Asset Prices with Heterogeneous Loss Averse Investors

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Job Market Paper

Abstract

This paper considers a general-equilibrium model with loss-aversion in consumption and heterogeneity: there is a continuum of agents, with s-shaped utility, who differ in the time-varying reference level of consumption. Heterogeneity in the reference level is crucial for the existence of the equilibrium, which cannot be obtained with a representative agent or a discrete number of agents. Loss-aversion in consumption induces a kink in the pricing kernel and consequently, jumps in the market price of risk, stock return, and volatility. An economy populated with only loss-averse agents produces one counterfactual property of asset prices: the return volatility and the market price of risk are higher in good times than in bad times. The coexistence of both loss-averse and risk-averse agents in the economy helps fixing this undesirable property and also explains the dynamics of trading volume and its correlation with asset prices.

Keywords: equilibrium, heterogeneity, loss aversion, trading-volume, local time.

JEL Classification: G11, G12

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1 Introduction

Loss-aversion refers to the idea of asymmetric attitudes with respect to gains and losses: agents are much more sensitive to losses than to gains and, in addition, they are risk-seeking over losses and risk-averse over gains rather than being purely risk-averse as postulated by classical financial theory. Following the seminal works of Kahneman and Tversky (1979, 1991, 1992), several articles have provided empirical and experimental evidence of loss-aversion in individual behavior, concerning both financial and non-financial choices. In addition, recent empirical evidence suggests a significant degree of heterogeneity in loss-aversion across individuals. Nonetheless, the general equilibrium implications of heterogeneous loss-averse agents are still non clear.

This paper presents a general equilibrium model with heterogeneous loss-averse agents which aims to provide a comprehensive study concerning the joint effect of loss-aversion and preference heterogeneity on financial markets. In particular, I analyze the general equilibrium implications of loss-aversion in consumption for asset prices, trading volume and their dynamic relation. The economy is populated with a fraction of loss-averse agents, and a fraction of standard risk-averse agents. Both groups of agents have utility defined over consumption: loss-averse agents are equipped with s-shaped utility over consumption relative to a time-varying reference level; risk-averse agents have standard power utility over consumption. Finally, the reference level of consumption differs across agents.

The idea of loss-aversion has been widely exploited in finance, producing a large literature with the aim of understanding the effect of loss aversion on asset prices and their properties. With few exceptions, this literature has focused only on the representative agent of Lucas (1978) or Cox Ingersol and Ross (1985). The representative agent can be interpreted as an aggregation of heterogeneous agents. However, it says nothing about the effect of individual behavior, such as individual asset holdings and consumption, on asset prices and trading volume which is, instead, of crucial importance in a model with loss-averse agents. Indeed, the attitude toward risk of loss-averse agents depends on economic conditions and individual preferences. As a result, heterogeneity implies the coexistence of risk-seeking and risk-averse agents who trade each other in the capital markets to hedge against fluctuations in the aggregate uncertainty. Agents with different risk attitudes have different impact on asset prices and, therefore, in order to understand the determination of prices, heterogeneity should be modeled explicitly. Besides the asset pricing implications, preference heterogeneity is a well documented feature of reality. For instance, in recent papers, Dimmock and Kouwenberg (2010) and Gill and Prowse (2012) find evidence of substantial heterogeneity in loss-aversion across individuals.

Another characteristic of the existing literature on loss aversion is the assumption that agents derive utility from fluctuations in financial wealth, around a certain reference point. As emphasized by Yogo (2008), the assumption that agents derive utility from of fluctuations in consumption, rather than from financial wealth, is much more in line with the notion of economic risk. The reason is that, when agents care about fluctuations in consumption, the dynamics of asset prices is explained by the agents' needs to hedge consumption risk, whereas, when agents care about fluctuations in financial wealth, the dynamics of asset prices is in general explained by psychological factors, unrelated to consumption. Yogo (2008) tests the moment conditions of a pure exchange economy with loss-averse agents over consumption and shows that the model is validated by empirical data. Similar evidence for the case of a production economy is provided by Rosenblatt - Wisch (2008).

To tackle these issues, I consider a standard asset-pricing model of the Lucas (1978) type with two extra ingredients: loss-aversion over consumption and preference heterogeneity, namely, agents differ from each others in their reference level of consumption. The interaction between these two ingredients produces the following remarkable results: first, a continuum distribution of reference levels is essential for the existence of the equilibrium which, on the contrary, may not obtain with one representative agent or with a discrete number of agents; second, an economy populated only with loss-averse agents produces counterfactual properties of asset prices, namely, prices of risk and return volatility tend to be higher in good states than in bad states; finally, an economy populated with a fraction of loss-averse agents and a fraction of standard risk-averse agents produces realistic properties for asset prices, the trading volume and the dynamic relation between the two. In particular, the model explains the high trading volume in bad times and the positive correlation between trading volume and stock return and trading volume and return volatility, amply documented by empirical researches.

Loss-aversion over consumption may be at odds with the existence of the equilibrium because the optimal consumption of loss-averse agents is discontinuous, that is, larger than the reference point or equal to zero. As a result, with a discrete distribution of reference levels, the demand curve of the consumption good jumps each time the consumption of loss-averse agents rises above (or fall below) the reference point, and the equilibrium fails to hold. Differently, with a continuum of reference levels, agents are infinitesimal and the demand curve of consumption adjusts smoothly to changes in consumption of loss-averse agents. As a result, the equilibrium exists. However, the discontinuity in the consumption policy introduces a kink in the pricing kernel which, in turn, implies that the equilibrium quantities adjust discontinuously to changes in economic conditions. For instance, when the economy enters a bad state, the consumption of some agents falls below their reference point, inducing risk-seeking behavior. This produces a down jump in the aggregate relative risk aversion which affects asset prices: return volatility, price of risk and equity premium immediately decrease following the down jump in the risk aversion. As a result, when the economy is only populated by loss-averse agents, risk aversion is higher in good times than bad times, leading to a counterfactual procyclical behavior of the equity premium and the return volatility. This counterfactual result is remedied through a fraction of standard risk-averse agents which keeps the risk aversion sufficiently high in bad times, thus, counterbalancing the effect of loss-averse agents. The interaction between risk-averse and risk-seeking agents also produces interesting implications for the dynamics of the trading volume which are explained by the dynamic hedging reasons of the loss-averse agents. When the consumption approaches their reference level, loss-averse agents become extremely unwilling to accept additional fluctuations in consumption, therefore, they sell the risky asset to the risk-averse agents, increasing the volume of trading.

The literature on loss-aversion has a long tradition in finance. Benartzi and Thaler (1995) find that loss-aversion with frequent evaluation of portfolios performance, helps explain the high empirical equity returns; Barberis Huang and Santos (2001) show that time-varying loss-aversion reproduces the high mean, the excess volatility and predictability of stock returns; McQueen and Workink (2004) provide evidence that loss-aversion gives rise to asymmetric stock market volatility; Yogo (2008) builds a model where agents evaluate consumption relative to slow-moving habits and finds that such a model explains the low real interest rate and the high equity premium. Andries (2011) considers an economy where agents are loss-averse with respect to the expectation of future consumption and they are equipped with Epstein and Zin (1989) utility and shows that loss-aversion predicts a negative premium for the skewness of stock returns and a security market line flatter than CAPM. My model differs on several points. First, these model analyze a representative agent economy while my paper considers the case of heterogenous preferences. Second, with the exception of Yogo (2008), and Andries (2011), agents of these models evaluate gains and losses in financial wealth, while, agents in my model derive utility from fluctuations in consumption.

Berkelaar and Kouwenberg (2008) introduce heterogeneity in a model with loss-aversion. They consider an economy with a standard risk-averse agent over consumption and two loss-averse agents over financial wealth who are identical except for their reference point and their initial wealth. My model differs in two important aspects. First, I assume there is a continuum of investors who differ from each other with respect to their reference level. Second, while Berkelaar and Kouwenberg (2008) assume that agents have power utility for consumption and s-shaped utility for financial wealth, I assume that agents are loss-averse in consumption. As a result, unlike the exchange economy in Berkelaar and Kouwenberg (2008), the market price of risk and the risk free rate are not constant in my model. Finally, Gomes (2005) studies the implications of loss-aversion over financial wealth for trading volume in a two-periods binomial model. Easley and Yang (2011) analyze the survival of a loss-averse investor over financial wealth in a 2-agents economy where the other agent is of the Epstein and Zin (1989) type.

The rest of the paper is organized as follow: in section 2, I introduce the model and present the assumption about the economy, the traded assets and the agent's preferences; in section 3, I study the competitive equilibrium; section 4 presents the numerical results; section 5 emphasize the importance of time variation in the reference level; section 6 concludes.

2 The Model

2.1 The Environment

I consider a standard pure exchange economy and a filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ on which is defined a standard one-dimensional Brownian motion B; Ω is the set of states of nature with generic element ω; $\mathcal{F}$ is the sigma algebra of observable elements and $\mathbb{P}$ is the probability measure common to all agents. The filtration $\mathbb{F} = \{\mathcal{F}_t\}$ is the augmentation under $\mathbb{P}$ of the filtration generated by the Brownian motion path. $\mathcal{F}_t$ has the interpretation of the information available at time t. The state of the world is determined by the realizations of the Brownian motion B. The economy has an infinite horizon. There is a single consumption good that serves as numeraire. The financial market consists of two assets: a locally riskless bond in zero net supply and a risky asset in positive supply of one unit. A share of the money market has price S_t at time t with $S_0 = 1$.

