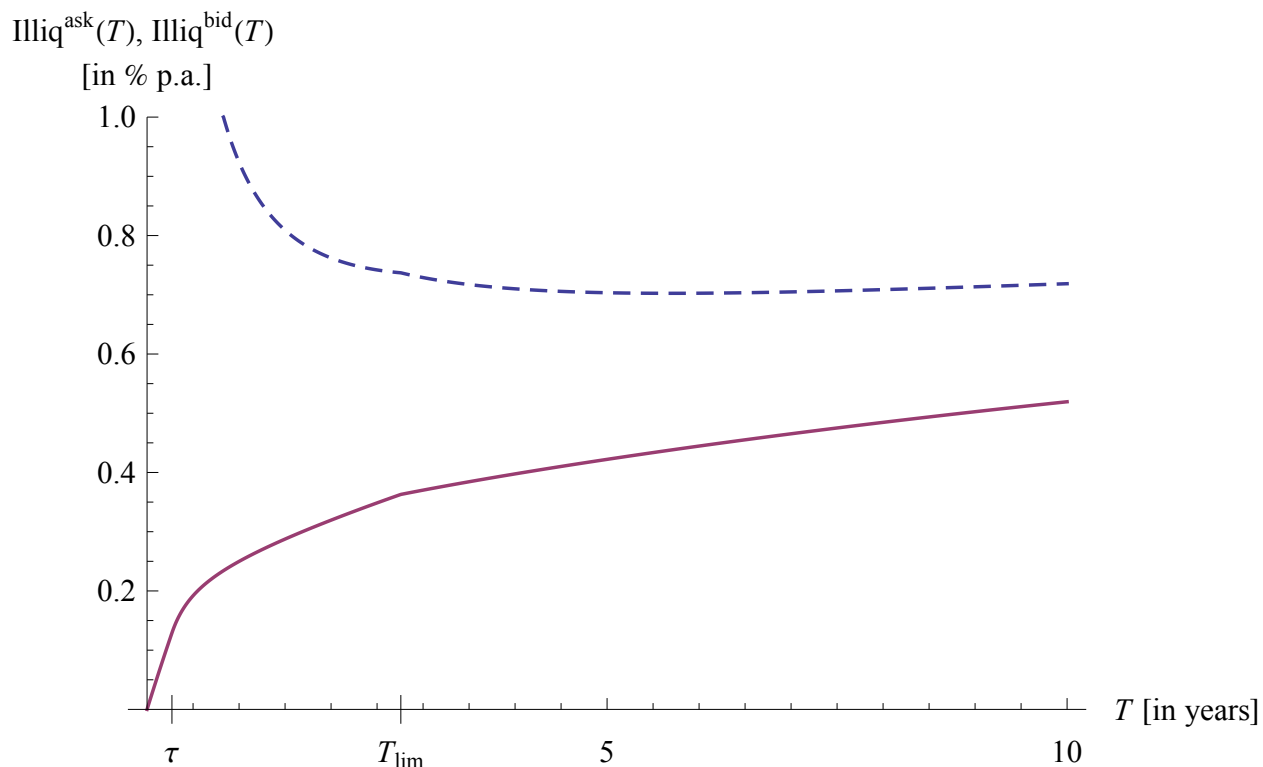
Figure 3: **Liquidity premia – Bond-specific spreads increase in maturity**
The figure presents liquidity premia for the case where we have calibrated bid-ask spreads to observed prices: $s(T) = 0.004396 + 0.024067 \cdot \left(1 - e^{-0.1014 \cdot T}\right)$ (see also Section 5.3 and Figure 4). The rate λ at which preference shocks occur equals 0.5 for high-risk investors and 0.25 for low-risk investors, the time preference rate increases from 0 to $b = 0.02$ if a preference shock occurs, the maximum bond maturity $T_{\max}$ equals 10 years, both investor types have the same aggregate wealth W of 1, and for each initial maturity, bonds are issued with a rate of $a = 0.025$, leading to a total bond supply of 1.25. In the resulting equilibrium allocation, high-risk investors invest in bonds with maturities up to $T_{\lim} = 2.758$ years, and only bonds with a maturity higher than $\tau = 0.270$ years are sold if a preference shock occurs. The solid line depicts $\text{Illiq}^{\text{ask}}(T)$, the dashed line $\text{Illiq}^{\text{bid}}(T)$.
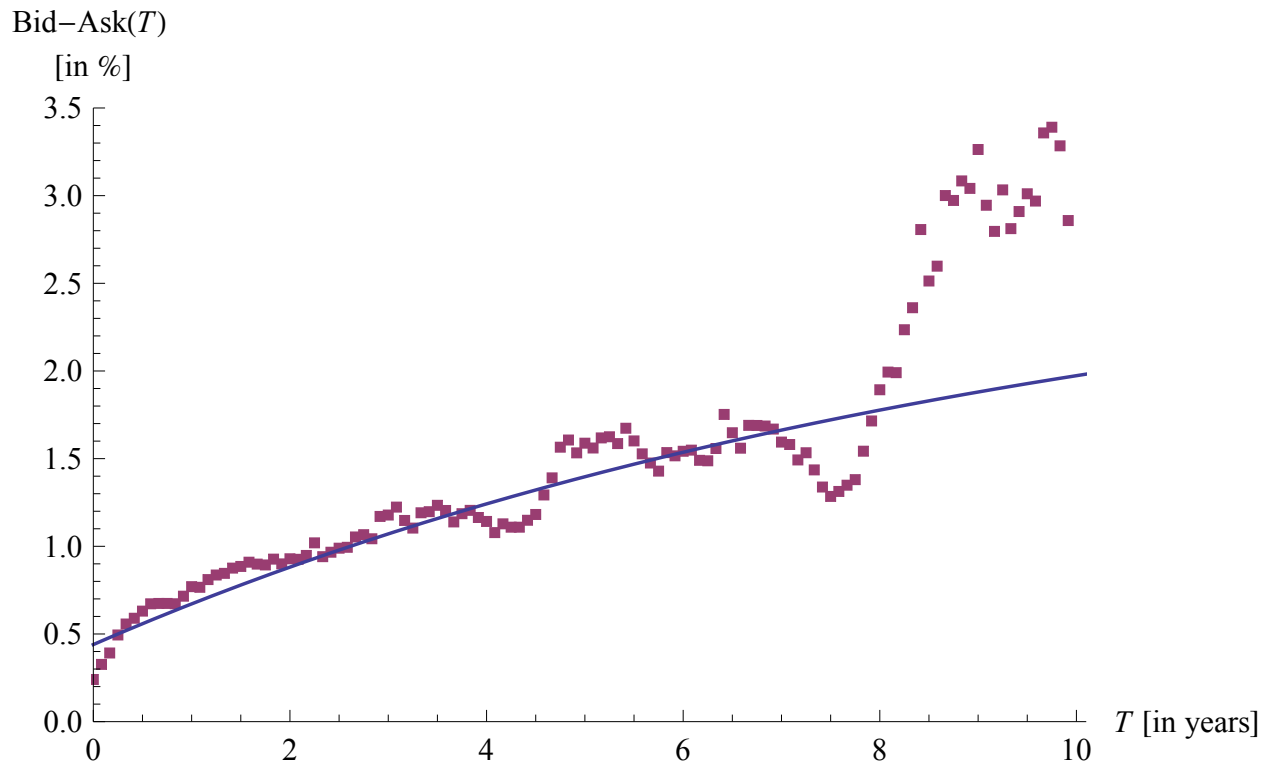
Figure 4: **Empirical term structure of bid-ask spreads**
The figure presents the average term structure of proportional bid-ask spreads (squares) together with the calibrated bid-ask spread function $s(T) = 0.004396 + 0.024067 \cdot \left(1 - e^{-0.1014 \cdot T}\right)$ (solid line). Bid-ask spreads are computed for each bond on days with trades on both