Competition, Markups and Predictable Returns

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Imperfect competition is a significant source of time-varying risk premia in asset markets. We embed a structural IO setup of imperfectly competitive industries into a general equilibrium production-based asset pricing model with recursive preferences. Movements in profit opportunities affect business formation and industry composition, and hence firms' competitive environment. We find that endogenous variation in industry concentration and competitive pressure amplifies and propagates macroeconomic risk asymmetrically. Endogenous countercyclical markups slow down demand and recoveries during downturns while procyclical profits render financial assets risky. The model predicts a sizeable and endogenously countercyclical equity premium which is forecastable with measures of markups and the intensity of new firm creation (entry), and a U-shaped term structure of equity returns. We find strong empirical support for these predictions in the data.

Keywords: Imperfect competition, markups, entry and exit, productivity, business cycle propagation, asset pricing, predictability, recursive preferences

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1 Introduction

Economists have long argued that the creation of new businesses is an important engine of growth. In fact, careful measurement reveals that the vast majority of productivity growth occurs as new establishments enter product markets (for recent evidence, see, e.g. Gourio, Messer and Siemer (2014)). The flipside of entry is that old establishments face increased competitive pressure that may eventually drive them out of business. Going back at least to Schumpeter, economists have labeled this process as 'creative destruction'. One striking stylized fact about the intensity of new business creation is that it is highly procyclical [1]. While procyclical entry likely reflects higher profit expectations in expansions, it also suggests that industry concentration and competitive pressure should adjust accordingly. Indeed, a long list of contributions documents empirically that markups are countercyclical[2] and that the degree of competitiveness in industries is strongly procyclical [3].

In this paper, we argue theoretically and empirically that cyclical movements in firm entry and the ensuing variation in competitive pressure not only are important drivers of macroeconomic fluctuations and growth, but also are qualitatively relevant and quantitatively significant determinants of risk premia in asset markets. Critically, we document that these movements give rise to an endogenously countercyclical, and thus predictable equity premium, as well as a U-shaped term structure of equity returns. Empirically, we show that measures of entry and time varying competitive pressure, such as markups, profit shares, or the intensity of net business formation, forecast aggregate stock returns over longer horizons.

To link firm competition and asset returns, we build a general equilibrium asset pricing model with imperfect competition and endogenous productivity growth. There are two sources of productivity growth in the model: Endogenous innovation impacts productivity growth because new firms enter, *product innovation* in the sense of Atkeson and Burstein (2014), and because incumbent firms invest in upgrading their own production technology, that is *process innovation*. Process innovation provides a powerful propagation mechanism for exogenous shocks. By generating endogenous persistence, this channel gives rise to significant low-frequency growth cycles akin to endogenous long-run risk as in Kung (2014) and Kung and Schmid (2014).

Product innovation, or the entry and exit of firms, on the other hand, implies a novel powerful amplification mechanism for shocks. Given its cyclical nature, this channel provides a significant

[1] See, e.g., Cooper and Chatterjee (1993), Portier (1995), Devereux, Head and Lapham (1996), Floetotto and Jaimovich (2008), Bilbiie, Ghironi and Melitz (2012)

[2] see, e.g., Bils (1987), Rotemberg and Woodford (1991, 1999), Chevalier, Kashyap and Rossi (2003)

[3] Bresnahan and Reiss (1991), Campbell and Hopenhayn (2005)

source of short-run risk. Critically, we show that this mechanism generates endogenous countercyclical macroeconomic volatility that implies excess stock return predictability. In economic downturns, profits fall, firms exit and industry concentration rises. As a consequence, surviving producers enjoy elevated market power and face steeper demand curves. While this allows firms to charge higher markups in our model, it also makes them more sensitive to aggregate shocks. Endogenous countercyclical markups thus come with asymmetric industry cycles, which are reflected in countercyclical aggregate volatility. In our model, agents have recursive preferences, so that they require a countercyclical risk premium as compensation for this elevated exposure.

Importantly, in the model, long and short-run risks are correlated. This is because countercyclical markups also amplify firms' investment in technology, as incumbent firms respond to changing entry threats. In bad times, markups are higher due to higher industry concentration which further depresses demand for goods. Lower demand reduces production and technology investment by firms which lowers expected growth rates. In good times, falling industry concentration reduces markups which further increases demand for goods. Rising demand raises production and innovation further, and boosts expected growth. This growth amplification mechanism not only raises aggregate growth risk, but also renders growth cycles and long-run risk endogenously heteroskedastic.

The calibrated model generates an equity premium of more than 5% on an annual basis, while simultaneously fitting basic data on aggregate and industry cycles. The model thus generates significant risks priced in asset markets. These two sources of risk are both tightly linked to movements in industry innovation and are reflected in endogenous dynamics of productivity growth. In line with earlier work by Kung and Schmid (2014), and Kung (2014), process innovation by means of R&D provides a source of endogenous low-frequency movements in productivity growth, in the sense of Bansal and Yaron (2004) and Croce (2014). Such endogenous long-run productivity risks operate at lower than business cycle frequency and generate long-run growth cycles, for which agents with recursive preferences require substantial compensation. Accounting for product innovation through entry, the ensuing variation in competitive pressure creates an additional, novel source of cyclical risk, or short-run risks priced in asset markets. Given its cyclical nature, agents require lower average compensation for exposure to such risk. However, the required compensation is time-varying as the corresponding fluctuations are endogenously heteroskedastic. Accordingly, the premium for this exposure is predictable. Since incumbent firms respond to impending product innovation by engaging in process innovation, short and long-run risks are endogenously correlated in the model, so that growth cycles equally inherit endogenous heteroskedasticity.

While it is well-known that exposure to long-run risk tends to generate term structures of expected equity returns that are upwarding sloping, recent empirical evidence (Brandt, Binsbergen and Koijen, 2012) suggests that the term structure is downward sloping at least in the short run. Our benchmark model produces a U-shaped term structure of equity returns. While for longer horizons, exposure to long-run risk starts to dominate, significant short run risks through cyclical variation through entry and exit and markups makes short horizon dividends highly risky, consistent with the empirical evidence.

Empirically, we show that aggregate stock returns are indeed forecastable by proxies for time-varying competitive pressure, consistent with the predictions of our model. We use markups, profit shares and measures of entry rates in our empirical work. These variables are correlated with price-dividend ratios. Our model thus contributes a novel and empirically relevant rationale for return predictability in the data.

1.1 Literature

Our work belongs to several strands of literature. The paper is related to the emerging literature on the links between risk premia and imperfect competition and connects it to research on sources of endogenous predictability. More generally, it contributes to the literature on general equilibrium asset pricing with production.

Our starting point is an innovation-driven model of stochastic endogenous growth along the lines of Kung (2014) and Kung and Schmid (2014). Methodologically, that work builds on the literature on medium-term cycles pioneered by Comin and Gertler (2006) and Comin, Gertler and Santacreu (2009), which in turn builds on the seminal work of Romer (1990). We extend this model to account for entry and exit along the lines of Bilbiie, Ghironi and Melitz (2012) and Floetotto and Jaimovich (2008) and examine the asset pricing implications. Following the latter paper's approach, we obtain endogenous countercyclical markups. Opp, Parlour and walden (2014) obtain time-varying markups in a model of strategic interactions at the industry level.

Imperfect competition and markups links our paper to a recent and growing literature on the links between product market competition and stock returns. Hou and Robinson (2006), Bustamante and Donangelo (2014), van Binsbergen (2014) and Loualiche (2014) examine the cross-sectional links between stock returns and competitive pressure. The paper most closely related to ours, Loualiche (2014), also looks at cross-sectional variation in stock returns, but through the lens of a general equilibrium asset pricing model with entry and exit related to our setup. He finds, theoretically and empirically, that aggregate shocks to entry rates are an important factor priced in the cross-section of returns. Our work differs from these papers by our focus on aggregate risk premia and their time-series behavior. Our aproach therefore provides distinct and novel empirical predictions.

Our paper shares its focus on asset pricing with the growing literature on asset pricing in general equilibrium models with production. Starting with the habit-based models of Jermann (1998) and Boldrin, Christiano, and Fisher (2001), recent examples, which include Tallarini (2000), Campanale, Castro, and Clementi (2008), Kuehn (2008, 2009), Ai (2010) and Kaltenbrunner and Lochstoer (2010) explore endogenous long-run consumption risks in real business cycle models with recursive preferences, while Gourio (2012, 2013) examines disaster risks. Particularly closely related are recent papers by Croce (2012), Backus, Routledge, and Zin (2007, 2010), Gomes, Kogan, and Yogo (2009), Kogan and Papanikolaou (2010), Garleanu, Panageas, and Yu (2011), Papanikolaou (2011) and Ai, Croce, and Li (2013) who examine the implications of long-run productivity risk and technological innovation for equity market returns. Our aproach differs from their work as technological progress and productivity growth arises endogenously in our model through process and product innovation. Similarly, these papers focus on unconditional asset pricing moments, while we consider return predictability.

In the latter respect, our work is related to a long list of papers examining mechanisms that endogenously generate time-varying and predictable risk premia. While some mechanisms work through preferences, such as habit preferences (as in Campbell and Cochrane (1999)) or time-varying risk aversion (a recent example is Dew-Becker (2012)), other papers have linked time-varying risk premia to endogenously time-varying volatility. A number of papers work through frictions in the labor markets, where wages effectively generate operating leverage (Favilukis and Lin (2014a, 2014b), Kuehn, Petrosky-Nadeau and Zhang (2014), Santos and Veronesi (2006)) and identify variables with predictive power for stock returns related to labor market conditions. Gomes and Schmid (2014) explicitly model financial leverage in general equilibrium and find that credit spreads forecast stock returns through countercyclical leverage. Our channel operating through time-varying markups is novel and quite distinct and allows us to empirically identify a new set of predictive variables linked to time-varying competitive pressure. Some of that work also addresses the term structure of equity returns (Ai, Croce, Diercks and Li (2014), Favilukis and Lin (2014a)), but these mechanisms are quite distinct (vintage capital and wage rigidity, respectively).

The paper is organized as follows. We describe our model in section 2 and examine the main economic mechanisms in section 3. The next section discusses quantitative implications by means of a calibration, and presents empirical evidence supporting our model predictions. Section 5 offers a few concluding remarks.

2 Model

In this section, we present a general equilibrium asset pricing model with imperfect competition and endogenous productivity growth. Endogenous innovation impacts productivity growth because of imperfect competition, as markups and the associated profit opportunities provide incentives for new firms to enter (product innovation) and for incumbent firms to invest in their own production technology (process innovation). Cyclical movements in profit opportunities affect the mass of active firms and thus competitive pressure and markups. We close the model with a representative household with recursive preferences.

Overall the model is a real version of the endogenous growth framework of Kung (2014), extended to allow for entry and exit with multiple industries and time-varying markups. We start by briefly describing the household sector, which is quite standard. Then we explain in detail the production sector and the innovation process in our economy, and define the general equilibrium. In the following, we describe the most important equilibrium conditions, while we defer the complete list of all relationships characterizing the equilibrium to the Appendix.

Notation In the following, we use the calligraphic letters to denote aggregate variables.

2.1 Household

The representative agent is assumed to have Epstein-Zin preferences over aggregate consumption $\mathcal{C}_t$ and labor $\mathcal{L}_t$[4]

$$U_t = u\left(\mathcal{C}_t, \mathcal{L}_t\right) + \beta \left(E_t[U_{t+1}^{1-\theta}] \right)^{\frac{1}{1-\theta}}$$

[4]Traditionally, Epstein-Zin preference are defined as $\tilde{U}_t = \left\{ u\left(\mathcal{C}_t, \mathcal{L}_t\right)^{1-1/\psi} + \beta \left(E_t[\tilde{U}_{t+1}^{1-\gamma}] \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}$ where γ is the coefficient of relative risk aversion and ψ is the intertemporal elasticity of substitution. The functional form above is equivalent when we define $U_t = \tilde{U}_t^{1-1/\psi}$ and $\theta = 1 - \frac{1-\gamma}{1-1/\psi}$ but has the advantage of admiting more general utility kernels $u\left(\mathcal{C}_t, \mathcal{L}_t\right)$ (see Rudebusch and Swanson (2012)).

where $\theta = 1 - \frac{1-\gamma}{1-1/\psi}$, γ captures the degree of risk aversion, ψ is the elasticity of intertemporal substitution, and β is the subjective discount rate. The utility kernel is assumed to be additively separable in consumption and leisure,

$$u(\mathcal{C}_t, \mathcal{L}_t) = \frac{\mathcal{C}_t^{1-1/\psi}}{1-1/\psi} + \mathcal{Z}_t^{1-1/\psi} \chi_0 \frac{(1-\mathcal{L}_t)^{1-\chi}}{1-\chi}$$

where χ captures the Frisch elasticity of labor[5], and χ_0 is a scaling parameter. Note that we multiply the second term by an aggregate productivity trend $\mathcal{Z}_t^{1-1/\psi}$ to ensure that utility for leisure does not become trivially small along the balanced growth path.

When $\psi \neq \frac{1}{\gamma}$, the agent cares about news regarding long-run growth prospects. We will assume that $\psi > \frac{1}{\gamma}$ so that the agent has a preference for early resolution of uncertainty and dislikes uncertainty about long-run growth rates.

The household maximizes utility by participating in financial markets and by supplying labor. Specifically, the household can take positions $\mathcal{Z}_t$ in the stock market, which pays an aggregate dividend $\mathcal{D}_t$, and in the bond market $\mathcal{B}_t$. Accordingly, the budget constraint of the household becomes

$$\mathcal{C}_t + \mathcal{Q}_t \mathcal{Z}_{t+1} + \mathcal{B}_{t+1} = \mathcal{W}_t \mathcal{L}_t + (\mathcal{Q}_t + \mathcal{D}_t)\mathcal{Z}_t + \mathcal{R}_{f,t}\mathcal{B}_t, \tag{1}$$

where $\mathcal{Q}_t$ is the stock price, $\mathcal{R}_{f,t}$ is the gross risk free rate and $\mathcal{W}_t$ is the wage rate.

These preferences imply the stochastic discount factor (intertemporal marginal rate of substitution)

$$\mathcal{M}_{t+1} = \beta \left(\frac{U_{t+1}}{E_t(U_{t+1}^{1-\theta})^{\frac{1}{1-\theta}}} \right)^{-\theta} \left(\frac{\mathcal{C}_{t+1}}{\mathcal{C}_t} \right)^{-\frac{1}{\psi}}$$

Additionally, the labor supply condition states that at the optimum the household trades off the wage rate against the marginal disutility of providing labor, so that

$$\mathcal{W}_t = \frac{\chi_0 (1-\mathcal{L}_t)^{-\chi}}{\mathcal{C}_t^{-1/\psi}} \mathcal{Z}_t^{1-1/\psi}.$$

[5] Given our assumption that the household works 1/3 of his time endowment in the steady state, the steady state Frisch labor supply elasticity is $2/\chi$.

2.2 Production Sector

The production sector is composed of three entities: final goods production, intermediate goods production, and the capital producers. The final good aggregates inputs from a continuum of industries, each of which uses differentiated intermediate goods as inputs. Stationary shocks drive stochastic fluctuations in the profits on intermediate goods. Changing profit opportunities lead to product innovation as new intermediate goods producers find it profitable to enter, and to process innovation as incumbent firms respond by upgrading their technology by investing in intangible capital in order to stay competitive. The capital sector produces both physical and intangible capital and rents it out to be used in the production of intermediate goods.

Sustained growth obtains in this economy because investment in technology is only partially appropriable: Firm-level innovation partially spills over to competitors thereby creating an aggregate externality. Process innovation thus fosters not only an intermediate goods firm's investment by raising its productive efficiency, but also that of competitors through spillovers, thus boosting growth even further.

Final Goods The final goods sector is modeled following Jaimovich and Floetotto (2008). The final good is produced by aggregating sectorial goods which are themselves aggregates of intermediate goods. We think of each sector as a particular industry and use these labels interchangeably. As we show later, this two-stage aggregation generates endogenous time-varying markups.

More specifically, a representative firm produces the final (consumption) goods in a perfectly competitive market. The firm uses a continuum of sectorial goods $Y_{i,t}$ as inputs in the following CES production technology

$$\mathcal{Y}_t = \left(\int_0^1 Y_{i,t}^{\frac{\nu_1 - 1}{\nu_1}} \, di \right)^{\frac{\nu_1}{\nu_1 - 1}}$$

where ν_1 is the elasticity of substitution between sectorial goods. The profit maximization problem of the firm yields the isoelastic demand for sector j goods,

$$Y_{j,t} = \mathcal{Y}_t \left(\frac{P_{j,t}}{P_{Y,t}} \right)^{-\nu_1}$$

where $P_{Y,t} = \left(\int_0^1 P_{j,t}^{1-\nu_1} \, dj \right)^{\frac{1}{1-\nu_1}}$ is the final goods price index (and the numeraire). We provide the derivations in the appendix.