Since agents are endowed with s-shaped utility, their optimal consumption may fall below the reference level and, as showed in Karatzas and Shreve (1991) and (1998), the money market price cannot be described solely in terms of an interest rate. More precisely, given that the process S_t is $\mathcal{F}_t$-adapted with finite total variation, it can be decomposed into absolutely continuous and singularly continuous parts, S_t^{ac} and S_t^{sc}, respectively. Thus, define

$$r_t = \frac{\frac{d}{dt} S_t^{ac}}{S_t}, \; A_t = \int_0^t \frac{\frac{d}{dt} S_t^{sc}}{S_t} \tag{1}$$

so that

$$S_t = \exp\left\{ \int_0^t r_u du + A_t \right\} \tag{2}$$

for some instantaneous interest rate process $r \in \mathbb{R}$ which is to be determined in equilibrium. A_t is a singularly continuous component with zero derivative, in the Lebesgue sense, almost every time. Karatzas and Shreve (1998) show that, if the process A_t is not given by Eq. (1) then an arbitrage opportunity exists in the model. When S_t is itself absolutely continuous, $A_t = 0$ and the price of the risk free asset evolves like the standard saving account with instantaneous riskless rate given by r. The process A_t has to be determined in equilibrium.

The cumulative dividends (or aggregate endowment), D_t, follows a geometric Brownian motion of the form:

$$dD_t = \mu_D D_t dt + \sigma_D D_t dB_t \tag{3}$$

where μ_D, σ_D and D_0 are positive constant. The ex dividend price of the risky asset, P_t, is continuous, strictly positive and satisfies the following stochastic differential equation

$$dP_t + D_t dt = +P_t[\mu_t dt + \sigma_t dB_t + dA_t] \tag{4}$$

where the processes μ_t and σ_t are endogenous quantities to be determined in equilibrium. The process:

$$\theta_t = \frac{\mu_t - r_t}{\sigma_t} \tag{5}$$

is the market price of risk associated to the unique source of risk in the model and is to be determined in equilibrium.

As the market is complete, it exists a unique state-price density, H_t, that satisfies

$$dH_t = -r_t H_t dt - \theta_t H_t dB_t - H_t dA_t. \tag{6}$$

Given the state price density, the price of the risky asset is given by

$$P_t = \mathbb{E}_t \int_t^{\infty} \frac{Hs}{H_t} D_s ds. \tag{7}$$

2.2 Preferences

The economy is populated with two categories of agents. A fraction λ is equipped with standard power-utility function, U_{RA}, given by

$$U_{RA}(c_t) = \frac{c_t^{1-\gamma_1}}{1-\gamma_1} \tag{8}$$

where c_t represents consumption and $\gamma_1 > 0$. Since utility in Eq. (8) is increasing and strictly concave, U_{RA} represents risk-averse agents. The remaining fraction $(1-\lambda)$ of agents is loss-averse with utility U_{LA} of the form

$$U_{LA}(c_t, z_t) = \begin{cases} -B\frac{(z_t-c_t)^{1-\gamma_2}}{1-\gamma_2} & if\ c_t < z_t \\ \frac{(c_t-z_t)^{1-\gamma_2}}{1-\gamma_2} & if\ c_t \geq z_t \end{cases} \tag{9}$$

where z_t denotes the time-t reference level of consumption and $\gamma_2 \in [0,1]$. The utility function in Eq. (9) is increasing, concave when consumption is above the reference level and convex otherwise. Finally, U_{LA} is steeper for losses (consumption below the reference point) than for gains (consumption above the reference point). Hence, agents equipped with utility are U_{LA} are risk-averse in the domain of gains and risk-seeking in the domain of losses. Figure 1 shows the utility function of loss-averse agents.

Insert Figure 1 about here

Each agent has a time-varying reference level $z_t = ze^{\alpha t}$ where z and α are strictly positive constant. To gain the economic intuition for this definition of the reference point, consider first the case $\alpha = 0$. z in this case defines the desired consumption class and U_{LA} represents an agent that does not tolerate to descend to consumption classes lower than z. $\alpha > 0$ captures the idea that, as time passes, agents wish to belong to classes of higher consumption. This assumption also ensures alternation between times when consumption exceeds the reference level and times where, instead, the reference level exceeds consumption. To the contrary, such alternation vanishes when $\alpha = 0$. Modelling the reference point as an increasing function of time could be interpreted as a compromise between a constant reference level, and a consumption-dependent reference level in the spirit of the internal habit formation literature (as, for instance, in Constantinides (1990)). Indeed, introducing a consumption-dependent reference level in a model where agents have s-shaped utility considerably complicates the analysis and the existence of an equilibrium is, at least, questionable. Differently, the assumption of deterministic reference, captures in a simple way the idea of loss-aversion, preserves the model tractability and allow for analytical solutions of most of the equilibrium quantities. Other models of loss-aversion, as Andries (2011) for instance, consider expecation-based reference levels. Gill and Prowse (2011) provide empirical evidence for this type of reference levels. Reference levels which increase over time may be consistent with reference levels based on expectations of future consumption. Indeed, since the aggregate consumption rises (on average) over time, conditional expectations of individual consumption should also rise (on average) over time, thus, leading to increasing reference levels.

The standard risk-averse agents are endowed with initial wealth $W_{RA,0}$ and $W_{LA,0}(z)$ represents the initial wealth of a loss-averse agent with reference level z. Given the initial wealth, risk-averse and loss-averse agents choose a non-negative consumption process $c_{i,t}$ and a proportion of wealth to be invested in stocks $\pi_{i,t}$ in a such a way to maximize

$$\mathbb{E}_0 \int_0^{\infty} e^{-\rho t} U_i dt$$

subject to

$$\begin{aligned} dW_{i,t} &= W_{i,t} r_t dt + W_{i,t} \pi_{i,t} (\mu_t - r_t) dt + W_{i,t} \pi_{i,t} \sigma_t dB_t - c_{i,t} dt + W_{i,t} dA_t \qquad (10) \\ c_{i,t} &\geq 0,\, W_{i,t} \geq 0 \end{aligned}$$

for $i = RA$, LA and $\forall t$.

2.3 Cross-sectional heterogeneity

Loss-averse agents have the same growth rate of the reference level α and the same parameter γ_2 but they differ with respect to their initial reference z. More precisely, I assume that there is a continuum of preference types parametrized by $z \in [0, \overline{z}]$ with $\overline{z} > 0$. Furthermore, I assume that $\overline{z} < \infty$ because this assumption rules out unrealistic equilibria where the consumption of a subset of loss-averse agents is always smaller than the reference point independently of the endowment of the economy. Indeed, it is easy to see that an agent endowed with reference $\overline{z} = \infty$ consumes only when the aggregate endowment goes to infinity. However, a detailed discussion on the role of the previous assumptions will follow in section 4 below. Risk-averse agents are characterized by the risk aversion parameter γ_1 which may differ from γ_2. Finally, risk-averse and loss-averse agents have the same subjective discount rate ρ.

3 Optimal consumption policy and equilibrium

In equilibrium, each agent solves his individual consumption-investment problem, while markets for the riskless asset, the stock and consumption have to clear at each time t. I determine the optimal consumption policy of the two types of agents by employing the martingale methodology of Cox and Huang (1989) as modified by Berkelaar & Kouwenberg (2000). This leads to the next Proposition.

Proposition 1 *Given the state price density process of Eq. (6), the optimal consumption policies c^*_{RA} and c^*_{LA} are given by*

$$c^*_{RA}(H_t) = \left(e^{\rho t} H_t \phi_{RA}\right)^{-\frac{1}{\gamma_1}},$$
$$c^*_{LA}(H_t, z) = \left(ze^{\alpha t} + \left(e^{\rho t} H_t \phi_{LA}(z)\right)^{-\frac{1}{\gamma_2}}\right) \mathbf{1}_{\{H_t \leq e^{-\rho t}\overline{x}(z)\}}$$

and $\overline{x}(z)$ is implicitly defined by

$$f(\overline{x}, z) = (\phi_{LA}(z)\overline{x})^{\frac{\gamma_2 - 1}{\gamma_2}} \frac{\gamma_2}{1-\gamma_2} - \phi_{LA}(z)\overline{x} z e^{\alpha t} + B\frac{(ze^{\alpha t})^{1-\gamma_2}}{1-\gamma_2} = 0, \tag{11}$$

where ϕ_{RA} and $\phi_{LA}(z)$ are Lagrange multipliers satisfying, respectively, the static budget constraints

$$\mathbb{E}_0 \int_0^\infty H_t c^*_{RA} dt = H_0 W_{RA,0},$$
$$\mathbb{E}_0 \int_0^\infty H_t c^*_{LA} dt = H_0 W_{LA,0}(z).$$

Proof. See Appendix ■

The standard risk-averse agent smooths consumption across time and states of the world. On the contrary, the optimal consumption profile of prospect theory is discontinuous: it is either above the reference point or equals zero depending on the value of the state price density. Indeed, in the states of the world where consumption is excessively expensive ($H_t > e^{-\rho t}\overline{x}(z)$), the risk-seeking behavior induces the loss-averse agent to abandon consumption and invest all wealth in the capital market. By doing so, the loss averse agent aims to maximize the probability of beating the reference level z. The position of $\overline{x}(z)$ depends on the agents' individual preferences. The higher the reference level z, the smaller is the threshold $\overline{x}(z)$ and, consequently, the larger is the zero-consumption region.[1].

3.1 Heterogeneity and existence of the equilibrium

This subsection aims to discuss the link between heterogeneity in the reference level and the existence of the equilibrium. For simplicity of exposition define the variable $x_t = e^{\rho t} H_t$. Note that x_t is just the state-price density multiplied by a non-random function of time. Therefore, any economic or mathematical property of H_t can be equivalently addressed to x_t and viceversa, without altering its meaning. Therefore, throughout the paper, I refer to x_t and H_t interchangeably.