sides as the difference between the average bid and ask transaction price. The depicted average spread is computed as the mean spread across all bonds of a given duration. The sample period is from October 1, 2004 to September 30, 2012.
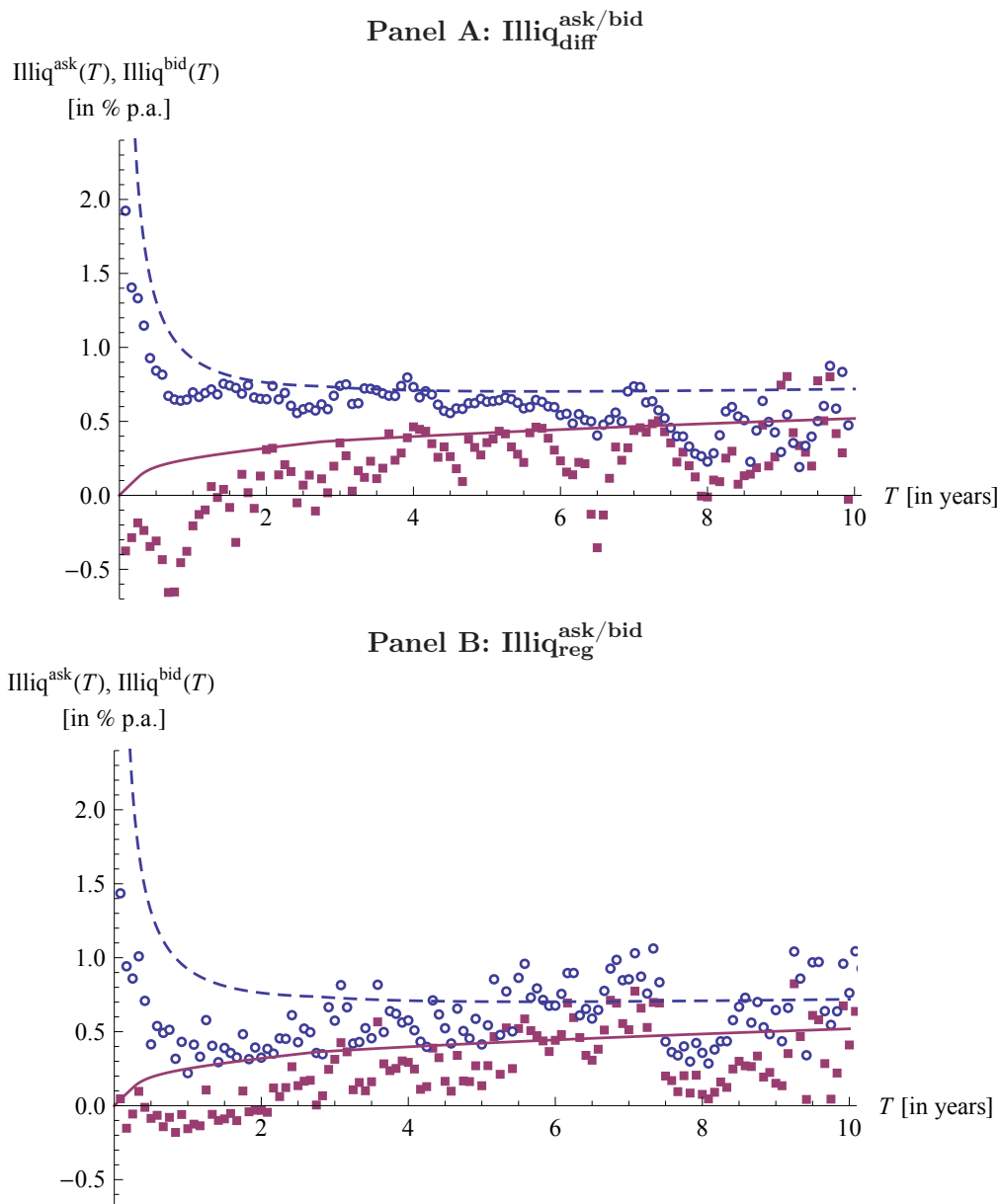
Figure 5: **Empirical term structure of ask and bid liquidity premia**
The figure presents the average term structure of ask and bid liquidity premia together with the predictions of our model (see Figure 3). In Panel A, the liquidity premium $\text{Illiq}_{\text{diff}}^{\text{ask/bid}}$ is determined as the difference of the bond yield and a theoretical credit adjusted yield calculated by discounting the bond's cash flows with a bootstrapped discount curve computed from Treasury yields and a CDS curve. In Panel B, the liquidity premium $\text{Illiq}_{\text{reg}}^{\text{ask/bid}}$ is determined as in Dick-Nielsen, Feldhütter, and Lando (2012) as the yield spread proportion explained by the liquidity measure lm in a linear regression, where lm is the equally weighted average of the Amihud (2002) liquidity measure, imputed roundtrip costs as in Feldhütter (2012), and the standard deviations of these measures. Squares indicate average ask liquidity premia, circles show average bid liquidity premia. The solid line depicts model implied $\text{Illiq}^{\text{ask}}(T)$, the dashed line shows model implied $\text{Illiq}^{\text{bid}}(T)$. The sample period is from October 1, 2004 to September 30, 2012.