In turn, each industry j produces sectorial goods using a finite number $N_{j,t}$ of differentiated goods $X_{i,j,t}$. Importantly, the number of differentiated goods in each industry is allowed to vary over time. Because each industry is atomistic, sectorial firms face an isoelastic demand curve with constant price elasticity ν_1. The sectorial goods are aggregated using a CES production technology

$$Y_{j,t} = N_{j,t}^{1-\frac{\nu_2}{\nu_2-1}} \left(\sum_{i=1}^{\mathcal{N}_t} X_{i,j,t}^{\frac{\nu_2-1}{\nu_2}} \right)^{\frac{\nu_2}{\nu_2-1}}$$

where $N_{j,t}$ is the number of firms and ν_2 is the elasticity of substitution between intermediate goods. The multiplicative term $N_{j,t}^{1-\frac{\nu_2}{\nu_2-1}}$ is added to eliminate the variety effect in aggregation.

The profit maximization problem of the firm yields the following demand schedule for intermediate firms in industry j (see the appendix for derivations):

$$X_{i,j,t} = \frac{Y_{j,t}}{N_{j,t}} \left(\frac{P_{i,j,t}}{P_{j,t}} \right)^{-\nu_2}$$

where $P_{i,j,t}$ is the price of intermediate good i in industry j and $P_{j,t} = N_{j,t}^{\frac{-1}{1-\nu_2}} \left(\sum_{i=1}^{N_{j,t}} P_{i,j,t}^{1-\nu_2} \right)^{\frac{1}{1-\nu_2}}$ is the sector j price index. In the following, we assume that the elasticity of substitution within industry is higher than across industries, i.e. $\nu_2 > \nu_1$.

Intermediate Goods Intermediate goods prodution in each industry is characterized by monopolistic competition. In each period, a proportion δ_n of existing firms becomes obsolete and leaves the economy. The specification of the production technology is similar to Kung (2014). Intermediate goods firms produce $X_{i,j,t}$ using a Cobb-Douglas technology defined over physical capital $K_{i,j,t}$, labor $L_{i,j,t}$, and technology $Z_{i,j,t}$. We think of technology as intangible capital, such as patents. Firms rent their physical and technology from capital producers at a period rental rate of $r^k_{j,t}$ and $r^z_{j,t}$, respectively. Labor input is supplied by the household. We assume that technology is only partially appropriable and that there are spillovers across firms. The production technology is

$$X_{i,j,t} = K_{i,j,t}^{\alpha} \left(A_t Z_{i,j,t}^{\eta} \mathcal{Z}_t^{1-\eta} L_{i,j,t} \right)^{1-\alpha}$$

where $\mathcal{Z}_t \equiv \int_0^1 \left(\sum_{i=1}^{N_{j,t}} Z_{i,j,t} \right) dj$ is the aggregate stock of technology in the economy and the parameter $\eta \in [0,1]$ captures the degree of technological appropriability. These spillover effects are crucial for generating sustained growth in the economy (e.g. Romer (1990)). Technology increases the efficiency of intermediate good production, so that we interpret that input as process innovation.

The variable A_t represents an aggregate technology shock that is common across firms and evolves in logs as an AR(1) process:

$$a_t = (1-\rho)a^\star + \rho a_{t-1} + \sigma\epsilon_t$$

where $a_t \equiv \log(A_t)$, $\epsilon_t \sim N(0,1)$ is i.i.d., and $a^\star$ is the unconditional mean of a_t.

Intermediate good firms' payout is then given by

$$D_{i,j,t} = \frac{P_{i,j,t}}{P_{Y,t}} X_{i,j,t} - W_{j,t}L_{i,j,t} - r^k_{j,t}K_{i,j,t} - r^z_{j,t}Z_{i,j,t}.$$

The demand faced by an individual firm depends on its relative price and the sectorial demand which in turn depends on the final goods sector. Expressing the inverse demand as a function of final goods variables,

$$X_{i,j,t} \quad = \quad \frac{\mathcal{Y}_t}{N_{j,t}} \left(\tilde{P}_{i,j,t}\right)^{-\nu_2} \left(\tilde{P}_{j,t}\right)^{\nu_2-\nu_1}$$

where tilde-prices are normalized by the numeraire, i.e. $\tilde{P}_{i,j,t} \equiv \frac{P_{i,j,t}}{P_{Y,t}}$ and $\tilde{P}_{j,t} \equiv \frac{P_{j,t}}{P_{Y,t}}$.

The objective of the intermediate goods firm is to maximize shareholder's wealth, taking input prices and the stochastic discount factor as given:

$$\begin{aligned} V_{i,j,t} = & \max_{\left\{L_{i,j,t},K_{i,j,t},Z_{i,j,t},\tilde{P}_{i,j,t}\right\}_{t\geqslant 0}} E_0\left[\sum_{s=0}^{\infty} \mathcal{M}_{t,t+s}(1-\delta_n)^s D_{i,j,s}\right] \\ \text{s.t.} \quad & X_{i,j,t} = \frac{\mathcal{Y}_t}{N_{j,t}} \left(\tilde{P}_{i,j,t}\right)^{-\nu_2} \left(\tilde{P}_{j,t}\right)^{\nu_2-\nu_1} \end{aligned}$$

where $\mathcal{M}_{t,t+s}$ is the marginal rate of substitution between time t and time $t+s$.

This market structure yields a symmetric equilibrium in the intermediate goods sector. Hence, we can drop the i subscripts in the equations above. As derived in the appendix, the corresponding first order necessary conditions are

$$\begin{aligned}
r^k_{j,t} &= \frac{\alpha}{\o_{j,t}} \frac{X_{j,t}}{K_{j,t}} \\
r^z_{j,t} &= \frac{\eta(1-\alpha)}{\o_{j,t}} \frac{X_{j,t}}{Z_{j,t}} \\
W_{j,t} &= \frac{(1-\alpha)}{\o_{j,t}} \frac{X_{j,t}}{L_{j,t}} \\
\o_{j,t} &= \frac{-\nu_2 \, N_{j,t} + (\nu_2 - \nu_1)}{-(\nu_2 - 1) \, N_{j,t} + (\nu_2 - \nu_1)}
\end{aligned}$$

where $\o_{j,t}$ is the price markup reflecting monopolistic competition. Note that the price markup depends on the number of active firms $N_{j,t}$ in each industry, and so can be time-varying. We describe how the evolution of the mass of active firms is endogenously determined below.

Capital producers Capital producers operate in a perfectly competitive environment and produce industry-specific capital goods. They specialize in the production of either physical capital or technology and are owned by the representative household.

Physical capital producers lease capital $\mathcal{K}^c_{j,t}$ to sector j for production in period t at a rental rate of $r^k_{j,t}$. At the end of the period, they retrieve $(1-\delta_k)\mathcal{K}^c_{j,t}$ of depreciated capital. They produce new capital by transforming $\mathcal{I}_{j,t}$ units of output bought from the final goods producers into new capital via the technology[6]:

$$\Phi_{k,j,t}\mathcal{K}^c_{j,t} = \left(\frac{\alpha_{1,k}}{1-\frac{1}{\zeta_k}} \left(\frac{\mathcal{I}_{j,t}}{\mathcal{K}^c_{j,t}} \right)^{1-\frac{1}{\zeta_k}} + \alpha_{2,k} \right) \mathcal{K}^c_{j,t}$$

Therefore, the evolution of aggregate physical capital in industry j is

$$\mathcal{K}^c_{j,t+1} = (1-\delta_k)\mathcal{K}^c_{j,t} + \Phi_{k,j,t}\mathcal{K}^c_{j,t}$$

and the period revenue is $r^k_{j,t}\mathcal{K}^c_{j,t} - \mathcal{I}_{j,t}$.

The optimization problem faced by the representative physical capital producer is to choose $\mathcal{K}^c_{j,t+1}$ and $\mathcal{I}_{j,t}$ in order to maximize the present value of revenues taking the stochastic discount

[6]This functional form for the capital adjustment costs is borrowed from Jermann(1998). The parameters $\alpha_{1,k}$ and $\alpha_{2,k}$ are set to values so that there are no adjustment costs in the deterministic steady state. Specifically, $\alpha_{1,k} = (\Delta\mathcal{Z} - 1 + \delta_k)^{\frac{1}{\zeta_k}}$ and $\alpha_{2,k} = \frac{1}{\zeta_k - 1}(1 - \delta_k - \Delta\mathcal{Z})$.

factor as given.

$$
\begin{aligned}
V^k_{j,t} = &\max_{\{\mathcal{I}_{j,t},\mathcal{K}^c_{j,t+1}\}_{t\geqslant 0}} E_0\left[\sum_{s=0}^{\infty}\mathcal{M}_{t,t+s}(r^k_{j,s}\mathcal{K}^c_{j,s}-\mathcal{I}_{j,s})\right]\\
\text{s.t. } &\mathcal{K}^c_{j,t+1}=(1-\delta_k)\mathcal{K}^c_{j,t}+\Phi_{k,j,t}\mathcal{K}^c_{j,t}
\end{aligned}
$$

As we show in the appendix, this optimization problem yields the following first order conditions:

$$
\begin{aligned}
Q^k_{j,t} &= \Phi'^{-1}_{k,j,t}\\
Q^k_{j,t} &= E_t\left[\mathcal{M}_{t,t+1}\left(r^k_{j,t+1}+Q^k_{j,t+1}\left(1-\delta_k-\Phi'_{k,j,t+1}\left(\frac{\mathcal{I}_{j,t}}{\mathcal{K}^c_{j,t}}\right)+\Phi_{k,j,t+1}\right)\right)\right]
\end{aligned}
$$

where Q^k_t is the Lagrange multiplier on the capital accumulation constraint.

The structure of the technology sector is similar. More specifically, this sector produces new intangible capital by transforming $\mathcal{S}_{j,t}$ units of output bought from the final goods producers into new technology via the technology[7]:

$$
\Phi_{k,j,t}\mathcal{Z}^c_{j,t}=\left(\frac{\alpha_{1,z}}{1-\frac{1}{\zeta_z}}\left(\frac{\mathcal{S}_{j,t}}{\mathcal{Z}^c_{j,t}}\right)^{1-\frac{1}{\zeta_z}}+\alpha_{2,z}\right)\mathcal{Z}^c_{j,t}.
$$

We think of $\mathcal{S}_{j,t}$ as investment in R&D. In the model, therefore, technology accumulates endogenously.

As with physical capital producers, the optimization problem of the representative technology producer is to maximize the present value of revenues, so that the first conditions are,

$$
\begin{aligned}
Q^z_{j,t} &= \Phi'^{-1}_{z,j,t}\\
Q^z_{j,t} &= E_t\left[M_{t,t+1}\left(r^z_{j,t+1}+Q^z_{j,t+1}\left(1-\delta_z-\left(\frac{\mathcal{S}_{j,t+1}}{\mathcal{Z}^c_{j,t+1}}\right)\Phi'_{z,j,t+1}+\Phi_{z,j,t+1}\right)\right)\right]\\
\mathcal{Z}^c_{j,t+1} &= (1-\delta_z)\mathcal{Z}^c_{j,t}+\Phi_{z,j,t}\mathcal{Z}^c_{j,t}.
\end{aligned}
$$

2.3 Entry & Exit

Each period, new firms contemplate entering intermediate good production. Entry into the intermediate goods sector entails the fixed cost $F_{E,j,t}\equiv\kappa_j\mathcal{Z}_t$. A newly created firm will start producing

[7] Similarly, the parameters $\alpha_{1,z}$ and $\alpha_{2,z}$ are set to values so that there are no adjustment costs in the deterministic steady state. Specifically, $\alpha_{1,z}=(\Delta\mathcal{Z}-1+\delta_z)^{\frac{1}{\zeta_z}}$ and $\alpha_{2,z}=\frac{1}{\zeta_z-1}(1-\delta_z-\Delta\mathcal{Z})$.

in the following period. Note that these costs are multiplied by the aggregate trend in technology to ensure that the entry costs do not become trivially small along the balanced growth path.

The evolution equation for the number of firms in the intermediate goods sector is

$$N_{j,t+1} = (1-\delta_n)N_{j,t} + N_{E,j,t}$$

where $N_{E,j,t}$ is the number of new entrants and δ_n is the fraction of firms, randomly chosen, that become obsolete after each period. The entry condition is:

$$E_t[\mathcal{M}_{t+1}V_{j,t+1}] = F_{E,j,t} \tag{2}$$

where $V_{j,t} = D_{j,t} + (1-\delta_n)E_t[\mathcal{M}_{t+1}V_{j,t+1}]$ is the market value of the representative firm in sector j. Movements in profit opportunities and valuations thus lead to fluctuations in the mass of entering firms.

2.4 Equilibrium

Symmetric Equilibrium We focus on a symmetric equilibrium, in which all sectors and intermediate firms make identical decisions, so that the i and j subscripts can be dropped. Given the symmetric equilibrium, we can express aggregate output as

$$\begin{aligned} \mathcal{Y}_t &= \mathcal{N}_t X_t \\ X_t &= K_t^{\alpha}(A_t Z_t^{\eta} \mathcal{Z}_t^{1-\eta} L_t)^{1-\alpha} \end{aligned}$$

Aggregation Aggregate macro quantities are defined as: $\mathcal{I}_t \equiv \sum_{j=1}^{\mathcal{N}_t} I_{j,t} = \mathcal{N}_t I_t$, $\mathcal{S}_t \equiv \sum_{j=1}^{\mathcal{N}_t} S_{j,t} = \mathcal{N}_t S_t$, $\mathcal{Z}_t \equiv \sum_{j=1}^{\mathcal{N}_t} Z_{j,t} = \mathcal{N}_t Z_t$, $\mathcal{K}_t \equiv \sum_{j=1}^{\mathcal{N}_t} K_{j,t} = \mathcal{N}_t K_t$. The aggregate dividend coming from the production sector is defined as

$$\mathcal{D}_t = \mathcal{N}_t D_t + (r_t^k \mathcal{K}_t - \mathcal{I}_t) + (r_t^z \mathcal{Z}_t - \mathcal{S}_t)$$

Note that the aggregate dividend includes dividends from the capital and technology sectors.

Market Clearing Imposing the symmetric equilibrium conditions, the market clearing condition for the final goods market is:

$$\mathcal{Y}_t = \mathcal{C}_t + \mathcal{I}_t + \mathcal{S}_t + N_{E,t} \cdot F_{E,t}$$

The market clearing condition for the labor market is:

$$\mathcal{L}_t = \sum_{j=1}^{\mathcal{N}_t} L_{j,t}$$

Imposing symmetry, the equation above implies

$$L_t = \frac{\mathcal{L}_t}{\mathcal{N}_t}$$

The market clearing condition for the capital markets implies that the amount of capital rented by firms equals the aggregate supply of capital:

$$\mathcal{K}_t = K_t^c$$
$$\mathcal{Z}_t = Z_t^c$$

Equilibrium We can thus define an equilibrium for our economy in a standard way. In a synmmetric equilibrium, there is one exogenous state variable, A_t, and three endogenous state variables, the physical capital stock $\mathcal{K}_t$, the intangible capital stock $\mathcal{Z}_t$, and the number of intermediate good firms, $\mathcal{N}_t$. Given an initial condition $\{A_0, \mathcal{K}_0, \mathcal{Z}_0, \mathcal{N}_0\}$ and the law of motion for the exogenous state variable A_t, an equilibrium is a set of sequences of quantities and prices such that (i) quantities solve producers' and the household's optimization problems and (ii) prices clear markets.

We interpret the stcok market as the claim to the entire stream of future aggregate dividends, $\mathcal{D}_t$, and define the return accordingly.

3 Economic mechanism

Our model departs in two significant ways from the workhorse stochastic growth model in macroeconomics. First, our setup incorporates imperfect competition and entry and exit of intermediate goods firms. By introducing new differentiated products available to consumers, such product innovation changes competitive pressure in goods markets. Second, rather than stemming from an exogenous drift in aggregate productivity, the trend growth path is endogenously determined by firms' investment in their technology, which we label as process innovation.

In this section, we qualitatively examine how both product and process innovation affect aggregate risk and its dynamics in the economy. In particular, in the language of Bansal and Yaron (2004), we document that product innovation creates short-run risks by *amplifying* shocks and process innovation generates long-run risks by *propagating* shocks. Moreover, by changing competitive pressure, in our model product innovation creates endogenous movements in factor shares and thus time-varying risks.

While we focus on a qualitative examination of our setup here, we provide a detailed quantitative analysis of the model in the next section.

Entry & Exit We start by examining the entry and product creation process through the lens of the free entry condition, equation (2). Rising expected market valuations makes entering product markets more attractive, given the fixed costs of entry $F_{E,t}$, suggesting procyclical entry dynamics. Indeed, suppose a positive technologyy shock. As firms become more productive, the value of intermediate goods firms increases. Attracted by profit opportunities, new firms enter the market. Firms will enter the market up until the entry condition is satisfied, generating procyclical entry. On the other hand, as the number of firms in the economy grows, product market competition intensifies. The model is thus is consistent with the empirical evidence that the degree of competitiveness in industries is pronouncedly procyclical, as documented e.g. in Bresnahan and Reiss (1991) and Campbell and Hopenhayn (2005).

We now show that in our model, movements in competitve pressure lead to time variation in markups.

Markups In the classic Dixit-Stiglitz CES aggregator, an individual firm is small relative to the rest of economy. Therefore, the single firm will not affect the sectorial price level, $P_{j,t}$ nor the aggregate price level, $P_{Y,t}$. The firm faces a constant price elasticity of demand and charges a constant markup equal to $\frac{\nu_2}{\nu_2-1}$.