Consider first a financial market with a representative agent endowed with utility function U_{LA} and reference level z. Proposition 1 implies that the optimal consumption of the representative agent is $z + \phi_{LA}^{-\frac{1}{\gamma}} x_t^{-\frac{1}{\gamma}}$ if $x_t \leq \overline{x}(z)$ and zero otherwise. Market clearing would imply that the total demand of consumption equals the exogenous supply, thus, there exist states of the world where the market clearing in the goods market fails. In particular, whenever $D_t < z + \phi_{LA}^{-\frac{1}{\gamma}} \overline{x}(z)^{-\frac{1}{\gamma}}$ the consumption market does not clear and the equilibrium fails to exist. Consider now a financial market populated with two agents: a loss-averse agent with utility U_{LA} and reference level z and a risk-averse agent with utility U_{RA}. Proposition 1 implies that when $x_t > \overline{x}(z)$ the consumption of the loss-averse agent is zero. Thus, in this case, the market clearing condition writes as

$$\left(e^{\rho t} H_t \phi_{RA}\right)^{-\frac{1}{\gamma_1}} = D_t \tag{12}$$

Let x^1 be the solution to Eq (12), thus, by individual optimally, we must have $x^1 > \overline{x}(z)$ which implies that x^1 is indeed the solution to the market clearing condition when $D_t <$

[1] A detailed analysis of the optimal consumption-portfolio policies with s-shaped utility is in Curatola (2011).

$\underline{D} = \phi_{RA}^{-\frac{1}{\gamma_1}} \overline{x}(z)^{-\frac{1}{\gamma_1}}$. When instead $x_t \le \overline{x}(z)$ the market clearing condition writes as

$$z + \left(e^{\rho t} H_t \phi_{LA}(z)\right)^{-\frac{1}{\gamma_2}} + \left(e^{\rho t} H_t \phi_{RA}\right)^{-\frac{1}{\gamma_1}} = D_t \tag{13}$$

Denote by x^2 the solution to Eq (13), thus, by individual optimally we must have $x^2 \le \overline{x}(z)$ which implies that x^2 solves the market clearing condition when $D_t \ge \overline{D} = z + \phi_{RA}^{-\frac{1}{\gamma_1}} \overline{x}(z)^{-\frac{1}{\gamma_1}} + \phi_{LA}(z)^{-\frac{1}{\gamma_2}} \overline{x}(z)^{-\frac{1}{\gamma_2}}$. Clearly, $\underline{D} < \overline{D}$ and therefore, if $\underline{D} < D_t < \overline{D}$ the state-price density may not be defined. The point here is that, with heterogeneity in the reference level, loss averse agents join the consumption market in different states of the world. More precisely, the larger their reference level, the bigger has to be the aggregate endowment to allow them to enter the market. In case of a discrete number of reference levels, the arrival of a new agent in the market produces a discontinuous adjustment of the total demand of consumption (the left hand side of Eq's (12) and (13)). As a result, the relation between demand and price of consumption good is discontinuous, preventing the equilibrium to exist. Note that, this reasoning can be easily extended to arbitrary number of agents. Thus, we conclude that any discrete number of agents, with different reference levels, precludes the existence of the equilibrium. Intuitively, with a continuum of reference levels, loss-averse agents enter the market in a continuous way and smooth the demand curve of consumption. The section below provides the details about the economy with a continuum of reference levels.

3.2 The economy with a continuum of agents

When the market is populated with a fraction λ of risk averse agents and a fraction $1-\lambda$ of loss averse agents with reference levels continuously distributed over $[0, \overline{z}]$ the market clearing condition writes

$$\lambda c_{RA}^* + (1-\lambda) \int_0^{\overline{z}} c_{LA}^* dz = D_t. \tag{14}$$

Eq (14) has to be solved for the state-price density H_t. This can be done by assuming a particular distribution for the initial wealth $W_{LA,0}(z)$ and $W_{RA,0}$, and then solving the static budget constraints and Eq (14) jointly for the Lagrange multipliers function $\phi_{LA}(z)$, ϕ_{RA} and the state-price density H_t. However, given the discontinuous consumption policy of the loss-averse agents, not all the distributions $W_{LA,0}(z)$ are compatible with the existence of the equilibrium. To overcome this difficulty, I select the function $\phi_{LA}(z)$ exogenously, in a such a way to guarantee the existence of the equilibrium, and then I determine endogenously the corresponding wealth distribution $W_{LA,0}(z)$. In a centralized economy, this corresponds to exogenous weighting function in the social planner allocation problem, in the spirit of Chan and Kogan (2002). The initial distribution of wealth provides an additional factor to judge the plausibility of model with heterogeneous loss-averse investors as an explanation of the main characteristics of the capital market. Thus, from now on I assume that the function $\phi_{LA}(z)$ satisfies the following condition.

Condition 1 $\frac{\partial \phi_{LA}(z)}{\partial z} \geq 0$

Economically, Condition 1 means that the change in the agents' value function, following changes in the initial wealth, increases with the reference level. Indeed, high reference agents tend to have zero consumption in most of the states of the world and the increase in the initial wealth increases their chances to consume above the reference level. Thus, it is natural to expect that the benefit of an increase in the initial wealth is larger for high-reference agents and, as a consequence, the Lagrange multiplier $\phi_{LA}(z)$ should not decrease with the reference level z. Condition 1 implies that the consumption policy of loss-averse agents can be written in term of the reference level z as follow.

Lemma 1 *The optimal consumption of an agent with reference level z is given by:*

$$c^*_{LA} = \begin{cases} 0 & if \;\; z > z^* \\ ze^{\alpha t} + (\phi_{LA}(z)x_t)^{-\frac{1}{\gamma_2}} & if \;\; z \leq z^* \end{cases} \tag{15}$$

where z^* solves $f(x_t, z^*) = 0$, $x_t = e^{\rho t} H_t$ and $f(.)$ is defined in Proposition 1.

Proof. see Appendix ■

Lemma 1 says that the heterogeneity in the reference level, associated with s-shaped utility, leads to heterogeneity in the agents' optimal consumption and, consequently, in their attitude toward risk: agents with reference level $z \leq z^*$ are-risk averse because their consumption exceeds their reference point; agents with reference level $z > z^*$ are, instead, risk-seeking. Thus, given the result in Lemma 1, the market clearing condition for consumption can be written as

$$\lambda \left(x_t \phi_{RA}\right)^{-\frac{1}{\gamma_1}} + (1-\lambda) \int_0^{z^*} \left(z e^{\alpha t} + \left(\phi_{LA}(z) x_t\right)^{-\frac{1}{\gamma_2}} \right) dz = D_t \tag{16}$$

where I only need to integrate up to z^* because reference levels bigger than z^* contribute nothing to the left hand side of Eq. (16). From Lemma 1 we also note that the reference point z^*, that separates risk-averse from risk-seeking agents, and x_t have to be determined simultaneously.

Proposition 2 *In the state of the world characterized by $D_t < D_t^*$, x_t and z^* solve*

$$\begin{cases} \lambda \left(x_t \phi_{RA}\right)^{-\frac{1}{\gamma_1}} + (1-\lambda) \int_0^{z^*} \left(z e^{\alpha t} + \left(\phi_{LA}(z) x_t\right)^{-\frac{1}{\gamma_2}} \right) dz = D_t \\ f(x_t, z^*) = 0 \end{cases}$$

with $z^* < \overline{z}$ and $\partial z^* / \partial D > 0$; in the state of the world characterized by $D_t \geq D_t^*$, $z = \overline{z}$ and x_t solves

$$\lambda \left(x_t \phi_{RA}\right)^{-\frac{1}{\gamma_1}} + (1-\lambda) \int_0^{\overline{z}} \left(z e^{\alpha t} + \left(\phi_{LA}(z) x_t\right)^{-\frac{1}{\gamma_2}} \right) dz = D_t$$

where

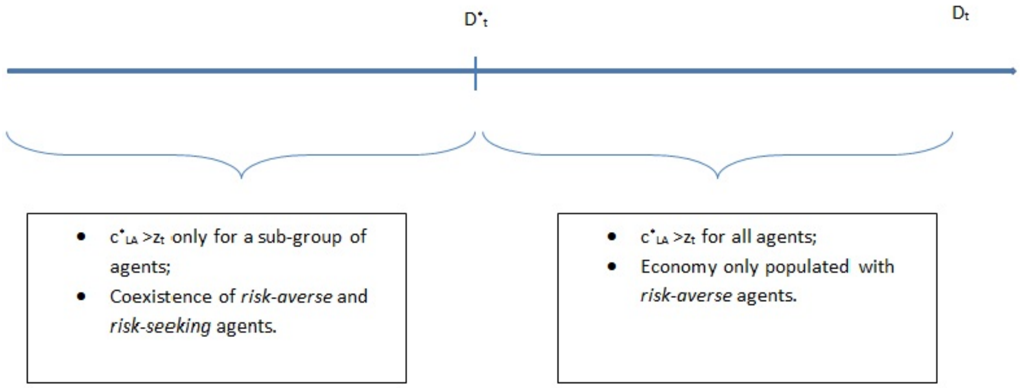
$$D_t^* = \lambda \left(\overline{x}(\overline{z}) \phi_{RA}\right)^{-\frac{1}{\gamma_1}} + (1-\lambda) \left[\frac{\overline{z}^2}{2} e^{\alpha t} + \overline{x}(\overline{z})^{-\frac{1}{\gamma_2}} \int_0^{\overline{z}} \phi_{LA}(z)^{-\frac{1}{\gamma_2}} dz \right].$$

In addition x_t has the following properties:

- *x_t is continuous in D_t;*

- $\partial x/\partial D$, $\partial^2 x/\partial D^2$ *and* $\partial x/\partial t$ *are discontinuous at* $D_t = D_t^*$.

The dynamic behavior of the economy with risk-averse and (a continuum of) loss-averse agents is summarized in the following scheme which illustrates the heterogeneity in consumption and risk-attitude as a function of the aggregate endowment.



3.3 Financial Assets

Market completeness and the absence of arbitrage opportunities imply that the unique state-price density in the economy, H_t, satisfies the dynamics given in Eq.(6). In addition, we have the identification $H_t = x_t e^{-\rho t}$. Proposition 2 implies that H_t is kinked at $D_t = D_t^*$ and therefore, the standard Ito's rule cannot be applied to H_t. Since the discontinuities in the state price density process arise along the curve D_t^*, I apply the result in Peskir (2005) which extends the Ito's rule allowing for local time on curves. This leads to the next Lemma.