Table I: **Sample description**
The table presents the procedure used to arrive at the final samples employed in our main analysis and in the robustness checks in Section 6, the number of trades, the number of bonds, and the traded notional value in billion USD. The sample period is from October 1, 2004 to September 30, 2012.

	Number of trades	Number of bonds	Traded notional value (in bn. USD)
All trade entries within the TRACE database	92,188,862	77,003	86,842
Subtotal after filtering out erroneous and duplicate trade entries with the procedures described in Dick-Nielsen (2009, 2014)	57,806,924	63,829	38,376
Turnover sample (Table III): excluding bonds with missing information (in Bloomberg, Reuters, or Markit), bonds with embedded call or put options (incl. make-whole call provisions, death puts, poison puts, ...), bonds with remaining time to maturity of more than 30 years, bonds with sinking funds, zero coupon bonds, convertible bonds, bonds with variable coupon payments, bonds with other non-standard cash flow or coupon structures, issues which do not have an investment grade rating from at least two rating agencies (i.e., Moody's, S&P, or Fitch) at the trading date, bonds which are not classified as senior unsecured, private placements, bonds with government guarantee, trades on days for which a Treasury curve is not available, trades that could not be matched to CDS data	10,483,321	2,786	4,783
Samples used to calculate liquidity premia: in addition to the turnover sample, we exclude interdealer trades, trades under non-standard terms (e.g., special settlement or sale conditions), trades for which we could not replicate the reported yield from the trade price, and bonds with durations of less than one month			
Main sample (Table II):			
Trades for which dealer is seller (ask)	3,543,343	2,637	1,516
Trades for which dealer is buyer (bid)	1,962,313	2,631	1,479
Swap-implied liquidity premia (Table IV): in contrast to main sample excluding trades on days without an available swap curve (instead of Treasury curve)			
Trades for which dealer is seller (ask)	3,482,571	2,636	1,495
Trades for which dealer is buyer (bid)	1,926,684	2,629	1,461
AAA bonds before financial crisis (Table V): in contrast to main sample not matched to CDS data, only trades until March 31, 2007 for which the bond is rated AAA from at least two rating agencies (i.e., Moody's, S&P, or Fitch) at the trading day			
Trades for which dealer is seller (ask)	116,404	163	49
Trades for which dealer is buyer (bid)	66,449	161	48

Table II: **Regression of ask and bid liquidity premia on duration**

The table presents the regression analysis of ask and bid liquidity premia (in percentage points) on the bond's duration and control variables for different breakpoints that separate the short end from longer maturities of the liquidity term structure:

$$\text{Illiq}^{\text{ask}}_{\text{diff/reg}}(T) = \alpha^a + \beta_1^a \cdot \mathbb{1}_{\{T \le y\}} \cdot (T - y) + \beta_2^a \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \gamma^a \cdot \text{Controls} + \varepsilon,$$

$$\text{Illiq}^{\text{bid}}_{\text{diff/reg}}(T) = \alpha^b + \beta_1^b \cdot \mathbb{1}_{\{T \le y\}} \cdot (T - y) + \beta_2^b \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \gamma^b \cdot \text{Controls} + \varepsilon.$$