In contrast, in our model the measure of firms within each sector is finite and the intermediate producer now takes its effect on the remaining firms into account. As we show in the appendix, firms' cost minimization problem implies that the price markup is[8]

$$\o_t = \frac{-\nu_2 \mathcal{N}_t + (\nu_2 - \nu_1)}{-(\nu_2 - 1)\mathcal{N}_t + (\nu_2 - \nu_1)}.$$

Importantly, equilibrium markups depend on the number of active firms and thus the degree of competition. Taking the derivative of the markup with respect to $\mathcal{N}_t$, we find

$$\frac{\partial \o_t}{\partial N_t} = \frac{\nu_1 - \nu_2}{\left[-(\nu_2 - 1)\mathcal{N}_t + (\nu_2 - \nu_1)\right]^2} < 0. \tag{3}$$

Assuming that the elasticity of substitution within industries is higher than across sectors, that is $\nu_2 > \nu_1$, implies that markups decrease as the number of firms increases, and thus are countercyclical in the model. This implication is consistent with the empirical evidence documented e.g. in Bils (1987), Rotemberg and Woodford (1991, 1999) and Chevalier, Kashyap and Rossi (2003).

Importantly, the expression for the derivative of the markup with respect to $\mathcal{N}_t$ implies that the sensitivity of markups to a marginal entrant depends on the number of firms in the industry. In particular, adding a new firm to an already highly competitive industry (high $\mathcal{N}_t$) will have little impact on product market competition. In contrast, a marginal entrant will have large effects on markups when the number of incumbents is low. Since $\mathcal{N}_t$ is procyclical, time-varying markups will make the economy more sensitive in recessions. This mechanism linked to product innovation is a source of business cycle asymmetry in our model.

Figure 1 illustrates this pattern. As the mass of firms $\mathcal{N}_t$ increases, the markup falls, as shown in the left panel. While it falls sharply when the mass of firms is low, it flattens out for higher values of $\mathcal{N}_t$. As the right panel demonstrates, this is reflected in the sensitivity of the markup with respect to the marginal entrant. That sensitivity is very high when the mass of incumbents is low, while it asymptotes out the more competitive an industry gets.

We next examine how the dynamics of entry and markups impact aggregate risk priced in financial markets and its dynamics in the model. To that end, we find it convenient to represent the aggregate production function in a form that permits straightforward comparison with the specification commonly used in the workhorse stochastic growth model. In the latter model, typically, the only source of risk is a single shock to an exogenously given process for measured productivity, that is, productivity risk. In contrast, in our model, productivity is endogenous, as we now document.

[8] The standard constant markup specification is a particular case in which $N_t \to \infty$.

Endogenous Productivity The aggregate production technology can be expressed in terms of aggregate variables as

$$\begin{aligned}
\mathcal{Y}_t &= \mathcal{N}_t K_t^{\alpha}(A_t Z_t^{\eta} \mathcal{Z}_t^{1-\eta} L_t)^{1-\alpha} \\
&= \mathcal{N}_t \left(\frac{\mathcal{K}_t}{\mathcal{N}_t}\right)^{\alpha} \left[A_t \left(\frac{\mathcal{Z}_t}{\mathcal{N}_t}\right)^{\eta} \mathcal{Z}_t^{1-\eta} \left(\frac{\mathcal{L}_t}{\mathcal{N}_t}\right)\right]^{1-\alpha} \\
&= \mathcal{K}_t{}^{\alpha} \left[\mathcal{Z}_{p,t}\mathcal{L}_t\right]^{1-\alpha}
\end{aligned}$$

where $\mathcal{Z}_{p,t} \equiv A_t \mathcal{Z}_t \mathcal{N}_t^{-\eta}$ is measured productivity. The equilibrium productivity process thus contains a component driven by the exogenous forcing process, A_t, and two endogenous components. These two components reflect $\mathcal{Z}_t$, the stock of intangible capital, accumulated through process innovation by means of R&D on the one hand, and the mass of active firms $\mathcal{N}_t$, created through product innovation, on the other.

In the model, aggregate fluctuations and thus risk reflect movements in measured productivity. Taking the log expected growth rate of $\mathcal{Z}_{p,t}$ we have

$$E_t[\Delta z_{p,t+1}] = E[\Delta a_{t+1}] + \Delta z_{t+1} - \eta\, \Delta n_{t+1}$$

The exogenous technological shock, a_t, is a highly persistent process. We thus consider that $\Delta a_t \approx 0$.

The decomposition allows us to identify two sources of movements in measured productivity growth. On the one hand, there are movements driven by the accumulation of aggregate technology through R&D, Δz_t. In earlier work, Kung and Schmid (2014) and Kung (2014) show empirically and theoretically that movements in R&D are highly persistent and give rise to predictable low frequency fluctuations in aggregate growth rates priced in asset markets. In other words, process innovation endogenously generates qualitatively and quantitatively significant *long-run risks*, in the sense of Bansal and Yaron (2004). Similar dynamics obtain in the present case, so that process innovation propagates shocks.

As a novel mechanism, our model identifies movements in the number of active firms induced by entry and exit, Δn_t, as an important driver of fluctuations in productivity growth. In contrast to the low-frequency movements in productivity induced by innovation of incumbent firms, entry and exit is highly cyclical. We will therefore associate the implied cyclical component in productivity with *short-run risks*. Importantly, note that in our model short- and long-run risks are correlated, as with time-varying markups entry impacts investment in technology, as we discuss in more detail below.

In sum, our model thus endogenously generates long and short-run components in productivity risk linked to process and product innovation:

$$E_t[\Delta z_{p,t+1}] \approx \Delta z_{t+1} - \eta \Delta n_{t+1}$$

Notably, in our model, short and long-run risks are correlated and reinforce one another, as we now explain.

The determinants of Δz_{t+1} In a model without entry & exit (E&E), intermediate firms increase their demand for technology in response to a positive productivity shock. This happens because A_t increases the marginal productivity of technology. This leads to sustained growth in the aggregate.

Allowing for E&E amplifies this effect. First consider the case of E&E without time-varying markups. Following a positive productivity shock, firms enter the economy. As each new firm needs technology to operate, the aggregate demand for technology increases, amplifying the initial effect of the technological shock.

Time-varing markups provide an additional transmission mechanism. As the number of firms increases, the price markup decreases and the final goods sector increases its demand for intermediate goods. To satisfy the higher demand, firms produce more and increase their demand for capital and technology, further amplifying macroeconomic fluctations. In particular, through this channel, entry of firms and investment in technology become positively correlated, so that higher entry rates are associated with growth in measured productivity, as we demonstrate quantitatively and empirically below.

With recursive preferences, both high and low-frequency fluctuations will be priced and require a sizeable risk premium. In the next section, we quantify the effect of each of these channels on the risk premium.

4 Quantitative Implications

In this section, we present quantitative results from a calibrated version of our model. We calibrate it to the replicate salient features of industry and business cycles and use it both to gauge the quantitative significance of our mechanisms for risk premia as well as to generate empirical predictions.

We provide empirical evidence supporting the model predictions alongside.

In order to quantitatively isolate the contributions of process innovation, product innovation and time-varying markups on aggregate risk and risk premia, we find it instructive to compare our benchmark model to two nested models. In the following, we refer to the benchmark model as model A. Model B refers to a specification where our industry aggregator is replaced by a standard CES aggregator. While this specification features entry and exit, this alternative aggregator implies a constant markup of $\frac{\nu_2}{\nu_2-1}$. Model C, finally, features a CES aggregator, and abstracts away from entry and exit, so that the mass of firms is constant.

The models are calibrated at quarterly frequency. The empirical moments correspond to the U.S. postwar sample from 1948 to 2013. The model is solved using third-order perturbation methods[9].

Calibration We calibrate our model to be consistent with salient features of industry and business cycles. To that end, we first need to construct empirical targets for some key variables in the model, such as entry rates, markups, R&D and the intangible capital stock.

Following Bils (1987), Rotemberg and Woodford (1999) and Campello (2003), we construct an empirical markup series by exploiting firms' first order condition with respect to L_t, imposing the symmetry condition,

$$\o_t = (1-\alpha)\frac{\mathcal{Y}_t}{\mathcal{L}_t W_t} = (1-\alpha)\frac{1}{S_{L,t}}$$

and adjusting for potential nonlinearities in the empirical counterparts. Here, $S_{L,t}$ is the labor share in the model. We discuss further details about the construction of the markup measure in the appendix.

For entry rates, we use two empirical counterparts. First, we use the index of net business formation (NBF). This index is one of the two series published by the BEA to measure the dynamics of firm entry and exit at the aggregate level. It combines a variety of indicators into an approximate index and is a good proxy for n_t. Since n_t is a stationary variable in our model, and the data counterpart is not, we linearly detrend the log of NBF before using the data in the forecasting regressions[10] The other is the number of new business incorporations (INC), obtained from the U.S. Basic Economics Database. While qualitatively bot series behave very similarly, quantitatively

[9] We prune simulations using the Kim, Kim, Schaumburg and Sims (2008) procedure to avoid generating explosive paths in simulations.

[10] We also tried to detrend using the Hodrick-Prescott filter and the log-difference and the results remain.

there are small differences. Below, we provide a number of robustness checks with respect to both measures.

Finally, our empirical series for $\mathcal{S}_t$ measures private business R&D investment and comes from the National Science Foundation (NSF). The Bureau of Labor Statistics (BLS) constructs the R&D stock by accumulating these R&D expenditures and allowing for depreciation, much in the same way as the physical capital stock is constructed. We thus use the R&D stock as our empirical counterpart for the stock of technology $\mathcal{Z}_t$. For consistency, we use the same depreciation rate δ_n in our calibration as does the BLS in its calculations.

The remaining empirical series are standard in the macroeconomics and growth literature. Additional details are collected in the appendix.

Table 1 presents the quarterly calibration. Panel A reports the values for the preference parameters. The elasticity of intertemporal substitution ψ is set to 2.0 and the coefficient of relative risk aversion γ is set to 10.0, both of which are standard values in the long-run risks literature (e.g. Bansal, Kiku, and Yaron (2008)). The labor elasticity parameter χ, is set to 3. This implies a Frisch elasticity of labor supply of 2/3, which is consistent with estimates from the microeconomics literature (e.g. Pistaferri (2003)). χ_0 is set so that the representative household works 1/3 of her time endownment in the steady state. The subjective discount factor β is calibrated to 0.9935 to be consistent with the level of the real risk-free rate.

Panel B reports the calibration of the technological parameters. The capital share α is set to 0.33, and the depreciation rate of capital δ_k is set to 2.0%. These two parameters are calibrated to standard values in the macroeconomics literature (e.g. Comin, and Gertler (2006)). The capital adjustment cost parameter ζ_k is set at 0.825 to match the relative volatility of investment growth to consumption growth. The parameters related to R&D are calibrated following Kung (2014). The depreciation rate of the R&D capital stock δ_z is set to 3.75%, implying an annualized depreciation rate of 15%. The R&D capital adjustment cost parameter ζ_z is set at 0.825 to match the relative volatility of R&D investment growth to consumption growth. The degree of technological appropriability η is calibrated to 0.075, in line with Kung (2014). The exogenous firm exit shock δ_n is set to 1%, slightly lower than in Bilbiie, Ghironi, and Melitz (2012). The price elasticity accross (ν_1) and within (ν_2) industries are calibrated to 1.05 and 75, respectively to be consistent with estimates from Jaimovich and Floetotto (2008). κ is set to ensure an aggregate price markup of 20% in the steady state.

Panel C reports the parameter values for the exogenous technology process. The volatility parameter σ is set at 1.15% to match the unconditional volatility of measured productivity growth. The persistence parameter ρ is calibrated to 0.99 to match the first autocorrelation of expected productivity growth. $a^\star$ is chosen to generate an average output growth of 2.0%.

4.1 Quantitative Results

We now report quantitative results based on our calibration. Risk premia in this economy are tightly linked to economic fluctuations. We therefore start by discussing the nature of macroeconomic movements and then present quantitative predictions for asset returns and empirical tests.

4.1.1 Implications for Growth and Cycles

Aggregate cycles in the model reflect movements at the industry level. New firms enter, obsolete products exit, competitive pressure and markups adjust, and measured productivity fluctuates. Productivity dynamics in turn shape macroeconomic cycles.

Industry Cycles Table 2 reports basic industry moments from the benchmark model. As targeted, the calibration is quantitatively in line with the average markup in the data, and is close to mean measured productivity and the mean profit share. Similarly, the model quantitatively captures industry cycles well by closely matching the volatilities of key variables such as markups, intangible capital growth, profit shares and net entry rates as well as their autocorrelations. Importantly, the table confirms our qualitative intuition from the last section that markups should fall with a growing mass of active firms. Indeed, the last panel shows that in the data the correlation between markups and both the mass of firms and entry rates are negative. The model reproduces this pattern qualitatively, and to some extent, quantitatively well.

Figure 2 illustrates the underlying dynamics by plotting the responses of key variables to a positive one standard deviation exogenous technology shock. We focus on two model specifications, namely the benchmark model and model C, featuring a constant mass of firms and a constant markup. In the benchmark model, a positive exogenous shock raises valuations and thus, by the free entry condition, equation (2), triggers entry, as shown in the top left panel, and the mass of firms increases, as documented in the top right panel. In our benchmark model, firms take their effect on competitor firms into account when setting prices, so that increasing competitive pressure leads to falling markups, as shown in the lower left panel. Importantly, as the lower right panel illustrates, the entry margin significantly amplifies investment in technology. This is because in response to falling markups, the final goods sector increases its demand for intermediate goods. To satisfy the higher demand, firms produce more and increase their investment in technology. This observation implies that measured productivity is positively related to entry, through the latter's effects on investment in technology.

Note that in line with the shorter lived nature of the entry and exit and product innovation cycle, movements in the growth rate of technology are considerably more persistent in the benchmark model, consistent with the notion of R&D driven *growth cycles* as discussed in earlier work by Kung and Schmid (2014). Accordingly, the entry margin not only amplifies, but also significantly propagates shocks by generating endogenous persistence.

We now provide some empirical evidence on the links between entry rates and productivity growth and aggregate growth rates, respectively. As shown in the impulse responses, the model predicts that both entry rates and the growth rate of technology are determinants of measured productivity growth. We confirm this prediction in tables 3 and 4, by projecting productivity growth on the growth rate of the R&D stock, our empirical proxy for technology growth, and the growth rate of the index of net business formation (NBF) and of the number of new business incorporations (INC), our empirical proxies for entry. We find positive and statistically significant point estimates, both in univariate and bivariate regressions. We obtain similar results in the model. In the light of our productivity growth decomposition, expression (4), this finding highlights the observation that entry and technology growth are strongly contemporaneously correlated.

In table 5, we report results from predictive regressions of aggregate growth rates on entry rates. Qualitatively, the model predicts that a rise in entry rates forecasts higher growth going forward. Indeed, we empirically find that entry growth positively forecasts higher growth rates of output, consumption, and investment, respectively. While the signs are consistent with the model prediction throughout, statistical significance obtains only for shorter horizons, consistent with the notion that entry rates are highly cyclical. This suggests that variations in entry rates are an important determinant of business cycles and business cycle risk, which we examine next.

Business Cycles Table 6 reports the main business cycle statistics throughout model specifications, models A, B and C. While all of them are calibrated to match the average consumption growth rate and the consumption volatility in our postwar sample, the cyclical behavior across models differs considerably.

The benchmark model quantitatively captures basic features of macroeconomic fluctuations in the data well. It produces consumption volatility, investment volatility and R&D volatility close to their empirical counterparts. While it matches consumption volatility by construction, generating sufficient investment volatility in general equilibrium asset pricing models is often quite challenging. The reason is that generating sufficient movement in the price of capital to make equity returns risky usually requires high capital adjustment costs, which dampens investment volatility. Our benchmark model in turn produces volatile investment and R&D. Similarly, the model generates volatile movements in labor markets, even overshooting the volatility of hours worked slightly. This is noteworthy, as standard macroeconomic models typically find it challenging to generate labor market fluctuations of the orders of magnitude observed in the data.

The quantitative success of the benchmark model is quite in contrast to the nested models (B) and (C). With a constant markup, investment and R&D volatility are essentially halved. Without entry and exit, volatilities are further reduced. Entry and exit and countercyclical markups thus serve as a quantitatively significant amplification mechanism for shocks, creating realistic volatility and risk at the business cycle frequency.

The amplification mechanism is illustrated in figure 3, which plots the impulse response functions of aggregate quantities. Upon impact of a positive exogenous productivity shock, output, investment and consumption all rise, and significantly more so than in a specification without the entry margin. The lower two panels show that both the responses of realized and and expected consumption growth are amplified in the benchmark model. Accordingly, the amplification mechanism increases the quantity of priced risk in the economy, since the stochastic discount factor in the model reflects both realized and predictable movements in consumption growth, given Epstein-Zin preferences.

The intuition for the amplification result is that business cycles in the model reflect industry dynamics. With procyclical entry, the model predicts countercyclical markups, so that falling markups in expansions trigger higher demand for intermediate goods from the final good producer, further stimulating investment in capital and technology, and thus output. Similarly, rising markups in downturns dampen the demand for intermediate goods, and slow down recoveries in the model.

Table 7 documents that the model's predictions about the cyclical behavior of entry rates and markups that are at the core of the amplification mechanism are empirically supported. The correlation of aggregate quantities and our empirical markup series is uniformly negative in the data, and that of quantities with the mass of firms and entry rates positive throughout. The model matches the countercyclical movements of markups and the procyclical fluctuations in firm mass and entry quantitatively well.