Lemma 2 *The function* x_t *satisfies the integral equation*

$$\begin{aligned} x_t \;=\;& x_0 + \int_0^t \left[\frac{\partial x_s}{\partial D}\mu_D D_s + \frac{1}{2}\frac{\partial^2 x_s}{\partial D^2}\sigma_D^2 D_s^2\right] ds + \int_0^t \frac{\partial x_s}{\partial D}\sigma_D D_s dB_t + \int_0^t \frac{\partial x_s}{\partial s} ds \\ &+ \int_0^t \left[\frac{\partial x_s}{\partial D}(s, D_{s+}^*) - \frac{\partial x_s}{\partial D}(s, D_{s-}^*)\right] dL_s(D_s^*) \end{aligned} \quad (17)$$

where $L_t(D_t^*)$ is the local time of D_t at D_t^* and it is given by

$$L_t(D_t^*) = \lim_{\varepsilon \downarrow 0} \frac{1}{\varepsilon} \int_0^t \mathbf{1}\left\{D_s^* \le D_s < D_s^* + \varepsilon\right\} d\left\langle D, D\right\rangle_s$$

and $dL_t(D_t^*)$ refers to the integration with respect to the function $L_t(D_t^*)$.

Proof. See Appendix ■

The correction term appearing in Eq (17) is expressed as an integral with respect to the local time of the aggregate endowment D_t at the curve D_t^*. Roughly speaking, the correction term accounts for all the discontinuities of $\partial x_t/\partial D$ up to time t that obtain when the consumption of a sub-group of loss-averse agents either falls bellow, or rises above, the reference point. Lemma 2 in conjunction with Eq. (6) implies that

$$\theta_t = -\frac{\partial x_t/\partial D}{x_t}\sigma_D D_t \tag{18}$$

$$r_t = \rho - \frac{\partial x_t/\partial t}{x_t} - \frac{\partial x_t/\partial D}{x_t}\mu_D D_t - \frac{1}{2}\frac{\partial^2 x_t/\partial D^2}{x_t}\sigma_D^2 D_t^2 \tag{19}$$

$$A_t = -\frac{\int_0^t \left[\frac{\partial x_t}{\partial D}(t, D_{t+}^*) - \frac{\partial x_t}{\partial D}(t, D_{t-}^*)\right] dL_t(D_t^*)}{x(t, D_t^*)}. \tag{20}$$

The market price of risk and the risk free rate have the usual components. The price of risk accounts for the remuneration that agents require for the risk associated to the aggregate endowment. The risk free is determined by the agent's impatience, the growth rate of aggregate endowment and the desire of precautionary saving due to the risk that consumption falls below the reference level. The singular continuous component represents the required correction in asset prices necessary for excluding arbitrage opportunities in the financial market[2].

Furthermore, the return volatility and the equity premium can be computed by differentiating the pricing equation (7).

[2] For a formal proof of this statement see Karatzas and Shreve (1991).

Proposition 3 *The return volatility, σ_t, is given by:*

$$\begin{aligned}\sigma_t &= \left[\frac{\partial Q_t/\partial D}{Q_t} - \frac{\partial x_t/\partial D}{x_t}\right] D_t \sigma_D \\ & \frac{\partial Q_t/\partial D}{Q_t} D_t \sigma_D + \theta_t\end{aligned} \tag{21}$$

where:

$$Q_t = \mathbb{E}_t \int_t^\infty e^{-\rho(s-t)} x_s D_s ds.$$

The equity premium is

$$\mu_t - r_t = \theta_t \sigma_t. \tag{22}$$

where θ_t is given by Eq.(18).

Proof. See Appendix ■

Thus, the stock market volatility consists of two components: the relative change in the risk-adjusted expectations of the aggregate endowment and the price of risk θ_t. Finally, we observe that all equilibrium quantities are discontinuous function of the aggregate endowment and they jump at $D_t = D_t^*$. This is because the fluctuations of the aggregate endowment around D_t^* produce discontinuous adjustments in the agent's optimal consumption which affect asset prices through their effect on the aggregate attitude toward risk.

3.4 Portfolio policies and trading volume

Agents' financial wealth is defined as the value of the asset paying the agent's lifetime consumption stream of Proposition 1. More formally,

$$W_{i,t} = \mathbb{E}_t \int_t^\infty e^{-\rho(s-t)} \frac{x_s}{x_t} c_{i,s}^* ds$$

for $i = RA, LA$. The dynamics of individual wealth is

$$dW_{i,t}/W_{i,t} = [...]\, dt + dA_t + \left[\frac{\partial V_{i,t}/\partial D}{V_{i,t}} - \frac{\partial x_t/\partial D}{x_t} \right] D_t \sigma_D dB_t$$

where $V_{i,t} = \mathbb{E}_t \int_t^\infty e^{-\rho(s-t)} x_s c_{i,s}^* ds$. Thus, the proportion of wealth $\pi_{i,t}$ invested in the risky asset is then

$$\begin{aligned} \pi_{i,t} &= \left[\frac{\partial V_{i,t}/\partial D}{V_{i,t}} - \frac{\partial x_t/\partial D}{x_t} \right] \frac{D_t \sigma_D}{\sigma_t} \\ &= \frac{\theta_t}{\sigma_t} + \frac{\partial V_{i,t}/\partial D}{V_{i,t}} \frac{D_t \sigma_D}{\sigma_t} \end{aligned}$$

where the last equality follows from the definition of market price of risk. Therefore, the optimal proportion of the risky asset decomposes into myopic part, θ_t/σ_t, and the agent-specific hedging demand $\frac{\partial V_{i,t}/\partial D}{V_{i,t}} \frac{D_t \sigma_D}{\sigma_t}$.

Agents trade in stocks to hedge the fluctuations in their consumption process. In particular, loss-averse agents have the objective to preserve their consumption above the reference level to avoid painful losses. To obtain this result they can either trade each other or with the risk-averse agent. However, in continuous-time economies, trading volume is not well defined (i.e. it is infinite). This can be circumvented by assuming discrete trading at some time interval. More formally, let (t_1, t_2) be two consecutive trading dates, and (D_1, D_2) the two associated realizations of the aggregate dividend at these two dates. Hence, one can define the trading volume between these two dates as

$$T(D_1, D_2) = \frac{1}{2} \left[\int_0^{\overline{z}} (1-\lambda) \left| n_{LA,t}(D_2) - n_{LA,t}(D_1) \right| dz + \lambda \left| n_{CRRA,t}(D_2) - n_{CRRA,t}(D_1) \right| \right] \tag{23}$$

where $n_{i,t}$ is the state-dependent number of the risky asset held by either loss-averse or power utility agents, that is, $n_{i,t} = \pi_{i,t} W_{i,t}/P_t$.

4 Asset Pricing Implications

In this section, I analyze the asset pricing implications of the model by plotting the equilibrium quantities as a function of the aggregate endowment of the economy. In order to simplify the exposition of results, I define "high-consumption" as a period in which all agent's consumption exceeds the reference point (i.e. $D_t \geq D_t^*$) and a "low-consumption" as a period in which the reference level exceeds consumption for a subset of agents (i.e. $D_t < D_t^*$). The model parameters are specified in Table 1 below.

Insert Table 1 about here

The preference parameter $B = 2.25$ corresponds to the value estimated by Kahneman and Tversky (1992). For μ_D and σ_D, the drift and volatility of the aggregate endowment process, I choose $\mu_D = 1.84\%$ and $\sigma_D = 3,5\%$. These numbers are similar to those used by Mehra and Prescott (1985), Constantinides (1990) and Barberis Huang and Santos (2001). α determines the growth rate of the reference points. In order to ensure alternation between high and low consumption periods I choose $\alpha = 0.85\%$. Finally I assume for simplicity that $\phi_{LA}(z) = \phi_{RA} = 1 \ \forall z$. This assumption satisfies the requirement of Condition 1. In a centralized economy, this corresponds to the assumption of uniform social weights among agents as, for instance, in Weinbaum (2010). Finally, in order to study the effect of different proportion of loss-averse agents I consider $\lambda = \{0.2, 0.4, 0.8\}$.

4.1 The initial distribution of wealth

Figure 2 shows the initial distribution of wealth which results from the assumption about the Lagrange multipliers $\phi_{LA}(z)$ and ϕ_{RA}. The risk-averse agents hold 8%, $16,4\%$ and 35% of the aggregate wealth when their proportion is 20%, 40% and 80%, respectively. Thus, in all cases, financial wealth is relatively more concentrated in the hands of loss-averse agents. Among the loss-averse agents, those with high reference level hold a larger fraction of financial wealth. Intuitively, the richer the loss-averse agent today, the higher is the consumption class to which she expects to belong in the future. This is economically plausible and holds for all values of λ. However, the distribution of financial wealth shifts toward the agents with high reference levels when the fraction of loss averse agents increases. In terms of cumulative distribution function, agents with reference level smaller or equal than .5 generally hold slightly less than 50% of the total wealth of loss-averse agents. In summary, the distribution of financial wealth shows two main features: 1) it is relatively more concentrated in the hands of loss-averse agents; 2) it is quite well dispersed among loss-averse agents.

Insert Figure 2 about here

4.2 Market price of risk and risk free rate

Figure 3 shows the market price of risk and the risk free rate as a function of the aggregate endowment. The market price of risk decreases with the aggregate endowment in both high-consumption and low-consumption periods. More precisely, in high-consumption periods, as the aggregate endowment declines toward D_t^*, the consumption of the loss-averse agents declines toward the reference level and their risk aversion rapidly increases which, in turn, leads to higher price of risk. Conversely, when the economy enters a low-consumption period, the loss-averse agents, in particular those with large reference level, become risk-seeking and thus, the price of risk immediately jumps down. However, since loss-averse agents do not consume anything in low-consumption periods, additional decreases in the aggregate endowment shift financial wealth toward the standard risk-averse agent. This increases the contribution of the risk-averse agent to the aggregate risk aversion and, thus, the equilibrium price of risk. Therefore, when the economy falls deeply in low-consumption periods (i.e. the aggregate endowment approaches zero) the market price of risk converges to that of an economy populated with only risk-averse agents

Figure 3 also reveals the effect of λ, the fraction of risk-averse agents. In general, λ has two distinct effects on the dynamic properties of the price of risk: first, it determines the level of the price of risk in both low-consumption and high-consumption periods; second, it governs the discontinuous adjustment in the price of risk around D_t^*. About the first effect, we observe that the larger is λ the lower is the price of risk. Indeed, since the risk aversion of the risk-averse agent is constant and, most of the time, larger than that of loss-averse agents, the fraction of risk-averse agents increases the aggregate risk aversion and, thus, the price of risk. About the second effect, the lower is λ (i.e. the larger is the proportion of loss-averse agents) the larger is the increase in the price of risk when the aggregate endowment falls to D_t^* and the larger (in absolute value) is the down jump in the price of risk when the economy enters a low-consumption period.