In Panel A, the liquidity premium $\text{Illiq}^{\text{ask/bid}}_{\text{diff}}$ is determined as the difference of the bond yield and a theoretical credit adjusted yield calculated by discounting the bond's cash flows with a bootstrapped discount curve computed from Treasury yields and a CDS curve. In Panel B, the liquidity premium $\text{Illiq}^{\text{ask/bid}}_{\text{reg}}$ is determined as in Dick-Nielsen, Feldhütter, and Lando (2012) as the proportion of the yield spread (in excess of the Treasury yield curve) explained by the liquidity measure lm in a linear regression, where lm is the equally weighted average of the Amihud (2002) liquidity measure, imputed roundtrip costs as in Feldhütter (2012), and the standard deviations of these measures. The explanatory variable is the duration T (in years) minus the breakpoint y for $T \le y$ and $T > y$. In Panel A, we additionally include the control variables age in years, the average numerical rating (Rating), and the logarithm of the outstanding amount (log(Amt)) and use firm and month fixed effects. The breakpoints y equal three months, six months, nine months, one year, two years, and three years. In Panel A, we present standard errors clustered at the firm level in parentheses. In Panel B, we use White (1982) standard errors. The sample period is from October 1, 2004 to September 30, 2012. *, ** indicate significance at the 5% or 1% level.

	Ask						Bid					
	$y = 0.25$	$y = 0.5$	$y = 0.75$	$y = 1$	$y = 2$	$y = 3$	$y = 0.25$	$y = 0.5$	$y = 0.75$	$y = 1$	$y = 2$	$y = 3$
						Panel A: Liquidity premium $\text{Illiq}^{\text{ask/bid}}_{\text{diff}}$						
Constant	0.1875	0.1978	0.2024	0.1840	0.2911	0.5952	0.0539	−0.0302	−0.0593	−0.0719	−0.0768	−0.0585
	(0.7548)	(0.7530)	(0.7522)	(0.7519)	(0.7266)	(0.6980)	(0.7032)	(0.7017)	(0.6978)	(0.6949)	(0.6863)	(0.6768)
$\mathbb{1}_{\{T \le y\}} \cdot (T - y)$	3.3402**	1.4416**	1.0140**	0.9013**	0.4800**	0.2968**	−8.7631**	−2.5747**	−1.2421**	−0.7039**	−0.1674**	−0.0735*
	(1.2911)	(0.4988)	(0.2840)	(0.2050)	(0.0963)	(0.0535)	(0.6059)	(0.2526)	(0.1496)	(0.1080)	(0.0502)	(0.0315)
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$	0.0573**	0.0550**	0.0518**	0.0464**	0.0273*	0.0142	−0.0030	0.0016	0.0042	0.0053	0.0062	0.0061
	(0.0110)	(0.0112)	(0.0113)	(0.0113)	(0.0129)	(0.0147)	(0.0094)	(0.0092)	(0.0092)	(0.0093)	(0.0103)	(0.0115)
						Controls						
Age [in years]	0.0004	0.0015	0.0031	0.0056	0.0125	0.0149	0.0284**	0.0266**	0.0255**	0.0251**	0.0252**	0.0260**
	(0.0070)	(0.0070)	(0.0070)	(0.0070)	(0.0077)	(0.0081)	(0.0057)	(0.0055)	(0.0053)	(0.0053)	(0.0051)	(0.0050)
Rating	−0.0139	−0.0118	−0.0096	−0.0069	0.0010	0.0014	0.1665**	0.1657**	0.1657**	0.1665**	0.1674**	0.1676**
	(0.0487)	(0.0484)	(0.0483)	(0.0481)	(0.0472)	(0.0470)	(0.0542)	(0.0538)	(0.0537)	(0.0537)	(0.0539)	(0.0542)
log(Amt)	0.0586	0.0584	0.0581	0.0591	0.0584	0.0515	−0.0429*	−0.0419	−0.0414	−0.0420	−0.0436*	−0.0433*
	(0.0321)	(0.0318)	(0.0316)	(0.0316)	(0.0308)	(0.0295)	(0.0217)	(0.0221)	(0.0221)	(0.0221)	(0.0219)	(0.0214)
Firm Fixed Effects						Yes						
Month Fixed Effects						Yes						
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$ $-\mathbb{1}_{\{T \le y\}} \cdot (T - y)$	−3.2830*	−1.3866**	−0.9621**	−0.8549**	−0.4527**	−0.2826**	8.7601**	2.5763**	1.2462**	0.7092**	0.1736**	0.0797*
	(1.2943)	(0.5030)	(0.2886)	(0.2097)	(0.1042)	(0.0639)	(0.6088)	(0.2546)	(0.1515)	(0.1101)	(0.0544)	(0.0371)
N			3,543,343						1,962,313			
R^2	0.3934	0.3946	0.3965	0.4003	0.4094	0.4099	0.3877	0.3907	0.3896	0.3872	0.3815	0.3796