While our benchmark model creates significant business cycles and risks unconditionally, we now document that it endogenously generates asymmetric movements in quantities in that it predicts recessions to be deeper than expansions. It therefore endogenously implies *countercyclical conditional risks.*

Asymmetric Cycles Fig. 4 plots the *difference* between the response of quantities to a positive shock, and to a negative shock of the same magnitude. Any deviation from a zero difference reflects an asymmetry in responses at some horizon. To start, let's observe that model C, with constant firm mass and markup, generates no differential response at any horizon. That specification thus predicts exactly symmetric cycles. This is quite in contrast to our benchmark model. It features differential responses at all horizons. The number of firms increases relatively more in expansions than it falls in recessions. Similarly, markups fall relatively more in upswings than they rise in downturns. On the other hand, investment, consumption and output rise by relatively less in good times than they fall in bad times, so that recessions are deeper in our benchmark economy.

The source of asymmetry in the model comes from time-varying markups. From equation (3), we know that the sensitivity of price markups depends on the number of incumbant firms $\mathcal{N}_t$. Because $\mathcal{N}_t$ is procyclical, the demand elasticity becomes steeper in recessions, increasing competitive distortions in downturns. In other words, the sensitivity of markups to a marginal entrant depends on the number of firms in the industry. In particular, on impact, adding a new firm has considerable impact on product market competition, which falls as more firms enter. In contrast, a marginal entrant will have small effects on markups when the number of incumbants is already high.

Figure 5 illustrates this pattern. It plots responses of quantities in the benchmark conditional on the prevailing mass of firms. To be specific, we plot responses in scenarios with a high and a low number of incumbent firms, respectively. As discussed previously, these situations most likely obtain in expansions and recessions, respectively. Suppose a negative exogenous technology shock. With a low mass of incumbent firms, although the mass of firms falls by less on impact (top right panel), the markup will increase relatively by significantly more. This dampens the demand for intermediate goods from the final goods sector more, so that investment in capital and technology fall more.

Importantly, the figure shows that both realized and expected consumption growth fall by relatively more in a scenario with a low mass of incumbent firms, such as in a recession. The quantities of priced risks therefore increase more as well, giving rise to endogenously time-varying conditional risks.

Table 8 provides some empirical evidence linking time varying markups to time varying risks. Using our markup series, we split the data sample into high and low markup episodes. This procedure allows us to compute moments conditional on markups. Given the countercyclicality of our markup measure, it is perhaps not surprising that average output, consumption and investment is lower in high markup episodes. More interestingly, however, we find that the volatilities conditional on high markups are also higher. In line with the discussion above, the model is consistent with these findings. These results thus provide empirical support to the notion of countercyclical risks driven by markup variation.

4.1.2 Asset Pricing Implications

We now examine how the risks inherent in our economy are priced in asset markets. Intuitively, we expect two effects. First, the entry margin endogenously increases movements in realized consumption growth through amplification, and creates endogenous persistence and thus movements in expected consumption growth through propagation. Since both realized and predictable components in consumption growth are priced with Epstein-Zin preferences, we expect these two mechanisms to give rise to a sizable *unconditional* equity premium. Second, since the quantities of priced risks are sensitive to the mass of firms, we expect a countercyclical *conditional* and thus predictable equity premium.

We discuss and quantify these implications in turn and present empirical evidence supporting the model predictions.

Equity Premium Table 9 reports the main asset pricing implications of the benchmark model and the alternative specifications.

A first striking feature of the model comparison is that in the specifications absent countercyclical markups, the risk free rate is about double its empirical counterpart, while the benchmark model replicates a low and stable risk free rate. The intuition for this finding is similar to that developed in earlier work by Kung and Schmid (2014). While we calibrate the endogenous average growth rate to coincide across all models, the amplification and the propagation mechanism working through the entry margin coupled with countercyclical markups creates persistent uncertainty about future growth prospects. As discussed previously, falling markups following elevated entry rates encourage technology investment and generate persistent swings in growth rates. This effect is strongest in the benchmark model. Absent that amplification of long-run risks, with a similar average growth rate, agents find themselves in a position to borrow agents future growth, thus driving up the risk free rate. Amplification and propagation of shocks with countercyclical markups raises the precautionary savings motive, driving down interest rates to realistic levels in our benchmark economy.

The different specifications also generate very different risk premia. Our calibration suggests that the representative agent demands an increase in the risk premium of more than a percent to compensate for the additional risk of fluctuations in the number of firms through entry and exit. Adding time-varying markups adds more amplification and propagation. The resulting increase in the levered risk premium is estimated to be around three percent. Our benchmark economy thus generates a realistic equity premium. Long-run risks associated with process innovation and short-run risks through product innovation therefore both make up around half of the premium, given their correlation. On the other hand, none of the models does generate sufficiently volatile stock returns, a feature it shares with many equilibrium asset pricing models with production. One potential explanation is that our model focuses exclusively on productivity risks, and abstracts away from other sources of risk priced in asset markets that likely contribute to return volatility. Indeed, Ai, Croce and Li (2013) report that empirically, the productivity driven fraction of return volatility is around just 6%, which is close to our quantitative finding.

The benchmark model also generates quantitatively realistic implications for the level and the volatility of the price-dividend ratio.

Competition and asset prices Imperfect competition and variations in competitive pressure is a key mechanism driving risk premia in our setup. We now provide some comparative statics of risk and risk premia with respect to the average competitive pressure. We do this by reporting some sensitivity analysis of simulated data with respect to the sectoral elasticity of substitution between goods, ν_2. Fig. 6 reports the results by plotting key industry, macro and asset pricing moments for different values of ν_2.

Raising the sectoral elasticity of substitution between goods, ν_2, has two main effects on markups. First, by facilitating substitution between intermediate goods, it increases competition and therefore, holding all else constant, lowers markups. Second, by the virtue of our expression for the markup, equation (3), it raises the sensitivity of markups with respect to the number of incumbent firms, and thus, all else equal, makes markups more volatile. The first effect is an important determinant of the average growth rate of the economy, while the latter affects the volatility of growth.

With respect to the first effect, increasing ν_2 has two opposing implications. First, decreasing the average markup, holding all else equal, lowers monopoly profits in the intermediate sector. Second, a lower average markup increases the demand for intermediate goods inputs, which raises monopoly profits. In our benchmark calibration, the second effect dominates, and therefore more intense competition, and a higher average markup raises steady-state growth. On the other hand, a more volatile demand for intermediate goods inputs triggered by increasingly volatile markups leads to a more volatile growth path. This effect is exacerbated by increasingly cyclical entry as profit opportunities become more sensitive to aggregate conditions. The net effect is a riskier economy, which translates into a higher risk premium.

Term structure of equity returns An emerging literature starting with van Binsbergen, Brandt, and Koijen (2012) provides evidence that the term structure of expected equity returns is downward sloping, at least in the short run. This is in contrast to the implications of the baseline long-run risk model (Bansal and Yaron (2004)) or the baseline habits model (Campbell and Cochrane (1999)). The empirical finding reflects the notion that dividends are very risky in the short-run.

Our benchmark model is qualitatively consistent with that notion. Figure 7 shows the term structures of equity returns and the term structures of equity volatilities for the benchmark model and model (C), without entry and exit and, accordingly, constant markups. Consistent with the long-run risk literature, the model absent entry and exit produces a slightly upward sloping term structure. This is consistent with the earlier discussion that process innovation in that model is a source of long-run risk. Similarly, product innovation associated with entry and exit, and the implied countercyclical variation in markups in the benchmark model provides additional, short run risk. In fact, this risk is quantitatively sufficiently important so as to render the equity term structure U-shaped. This short-run risk makes dividends highly risky at shorter horizons, consistent with the empirical evidence.

Return predictibility So far, we have shown how the endogenous short and long run risks in our economy are sufficient to require a realistic unconditional aggregate equity premium. We now document that the countercyclical volatility stemming from endogenous markups documented above requires a realistic time-varying risk premium. We show that risk premia are endogenously countercyclical and can be forecasted using model-implied measures and verify this prediction empirically.

Table 10 presents our main predictability results. Panel A first verifies standard long-horizon predictability regressions projecting future aggregate returns on current log price-dividend ratios in our data sample, and shows statistically significant and negative slope coefficients, and R^2's increasing with horizons up to 5 years. Perhaps more interestingly, we run the same regressions with simulated data from our benchmark model using a sample of equal length as the empirical counterpart. The top right panel reports the results. Consistent with the data, we find statistically significant and negative slope coefficients, with R^2's increasing with horizons up to 5 years. Accordingly, the model generates a time-varying and predictable risk premium.

These predictability results in the model imply that it exhibits conditional heteroskedasticity endogenously, because the single exogenous forcing process, A_t, is specified to be homoskedastic. Fig. 8 confirms this. It shows the impulse response functions of the conditional risk premium and the conditional variance of excess returns to a positive exogenous technology shock, both in the benchmark model and in model specification (C), absent entry and exit and time-varying markups. While in model (C) neither the risk premium nor the conditional variance respond, they both persistently fall on impact in the benchmark model. With the entry margin and countercyclical markups, the risk premium and its variance are thus sharply countercyclical. In contrast, absent these mechanisms, the risk premium and its variance are constant, reflecting the homoskedastic nature of the shocks.

Intuitively, countercyclical risk premia in the benchmark economy mirror the asymmetric nature of cycles. As discussed above, conditioning on markups, the conditional volatility of consumption is elevated in high markup episodes, in both data and model. With countercyclical markups, consumption risk is endogenously higher in downturns, leading to countercyclical and predictable risk premia.

Since our predictability results stem from movements in competitive pressure through entry and exit, we expect that variables capturing such fluctuations forecast risk premia in the model as well. The remaining panels in table 10 confirm this intuition. Moreover, we present empirical evidence supporting this novel prediction in the data.

We use two measures of entry, namely the index of net business formation (IBF) and the growth index of new incorporations (INC), our markup series and the profit share as predictive variables. All of them capture movements in competitive pressure in the model. Panels B to E report the results from projecting future aggregate returns on these variables over horizons of up to 5 years, in the model and in the data. In the model, the proxies for entry forecast aggregate returns with a statistically significant negative sign, while markups and profit shares forecast them with a statistically significant positive sign. We verify this empirical prediction in the data. The empirically estimated slope coefficients all have the predicted sign, and except for the profit share regressions, are statistically significant. We thus provide novel evidence on return predictability related to time-varying competitive pressure.

It is well-known that statistical inference in predictive regressions is complicated through small sample biases. In order to provide evidence on the sources of predictability in our model robust to these concerns, we repeat the predictability regressions in simulated samples with 150,000 observations. For simplicity, we only report evidence from projecting returns on log price-dividend ratios. Table 11 shows the results from these regressions across model specifications. While all slope coefficients casually replicate the empirical negative relationship between price-dividend ratios and future returns, statistically there are substantial differences between model specifications. In case of model (C) and a constant firm mass and markup, the slope coefficients are not significantly different from zero and the explanatory power of the regressions identically equal to zero. Allowing for entry and exit with constant markups (model B) yields statistically significant slope coefficients, but the explanatory power of the regressions is tiny. In our benchmark economy, in contrast, we find statistically significant evidence for negative slope coefficient and explanatory power increasing with horizon. Only our benchmark model with countercyclical markups thus provides robust evidence of countercyclical and predictable risk premia.

4.2 Extensions

Given the importance of markup dynamics for our asset pricing results, we next consider two extensions of the model that address properties of markups recently emphasized in the literature. Countercyclical movements in both price and wage markups are often recognized as the main source of fluctuations at higher frequency (e.g. Christiano, Eichenbaum and Evans (2005)). The objective of this section is to investigate which features of markups appear relevant through the lens of asset pricing. In a first extension, we consider price markup shocks, in a way often considered in the DSGE literature (e.g. Smets and Wouters (2003), Justiniano, Primiceri, and Tambalotti (2010)). Second, in addition to price markups, we consider wage markups, whose relevance has recently been pointed out in the context of New Keynesian macroeconomic models (e.g. Gali, Gertler, and Lopez-Salido (2007)). The two extensions also allow us to gain further intuition about the mechanisms underlying the risk premia and predictability results in the benchmark model.

4.2.1 Markup Shocks

In this section, we show that we need two ingredients to jointly generate a high risk premium and predictability in excess returns; markups need to be countercyclical and to generate conditional heteroskedasticity.

We start by considering exogenously stochastic price markups. To that end, we solve the version of the model without entry and exit and specify the markup process as

$$\log(\o_t) = (1-\rho_{\o})\log(\o) + \rho_{\o}\log(\o_{t-1}) + \sigma_{\o}u_t$$

where u_t is a standard normal i.i.d. shock that has a contemporaneous correlation of ϱ with ϵ_t. We investigate three cases, (i) constant price markups, (ii) uncorrelated time-varying markups, and (iii) countercyclical markups. We set $\o$, $\rho_{\o}$, and $\sigma_{\o}$ to match the unconditional mean, first autocorrelation, and unconditional standard deviation of $\o_t$ in the benchmark model.

Panels A, B, and C in table 12 report the main quantitative implications for asset returns and price-dividend ratios. The results are instructive. Panel B shows that while adding risk, introducing uncorrelated stochastic markups has a negligible impact on risk premia but increases significantly the volatility of the price dividend ratio. The additional risk is thus not priced. Consistent with the intuition developed earlier, the additional risk raises the precautionary savings motive and lowers the risk-free rate, but not sufficiently so. When markups are exogenously countercyclical, panel C shows that the risk premium goes up by close to one percent. In line with the intuition explained in the benchmark case, countercyclical markups give rise to long-run risks through their impact on firms' investment in technology.

A natural question to ask is whether exogenous countercyclical markups also generate return predictability, as with the endogenous markups in the benchmark model? Interestingly, the answer is no. Table 13 documents this by reporting the results from projecting future returns on log price-dividend ratios in models with exogenous stochastic models. The results in panels A,B, and C show that none of these specifications generate any predictability in long samples, even when accounting for countercyclicality. This is quite in contrast to the benchmark model with endogenous countercyclical markups.

Why is this so? The reason is that all exogenous markup specifications have symmetric effects on the economy. In our benchmark model, markups endogenously generate business cycle asymmetry. To further illustrate the importance of this property for predictibility, we solve a version of the model where the volatility of technology shocks is affected by the level of markups. In particular, we assume

$$\begin{aligned} a_t &= (1-\rho_a)a^\star + \rho_a a_{t-1} + \sigma_t \epsilon_t \\ \sigma_t &= \sigma(1+\kappa_{\o}\hat{\o}_t) \end{aligned}$$

where $\kappa_{\o} > 0$ captures the effects of markups on the conditional volatility of productivity shocks. We choose $\kappa_{\o}$ to approximately replicate the asymmetry generated by the benchmark model. Results from the simulation are reported in Tables 12 and 13, panel D. While the average risk premium is barely affected, markup induced heteroskedasticity generates excess stock return predictibility in the long sample.

4.2.2 Wage Markups

In addition to price markups, imperfect competition in labor markets reflected in wage markups increasingly plays an important role in current DSGE models. The dynamics of such wage markups is currently subject to a debate after an influential paper by Gali, Gertler, and Lopez-Salido (2007) which has argued that they should be countercyclical. In this section, we quantitatively explore the implications of dynamic wage markups for asset returns.

Formally, the wage markup is defined as the ratio of the real wage to the households marginal rate of substitution between labor and consumption,

$$\log(\o^w_t) = \log(W_t) - \log\left(\frac{\chi_0(1-\mathcal{L}_t)^{-\chi}}{\mathcal{C}_t^{-1/\psi}}\mathcal{Z}_t^{1-1/\psi}\right)$$

reflecting imperfect competition in the labor supply market. We specify the wage markup process exogenously as an AR(1) process in logs

$$\log(\o_t^w) \quad = \quad (1-\rho_\o^w)\log(\o^w) + \rho_\o^w \log(\o_{t-1}^w) + \sigma_\o^w u_t^w$$

where u_t^w is a standard normal i.i.d. shock that has a contemporaneous correlation of ϱ^w with ϵ_t.

We augment the benchmark model with wage markups and compare asset pricing moments and predictability results for two specifications: (i) uncorrelated time-varying markups, and (ii) countercyclical wage markup. We also investigate a third specification where the price markup is constant and the wage markups is countercyclical. We calibrate the markup process to match the standard deviation and first autocorrelation of the wage markup reported in Gali, Jordi, Gertler, and Lopez-Salido (2007): $\rho_\o^w = 0.96$, and $\sigma_\o^w = 2.88\%$. Whenever applicable, we set $\varrho^w = -0.45$ in order to replicate the -0.79 correlation between wage markups and output documented in Gali, Jordi, Gertler, and Lopez-Salido (2007). The steady state markup is set to 1.2 (see e.g., Comin and Gertler, 2006).

The main asset pricing implications are collected in table 14 and predictability results are reported in table 15. Accounting for wage markups in addition to endogenous countercyclical price markups amplifies priced risk and raises risk premia by around half a percentage point each. On the other hand, introducing wage markups only sharpens predictability when these are countercyclical. Similar to the case of exogenously countercyclical price markups considered above, with constant price markups and exogenously countercyclical wage markups generate additional priced risk, but do not create return predictability. This further highlights the importance of asymmetric cycles induced by endogenously countercyclical markups for predictability.