Finally, when the loss-averse agents dominate the economy (i.e. for low values of λ), the price of risk in low-consumption periods is almost constant and always smaller than the price of risk in high-consumption periods[3]. This suggests that an economy populated with only loss-averse agents counterfactually produces prices of risk higher in good times than in bad times. Economically, this is explained by the effect of the risk-seeking agents in low-consumption periods which push down the price of risk. Instead, the presence of the standard risk-averse agent counterbalances the effect of loss-aversion and keeps the aggregate risk aversion sufficiently high, even in low-consumption periods, inducing a reasonable dynamics of the price of risk.

The risk free rate decreases with the aggregate endowment in high-consumption periods, while it is almost constant in low-consumption periods. Economically, this is explained by the dynamic behavior of the marginal utility. In high-consumption periods, all agents are risk-averse and their marginal utility increases when the aggregate endowment falls to D_t^*. As a result all investors are motivated to borrow, driving up the equilibrium interest rate. Differently, low-consumption periods are characterized by the coexistence of risk-averse and risk-seeking agents, therefore, a fall in the aggregate endowment decreases the marginal utility of risk-seeking agents and increases the marginal utility of the risk-averse agents. These two effects tend to offset each other keeping the risk free rate almost constant.

[3] Indeed, one can show that, with $\lambda = 0$, the price of risk in low-consumption periods is $\theta = .5\gamma_2\sigma_D$ while in high-consumption periods $\theta = \frac{D_t}{D_t - .5e^{\alpha t}\overline{z}^2}\gamma_2\sigma_D > .5\gamma_2\sigma_D$.

Insert Figure 3 about here

4.3 Return volatility and equity premium

Figure 4 shows the return volatility and the equity premium as a function of the aggregate endowment. We observe that, as the aggregate endowment declines toward D_t^*, the return volatility rapidly increases. When the economy enters low-consumption period, the return volatility initially falls down because of the effect of risk-seeking agents on the price of risk. Then, following additional decreases in the aggregate endowment, financial wealth shifts toward the risk-averse agent, the price of risk increases and the return volatility rises again. The level of the return volatility and its discontinuous adjustment around D_t^* are governed by the fraction of loss-averse agents in the economy: 1) the larger is the fraction of loss-averse agents the lower is the return volatility over most of the range of D_t; 2) the larger is the fraction of loss-averse agents the higher is the stock market volatility around D_t^* and the larger is the down jump when the economy enters low-consumption periods; 3) The interaction between loss-averse and risk-averse agents generates excess volatility in stock returns. These results are explained by the effect of λ on the dynamics of the price of risk. Differently, when the loss-averse agents dominate the economy, the return volatility in low-consumption periods tends to remain constant at the level of the endowment volatility. A reasonable level of excess volatility is generated only in high-consumption periods. This suggests again that an economy populated with only loss-averse agents cannot reproduce the stylized facts of asset prices.

Figure 4 also shows the equity premium as a function of the aggregate endowment. Naturally, the equity premium is positively correlated with the market volatility: it generally decreases with the aggregate endowment in both low- consumption and high-consumption periods with a peak at D_t^* when the consumption of the loss-averse agents declines toward the reference level; when the economy moves to low-consumption periods the equity premium has a down jump with size that depends on the fraction of loss-averse agents and, after that, it rises again when the economic conditions additionally deteriorate. Thus, positive stock returns are accompanied by increased volatility of returns as empirically observed.

Insert Figure 4 about here

4.4 Portfolios and trading volume

In order to correctly analyze trading volume and in particular, the relation between prices and trading, asset prices need to follow a realistic processes. For this reason, I choose $\lambda = 0.8$ which produces a realistic dynamics of the the price of risk and the return volatility. Figure 5 shows the number of risky asset held by loss-averse and risk-averse agents. We observe that, when the aggregate endowment falls to D_t^* loss-averse agents decumulate stocks. Indeed, in those states of the world, stocks become unacceptably risky for the loss-averse agents because they deliver low dividends when consumption approaches the reference level. As a result, they transfer the stocks to the risk-averse agent. When, instead, the economy enters low-consumption periods, loss-averse agents gradually become risk-seeking and, therefore, the number of assets that they hold has a positive jump. By following this strategy, loss-averse agents aim to increase the probability of pushing their consumption above the reference point. The third panel of Figure 5 analyzes the dynamic re-distribution of stocks among loss-averse agents. We observe that, when the dividend decreases, the percentage of the risky asset held by agents with reference level larger than 0.5 increases, meaning the stock becomes more concentrated around agents with large reference level. In high-consumption periods, this is explained by the reluctance of high-reference agents to sell stocks that pay low dividends (i.e. the disposition effect). In low-consumption periods, instead, agents with high reference become risk-seeking and accumulate more stocks than low-reference agents.

Insert Figure 5 about here

The transfer of stocks among agents produces trading volume in equilibrium. Figure 6 plots the expected trading volume conditional on the current aggregate endowment, that is

$\mathbb{E}\left(T\left(D_1, D_2\right)\middle| D_1\right)$. We observe that the trading volume is negatively related to the state of the economy, as defined by D_t, with a down jump when the aggregate endowment falls below D_t^*. Economically, this is explained by the dynamics of the risk aversion. Indeed, when the aggregate endowment declines, the consumption of loss-averse agents approaches the reference level and their risk aversion drastically increases, producing a high incentive to sell the risky asset. This effect is stronger for high-reference agents because they are more subject to the risk that their consumption falls below the reference level when economic conditions deteriorate. Therefore, much of the trading volume is determined by the effect of high reference agents. When, the aggregate endowment falls below D_t^*, high-reference agents have less hedging needs and the trading volume initially decreases. However, when the economic conditions additionally deteriorate, the hedging necessity of intermediate and low reference agents rises the trading volume again.

Insert Figure 6 about here

After having analyzed the determinants of the trading volume, I study the relation between trading volume and asset prices. Empirically, trading volume and asset prices are dynamically related. In particular, there are two stylized facts: a positive relation between volume and return volatility and a positive relation between volume and stock return. This empirical evidence is summarized, for instance, in Xiouros (2012). In order to study the dynamic relation between trading volume and volatility and between trading volume and stock return I need a measure of returns and return volatility. It may seems obvious to take the quantity estimated in the previous paragraph, that is σ_t and μ_t. However, σ_t and μ_t are instantaneous quantities and depend only on the contemporaneous endowment D_t, while, the trading volume is obtained through discretization and therefore it depends on the aggregate endowment at the two extremes of the trading period (that is, D_1 and D_2). Thus, it would be better to compute volatility and stock return using the same discretization scheme as in the computation of the trading volume. Let R_{t+1} be the discrete return from holding the risky asset between time t and $t+1$. As a proxy for the stock market volatility I consider the absolute change in the log price-dividend ratio, that is $|\Delta pd| = \left|\log\left(\frac{P_{t+1}}{D_{t+1}}\right) - \log\left(\frac{P_t}{D_t}\right)\right|$, as suggested by Xiouros (2012). Figure 7 shows a strong positive correlation between trading volume and volatility (i.e. changes in the price-dividend ratio) and between trading volume and stock return. To understand this assume, for instance, a negative shock on the aggregate endowment: the consumption of loss-averse agents approaches the reference level and, consequently, returns and volatility rise; then, loss-averse agents readily sell the risky asset to risk-averse agents, thus, increasing the volume of trading. The opposite holds in case of a positive shock of the aggregate endowment. This mechanism explains the positive correlation between trading volume and volatility and between trading volume and stock returns. Finally, we observe that the correlations between trading volume, stock return and volatility are higher in intermediate states (either bad or good) than in very good or very bad states. Indeed, the positive correlations are explained by the hedging reasons of loss-averse agents. In very good states, hedging reasons are minimal and the effect of loss-aversion tends to disappear, decreasing the dynamic correlations involving trading volume. On the contrary, in very bad states, financial wealth shifts toward the standard risk-averse agent, decreasing the contribution of the loss-averse agents to the dynamic correlations of trading volume. Interestingly, the explanations for the dynamics of the trading volume and its correlation with asset prices are purely based on time-varying hedging reasons and does not require any type of market friction or incompleteness[4].

Insert Figure 7 about here

5 A note on the time-varying reference level

Besides its implications for the existence of the equilibrium, the time-varying reference level of consumption also determines the economic fluctuations over time and the alteration between high-consumption and low-consumption periods. To understand this, assume that the

[4] For a review of different theories related to trading see, for instance, Xiouros (2012).

reference level reference of consumption is constant instead of time-varying, that is, $\alpha = 0$. In this case, the threshold of the aggregate endowment equals the constant value D^* given by

$$D^* = \lambda \left(\overline{x}(\overline{z})\phi_{RA}\right)^{-\frac{1}{\gamma_1}} + (1-\lambda)\int_0^{\overline{z}} \left(z + \left(\phi_{LA}(z)\overline{x}(\overline{z})\right)^{-\frac{1}{\gamma_2}}\right) dz$$

and the singular continuous component[5] is

$$A_t = -\frac{\left[x_D(t, D^*_+) - x_D(t, D^*_-)\right] L_t(D^*)}{x(t, D^*)}. \quad (24)$$

Since the aggregate endowment has a positive drift, it tends to increase away from the constant D^*. This implies that high-consumption periods tend to dominate the economy when simulating over a long time horizon, reducing the effect of loss aversion on asset prices. Instead, with a time-varying reference level z_t and the appropriate α, one obtains the desired alternation between high-consumption and low-consumption periods.