	Ask						Bid					
	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$
	Panel B: Liquidity premium Illiq$_{\mathbf{reg}}^{\mathbf{ask/bid}}$											
Constant	0.1251**	0.1379**	0.1517**	0.1673**	0.2344**	0.2963**	0.5736**	0.5599**	0.5567**	0.5584**	0.5886**	0.6277**
	(0.0300)	(0.0303)	(0.0304)	(0.0301)	(0.0281)	(0.0270)	(0.0315)	(0.0300)	(0.0305)	(0.0309)	(0.0312)	(0.0311)
$\mathbb{1}_{\{T\leq y\}}\cdot(T-y)$	1.0438	0.5310*	0.3875**	0.3306**	0.2268**	0.1720**	−5.0205**	−1.7129**	−0.8167**	−0.4475**	−0.0390	0.0320
	(0.6812)	(0.2082)	(0.1121)	(0.0793)	(0.0339)	(0.0193)	(0.2827)	(0.2859)	(0.1970)	(0.1532)	(0.0718)	(0.0414)
$\mathbb{1}_{\{T> y\}}\cdot(T-y)$	0.0254**	0.0248**	0.0242**	0.0233**	0.0192**	0.0149**	0.0063	0.0075	0.0080*	0.0080	0.0058	0.0024
	(0.0035)	(0.0036)	(0.0037)	(0.0037)	(0.0038)	(0.0040)	(0.0039)	(0.0039)	(0.0040)	(0.0041)	(0.0044)	(0.0047)
$\mathbb{1}_{\{T> y\}}\cdot(T-y)$	−1.0185	−0.5062*	−0.3633**	−0.3072**	−0.2075**	−0.1571**	5.0268**	1.7205**	0.8247**	0.4554**	0.0449	−0.0296
$-\mathbb{1}_{\{T\leq y\}}\cdot(T-y)$	(0.6822)	(0.2098)	(0.1139)	(0.0812)	(0.0362)	(0.0220)	(0.2854)	(0.2871)	(0.1983)	(0.1545)	(0.0736)	(0.0438)
N	221						221					
R^2	0.2649	0.2684	0.2732	0.2806	0.3211	0.3583	0.0678	0.0818	0.0684	0.0520	0.0117	0.0124

Table III: **Regression of turnover on duration and age**

The table presents the regression analysis of turnover for the two subsamples on duration, age, and control variables for different breakpoints:

$$\text{Turnover}(T) = \alpha + \beta_1 \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta_2 \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \beta_3 \cdot \text{Age} + \gamma \cdot \text{Controls} + \varepsilon,$$

where $\text{Turnover}(T)$ is calculated as the average daily turnover for each bond and each calendar month. The left panel contains the regression results for the full sample, the right panel contains the regression results for the subsample that excludes bonds two months prior to and six months after changes in their outstanding amount. The explanatory variables are the bond's duration T (in years) minus the breakpoint y for $T \leq y$ and $T > y$ as well as age (in years). The control variables are the average numerical rating (Rating) and the logarithm of the outstanding amount (log(Amt)). The breakpoints y are given by three months, six months, nine months, one year, two years, and three years. We use month fixed effects. Clustered standard errors at the firm level are presented in parentheses. Parameter estimates and standard errors are multiplied by 1,000. The sample period is from October 1, 2004 to September 30, 2012. *, ** indicate significance at the 5% or 1% level.

	All						Excl[-2, +6]					
	$y = 0.25$	$y = 0.5$	$y = 0.75$	$y = 1$	$y = 2$	$y = 3$	$y = 0.25$	$y = 0.5$	$y = 0.75$	$y = 1$	$y = 2$	$y = 3$
Constant	−4.6270**	−4.5999**	−4.5866**	−4.5912**	−4.6515**	−4.6348**	−4.8264**	−4.8117**	−4.8104**	−4.8276**	−4.9551**	−5.0349**
	(0.8140)	(0.8085)	(0.8056)	(0.8066)	(0.8203)	(0.8133)	(0.8894)	(0.8833)	(0.8802)	(0.8821)	(0.9077)	(0.9179)
$\mathbb{1}_{\{T \leq y\}} \cdot (T - y)$	5.7766**	2.1746**	1.1631**	0.6521**	0.0853	0.0602	5.6439**	2.0882**	1.0883**	0.5754**	−0.0130	−0.0569
	(0.4585)	(0.2151)	(0.1452)	(0.1077)	(0.0505)	(0.0309)	(0.4470)	(0.2056)	(0.1369)	(0.1006)	(0.0485)	(0.0310)
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$	−0.0214	−0.0263	−0.0292*	−0.0287	−0.0198	−0.0263	−0.0671**	−0.0720**	−0.0747**	−0.0738**	−0.0595**	−0.0542**
	(0.0133)	(0.0137)	(0.0141)	(0.0146)	(0.0167)	(0.0182)	(0.0090)	(0.0095)	(0.0099)	(0.0102)	(0.0113)	(0.0127)
						Controls						
Age [in years]	−0.2040**	−0.2031**	−0.2027**	−0.2029**	−0.2048**	−0.2043**	−0.1318**	−0.1309**	−0.1305**	−0.1308**	−0.1335**	−0.1343**
	(0.0169)	(0.0168)	(0.0168)	(0.0168)	(0.0171)	(0.0171)	(0.0119)	(0.0117)	(0.0116)	(0.0115)	(0.0118)	(0.0120)
Rating	0.0424	0.0425	0.0425	0.0425	0.0423	0.0425	0.0634*	0.0635*	0.0635*	0.0634*	0.0630*	0.0629*
	(0.0280)	(0.0280)	(0.0280)	(0.0280)	(0.0281)	(0.0281)	(0.0248)	(0.0248)	(0.0248)	(0.0248)	(0.0249)	(0.0249)
log(Amt)	0.4312**	0.4310**	0.4308**	0.4306**	0.4306**	0.4304**	0.4011**	0.4009**	0.4007**	0.4005**	0.4004**	0.4005**
	(0.0333)	(0.0330)	(0.0328)	(0.0328)	(0.0335)	(0.0332)	(0.0383)	(0.0379)	(0.0377)	(0.0378)	(0.0389)	(0.0392)
Firm Fixed Effects						No						
Month Fixed Effects						Yes						
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$	−5.7980**	−2.2009**	−1.1922**	−0.6808**	−0.1051	−0.0865*	−5.7110**	−2.1602**	−1.1630**	−0.6492**	−0.0465	0.0027
$-\mathbb{1}_{\{T \leq y\}} \cdot (T - y)$	(0.4630)	(0.2210)	(0.1520)	(0.1155)	(0.0614)	(0.0425)	(0.4508)	(0.2103)	(0.1421)	(0.1063)	(0.0554)	(0.0394)
N			102,304						96,458			
R^2	0.1548	0.1548	0.1545	0.1539	0.1527	0.1528	0.1482	0.1482	0.1477	0.1466	0.1447	0.1446