5 Conclusion

We embed a structural IO setup of imperfectly competitive industries into a general equilibrium production-based asset pricing model with recursive preferences. In the model, productivity growth is endogenously impacted through two channels. Endogenous innovation impacts productivity growth because new firms enter, that is *product innovation*, and because incumbent firms invest in upgrading their own production technology, that is *process innovation.* Entry and exit changes the number of active firms and industry concentration, and therefore competitive pressure, rendering markups countercyclical.

We show that process innovation generates significant long-run risks, while product innovation produces substantial short-run risks compensated in asset markets. Higher concentration raises firms' market power in downturns and makes industries more vulnerable to shocks, rendering aggregate risk markedly countercyclical. Our model implies a high and endogenously countercyclical equity premium in compensation for these risks, and a U-shaped term structure of equity returns. Empirically, we find that measures of entry and competitive pressure, such as markups, the net business formation and profit shares forecast equity returns over longer horizons, as predicted by the model.

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Table 1: Quarterly Calibration

Parameter	Description	Model
A. Preferences		
β	Subjective discount factor	0.9935
ψ	Elasticity of intertemporal substitution	2
γ	Risk aversion	10
χ	Labor elasticity	3
B. Production		
α	Capital share	0.33
η	Degree of technological appropriability	0.075
δ_k	Depreciation rate of capital stock	2.0%
δ_k	Depreciation rate of R&D stock	3.75%
δ_n	Firm obsolescence rate	1.0%
ζ_k	Capital adjustment cost parameter	0.825
ζ_z	R&D capital adjustment cost parameter	0.825
ν_1	Price elasticity accross industries	1.05
ν_2	Price elasticity within industries	75
C. Productivity		
ρ	Persistence of a_t	0.99
σ	Conditional volatility of a_t	1.15%

This table reports the parameter values used in the benchmark quarterly calibration of the model. The table is divided into three categories: Preferences, Production, and Productivity parameters.

Table 2: Industry moments

	Data	Model
A. Means		
$E[\log(\o)]$ (%)	13.39	13.53
$E[\Delta z_p]$ (%)	1.26	2.00
E[Profit Share] (%)	7.04	8.33
B. Standard deviations		
$\sigma[\log(\o)]$ (%)	2.30	2.14
$\sigma[\Delta z_p]$ (%)	1.74	2.37
$\sigma[\Delta z]$ (%)	1.05	0.82
σ[Profit Share] (%)	2.18	1.97
$\sigma[NE]$	0.06	0.05
C. Autocorrelations		
$AC1[\log(\o)]$	0.900	0.998
$AC1[\Delta z_p]$	0.159	0.123
$AC1[\Delta z]$	0.958	0.988
$AC1$[Profit Share]	0.955	0.998
$AC1[NE]$	0.701	0.686
D. Correlations		
$\text{corr}(\log(\o), N)$	-0.139	-0.200
$\text{corr}(\log(\o), NE)$	-0.101	-0.016

This table presents the means, standard deviations, autocorrelations, for key macroeconomic variables from the data and the model. The model is calibrated at a quarterly frequency using the benchmark calibration. The growth rate of technology has been annualized (Δz_p). To obtain a stationary, unit-free measure of entry, $\log(NE)$ is filtered using a Hodrick-Prescott filter with a smoothing parameter of 1,600.

Table 3: Productivity growth decomposition

	(1)		(2)		(3)	
	Data	Model	Data	Model	Data	Model
Δz_t	0.317	1.465			0.376	1.432
	0.100	0.265			0.134	0.107
$\Delta \log(INC_t)$			0.072	0.238	0.074	0.238
			0.018	0.007	0.018	0.004
Constant	-0.002	-0.002	0.003	0.004	-0.004	-0.002
	0.002	0.001	0.001	0.001	0.002	0.000
R^2	0.038	0.137	0.075	0.823	0.125	0.959

This table presents quarterly productivity growth decomposition by estimating the regressions $\Delta z_{p,t} = \beta_0 + \beta_z \Delta z_t + \beta_{ne} \Delta \log(NE_t) + \epsilon_t$. $\Delta z_{p,t}$ is measured productivity, Δz_t is the growth rate in the stock of technology, and $\Delta \log(NE_t)_t$ is measured as the growth in new business incorporations ($\Delta \log(INC_t)$).

Table 4: Productivity growth decomposition

	(1)		(2)		(3)	
	Data	Model	Data	Model	Data	Model
Δz_t	0.317	1.465			0.380	0.612
	0.100	0.265			0.135	0.232
$\Delta \log(NBF_t)$			0.126	8.257	0.114	7.489
			0.030	0.908	0.031	0.850
Constant	-0.002	-0.002	0.003	0.005	-0.003	0.002
	0.002	0.001	0.001	0.001	0.002	0.001
R^2	0.038	0.137	0.086	0.318	0.111	0.356

This table presents quarterly productivity growth decomposition by estimating the regressions $\Delta z_{p,t} = \beta_0 + \beta_z \Delta z_t + \beta_n \Delta n_t + \epsilon_t$. $\Delta z_{p,t}$ is measured productivity, Δz_t is the growth rate in the stock of technology, and Δn_t is measured as the growth in net business formation ($\Delta \log(NBF_t)$).

Table 5: Forecasts with growth of new incorporations

	Data			Model		
	Horizon (in quarters)					
	1	4	8	1	4	8
A. Output						
β	0.235	0.429	0.108	0.835	3.386	6.870
S.E.	0.034	0.107	0.165	0.446	1.686	3.633
R^2	0.242	0.118	0.004	0.006	0.016	0.024
B. Consumption						
β	0.071	0.206	0.157	2.641	9.401	16.380
S.E.	0.013	0.049	0.064	0.192	1.229	3.086
R^2	0.012	0.125	0.036	0.198	0.192	0.160
C. Investment						
β	1.277	1.980	0.225	1.329	4.765	8.572
S.E.	0.213	0.549	0.785	0.694	2.369	4.767
R^2	0.268	0.119	0.001	0.006	0.016	0.021

This table presents output growth, consumption growth, and investment growth forecasts for horizons of one, four, and eight quarters using the growth in net business formation from the data and the model. The n-quarter regressions, $\frac{1}{n}(x_{t,t+1} + \cdots + x_{t+n-1,t+n}) = \alpha + \beta \Delta n_t + \epsilon_{t+1}$, are estimated using overlapping quarterly data and Newey-West standard errors are used to correct for heteroscedasticity.

Table 6: Business cycle moments

	Data	A.	B.	C.
First Moment				
$E(\Delta c)$	2.00	2.00	2.00	2.00
Second Moment				
$\sigma_{\Delta c}/\sigma_{\Delta y}$	0.64	0.49	0.43	1.01
$\sigma_{\Delta i}/\sigma_{\Delta c}$	4.38	3.19	1.58	1.31
$\sigma_{\Delta s}/\sigma_{\Delta c}$	3.44	2.88	1.56	1.19
$\sigma(\Delta c)$	1.11	1.11	1.11	1.11
$\sigma(l)$	1.52	2.16	1.90	0.63

This table reports simulated moments for three specifications of the model. Column A reports model moments for the benchmark model. Column B reports model moments for the benchmark model where the price markup is constant and set to its steady state value. Column C reports model moments for the model without entry and exit. To keep the comparison fair, we recalibrate $a^{\star}$ and σ to match the first and second moments of realized consumption growth. The risk premiums are levered following Boldrin, Christiano, and Fisher (2001). Growth rate moments are annualized percentage. Moments for log-hours (l) are reported in percentage of total time endowment.

Table 7: Industry cycles

	Data	Model
A. Markups		
corr($\o, Y$)	-0.174	-0.133
corr($\o, C$)	-0.283	-0.203
corr($\o, I$)	-0.164	-0.132
B. Number of firms		
corr(N, Y)	0.708	0.696
corr(N, C)	0.638	0.947
corr(N, I)	0.701	0.682
C. Entry		
corr(NE, Y)	0.449	0.811
corr(NE, C)	0.397	0.202
corr(NE, I)	0.487	0.824

This table reports correlations for key macro variables with aggregate markups ($\o$), the number of firms (NBF: Index of net business formation, and entry (INC: total number of new incorporations) for the data and the model. The model is calibrated at a quarterly frequency and all reported statistics are computed after applying an Hodrick-Prescott filter with a smoothing parameter of 1,600 to the log of all non-stationary variables.

Table 8: Summary statistics sorted on markups

	Data		Model	
	low $\o_t$	high $\o_t$	low $\o_t$	high $\o_t$
A. Output				
mean	0.436	-0.019	0.163	-0.312
std	1.030	1.970	1.255	1.333
min	-1.275	-3.798	-4.796	-4.943
max	2.319	3.536	4.726	4.445
B. Consumption				
mean	0.450	-0.158	0.175	-0.291
std	0.748	0.805	0.575	0.750
min	-0.543	-1.406	-1.569	-2.976
max	1.820	1.083	2.021	2.071
C. Investment				
mean	1.335	-0.753	0.256	-0.503
std	4.434	9.411	1.920	2.392
min	-9.264	-21.177	-7.197	-8.808
max	8.562	11.827	7.047	7.905

This table presents summary statistics for output, consumption, and investment by sorting the data on the level of markup. All non-stationary data are detrended using a Hodrick-Prescott filter with a smoothing parameter of 1,600. All units are percentage deviation from trend.

Table 9: Asset pricing moments

	Data	A.	B.	C.
First Moment				
$E(r_f)$	1.62	1.36	3.07	3.11
$E(r_d - r_f)$	5.84	5.26	2.21	1.30
$E[pd]$	3.43	3.72	3.76	3.98
Second Moment				
$\sigma(r_f)$	0.67	0.50	0.36	0.12
$\sigma(r_d - r_f)$	17.87	6.06	5.54	5.59
$\sigma[pd]$	0.37	0.22	0.07	0.04

This table reports simulated moments for three specifications of the model. Column A reports model moments for the benchmark model. Column B reports model moments for the benchmark model where the price markup is constant and set to its steady state value. Column C reports model moments for the model without entry and exit. To keep the comparison fair, we recalibrate $a^\star$ and σ to match the first and second moments of realized consumption growth. The risk premiums are levered following Boldrin, Christiano, and Fisher (2001). Returns are in annualized percentage units.

Table 10: Stock Return Predictability

	Data					Model				
	Horizon (in years)									
	1	2	3	4	5	1	2	3	4	5
A. Log Price-Dividend Ratio										
$\beta^{(n)}$	-0.132	-0.231	-0.292	-0.340	-0.430	-0.061	-0.116	-0.170	-0.221	-0.268
S.E.	0.041	0.078	0.099	0.112	0.135	0.032	0.056	0.076	0.092	0.107
R^2	0.090	0.157	0.193	0.214	0.254	0.044	0.087	0.128	0.168	0.205
B. Net Business Formation										
$\beta^{(n)}$	-0.770	-1.006	-1.059	-1.255	-1.790	-0.638	-1.220	-1.783	-2.324	-2.815
S.E.	0.248	0.385	0.375	0.411	0.590	0.348	0.619	0.836	1.013	1.172
R^2	0.121	0.122	0.108	0.121	0.166	0.042	0.081	0.120	0.158	0.193
C. Growth in New Incorporations										
$\beta^{(n)}$	-0.396	-0.866	-1.049	-1.219	-1.323	-0.638	-1.220	-1.783	-2.324	-2.815
S.E.	0.248	0.449	0.634	0.656	0.717	0.348	0.619	0.836	1.013	1.172
R^2	0.008	0.024	0.028	0.031	0.024	0.042	0.081	0.120	0.158	0.193
D. Markup										
$\beta^{(n)}$	1.516	2.571	2.747	3.529	4.185	0.482	0.928	1.356	1.760	2.128
S.E.	0.651	1.052	1.301	1.577	2.016	0.266	0.467	0.626	0.761	0.891
R^2	0.043	0.075	0.071	0.102	0.116	0.040	0.078	0.116	0.154	0.187
E. Profit Share										
$\beta^{(n)}$	0.298	0.733	0.943	1.678	2.135	0.444	0.847	1.238	1.607	1.944
S.E.	0.620	1.056	1.554	2.375	3.060	0.233	0.412	0.553	0.666	0.765
R^2	0.001	0.005	0.007	0.018	0.022	0.043	0.085	0.126	0.166	0.203

This table reports excess stock return forecasts for horizons of one to five years, i.e. $r^{ex}_{t,t+n} - y^{(n)}_t = \alpha_n + \beta\ x_t + \epsilon_{t+1}$, where x_t is the predicting variables. The different panels present forecasting regressions using different predicting variables: the log price-dividend ratio (panel A), the linearly detrended index of net business formation (panel B), the growth in new incorporations (panel C), price markups (panel D), and the profit share (panel E). The forecasting regressions use overlapping quarterly data. Newey-West standard errors are used to correct for heteroscedasticity. The estimates from the model regression are averaged across 100 simulations that are equivalent in length to the data sample. The sample is 1948-2013 for Panel A and E, 1948-1993 for panel B and C, and 1964-2013 for panel D. The risk premiums are levered following Boldrin, Christiano, and Fisher (2001).

Table 11: Stock Return Predictability in the Long Sample

	Data				
	Horizon (in years)				
	1	2	3	4	5
A. Benchmark					
$\beta^{(n)}$	-0.013	-0.025	-0.036	-0.047	-0.058
S.E.	0.003	0.005	0.008	0.011	0.013
R^2	0.005	0.010	0.015	0.019	0.023
B. Constant Markup					
$\beta^{(n)}$	-0.010	-0.018	-0.027	-0.036	-0.043
S.E.	0.003	0.005	0.008	0.010	0.013
R^2	0.000	0.000	0.001	0.001	0.001
C. No Entry/Exits					
$\beta^{(n)}$	-0.002	-0.002	-0.003	-0.004	-0.007
S.E.	0.004	0.007	0.010	0.014	0.017
R^2	0.000	0.000	0.000	0.000	0.000

This table reports excess stock return forecasts for horizons of one to five years using the log-price-dividend ratio: $r^{ex}_{t,t+n} - y^{(n)}_t = \alpha_n + \beta \ \log(P_t/D_t) + \epsilon_{t+1}$. Panel A presents the forecasting regressions for the benchmark model with time-varying markup, panel B presents the regression results for the model with entry and exit dynamics but constant price markup, and panel C presents regression results for the model without entry and exit and constant price markup. The forecasting regressions use overlapping quarterly data. The risk premiums are levered following Boldrin, Christiano, and Fisher (2001). The estimates are obtained using 150,000 simulated observations.

Table 12: Asset pricing moments: exogenous markups

	A.	B.	C.	D.
First Moment				
$E(r_f)$	3.12	2.76	2.15	2.08
$E(r_d - r_f)$	1.30	1.47	2.39	2.41
$E(pd)$	3.98	3.99	3.89	3.87
Second Moment				
$\sigma(r_f)$	0.13	0.20	0.25	0.35
$\sigma(r_d - r_f)$	3.73	6.44	5.54	5.94
$\sigma(pd)$	0.04	0.18	0.19	0.21

This table reports asset pricing moments for four specifications of the model with exogenous markups. Column A reports model moments for the model with constant markups ($\sigma_{\o} = 0$, $\varrho = 0$, and $\kappa_{\o} = 0$). Column B reports model moments for the time-varying markup model ($\sigma_{\o} = 0.25\%$, $\varrho = 0$, and $\kappa_{\o} = 0$). Column C reports model moments for the model with countercyclical markups ($\sigma_{\o} = 0.25\%$, $\varrho = -0.9$, and $\kappa_{\o} = 0$). Column D reports moment for the model with countercyclical markups and business cycle asymmetry ($\sigma_{\o} = 0.25\%$, $\varrho = -0.9$, and $\kappa_{\o} = 5$). The risk premiums are levered following Boldrin, Christiano, and Fisher (2001).

Table 13: Stock return predictability: exogenous markups

	Horizon (in years)				
	1	2	3	4	5
A. Constant markup					
$\beta^{(n)}$	-0.002	-0.002	-0.003	-0.004	-0.007
S.E.	0.004	0.007	0.010	0.014	0.017
R^2	0.000	0.000	0.000	0.000	0.000
B. Time-varying, uncorrelated $\o_t$					
$\beta^{(n)}$	0.001	0.001	0.001	0.001	0.001
S.E.	0.001	0.002	0.013	0.004	0.004
R^2	0.000	0.000	0.000	0.000	0.000
C. Countercyclical $\o_t$					
$\beta^{(n)}$	0.001	0.003	0.004	0.004	0.005
S.E.	0.001	0.001	0.002	0.003	0.004
R^2	0.000	0.000	0.000	0.000	0.000
D. Countercyclical and heteroskedastic $\o_t$					
$\beta^{(n)}$	-0.019	-0.037	-0.056	-0.074	-0.092
S.E.	0.001	0.002	0.002	0.029	0.004
R^2	0.010	0.020	0.030	0.040	0.049

This table reports excess stock return forecasts in the model with exogenous price markup for horizons of one to five years using the log-price-dividend ratio: $r^{ex}_{t,t+n} - y^{(n)}_t = \alpha_n + \beta \, \log(P_t/D_t) + \epsilon_{t+1}$. Panel A reports the forecasting regressions for the model with constant markup ($\sigma_{\o} = 0$, $\rho_{\epsilon,u} = 0$, $\kappa_{\o} = 0$). Panel B reports the forecasting regressions for the model with uncorrelated time-varying markup ($\sigma_{\o} = 0.25\%$, $\rho_{\epsilon,u} = 0$, $\kappa_{\o} = 0$). Panel C reports the forecasting regressions for the model with countercyclical markup ($\sigma_{\o} = 0.25\%$, $\rho_{\epsilon,u} = -0.9$, $\kappa_{\o} = 0$). Panel D reports the forecasting regressions for the model with countercyclical and heterosketasticity-inducing markups ($\sigma_{\o} > 0$, $\rho_{\epsilon,u} = -0.9$, $\kappa_{\o} = 5$). The risk premiums are levered following Boldrin, Christiano, and Fisher (2001). The estimates are obtained using 150,000 simulated observations.