The other crucial assumption about the distribution of reference levels concerns the upper bound $\overline{z}$ which I assume to be finite. Assume instead that the upper bound of the reference level distribution is $\overline{z} = \infty$. It is easy to see that, in this case, we have $D^*_t = \infty$ and $A_t = 0$ $\forall t$, meaning that the economy is always in low-consumption periods independently of the aggregate endowment.

6 Conclusion

In this article I study an economy where agents are loss-averse in consumption and they are equipped with power, s-shaped utility relative to time-varying reference level. In addition, agents are heterogeneous in the reference level. The heterogeneity in the reference level, besides being economically meaningful, allows for a deeper understanding of the effect of loss-aversion on asset prices which, to the contrary, is not possible in models characterized

[5] In this case the computation of the dynamics of x only requires the application of the generalized Ito's rule for convex function of semimartingale.

by a representative investor. The main results of the paper can be listed as follow:

- The optimal consumption related to s-shaped utility is discontinuous, either above the reference level or zero. This discontinuity is at odds with the existence of the equilibrium unless we assume a continuum of reference levels among agents.
- Loss-aversion in consumption implies, counterfactually, that the price of risk and the return volatility are higher in good times than in bad times.
- The interaction between loss-averse and standard risk-averse agents produces reasonable dynamics of the price of risk, the return volatility and explains the dynamic of the trading volume and its correlation with asset prices.

In general, dynamic economies based on risk-averse agents produces reasonable properties only for asset prices, while they do not explain well the trading volume and its correlation with asset prices. Instead, this paper suggests that, dynamic economies with only loss-averse agents do not explain well the dynamics of asset prices while they can potentially explain the dynamics of the trading volume through the hedging reasons of loss-averse agents. One possibility to explain the dynamics of asset prices and trading volume at the same time, is to consider a dynamic economy populated with a fraction of risk-averse agents and a fraction of loss-averse agents over consumption.

The framework considered in this paper could be extended by allowing for more general (and maybe more realistic) dynamics for the reference of consumption. For example one could model the reference level of consumption using the internal habit specification of Constantinides (1990) and Detemple and Zapatero (1991) that makes utility function no longer time-separable and forces investors to consider the effect of current consumption on the future reference level. This would change, almost completely, the economic mechanism that drives the results of this paper, therefore, it could potentially produce more complex dynamics of optimal consumption and asset prices. Finally, an extremely challenging and interesting problem would be the analysis of a production economy, in the spirit of Cox

Ingersol and Ross (1985). On the one hand, in a production economy, the absence of market clearing conditions for consumption, rules out the existence problem simplifying the solution method; on the other hand, the s-shaped utility makes the value function of the loss-averse agent potentially non-concave with additional difficulties in the consumption/portfolio problem.

References

[1] Barberis N., Huang M., Santos T, 2001, Prospect theory and asset prices, *Quarterly Journal of Economics* 116, 1–53.

[2] Benartzi, S., Thaler, R., 1995, Myopic loss aversion and the equity premium puzzle, *Quarterly Journal of Economics* 110, 73–92.

[3] Berkelaar A. B., Kouwenberg R., 2009, From boom 'till boost: how loss aversion affects asset prices, *Journal of Banking & Finance* 33, 6, 1005-1013.

[4] Berkelaar A. B., Kouwenberg R., Post, T., 2004, Optimal portfolio choice under loss aversion, *The Review of Economics and Statistics*, 86, 973–987.

[5] Chan Y. L., Kogan L. 2002, Catching up with the Joneses: heterogenous preferences and the dynamics of asset prices, *Journal of Political Economy* 110, 6, 1255-1285.

[6] Constantinides G., 1982, Intertemporal asset pricing with heterogeneous consumers and without demand aggregation, Journal of Business, 55, 253-267.

[7] Constantinides G., 1990, Habit formation: a resolution of the equity premium puzzle, *Journal of Political Economy* 98, 519–543.

[8] Cox J. C., Huang C. F., 1989, Optimal consumption and portfolio policies when asset prices follow a diffusion process, *Journal of Economic Theory* 49, 33-83.

[9] Cox J. C., Ingersoll, J., Ross, S., An intertemporal general equilibrium model of asset prices, Econometrica, 53, 363-384.

[10] Curatola, G., 2011, Optimal stock allocation under loss aversion, working paper.

[11] Detemple J. B., F. Zapatero, 1991, Asset prices in an exchange economy with habit formation, *Econometrica,* 59, 1633-1657.

[12] Dimmock, S. G., Kouwenberg, R., 2010, " Loss-aversion and household portfolio choices, *Journal of Empirical Finance, 17, 441-459.*

[13] Easley D., Yang L., 2011, Loss aversion, survival and asset prices, working paper.

[14] Gill, D., Prowse, V., 2012, A structural analysis of disappointment aversion in a real effort competition, *American Economic Review*, 102, 469–503.

[15] Gomes, F. J., 2005, Portfolio choice and trading volume with loss-averse investors, *The Journal of Business*, 78, 2, 675-706.

[16] Kahneman, D., Tversky, A., 1979, Prospect Theory: an analysis of decision under risk, *Econometrica* 47, 263–290.

[17] Karatzas, I., Shreve, S., 1991, Equilibrium model with singular asset prices, *Mathematical Finance* 1, 11-29.

[18] Karatzas, I., Shreve, S., 1998, Methods of mathematical finance, *Springer-Verlag: New York.*

[19] Lucas, R., 1978, Asset prices in an exchange economy, *Econometrica* 46, 1429–1445.

[20] McQueen, G., Vorkink, K., 2004, Whence garch? A preference-based explanation for conditional volatility*, Review of Financial Studies* 17, 915–949.

[21] Yogo M., 2008, Asset prices under habit formation and reference-dependent preferences, *Journal of Business and Economic Statistics* 26, 131-143.

[22] Peskir, G., 2005, A change of variable formula with local time on curves, J*ournal of Theoretical Probability* 18, 499-535.

[23] Rosenbatt-Wisch, R., 2008, Loss aversion in aggregate macroeconomic time series, *European Economic Review*, 52, 1140-1149.

[24] Tversky, A, Kahneman, D., 1991, Loss aversion in riskless choice: a reference-dependent Model, *The Quarterly Journal of Economics*, V, 297–323.

[25] Tversky, A, Kahneman, D., 1992, Advances in prospect theory: cumulative representation of uncertainty, *Journal of Risk and Uncertainty*, 106, 1039–1061.

[26] Xiouros, C., 2011, Disagreement, habit and the dynamic relation between volume and prices, working paper.

[27] Weinbaum D., 2010, Preference heterogeneity and asset prices: an exact solution*, Journal of Banking and Finance* 34 (9), 2238-2246.

Appendix A: Proof

6.0.1 Proof of Propositions

Proof of Proposition 1. The optimal consumption policy of the power utility agent follows from the standard martingale method. To solve for the optimal consumption policy of the loss-averse agents I follow the same technique in Berkelaar, Kouwenberg and Post (2004). Since markets are complete, this optimization reduces to a static problem. In other words, for any t, the loss-averse agent maximizes:

$$\mathbb{E}U_{LA}(c_t, z_t)$$

subject to the static budget constraint

$$\mathbb{E}\int_0^\infty e^{-\rho t}x_t c_t \leq x_0 W_0$$

where $x_t = e^{\rho t}H_t$. The Legendre-Fenchel transform (or convex conjugate) for the previous maximization problem is defined by

$$L(x_t) = \max_{c_t \geq 0}\left\{U_{LA}(c_t, z_t) - \phi_{LA}x_t c_t\right\}.$$

$U_{LA}(c_t, z_t)$ is not quasi concave and so the first order conditions may only describe local maxima. In particular, if $c_t < z_t$ the utility function is convex and we know from Weirestrass theorem that the maximum must lie on the boundary, 0 or z_t. If $c_t \geq z_t$ the utility function is concave and we can apply standard Kuhn Tucker (KT) theorem. Taking the KT conditions we obtain:

$$\begin{aligned}&(c_t - z_t)^{-\gamma_2} = \phi_{LA}x_t - y,\ c_t \geq 0\\ &yc_t = 0,\ y \geq 0\end{aligned}$$

Where ϕ_{LA} is the Lagrange multiplier associated to the budget constraint and y is the multiplier associate to the non-negativitiy constraint. After solving the KT conditions I obtain three candidate solutions for optimal consumption plan c_t^*. Let c_t^1, c_t^2 and c_t^3 be these candidate solutions, then

$$\begin{cases} c_t^1 = 0 \\ c_t^2 = z_t \\ c_t^3 = z_t + (\phi_{LA}x_t)^{-1/\gamma_2} \end{cases} \tag{25}$$

We need to compare the local maxima to determine a global maximum. First, $U(c_t^3, z_t) - \phi_{LA} x_t c_t^3 > U(c_t^2, z_t) - \phi_{LA} x_t c_t^2$ if

$$\frac{(\phi_{LA} x_t)^{\frac{\gamma_2 - 1}{\gamma_2}}}{1-\gamma_2} - \phi_{LA} x_t \left(z_t + (\phi_{LA} x_t)^{-1/\gamma_2} \right) > -x_t z_t$$

that is,

$$\frac{1}{1-\gamma_2} - 1 > 0.$$

Since $\gamma_2 < 1$, the above inequality is always true; second, $U(c_t^1, z_t) - \phi_{LA} x_t c_t^1 > U(c_t^2, z_t) - \phi_{LA} x_t c_t^2$ if

$$-B \frac{z_t^{1-\gamma_2}}{1-\gamma_2} > -x_t z_t \tag{26}$$

that is,

$$x_t > \frac{B z_t^{-\gamma_2}}{1-\gamma_2}.$$

When the above inequality holds true we need to compare c_t^1 with c_t^3, otherwise c_t^3 is the optimal solution. Finally, $U(c_t^3, z_t) - \phi_{LA} x_t c_t^3 > U(c_t^1, z_t) - \phi_{LA} x_t c_t^1$ if

$$\frac{(\phi_{LA} x_t)^{\frac{\gamma_2 - 1}{\gamma_2}}}{1-\gamma_2} - \phi_{LA} x_t \left(z_t + (\phi_{LA} x_t)^{-1/\gamma_2} \right) \geq -B \frac{z_t^{1-\gamma_2}}{1-\gamma_2}$$

that is,

$$(\phi_{LA} x_t)^{\frac{\gamma_2 - 1}{\gamma_2}} \frac{\gamma_2}{1-\gamma_2} - \phi_{LA} x_t z_t + B \frac{z_t^{1-\gamma_2}}{1-\gamma_2} \geq 0.$$