Table IV: **Regression of swap-implied ask and bid liquidity premia on duration**

The table presents the regression analysis of swap-implied ask and bid liquidity premia (in percentage points) on the bond's duration and control variables for different breakpoints that separate the short end from longer maturities of the liquidity term structure:

$$\text{Illiq}^{\text{ask}}_{\text{diff/reg}}(T) = \alpha^a + \beta_1^a \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta_2^a \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \gamma^a \cdot \text{Controls} + \varepsilon,$$

$$\text{Illiq}^{\text{bid}}_{\text{diff/reg}}(T) = \alpha^b + \beta_1^b \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta_2^b \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \gamma^b \cdot \text{Controls} + \varepsilon.$$

In Panel A, the liquidity premium $\text{Illiq}^{\text{ask/bid}}_{\text{diff}}$ is determined as the difference of the bond yield and a theoretical credit adjusted yield calculated by discounting the bond's cash flows with a bootstrapped discount curve computed from swap rates and a CDS curve. In Panel B, the liquidity premium $\text{Illiq}^{\text{ask/bid}}_{\text{reg}}$ is determined as in Dick-Nielsen, Feldhütter, and Lando (2012) as the proportion of the yield spread (in excess of the swap curve) explained by the liquidity measure lm in a linear regression, where lm is the equally weighted average of the Amihud (2002) liquidity measure, imputed roundtrip costs as in Feldhütter (2012), and the standard deviations of these measures. The explanatory variable is the duration T (in years) minus the breakpoint y for $T \leq y$ and $T > y$. In Panel B, we additionally include the control variables age in years, the average numerical rating (Rating), and the logarithm of the outstanding amount (log(Amt)) and use firm and month fixed effects. The breakpoints y equal three months, six months, nine months, one year, two years, and three years. In Panel A, we present standard errors clustered at the firm level in parentheses. In Panel B, we use White (1982) standard errors. The sample period is from October 1, 2004 to September 30, 2012. *, ** indicate significance at the 5% or 1% level.

	Ask						Bid					
	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$
	Panel A: Liquidity premium $\text{Illiq}^{\text{ask/bid}}_{\text{diff}}$											
Constant	−0.2994	−0.2774	−0.2580	−0.2551	−0.1169	0.1526	−0.0619	−0.1589	−0.1950	−0.2164	−0.2546	−0.2369
	(0.7641)	(0.7634)	(0.7637)	(0.7644)	(0.7430)	(0.7129)	(0.7418)	(0.7396)	(0.7343)	(0.7307)	(0.7185)	(0.6990)
$\mathbb{1}_{\{T \leq y\}} \cdot (T - y)$	1.1274	0.8630	0.6683*	0.6589**	0.3906**	0.2622**	−10.4996**	−3.1447**	−1.5796**	−0.9406**	−0.2653**	−0.1229**
	(1.3854)	(0.5602)	(0.3157)	(0.2252)	(0.1038)	(0.0569)	(0.6294)	(0.2641)	(0.1560)	(0.1126)	(0.0528)	(0.0334)
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$	0.1034**	0.1019**	0.0999**	0.0958**	0.0825**	0.0743**	0.0314**	0.0371**	0.0411**	0.0437**	0.0512**	0.0573**
	(0.0120)	(0.0124)	(0.0128)	(0.0131)	(0.0155)	(0.0177)	(0.0108)	(0.0105)	(0.0104)	(0.0103)	(0.0111)	(0.0122)
	Controls											
Age [in years]	−0.0001	0.0006	0.0017	0.0035	0.0083	0.0098	0.0307**	0.0284**	0.0267**	0.0257**	0.0235**	0.0231**
	(0.0078)	(0.0080)	(0.0081)	(0.0083)	(0.0092)	(0.0097)	(0.0060)	(0.0057)	(0.0055)	(0.0054)	(0.0051)	(0.0051)
Rating	−0.0091	−0.0078	−0.0064	−0.0043	0.0012	0.0012	0.1627**	0.1617**	0.1616**	0.1626**	0.1637**	0.1644**
	(0.0497)	(0.0494)	(0.0493)	(0.0491)	(0.0482)	(0.0481)	(0.0545)	(0.0540)	(0.0538)	(0.0537)	(0.0538)	(0.0540)
log(Amt)	0.0682*	0.0681*	0.0679*	0.0684*	0.0679*	0.0633*	−0.0420	−0.0407	−0.0400	−0.0405	−0.0423	−0.0414
	(0.0319)	(0.0318)	(0.0316)	(0.0316)	(0.0308)	(0.0294)	(0.0233)	(0.0237)	(0.0238)	(0.0239)	(0.0240)	(0.0234)
Firm Fixed Effects						Yes						
Month Fixed Effects						Yes						
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$ $-\mathbb{1}_{\{T \leq y\}} \cdot (T - y)$	−1.0240	−0.7611	−0.5684	−0.5631*	−0.3081**	−0.1878**	10.5310**	3.1818**	1.6207**	0.9843**	0.3165**	0.1802**
	(1.3907)	(0.5663)	(0.3222)	(0.2319)	(0.1142)	(0.0697)	(0.6335)	(0.2667)	(0.1581)	(0.1148)	(0.0572)	(0.0390)
N	3,482,571						1,926,684					
R^2	0.4040	0.4045	0.4052	0.4071	0.4114	0.4112	0.3375	0.3434	0.3437	0.3417	0.3358	0.3328