Table 14: Asset pricing moments: wage markup

	A.	B.	C.	D.
First Moment				
$E(r_f)$	1.36	0.99	0.03	2.68
$E(r_d - r_f)$	5.26	5.51	6.63	2.33
$E(pd)$	3.72	3.63	3.44	3.87
Second Moment				
$\sigma(r_f)$	0.50	0.56	0.66	0.12
$\sigma(r_d - r_f)$	6.06	8.59	11.61	4.07
$\sigma(pd)$	0.22	0.23	0.24	0.41

This table reports asset pricing moments for four specifications of the model with wage markups as well as the benchmark model. Column A reports moments for the benchmark model. Column B reports model moments for the benchmark model with time-varying, uncorrelated wage markup ($\sigma_{\o}^{w} = 2.88\%$, $\rho_{\o}^{w} = 0.96$, and $\varrho^{w} = 0$). Column C reports moments for the benchmark model with countercyclical wage markup ($\sigma_{\o}^{w} = 2.88\%$, $\rho_{\o}^{w} = 0.96$, and $\varrho^{w} = -0.45$). Column D reports model moments for the model with constant price markup and countercyclical wage markup ($\sigma_{\o}^{w} = 2.88\%$, $\rho_{\o}^{w} = 0.96$, and $\varrho^{w} = -0.45$). The risk premiums are levered following Boldrin, Christiano, and Fisher (2001).

Table 15: Stock return predictability: wage markup

	Horizon (in years)				
	1	2	3	4	5
A. Benchmark					
$\beta^{(n)}$	-0.013	-0.025	-0.036	-0.047	-0.058
S.E.	0.003	0.005	0.008	0.011	0.013
R^2	0.005	0.010	0.015	0.019	0.023
B. Time-varying, uncorrelated $ø_t^w$					
$\beta^{(n)}$	-0.014	-0.029	-0.043	-0.057	-0.068
S.E.	0.003	0.006	0.009	0.013	0.015
R^2	0.005	0.011	0.016	0.021	0.024
C. Countercyclical $ø_t^w$					
$\beta^{(n)}$	-0.028	-0.056	-0.081	-0.104	-0.125
S.E.	0.004	0.008	0.011	0.014	0.017
R^2	0.014	0.028	0.040	0.051	0.061
D. Countercyclical $ø_t^w$, constant $ø_t$					
$\beta^{(n)}$	-0.065	-0.142	-0.205	-0.239	-0.263
S.E.	0.051	0.096	0.137	0.167	0.197
R^2	0.000	0.001	0.001	0.001	0.001

This table reports excess stock return forecasts in the model with exogenous wage markup for horizons of one to five years using the log-price-dividend ratio: $r^{ex}_{t,t+n} - y^{(n)}_t = \alpha_n + \beta\ \log(P_t/D_t) + \epsilon_{t+1}$. Panel A reports the forecasting regressions for the benchmark model. Panel B reports the forecasting regressions for the benchmark model with time-varying, uncorrelated wage markup ($\sigma^w_ø = 2.88\%$, $\rho^w_ø = 0.96$, and $\varrho^w = 0$). Panel C reports the forecasting regressions for the benchmark model with countercyclical wage markup ($\sigma^w_ø = 2.88\%$, $\rho^w_ø = 0.96$, and $\varrho^w = -0.45$). Panel D reports the forecasting regressions for the model with constant price markup and countercyclical wage markup ($\sigma^w_ø = 2.88\%$, $\rho^w_ø = 0.96$, and $\varrho^w = -0.45$). The risk premiums are levered following Boldrin, Christiano, and Fisher (2001). The estimates are obtained using 150,000 simulated observations.

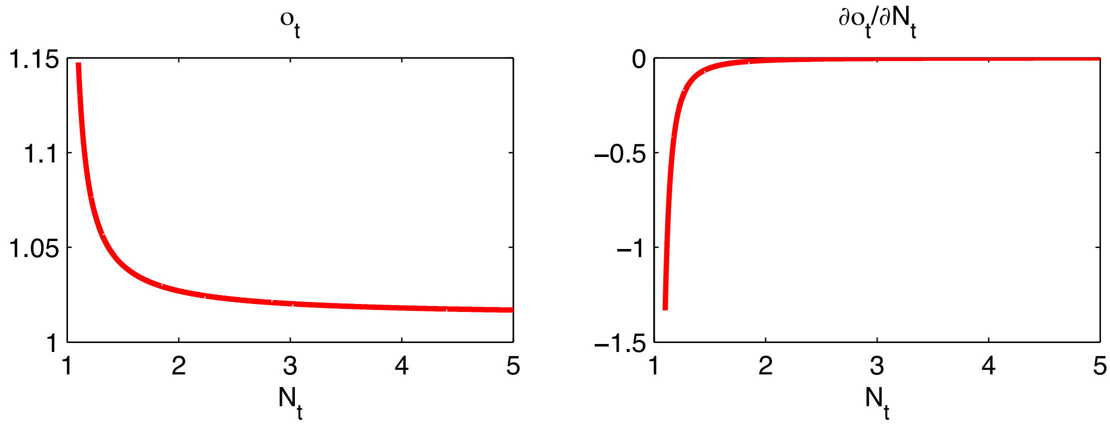

Figure 1: This figure plots the markup (left) and the first derivative of the markup with respect to $\mathcal{N}_t$ (left) as a function of the number of firms ($\mathcal{N}_t$) for the benchmark calibration of the model.

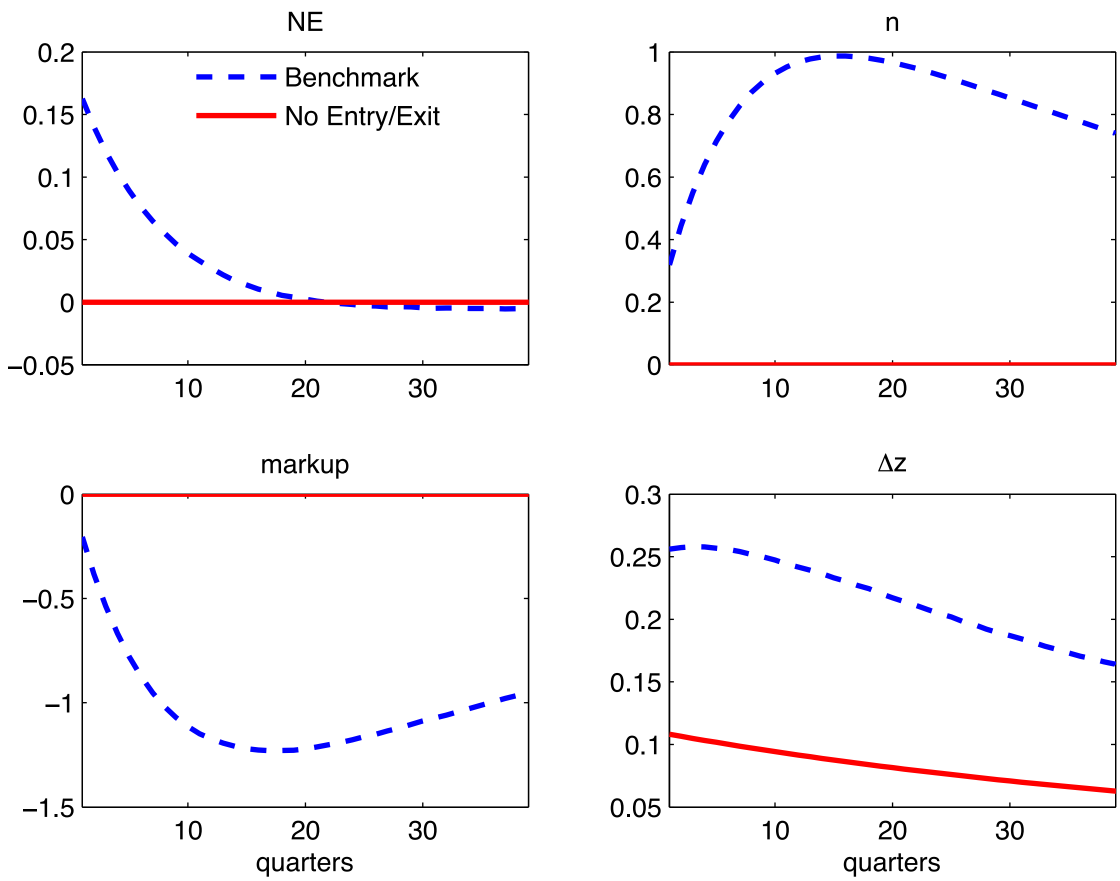

Figure 2: This figure plots the impulse response functions for entry (NE), the number of firms (n), the price markup, and the growth of technology (Δz) to a positive one standard deviation productivity shock for the benchmark model (dashed line), and the model without entry and exit (solid line). The parameters used to solve the no entry/exit model are the same as the benchmark model except for $a^{\star}$ that is modified to ensure an average growth rate of 2%, and σ that is modified to get a consumption growth volatility of 1.11%. All values on the y-axis are in annualized percentage log-deviation from the steady state.

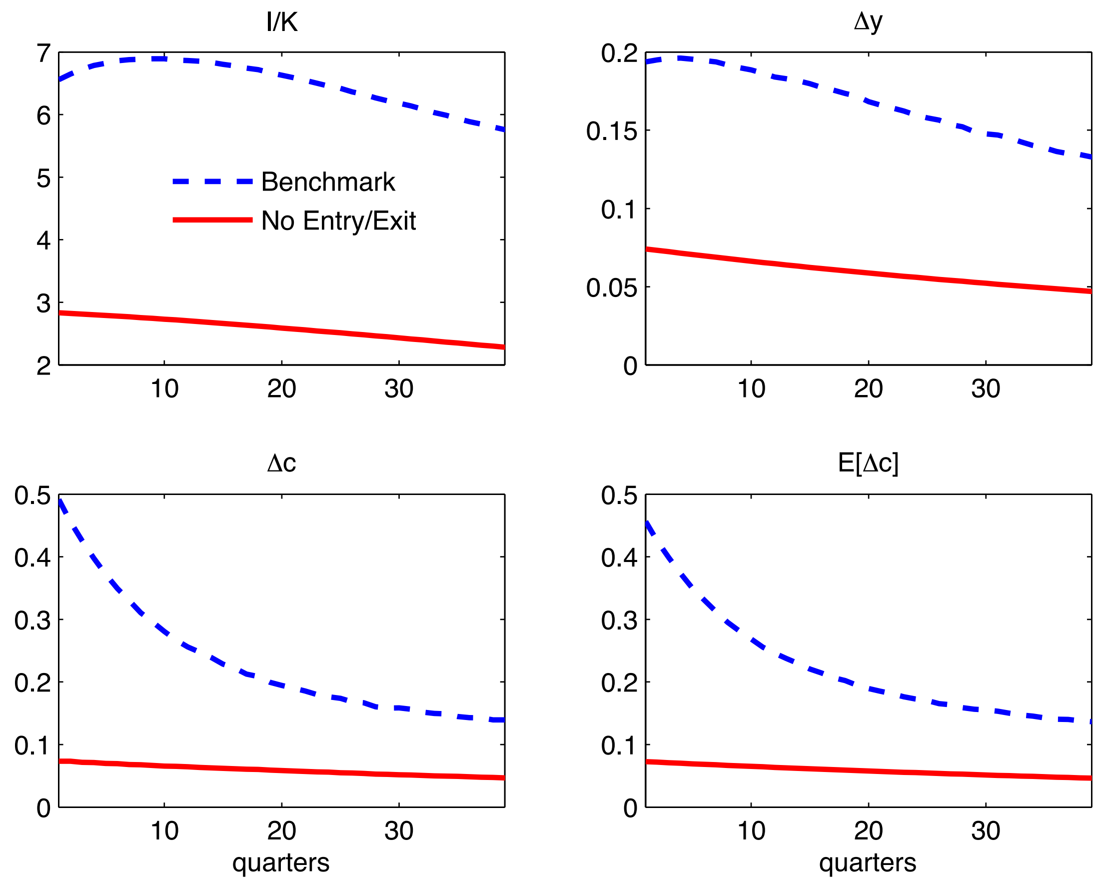

Figure 3: This figure plots the impulse response functions for the investment-to-capital ratio (I/K), output growth (Δy), consumption growth (Δc), and expected consumption growth ($E[\Delta c]$) to a positive one standard deviation productivity shock for the benchmark model (dashed line), and the model without entry and exit (solid line). The parameters used to solve the no entry/exit model are the same as the benchmark model except for $a^{\star}$ that is modified to ensure an average growth rate of 2%, and σ that is modified to get a consumption growth volatility of 1.11%. All values on the y-axis are in annualized percentage log-deviation from the steady state.

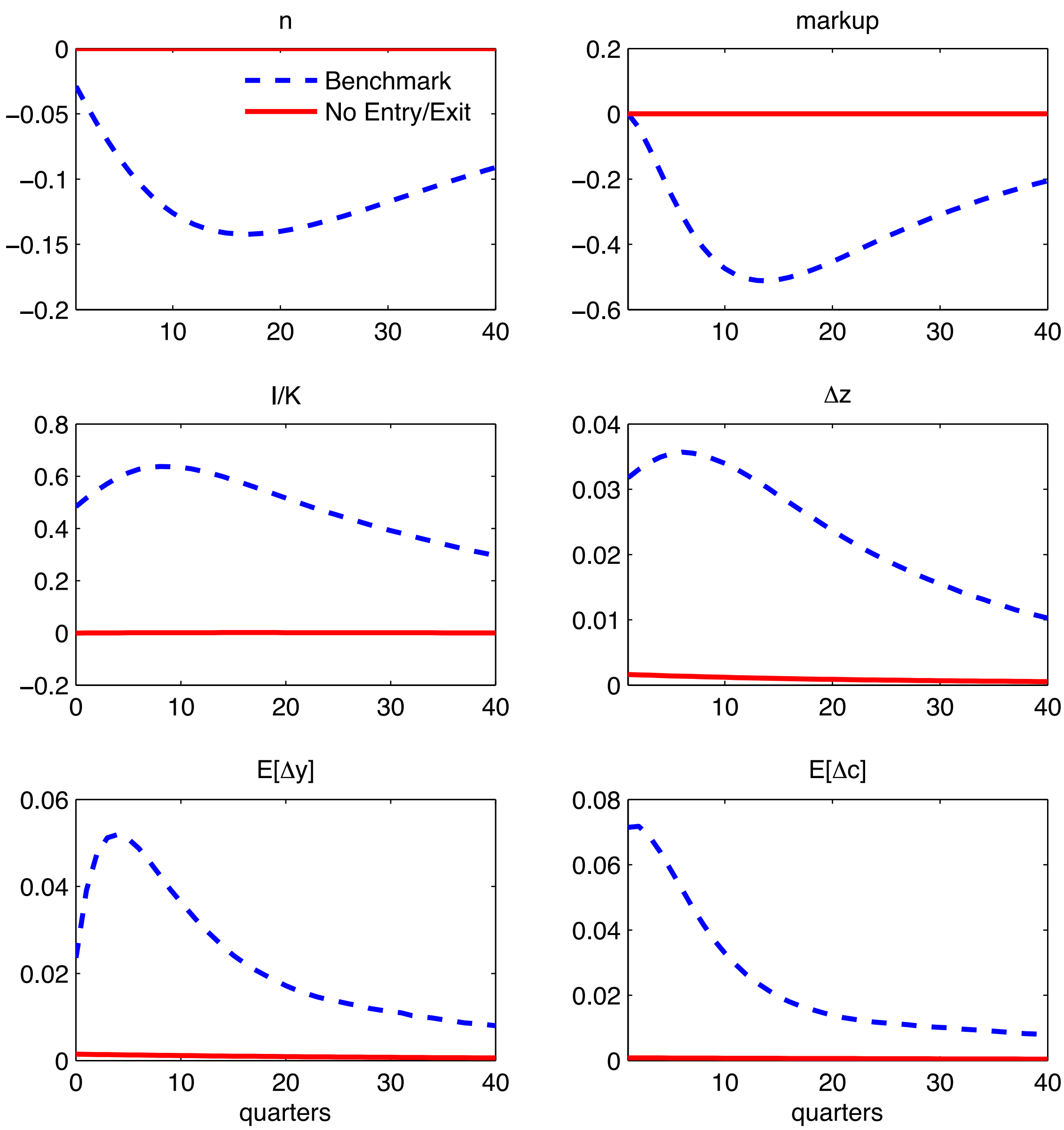

Figure 4: This figure plots the asymmetry in impulse response functions for the number of firms (n_t), the price markup, the investment-to-capital ratio (I/K), the growth in technology (Δz), and the expected growth rate of output ($E[\Delta y]$) and consumption ($E[\Delta c]$) in the benchmark model (dashed line), and the model without entry and exit (solid line). The graphs are obtained by taking the difference between minus the response to a two standard deviation negative productivity shock and the response to a positive two standard devitation shock. The parameters used to solve the no entry/exit model are the same as the benchmark model except for $a^\star$ that is modified to ensure an average growth rate of 2%, and σ that is modified to get a consumption growth volatility of 1.11%. All values on the y-axis are in annualized percentage log-deviation from the steady state.