In addition,

$$f\left(\frac{B z_t^{-\gamma_2}}{1-\gamma_2}, z_t \right) = \left(\frac{B z_t^{-\gamma_2}}{1-\gamma_2} \right)^{\frac{\gamma_2 - 1}{\gamma_2}} \frac{\gamma_2}{1-\gamma_2} > 0 \tag{27}$$

and

$$\frac{df(x_t, z_t)}{dx_t} = -\phi_{LA} \left[(\phi_{LA} x_t)^{-\frac{1}{\gamma_2}} + z_t \right] < 0 \tag{28}$$

and $\lim_{x \to \infty} f(x_t, z_t) = -\infty$. Putting all this information about $f(x_t, z_t)$ together we can conclude that $f(x_t, z_t)$ has exactly one zero in the interval $\left(\frac{B z_t^{-\gamma_2}}{1-\gamma_2}, \infty \right)$. Let $\overline{x}$ be such point with $f(\overline{x}, z) = 0$. $f(x_t, z_t)$ is strictly decreasing, $f(x_t, z_t) > 0$ for all $x_t < \overline{x}$ and $f(x_t, z_t) \leq 0$ for all $x_t \geq \overline{x}$. Hence, c_t^3 is optimal for $x_t < \overline{x}$ and c_t^1 is optimal for $x_t \geq \overline{x}$. Finally, let $\widehat{c}_t$ be a candidate solution, in alternative of c_t^* satisfying the budget constraint either with equality or inequality. Then,

$$\begin{aligned}
& \mathbb{E}\int_0^\infty e^{-\rho t}U(c_t^*,z)dt - \mathbb{E}\int_0^\infty e^{-\rho t}U(\widehat{c}_t,z)dt \\
= \; & \mathbb{E}\int_0^\infty e^{-\rho t}U(c_t^*,z)dt - \mathbb{E}\int_0^\infty e^{-\rho t}U(\widehat{c}_t,z)dt - \phi_{LA}x_0W_0 + \phi_{LA}x_0W_0 \\
\geq \; & \mathbb{E}\int_0^\infty e^{-\rho t}U(c_t^*,z) - \mathbb{E}\int_0^\infty e^{-\rho t}U(\widehat{c}_t,z) - \phi_{LA}\mathbb{E}\int_0^\infty e^{-\rho t}x_t c_t^* dt + \phi_{LA}\mathbb{E}\int_0^\infty e^{-\rho t}x_t\widehat{c}_t dt \\
= \; & \mathbb{E}\int_0^\infty e^{-\rho t}\left(U(c_t^*,z) - x_t c_t^*\right)dt - \mathbb{E}\int_0^\infty e^{-\rho t}\left(U(\widehat{c}_t,z) - x_t\widehat{c}_t\right)dt \geq 0
\end{aligned}$$

where, the first inequality follows because the budget constraint holds with equality for c_t^* while it holds with inequality for $\widehat{c}_t$ and the last inequality follows because c_t^* maximizes $L(x_t)$. This shows that the consumption policy of Proposition 1 solves the maximization problem of the loss-averse agent. ■

Proof of Lemma 1. The result in the lemma follows from the fact that, thanks to Assumption 1, $\overline{x}$ is monotonic decreasing function of z_t. An application of the implicit function theorem on Eq (11) in Proposition 1 yields

$$\frac{\partial \overline{x}}{\partial z_t} = \frac{-\frac{\partial \phi_{LA}}{\partial z}\left((\phi_{LA}\overline{x})^{-1/\gamma_2} + x_t z_t\right) + B z_t^{-\gamma_2} - \phi_{LA}\overline{x}}{\phi_{LA}\left((\phi_{LA}\overline{x})^{-1/\gamma_2} + z_t\right)} \tag{29}$$

Since the denominator is positive, the sign of $\partial\overline{x}/\partial z_t$ is determined by the numerator only. Using Eq. (11) we obtain

$$B z_t^{-\gamma_2} - \phi_{LA}(z)\overline{x} = -\frac{\gamma_2}{z_t}\left[(\phi_{LA}(z)\overline{x})^{\frac{\gamma_2-1}{\gamma_2}} + \phi_{LA}(z)\overline{x}z_t\right]$$

The right hand side is negative, thus, the left hand side is negative too, which, in conjunction with $\frac{\partial \phi_{LA}}{\partial z} \geq 0$ implies $\partial\overline{x}/\partial z_t < 0$. ■

Proof of Proposition 2. Lemma 1 implies that the market clearing condition writes

$$\lambda\left(x_t\phi_{RA}\right)^{-\frac{1}{\gamma_1}} + (1-\lambda)\int_0^{z^*}\left(z e^{\alpha t} + (\phi_{LA}(z)x_t)^{-\frac{1}{\gamma_2}}\right)dz = D_t \tag{30}$$

where z^* is the reference point of the agent with $\overline{x}(z^*) = x_t$. Thus, the couple (x_t, z^*) also satisfies

$$f(x_t, z^*) = (\phi_{LA}^* x_t)^{\frac{\gamma_2-1}{\gamma_2}}\frac{\gamma_2}{1-\gamma_2} - \phi_{LA}^* x_t z^* e^{\alpha t} + B\frac{(z^* e^{\alpha t})^{1-\gamma_2}}{1-\gamma_2} = 0. \tag{31}$$

where $\phi_{LA}^* = \phi_{LA}(z^*)$ is the Lagrange multiplier of the agent with reference level $z_t^* =$

$z^* e^{\alpha t}$. Therefore, the proof of the Proposition consists in demonstrating that the systems of continuous equations (30)-(31) admits a unique solution (x_t^*, z^*). Consider an arbitrary reference level $\widetilde{z} \in [0, \overline{z})$. From the proof of Proposition 1, we have that for any $\widetilde{z} \in [0, \overline{z})$ it exists a unique $\overline{x}(\widetilde{z}) \in \mathbb{R}^+$ satisfying $f(\overline{x}, \widetilde{z}) = 0$. The total demand for consumption corresponding to $\widetilde{z}$ is then (i.e. the LHS of Eq (30))

$$\lambda \left(x_t \phi_{RA}\right)^{-\frac{1}{\gamma_1}} + (1-\lambda) \int_0^{\widetilde{z}} \left(z e^{\alpha t} + \left(\phi_{LA}(z) x_t\right)^{-\frac{1}{\gamma_2}} \right) dz$$
$$\lambda \left(x_t \phi_{RA}\right)^{-\frac{1}{\gamma_1}} + (1-\lambda) \left(\frac{\widetilde{z}^2 e^{\alpha t}}{2} + x_t^{-\frac{1}{\gamma_2}} \int_0^{\widetilde{z}} \phi_{LA}(z)^{-\frac{1}{\gamma_2}} dz \right)$$

is monotonic increasing in $\widetilde{z}$, monotonic decreasing in $\overline{x}$ and ranges from 0 to $\lambda \left(\overline{x}(\overline{z}) \phi_{RA}\right)^{-\frac{1}{\gamma_1}} + (1-\lambda) \left(\frac{\overline{z}^2 e^{\alpha t}}{2} + \overline{x}(\overline{z})^{-\frac{1}{\gamma_2}} \int_0^{\overline{z}} \phi_{LA}(z)^{-\frac{1}{\gamma_2}} dz \right) = D_t^*$. Putting together all this information we conclude that the systems (30)-(31) admits at least one solution when $D_t < D_t^*$. Let (x_t^*, z^*) be such a solution and assume that $(\widehat{x}, \widehat{z})$ is an alternative solution satisfying the systems of equations (30)-(31). If $\widehat{z} = z^*$, then clearly we must have $\widehat{x} = x_t^*$ because Eq. (31) admits only one solution. Correspondingly, if $\widehat{x} = x_t^*$ then, necessarily, $\widehat{z} = z^*$ otherwise Eq. (30) does not hold. Moreover, we cannot have $\widehat{x} \geq (\leq) x_t^*$ and $\widehat{z} \geq (\leq) z^*$ because Eq. (31) does not hold in this case. Finally, the case $\widehat{x} \geq (\leq) x_t^*$ and $\widehat{z} \leq (\geq) z^*$ is impossible because it violates Eq. (30). Then, the system of equations (30)-(31) admits a unique solution when $D_t < D_t^*$. When instead $D_t \geq D_t^*$, $z^* = \overline{z}$ and x_t satisfies

$$\lambda \left(x_t \phi_{RA}\right)^{-\frac{1}{\gamma_1}} + (1-\lambda) \int_0^{\overline{z}} \left(z e^{\alpha t} + \left(\phi_{LA}(z) x_t\right)^{-\frac{1}{\gamma_2}} \right) dz = D_t$$

which admits a, possibly numerical, unique solution. By definition, the solution of the system is continuous at $D_t = D_t^*$. An application of the implicit function theorem on Eq (30) produces

$$\frac{\partial x_t}{\partial D} = \frac{1}{-\frac{\lambda \phi_{RA}}{\gamma_1} \left(x_t \phi_{RA}\right)^{-\frac{1}{\gamma_1}-1} + (1-\lambda) \left(z_t^* z_x^* + z_x^* \left(\phi_{LA}^* x\right)^{-\frac{1}{\gamma_2}} - \frac{1}{\gamma_2} x_t^{-\frac{1}{\gamma_2}-1} \int_0^{z^*} \phi_{LA}(z)^{-\frac{1}{\gamma_2}} dz \right)} \tag{32}$$