	Ask						Bid					
	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$
						Panel B: Liquidity premium Illiq$_{\mathbf{reg}}^{\mathbf{ask/bid}}$						
Constant	−0.0228	−0.0080	0.0073	0.0245	0.0993**	0.1730**	0.4215**	0.4105**	0.4090**	0.4126**	0.4512**	0.5018**
	(0.0334)	(0.0337)	(0.0339)	(0.0339)	(0.0338)	(0.0341)	(0.0370)	(0.0350)	(0.0351)	(0.0355)	(0.0365)	(0.0374)
$\mathbb{1}_{\{T\leq y\}}\cdot(T-y)$	0.1322	0.1535	0.1434	0.1513*	0.1463**	0.1278**	−6.0137**	−2.0990**	−1.0647**	−0.6285**	−0.1195	−0.0131
	(0.5288)	(0.1881)	(0.1031)	(0.0722)	(0.0333)	(0.0208)	(0.2664)	(0.2756)	(0.1864)	(0.1436)	(0.0689)	(0.0410)
$\mathbb{1}_{\{T> y\}}\cdot(T-y)$	0.0517**	0.0516**	0.0513**	0.0510**	0.0488**	0.0463**	0.0332**	0.0348**	0.0356**	0.0360**	0.0357**	0.0342**
	(0.0049)	(0.0050)	(0.0051)	(0.0052)	(0.0056)	(0.0061)	(0.0053)	(0.0053)	(0.0054)	(0.0056)	(0.0061)	(0.0066)
$\mathbb{1}_{\{T> y\}}\cdot(T-y)$ $-\mathbb{1}_{\{T\leq y\}}\cdot(T-y)$	−0.0805	−0.1019	−0.0920	−0.1003	−0.0975**	−0.0815**	6.0468**	2.1338**	1.1003**	0.6646**	0.1553*	0.0472
	(0.5308)	(0.1908)	(0.1064)	(0.0758)	(0.0374)	(0.0256)	(0.2712)	(0.2776)	(0.1885)	(0.1457)	(0.0716)	(0.0446)
N	225						225					
R^2	0.4601	0.4602	0.4604	0.4609	0.4654	0.4708	0.2415	0.2537	0.2510	0.2446	0.2190	0.2069

Table V: **Regression of ask and bid liquidity premia on duration for AAA rated bonds**

The table presents the regression analysis of ask and bid liquidity premia (in percentage points) on the bond's duration and control variables for different breakpoints that separate the short end from longer maturities of the liquidity term structure:

$$\text{Illiq}^{\text{ask}}_{\text{diff/reg}}(T) = \alpha^a + \beta^a_1 \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta^a_2 \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \gamma^a \cdot \text{Controls} + \varepsilon,$$

$$\text{Illiq}^{\text{bid}}_{\text{diff/reg}}(T) = \alpha^b + \beta^b_1 \cdot \mathbb{1}_{\{T \leq y\}} \cdot (T - y) + \beta^b_2 \cdot \mathbb{1}_{\{T > y\}} \cdot (T - y) + \gamma^b \cdot \text{Controls} + \varepsilon,$$

In Panel A, the liquidity premium $\text{Illiq}^{\text{ask/bid}}_{\text{diff}}$ is determined as the difference of the AAA-rated bond's yield and a theoretical risk free yield calculated by discounting the bond's cash flows with the Treasury curve. In Panel B, the liquidity premium $\text{Illiq}^{\text{ask/bid}}_{\text{reg}}$ is determined as in Dick-Nielsen, Feldhütter, and Lando (2012) as the proportion of the yield spread of a AAA-rated bond (in excess of the Treasury curve) explained by the liquidity measure lm in a linear regression, where lm is the equally weighted average of the Amihud (2002) liquidity measure, imputed roundtrip costs as in Feldhütter (2012), and the standard deviations of these measures (we exclude CDS quotes and ratings in the first-step regression, see Equation (4), for the AAA analysis). The explanatory variable is the duration T (in years) minus the breakpoint y for $T \leq y$ and $T > y$. In Panel B, we additionally include the control variables age in years and the logarithm of the outstanding amount ($\log(\text{Amt})$) and use firm and month fixed effects. The breakpoints y equal three months, six months, nine months, one year, two years, and three years. In Panel A, we present standard errors clustered at the firm level in parentheses. In Panel B, we use White (1982) standard errors. The sample period is from October 1, 2004 to September 30, 2012. *, ** indicate significance at the 5% or 1% level.