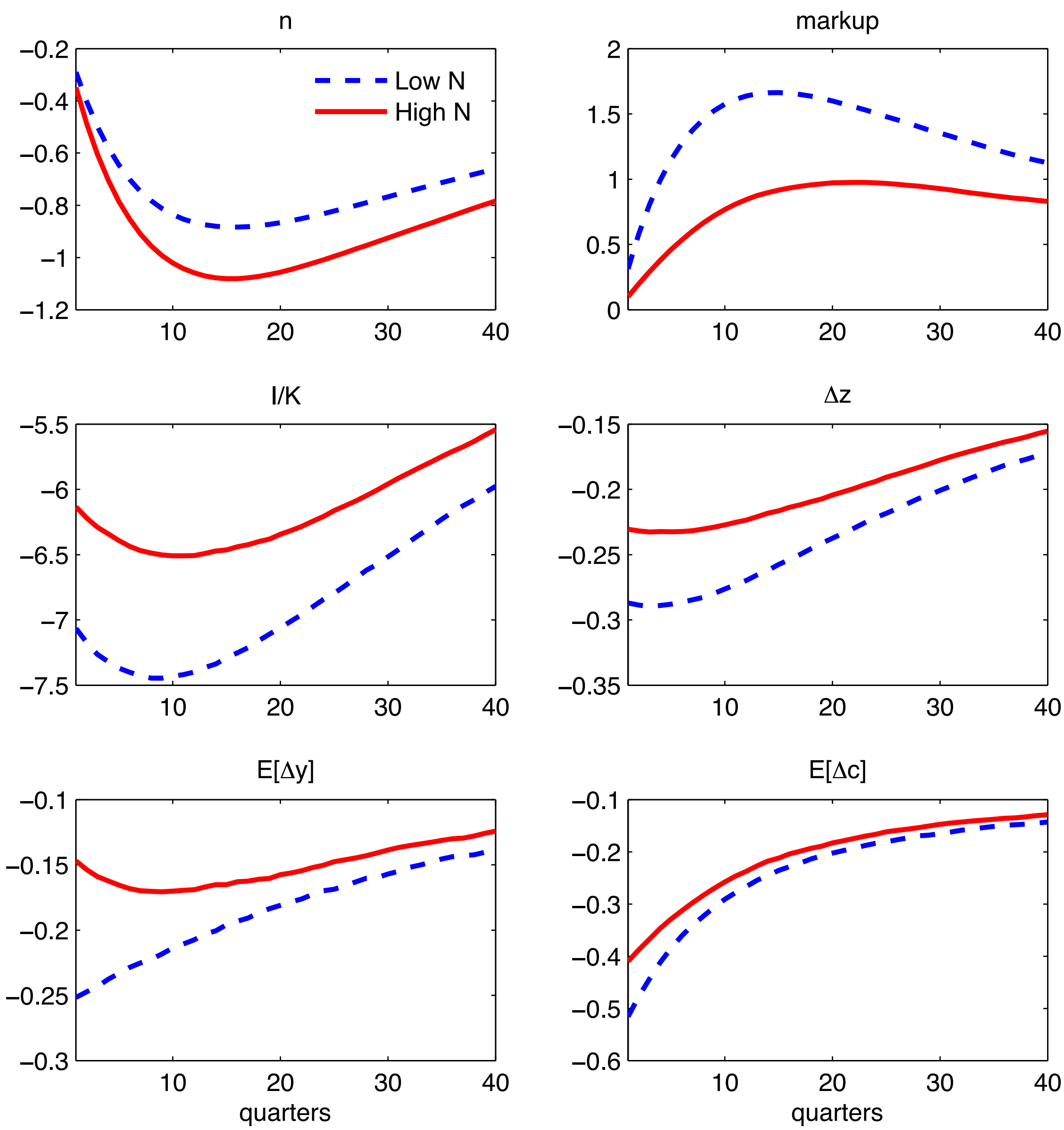

Figure 5: This figure plots the impulse response functions for the number of firms (n_t), the price markup, the investment-to-capital ratio (I/K), the growth in technology (Δz), and the expected growth rate of output ($E[\Delta y]$) and consumption ($E[\Delta c]$) in the benchmark model to a negative one standard deviation technology shock as a function of the number of firms in the economy, $\mathcal{N}_t$. The high N (low N) case corresponds to the average responses accross 250 draws in the highest (lowest) quintile sorted on $\mathcal{N}_t$. The data for the sorting is obtained by simulating the economy for 50 periods prior to the realization of the negative technology shock. All values on the y-axis are in annualized percentage log-deviation from the steady state.

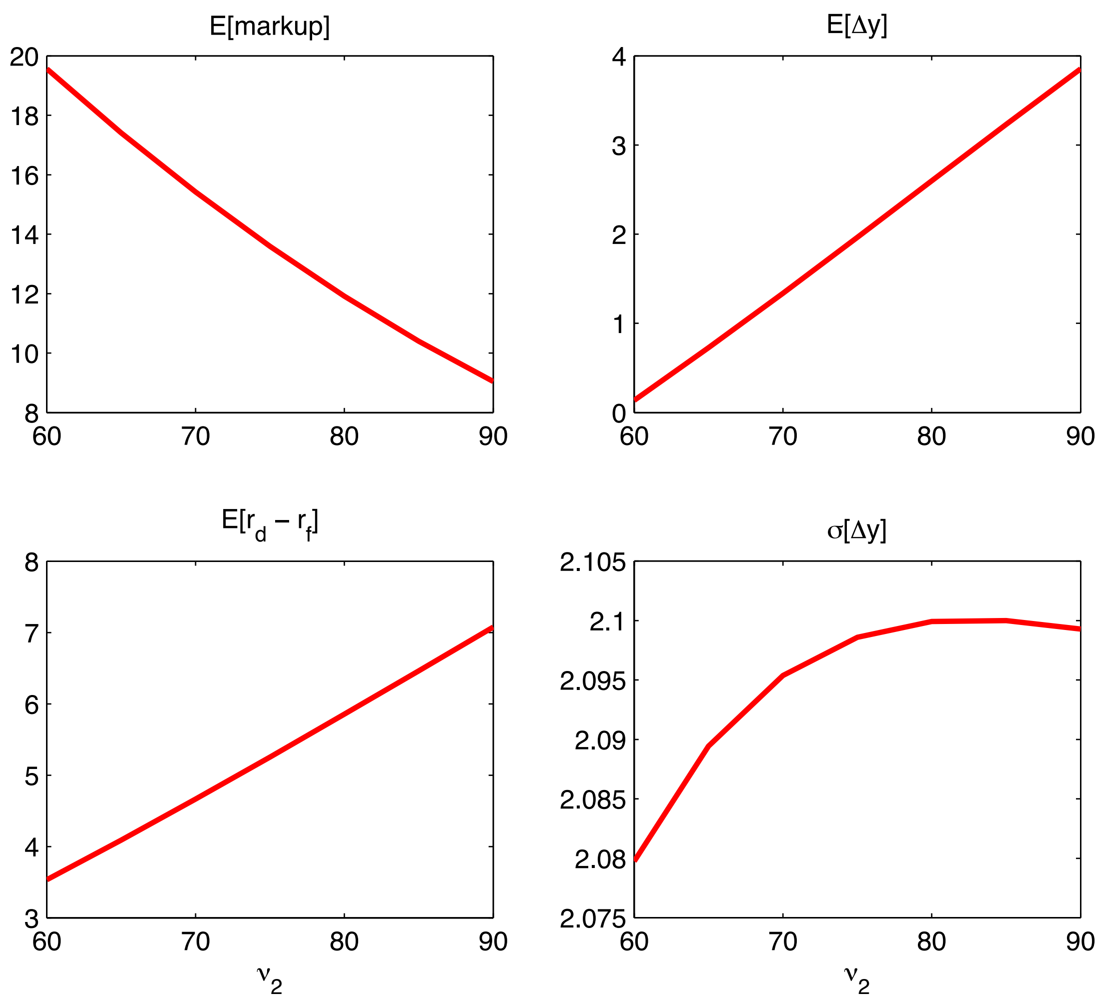

Figure 6: This figure plots the impact of varying the degree of competition within industry ν_2 on the average markup, the average output growth, the average equity premium, and the volatility of output growth. Values on y-axis are in annualized percentage units for expected consumption growth and the equity premium and in percentage units for the price markup.

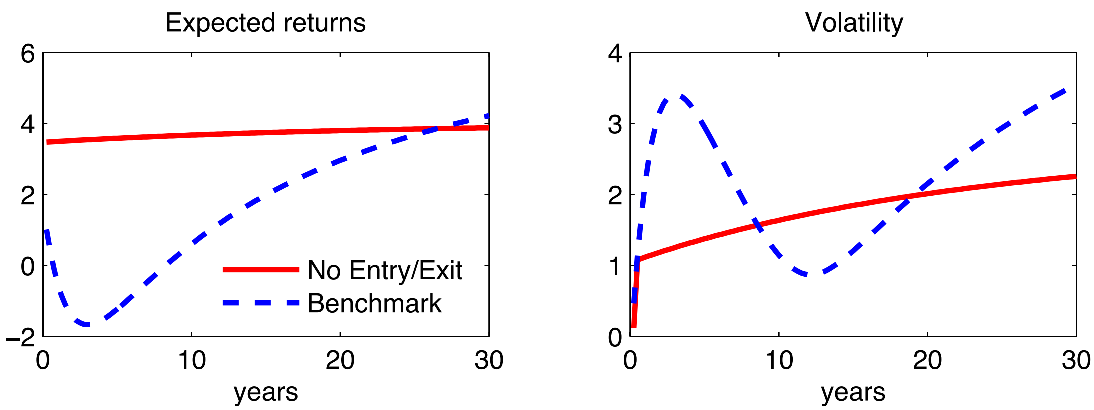

Figure 7: This figure plots the term structure of equity returns and of equity volatility in the benchmark model and in the model without entry and exit (constant markups). The parameters used to solve the no entry/exit model are the same as the benchmark model except for $a^\star$ that is modified to ensure an average growth rate of 2%, and σ that is modified to get a consumption growth volatility of 1.11%. All values on the y-axis are in annualized percentage.

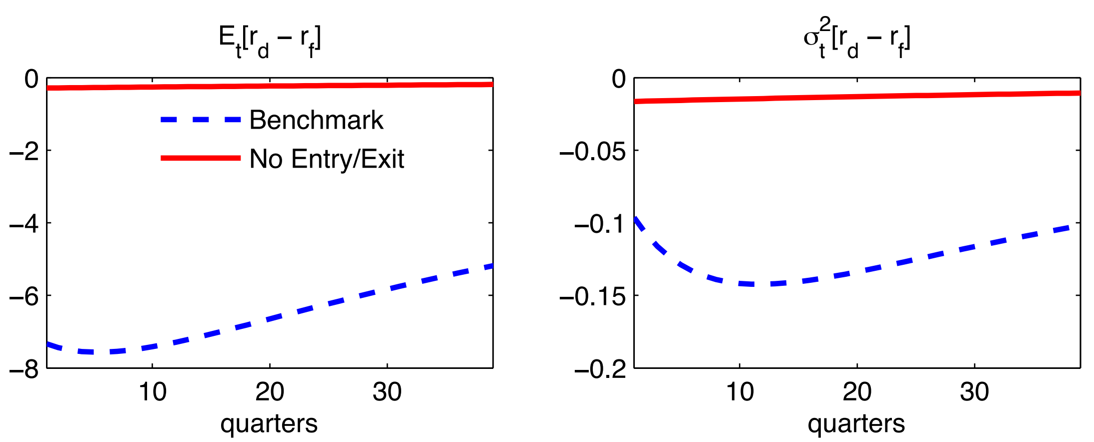

Figure 8: This figure plots the impulse response functions for the conditional risk premium ($E_t[r_d - r_f]$), and the conditional variance of the risk premium ($\sigma^2[r_d - r_f]$) to a positive one standard deviation productivity shock for the benchmark model (dashed line), and the model without entry and exit (solid line). The parameters used to solve the no entry/exit model are the same as the benchmark model except for $a^\star$ that is modified to ensure an average growth rate of 2%, and σ that is modified to get a consumption growth volatility of 1.11%. All values on the y-axis are in annualized percentage log-deviation from the steady state.

7 Appendix A: Data Sources

Quarterly data for consumption, capital investment, and GDP are from the Bureau of Economic Analysis (BEA). Annual data on private business R&D investment are from the survey conducted by the National Science Foundation. Annual data on the stock of private business R&D are from the Bureau of Labor Statistics. Real annual capital stock data is obtained from the Penn World Table. Quarterly productivity data are from Fernald (2009) (Federal Reserve Bank of San Francisco) and is measured as Business sector total factor productivity. The labor share and average weekly hours are obtained from the Bureau of Labor Statistics (BLS). The monthly index of net business formation (NBF) and number of new business incorporations (INC) are from the U.S. Basic Economics Database. Consumption is measured as expenditures on nondurable goods and services. Capital investment is measured as private fixed investment. Output is measured as GDP. The labor share is defined as the business sector labor share. Average weekly hours is measured for production and nonsupervisory employees of the total private sector. The variables are converted to real using the Consumer Price Index (CPI), which is obtained from the Center for Research in Security Prices (CRSP). Annual data are converted into quarterly data by linear interpolation. The inflation rate is computed by taking the log return on the CPI index. The sample period is for 1948-2013, except for the average weekly hours series which starts in 1964 and the NBF and INC series that were discontinued in 1993.

Monthly nominal return and yield data are from CRSP. The real market return is constructed by taking the nominal value-weighted return on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX) and deflating it using the CPI. The real risk-free rate is constructed by using the nominal average one-month yields on treasury bills and taking out expected inflation[11]. Aggregate market and dividend values are from CRSP. The price dividend ratio is constructed by dividing the current aggregate stock market value by the sum of the dividends paid over the preceding 12 months.

[11] The monthly time series for expected inflation is obtained using an AR(4).

7.1 Markup measure

Solving the intermediate producer problem links the price markup to the inverse of the marginal cost of production MC_t,

$$\text{\o}_t = \frac{1}{MC_t}$$

In equilibrium, MC_t is equal to the ratio of marginal cost over marginal product of each production input (see the cost mininization problem). Since data on wages are available at the aggregate level, the labor input margin has been the preferred choice in the literature. Using the first order condition with respect to L_t and imposing the symmetry condition,

$$\text{\o}_t = (1-\alpha)\frac{\mathcal{Y}_t}{\mathcal{L}_t W_t} = (1-\alpha)\frac{1}{S_{L,t}}$$

where $S_{L,t}$ is the labor share.

The inverse of the labor share should thus be a good proxy for the price markup. However, there are many reasons why standard assumptions may lead to biased estimates of the markup (see Rotemberg and Woodford (1999)). In this paper, we follow Campello (2003) by focusing on non-linearities in the cost of labor[12]. More specifically, when deriving the cost function, we assumed that the firm was able to hire all workers at the marginal wage. In practice however, the total wage paid $W(L_t)$, is likely to be convex in hours (e.g. Bils (1987)). This creates a wedge between the average and marginal wage that makes the labor share a biased estimate of the real marginal cost. Denoting this wedge by $\omega_t = W'(L_t)/(W(L_t)/L_t)$, the markup becomes,

$$\text{\o}_t = (1-\alpha)\frac{1}{S_{L,t}}\,\omega_t^{-1}$$

Log-linearizing this expression around the steady state,

$$\hat{\text{\o}}_t = -\hat{s}_{L,t} - \omega_L \hat{l}_t$$

where ω_L is the steady state elacticity of ω_t with respect to average hours. Bils (1987) proposes a

[12]Rotemberg and Woodford (1999) presents several other reasons that makes marginal costs more procyclical than the labor share (e.g. non-Cobb-Douglas production technology, overhead labor, etc.). For robustness, we tried additional corrections. Overall, they make markups even more countercyclical, and further strengthen our empirical results.

simple model of overtime. Assuming a 50% overtime premium[13] he estimates the elasticity ω_L to be 1.4. We use this value to build our overtime measure of the price markups. We set the steady state values for L_t and $S_{L,t}$ to 40 hours and 100[14], respectively and linearly detrend the series.