	Ask						Bid					
	$y = 0.25$	$y = 0.5$	$y = 0.75$	$y = 1$	$y = 2$	$y = 3$	$y = 0.25$	$y = 0.5$	$y = 0.75$	$y = 1$	$y = 2$	$y = 3$
						Panel A: Liquidity premium $\text{Illiq}^{\text{ask/bid}}_{\text{diff}}$						
Constant	−0.4896	−0.4504	−0.4093	−0.3780	−0.3220	−0.2244	0.2211	0.1850	0.1656	0.1603	0.2279	0.2544*
	(0.2540)	(0.2497)	(0.2431)	(0.2360)	(0.2017)	(0.1778)	(0.1337)	(0.1329)	(0.1315)	(0.1338)	(0.1265)	(0.1198)
$\mathbb{1}_{\{T \leq y\}} \cdot (T - y)$	0.7061**	0.4063**	0.3220**	0.2654**	0.1695**	0.1328**	−3.2910**	−1.0781**	−0.5755**	−0.3528**	−0.0701**	−0.0090
	(0.1686)	(0.0644)	(0.0407)	(0.0296)	(0.0124)	(0.0076)	(0.0741)	(0.0259)	(0.0180)	(0.0163)	(0.0114)	(0.0084)
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$	0.0626**	0.0615**	0.0596**	0.0574**	0.0462**	0.0326**	0.0262**	0.0305**	0.0338**	0.0362**	0.0392**	0.0373**
	(0.0113)	(0.0113)	(0.0112)	(0.0110)	(0.0098)	(0.0071)	(0.0058)	(0.0063)	(0.0068)	(0.0074)	(0.0095)	(0.0111)
						Controls						
Age [in years]	0.0076*	0.0083*	0.0092**	0.0100**	0.0113**	0.0108**	0.0328**	0.0303**	0.0291**	0.0288**	0.0314**	0.0337**
	(0.0033)	(0.0033)	(0.0033)	(0.0032)	(0.0028)	(0.0021)	(0.0024)	(0.0020)	(0.0019)	(0.0020)	(0.0028)	(0.0033)
$\log(\text{Amt})$	0.0130*	0.0119*	0.0108*	0.0106*	0.0142**	0.0166**	−0.0152**	−0.0137**	−0.0128**	−0.0131**	−0.0178**	−0.0182**
	(0.0050)	(0.0050)	(0.0050)	(0.0050)	(0.0048)	(0.0058)	(0.0038)	(0.0036)	(0.0035)	(0.0034)	(0.0029)	(0.0030)
Firm Fixed Effects						Yes						
Month Fixed Effects						Yes						
$\mathbb{1}_{\{T > y\}} \cdot (T - y)$	−0.6434**	−0.3449**	−0.2624**	−0.2080**	−0.1233**	−0.1002**	3.3172**	1.1086**	0.6093**	0.3890**	0.1093**	0.0463**
$-\mathbb{1}_{\{T \leq y\}} \cdot (T - y)$	(0.1761)	(0.0714)	(0.0469)	(0.0357)	(0.0172)	(0.0094)	(0.0776)	(0.0309)	(0.0214)	(0.0193)	(0.0158)	(0.0141)
N			116,404						66,449			
R^2	0.4443	0.4472	0.4523	0.4574	0.4787	0.4943	0.2003	0.2156	0.2208	0.2191	0.1964	0.1837

	Ask						Bid					
	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$	$y=0.25$	$y=0.5$	$y=0.75$	$y=1$	$y=2$	$y=3$
	Panel B: Liquidity premium Illiq$_{\mathbf{reg}}^{\mathbf{ask/bid}}$											
Constant	-0.1128^{**}	-0.1060^{**}	-0.0973^{**}	-0.0877^{**}	-0.0440^{**}	0.0006	0.1780^{**}	0.1693^{**}	0.1657^{**}	0.1640^{**}	0.1728^{**}	0.1907^{**}
	(0.0177)	(0.0181)	(0.0178)	(0.0178)	(0.0156)	(0.0128)	(0.0163)	(0.0146)	(0.0139)	(0.0138)	(0.0142)	(0.0137)
$\mathbb{1}_{\{T\leq y\}}\cdot(T-y)$	0.5345	0.2164^{*}	0.1717^{**}	0.1512^{**}	0.1166^{**}	0.0974^{**}	-2.7170^{**}	-0.9499^{**}	-0.4920^{**}	-0.3059^{**}	-0.0735^{*}	-0.0198
	(0.2772)	(0.0871)	(0.0582)	(0.0456)	(0.0204)	(0.0117)	(0.1653)	(0.1626)	(0.1109)	(0.0796)	(0.0326)	(0.0190)
$\mathbb{1}_{\{T>y\}}\cdot(T-y)$	0.0171^{**}	0.0168^{**}	0.0163^{**}	0.0157^{**}	0.0122^{**}	0.0078^{*}	0.0092^{**}	0.0104^{**}	0.0111^{**}	0.0116^{**}	0.0118^{**}	0.0109^{**}
	(0.0029)	(0.0030)	(0.0030)	(0.0030)	(0.0031)	(0.0032)	(0.0031)	(0.0031)	(0.0031)	(0.0032)	(0.0035)	(0.0038)
$\mathbb{1}_{\{T>y\}}\cdot(T-y)$	-0.5174	-0.1996^{*}	-0.1554^{*}	-0.1356^{**}	-0.1044^{**}	-0.0896^{**}	2.7262^{**}	0.9603^{**}	0.5031^{**}	0.3175^{**}	0.0853^{*}	0.0306
$-\mathbb{1}_{\{T\leq y\}}\cdot(T-y)$	(0.2783)	(0.0887)	(0.0597)	(0.0471)	(0.0219)	(0.0133)	(0.1671)	(0.1634)	(0.1117)	(0.0805)	(0.0339)	(0.0206)
N	225						225					
R^2	0.2315	0.2328	0.2369	0.2430	0.2851	0.3400	0.1177	0.1383	0.1351	0.1294	0.0875	0.0611