8 Appendix B: Derivation of demand schedule

Final goods sector The final goods firm solves the following profit maximization problem

$$\max_{\{Y_{j,t}\}_{j\in[0,1]}} P_{Y,t}\left(\int_0^1 Y_{j,t}^{\frac{\nu_1-1}{\nu_1}}\,dj\right)^{\frac{\nu_1}{\nu_1-1}} - \int_0^1 P_{j,t}Y_{j,t}\,dj$$

where $P_{Y,t}$ is the price of the final good (taken as given), $Y_{j,t}$ is the input bought from sector j and $P_{j,t}$ is the price of that input $j \in [0,1]$,

The first-order condition with respect to $Y_{j,t}$ is

$$P_{Y,t}\left(\int_0^1 Y_{j,t}^{\frac{\nu_1-1}{\nu_1}}\,dj\right)^{\frac{\nu_1}{\nu_1-1}-1} Y_{j,t}^{-\frac{1}{\nu_1}} - P_{j,t} = 0$$

which can be rewritten as

$$Y_{j,t} = \mathcal{Y}_t\left(\frac{P_{j,t}}{P_{Y,t}}\right)^{-\nu_1} \tag{4}$$

Using the expression above, for any two intermediate goods $j,k \in [0,1]$,

$$Y_{j,t} = Y_{k,t}\left(\frac{P_{j,t}}{P_{k,t}}\right)^{-\nu_1} \tag{5}$$

Since markets are perfectly competitive in the final goods sector, the zero profit condition must hold:

$$P_{Y,t}Y_t = \int_0^1 P_{j,t}Y_{j,t}\,dj \tag{6}$$

[13]This is the statutory premium in the United States.

[14]The Bureau of labor statistics use 100 as the index for the labor share in 2009. Our results stay robust to change in this value.

Substituting (9) into (6) gives

$$Y_{j,t} = P_{Y,t}\mathcal{Y}_t \frac{P_{j,t}^{-\nu_1}}{\int_0^1 P_{j,t}^{1-\nu_1}\, dj} \tag{7}$$

Substitute (8) into (7) to obtain the price index

$$P_{Y,t} = \left(\int_0^1 P_{j,t}^{1-\nu_1}\, dj\right)^{\frac{1}{1-\nu_1}}$$

Since each sector is atomistic, their actions will not affect $\mathcal{Y}_t$ nor $P_{Y,t}$. Thus, each of these sectors will face an isoelastic demand curve with price elasticity ν_1.

Sectorial goods sector The representative sectorial firm j solves the following profit maximization problem

$$\max_{\{X_{i,j,t}\}_{i=1,N_{j,t}}} P_{j,t} N_{j,t}^{1-\frac{\nu_2}{\nu_2-1}} \left(\sum_{i=1}^{N_{j,t}} X_{i,j,t}^{\frac{\nu_2-1}{\nu_2}}\right)^{\frac{\nu_2}{\nu_2-1}} - \sum_{i=1}^{N_{j,t}} P_{i,j,t} X_{i,j,t}$$

where $P_{j,t}$ is the aggregate price in sector j (taken as given by the firm), $X_{i,j,t}$ is intermediate good input produced by firm i in sector j, and $N_{j,t}$ is the number of firms in sector j.

The first-order condition with respect to $X_{i,j,t}$ is

$$P_{j,t} N_{j,t}^{1-\frac{\nu_2}{\nu_2-1}} \left(\sum_{i=1}^{N_{j,t}} X_{i,j,t}^{\frac{\nu_2-1}{\nu_2}}\right)^{\frac{\nu_2}{\nu_2-1}-1} X_{i,j,t}^{-\frac{1}{\nu_2}} - P_{i,j,t} = 0$$

which can be rewritten as

$$X_{i,j,t} = \frac{Y_{j,t}}{N_{j,t}} \left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\nu_2} \tag{8}$$

Using the expression above, for any two intermediate goods i, and k,

$$X_{i,j,t} = X_{k,j,t} \left(\frac{P_{i,j,t}}{P_{k,j,t}}\right)^{-\nu_2} \tag{9}$$

Now, raising both sides of the equation to the power of $\frac{\nu_2-1}{\nu_2}$, summing over i and raising both sides to the power of $\frac{\nu_2}{\nu_2-1}$, we get

$$\left(\sum_{i=1}^{N_{j,t}} X_{i,j,t}^{\frac{\nu_2-1}{\nu_2}}\right)^{\frac{\nu_2}{\nu_2-1}} = X_{k,j,t}\frac{\left(\sum_{i=1}^{N_{j,t}} P_{i,j,t}^{1-\nu_2}\right)^{\frac{\nu_2}{\nu_2-1}}}{P_{k,j,t}^{-\nu_2}} \tag{10}$$

Substituting for the production function in the left-hand side and rearranging the terms,

$$\frac{Y_{j,t}}{\mathcal{N}_t}\frac{P_{k,j,t}^{-\nu_2}}{X_{k,j,t}} = \mathcal{N}_t^{-\frac{\nu_2}{\nu_2-1}}\left(\sum_{i=1}^{N_{j,t}} P_{i,j,t}^{1-\nu_2}\right)^{\frac{-\nu_2}{\nu_2-1}} \tag{11}$$

Using the first order condition with respect to $X_{i,j,t}$, the left-hand side is equal to $P_{j,t}^{-\nu_2}$. Therefore, the sectoral price index is

$$P_{j,t} = N_{j,t}^{\frac{-1}{1-\nu_2}}\left(\sum_{i=1}^{N_{j,t}} P_{i,j,t}^{1-\nu_2}\right)^{\frac{1}{1-\nu_2}}$$

8.1 Individual firm problem

Using the demand faced by an individual firm i in sector j, and the demand faced by sector j, the demand faced by firm (i,j) can be expressed as

$$X_{i,j,t} = \frac{\mathcal{Y}_t}{N_{j,t}}\left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\nu_2}\left(\frac{P_{j,t}}{P_{Y,t}}\right)^{-\nu_1} \tag{12}$$

$$= \frac{\mathcal{Y}_t}{N_{j,t}}\left(\tilde{P}_{i,j,t}\right)^{-\nu_2}\left(\tilde{P}_{j,t}\right)^{\nu_2-\nu_1} \tag{13}$$

where $\tilde{P}_{i,j,t} \equiv \frac{P_{i,j,t}}{P_{Y,t}}$ and $\tilde{P}_{j,t} \equiv \frac{P_{j,t}}{P_{Y,t}}$.

The (real) source of funds constraint is

$$D_{i,j,t} = \tilde{P}_{i,j,t}X_{i,j,t} - W_{j,t}L_{i,j,t} - r_t^k K_{i,j,t} - r_t^z Z_{i,j,t}$$

Taking the input prices and the pricing kernel as given, intermediate firm (i,j)'s problem is to maximize shareholder's wealth subject to the firm demand emanating from the rest of the economy:

$$
\begin{aligned}
V_{i,j,t} &= \max_{\left\{L_{i,j,t},K_{i,j,t},Z_{i,j,t},\tilde{P}_{i,j,t}\right\}_{t\geqslant 0}} E_0\left[\sum_{s=0}^{\infty} M_{t,t+s}(1-\delta_n)^s D_{i,j,s}\right] \\
\text{s.t. } X_{i,j,t} &= \frac{\mathcal{Y}_t}{N_{j,t}}\left(\tilde{P}_{i,j,t}\right)^{-\nu_2}\left(\tilde{P}_{j,t}\right)^{\nu_2-\nu_1}
\end{aligned}
$$

where $M_{t,t+s}$ is the marginal rate of substitution between time t and time $t+s$. Note that each sector is atomistic and take the final goods price as given. However, the measure of each firm within a sector is not zero and individual firms will take into account the impact of their price setting on the sectorial price. Further, note that there is no intertemporal decisions. The objective of the firm thus simplifies to a profit maximixation problem with constraint.

The Lagrangian of the problem is

$$
\begin{aligned}
\mathcal{V}_{i,j,t} &= \tilde{P}_{i,j,t}K_{i,j,t}^{\alpha}\left(A_t Z_{i,j,t}^{\eta}\mathcal{Z}_t^{1-\eta}L_{i,j,t}\right)^{1-\alpha} - W_{j,t}L_{i,j,t} - r_{j,t}^{k}K_{i,j,t} - r_{j,t}^{z}Z_{i,j,t} \\
&+\Lambda_{j,t}^{d}\left(K_{i,j,t}^{\alpha}\left(A_t Z_{i,j,t}^{\eta}\mathcal{Z}_t^{1-\eta}L_{i,j,t}\right)^{1-\alpha} - \frac{\mathcal{Y}_t}{N_{j,t}}\left(\tilde{P}_{i,j,t}\right)^{-\nu_2}\left(\tilde{P}_{j,t}\right)^{\nu_2-\nu_1}\right)
\end{aligned}
$$

The corresponding first order necessary conditions are

$$
\begin{aligned}
r_{j,t}^{k} &= \alpha\frac{X_{i,j,t}}{K_{i,t}}(\tilde{P}_{i,j,t}+\Lambda_t^d) \\
r_{j,t}^{z} &= \eta(1-\alpha)\frac{X_{i,j,t}}{Z_{i,j,t}}(\tilde{P}_{i,j,t}+\Lambda_t^d) \\
W_{j,t} &= (1-\alpha)\frac{X_{i,j,t}}{L_{i,j,t}}(\tilde{P}_{i,j,t}+\Lambda_t^d) \\
X_{i,j,t} &= \Lambda_{j,t}^{d}\frac{\mathcal{Y}_t}{N_{j,t}}\left[-\nu_2\tilde{P}_{i,j,t}^{-\nu_2-1}\tilde{P}_{j,t}^{\nu_2-\nu_1}+(\nu_2-\nu_1)\tilde{P}_{i,j,t}^{-\nu_2}\tilde{P}_{j,t}^{\nu_2-\nu_1-1}\frac{\partial\tilde{P}_{j,t}}{\partial P_{i,j,t}}\right]
\end{aligned}
$$

where $\Lambda_{j,t}^{d}$ is the Lagrange multiplier on the inverse demand function.

In the standard Dixit-Stiglitz aggregator, $\frac{\partial\tilde{P}_{j,t}}{\partial P_{i,j,t}}=0$. This happens because each individual firm is atomistic and has no influence on the aggregate price. In our setup, it will be non-zero because the the measure of firm within an industry is strictly positive. Using the definition of the price index,

$$
\frac{\partial\tilde{P}_{j,t}}{\partial P_{i,j,t}} = \frac{1}{N_{j,t}}\left(\frac{P_{i,j,t}}{P_{j,t}}\right)^{-\nu_2}
$$

Imposing the symmetry condition, i.e. $\tilde{P}_{j,t} = \tilde{P}_{i,j,t} = 1$, and $\mathcal{Y}_t = N_{j,t}X_{j,t}$, our set of equilibrium conditions simplifies to:

$$
\begin{aligned}
r^k_{j,t} &= \alpha \frac{X_{j,t}}{K_{j,t}}(1 + \Lambda^d_{j,t}) \\
r^z_{j,t} &= \eta(1-\alpha)\frac{X_{j,t}}{Z_{j,t}}(1 + \Lambda^d_{j,t}) \\
W_{j,t} &= (1-\alpha)\frac{X_{j,t}}{L_{j,t}}(1 + \Lambda^d_{j,t}) \\
\Lambda^d_{j,t} &= \left[-\nu_2 + (\nu_2 - \nu_1)\frac{1}{N_{j,t}}\right]^{-1}
\end{aligned}
$$

The price markup is defined as the ratio of the optimal price set by the firm over the marginal cost of production. The marginal cost of production is obtained by solving the following cost minimization problem:

$$
\begin{aligned}
\min_{K_{i,j,t}, Z_{i,j,t}, L_{i,j,t}} \quad & r^k_{j,t}K_{i,j,t} + r^z_{j,t}Z_{i,j,t} + W_{j,t}L_{i,j,t} \\
\text{s.t.} \quad & K^{\alpha}_{i,j,t}(A_t Z^{\eta}_{i,j,t}\mathcal{Z}^{1-\eta}_t L_{i,j,t})^{1-\alpha} = X^{\star}
\end{aligned}
$$

In Lagrangian form,

$$
\mathcal{V}_{i,j,t} = r^k_{j,t}K_{i,j,t} + r^z_t Z_{i,j,t} + W_t L_{i,j,t} + \lambda_{i,j,t}\left(X^{\star} - K^{\alpha}_{i,j,t}(A_t Z^{\eta}_{i,j,t}\mathcal{Z}^{1-\eta}_t L_{i,j,t})^{1-\alpha}\right)
$$

where $\lambda_{i,j,t}$ is the Lagrange multiplier on the production objective. It is also the marginal cost of production of intermediate firms. Taking the first order conditions,

$$
\begin{aligned}
r^k_{j,t} &= \alpha\lambda_{i,j,t}\frac{X_{i,j,t}}{K_{i,j,t}} \\
r^z_{j,t} &= \eta(1-\alpha)\lambda_{i,j,t}\frac{X_{i,j,t}}{Z_{i,j,t}} \\
W_{j,t} &= (1-\alpha)\lambda_{i,j,t}\frac{X_{i,j,t}}{L_{i,j,t}}
\end{aligned}
$$

From the individual firm problem (FOC w.r.t. $L_{i,j,t}$), we know that

$$
W_{j,t} = (1-\alpha)\frac{X_{i,j,t}}{L_{i,j,t}}(\tilde{P}_{i,j,t} + \Lambda^d_{i,j,t})
$$

Putting the two FOCs w.r.t. to labour together and defining the price markup $\o_{i,j,t}$ as $\tilde{P}_{i,j,t}/\lambda_{i,j,t}$,

$$\o_{i,j,t} = \left(1 + \frac{\Lambda^d_{i,j,t}}{\tilde{P}_{i,j,t}}\right)^{-1}$$

Imposing the symmetry condition $\tilde{P}_{j,t} = 1$ and using the expression for $\Lambda^d_{j,t}$, the price markup is

$$\o_{i,j,t} = \frac{-\nu_2 \, N_{j,t} + (\nu_2 - \nu_1)}{-(\nu_2 - 1) \, N_{j,t} + (\nu_2 - \nu_1)}$$

8.2 Capital producer problem

The period profit of capital producers is $r^k_{j,t}\mathcal{K}^c_{j,t} - \mathcal{I}_{j,t}$. The optimization problem faced by the representative physical capital producer is to choose $\mathcal{K}^c_{j,t+1}$ and $\mathcal{I}_{j,t}$ in order to maximize the present value of revenues, given the capital accumulation constraint:

$$\begin{aligned} V^k_{j,t} = & \max_{\{\mathcal{I}_{j,t},\mathcal{K}^c_{j,t+1}\}_{t\geqslant 0}} E_0 \left[\sum_{s=0}^{\infty} M_{t,t+s}(r^k_{j,s}\mathcal{K}^c_{j,s} - \mathcal{I}_{j,s})\right] \\ \text{s.t.} \ \ & \mathcal{K}^c_{j,t+1} = (1-\delta_k)\mathcal{K}^c_{j,t} + \Phi_{k,j,t}\mathcal{K}^c_{j,t} \end{aligned}$$

The Lagrangian in recursive form is,

$$\mathcal{V}_{j,t} = r^k_{j,t}\mathcal{K}^c_{j,t} - \mathcal{I}_{j,t} + E_t\left[M_{t,t+1}\mathcal{V}_{j,t+1}\right] + Q^k_{j,t}\left((1-\delta_k)\mathcal{K}^c_{j,t} + \Phi_{k,j,t}\mathcal{K}^c_{j,t} - \mathcal{K}^c_{j,t+1}\right)$$

The first order conditions are:

$$\begin{aligned} Q^k_{j,t} &= \Phi'_k\left(\frac{\mathcal{I}_{j,t}}{\mathcal{K}_{j,t}}\right)^{-1} \\ Q^k_{j,t} &= E_t\left[M_{t,t+1}\frac{\partial \mathcal{V}_{j,t+1}}{\partial \mathcal{K}^c_{j,t+1}}\right] \end{aligned}$$

Using the enveloppe theorem,

$$\frac{\partial \mathcal{V}_{j,t}}{\partial \mathcal{K}^c_{j,t}} = \left(r^k_{j,t} + Q^k_{j,t}\left(1 - \delta_k - \left(\frac{\mathcal{I}_{j,t}}{\mathcal{K}^c_{j,t}}\right)\Phi'_{k,j,t} + \Phi_{k,j,t}\right)\right)$$

The set of equilibrium conditions for the representative capital producer is

$$\begin{aligned}
Q^k_{j,t} &= \Phi'^{-1}_{k,j,t} \\
Q^k_{j,t} &= E_t\left[M_{t,t+1}\left(r^k_{j,t+1} + Q^k_{j,t+1}\left(1-\delta_k - \left(\frac{\mathcal{I}_{j,t+1}}{\mathcal{K}^c_{j,t+1}}\right)\Phi'_{k,j,t+1} + \Phi_{k,j,t+1}\right)\right)\right] \\
\mathcal{K}^c_{j,t+1} &= (1-\delta_k)\mathcal{K}^c_{j,t} + \Phi_{k,j,t}\mathcal{K}^c_{j,t}
\end{aligned}$$

The equilibrium conditions for the technology sector are derived is the same way,

$$\begin{aligned}
Q^z_{j,t} &= \Phi'^{-1}_{z,j,t} \\
Q^z_{j,t} &= E_t\left[M_{t,t+1}\left(r^z_{j,t+1} + Q^z_{j,t+1}\left(1-\delta_z - \left(\frac{\mathcal{S}_{j,t+1}}{\mathcal{Z}^c_{j,t+1}}\right)\Phi'_{z,j,t+1} + \Phi_{z,j,t+1}\right)\right)\right] \\
\mathcal{Z}^c_{j,t+1} &= (1-\delta_z)\mathcal{Z}^c_{j,t} + \Phi_{z,j,t}\mathcal{Z}^c_{j,t}
\end{aligned}$$

where $\mathcal{S}_{j,t}$ is the aggregate investment in R&D in sector j.