where

$$\begin{aligned} z_x^* &= \frac{\partial z_t^*}{\partial x_t} e^{-\alpha t} \\ &= \frac{\phi_{LA}^* \left(\left(\phi_{LA}^* x_t\right)^{-1/\gamma_2} + z_t^* \right)}{-\phi_{z,LA}^* \left(\left(\phi_{LA}^* \overline{x}\right)^{-1/\gamma_2} + x_t z_t^* \right) + B z_t^{-\gamma_2} - \phi_{LA}^* x_t} e^{-\alpha t} \leq 0 \end{aligned}$$

is obtained by applying the implicit function theorem on Eq (11) and $\phi^*_{z,LA} = \frac{\partial \phi_{LA}(z)}{\partial z}\Big|_{z=z^*}$. In particular, from the proof of Lemma 1, we have that $z^*_x < 0$ when $D_t < D^*_t$; when instead $D_t \geq D^*_t$, $z^* = \overline{z}$ and therefore $z^*_x = 0$. This implies that

$$\lim_{D_t \to D^*_{t+}} \frac{\partial x_t}{\partial D} = \frac{1}{-\frac{\lambda \phi_{RA}}{\gamma_1} \overline{x} \phi_{RA}^{-\frac{1}{\gamma_1}-1} - (1-\lambda) \frac{1}{\gamma_2} \overline{x}^{-\frac{1}{\gamma_2}-1} \int_0^{z^*} \phi_{LA}(z)^{-\frac{1}{\gamma_2}} dz}$$

On the contrary, $\lim_{D_t \to D^*_-} z^*_x \neq 0$, thus

$$\lim_{D_t \to D^*_-} \frac{\partial x_t}{\partial D} \neq \lim_{D_t \to D^*_{t+}} \frac{\partial x_t}{\partial D}$$

which implies that $\frac{\partial x_t}{\partial D}$ is discontinuous at $D_t = D^*_t$. Moreover, $z^*_x \leq 0$ implies that $\frac{\partial x_t}{\partial D} < 0$ as can be verified by inspection of Eq (32). Double differentiation of the market clearing condition (30) yields

$$\xi(x_t, t) \left(\frac{\partial x_t}{\partial D} \right)^2 + \left(\frac{\lambda \phi_{RA}}{\gamma_1} \overline{x} \phi_{RA}^{-\frac{1}{\gamma_1}-1} + (1-\lambda) \frac{1}{\gamma_2} \overline{x}^{-\frac{1}{\gamma_2}-1} \int_0^{z^*} \phi_{LA}(z)^{-\frac{1}{\gamma_2}} dz \right) \frac{\partial^2 x_t}{\partial D^2} = 0 \quad (33)$$

which can be solved for $\frac{\partial^2 x_t}{\partial D^2}$. The functions appearing in Eq (33) are:

$$\begin{aligned}
\xi(x_t, t) &= (1-\lambda) \left(\frac{1}{\gamma_2} \left(\frac{1}{\gamma_2} + 1 \right) x_t^{-\frac{1}{\gamma_2}-2} \int_0^{z^*} \phi_{LA}(z)^{-\frac{1}{\gamma_2}} dz - \frac{2}{\gamma_2} x_t^{-\frac{1}{\gamma_2}-1} z^*_x + e^{\alpha t} z^{*2}_x + z^*_t z^*_{xx} + z^*_{xx} x^{-\frac{1}{\gamma_2}} \right) \\
&\quad - \frac{\lambda}{\gamma_1} x_t^{-\frac{1}{\gamma_1}-1} \phi_{CRRA}^{-\frac{1}{\gamma_1}}, \\
z^*_{xx} &= \frac{\partial^2 z^*_t}{\partial x_t^2} e^{-\alpha t} \\
&= \frac{z^*_x \Lambda_1 + (z^*_x)^2 \Lambda_2 - \phi_{LA}(z^*)^{1-\frac{1}{\gamma_2}} x_t^{-\frac{1}{\gamma_2}-1}}{-\phi^*_{z,LA} \left((\phi_{LA} \overline{x})^{-1/\gamma_2} + x_t z^*_t \right) + B z_t^{*-\gamma_2} - \phi^*_{LA} x_t}, \\
\Lambda_1 &= \phi^*_{z,LA} \left(- \frac{\phi_{LA}(z^*)^{-\frac{1}{\gamma_2}}}{\gamma_2} x_t^{-\frac{1}{\gamma_2}-1} \phi^*_{z,LA} + z^*_t \right) + 2\phi^*_{LA} + \left(1 - \frac{1}{\gamma_2} \right) \phi_{LA}^{*-\frac{1}{\gamma_2}} x_t^{-\frac{1}{\gamma_2}} \phi^*_{z,LA} \\
\Lambda_2 &= \phi^*_{zz,LA} \left((\phi^*_{LA} \overline{x})^{-1/\gamma_2} + x_t z^*_t \right) + \phi^*_{z,LA} \left(- \frac{\phi_{LA}^{*\,-\frac{1}{\gamma_2}-1}}{\gamma_2} x_t^{-\frac{1}{\gamma_2}} + x_t \right)
\end{aligned}$$

where $\phi^*_{zz,LA} = \frac{\partial^2 \phi_{LA}(z)}{\partial z^2}\Big|_{z=z^*}$. Finally, differentiating the market clearing condition w.r.t.

time we obtain

$$\frac{\partial x_t}{\partial t} = \frac{(1-\lambda)\frac{1}{2}z^* e^{\alpha t}}{-\frac{\lambda\phi_{RA}}{\gamma_1}\left(x_t\phi_{RA}\right)^{-\frac{1}{\gamma_1}-1} + (1-\lambda)\left(z_t^* z_x^* + z_x^*\left(\phi_{LA}^* x\right)^{-\frac{1}{\gamma_2}} - \frac{1}{\gamma_2}x_t^{-\frac{1}{\gamma_2}-1}\int_0^{z^*}\phi_{LA}(z)^{-\frac{1}{\gamma_2}}dz\right)}.$$

Since $\frac{\partial^2 x_t}{\partial D^2}$ and $\frac{\partial x_t}{\partial t}$ are function of z_x^* and z_{xx}^* they are discontinuous at $D_t = D_t^*$ for the same reasons explained before. ■

Proof of Lemma 2. It follows directly from the application of the Ito's rule with local time on curves on x_t. ■

Proof of Proposition 3. Given the definition of the function Q_t in Proposition 4 we can write the price of the stock as

$$P_t = \frac{Q_t}{x_t} \tag{34}$$

then,

$$dP_t = [...]\,dt + P_t dA_t + \left[\frac{\partial Q_t/\partial D}{x_t} - \frac{\partial x_t/\partial D}{x_t^2}Q_t\right]D_t\sigma_D dB_t$$

and

$$\frac{dP_t}{P_t} = [...]\,dt + dA_t + \left[\frac{\partial Q_t/\partial D}{Q_t} - \frac{\partial x_t/\partial D}{x_t}\right]D_t\sigma_D dB_t$$

where, for simplicity, I have omitted the drift term. Finally, by differentiating Q_t we obtain

$$\frac{\partial Q_t}{\partial D} = \frac{1}{D_t}\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{\partial x_t}{\partial D}D_s^2 ds + \int_t^\infty e^{-\rho(s-t)}x_t D_s ds\right]$$

■

6.1 Numerical Method

The equilibrium quantities in this paper involve expectations of the type

$$\mathbb{E}_t\int_t^\infty e^{-\rho(s-t)}x_s D_s ds$$

or

$$\mathbb{E}_t\int_t^\infty e^{-\rho(s-t)}\frac{\partial x_s}{\partial D}D_s^2 ds.$$

After the change of variable $y_t = log(D_t)$ these expectations can be written as

$$\int_{-\infty}^\infty\int_t^\infty e^{-\rho(s-t)}x_s e^{y_s}f(y_s)dy_s ds$$

and

$$\int_{-\infty}^{\infty}\int_{t}^{\infty} e^{-\rho(s-t)}\frac{\partial x_s}{\partial D}e^{2y_s}f(y_s)dy_s ds$$

where $f(y_s)$ is the density of the log of D_s, that is normal with mean $log(D_t)+(\mu_D-\frac{1}{2}\sigma^2)(s-t)$ and variance $\sigma^2(s-t)$. To compute this type of expectation I follow the steps listed below: first, I discretize the dividend and the time space using a 100×100 grid of discretization points; second, I solve the market clearing condition for any dividend and time in the grid of points; I interpolate the results in order to get x_t and $\frac{\partial x_t}{\partial D}$ as a function of aggregate dividend and time. Armed with the interpolated function I evaluate numerically the previous double integrals using standard quadrature technique.

Appendix B: Figures and Tables

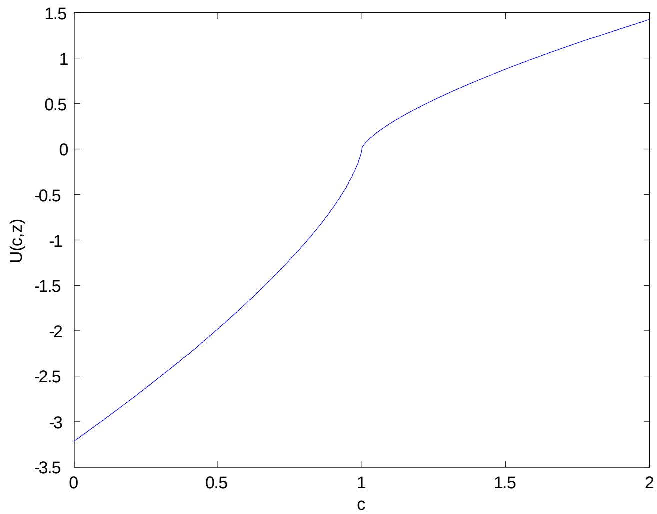


Figure 1: Utility of prospect theory. The s-shaped utility of prospect theory is plotted as a function of consumption. Preference parameters from Table 1, $z = 1$.

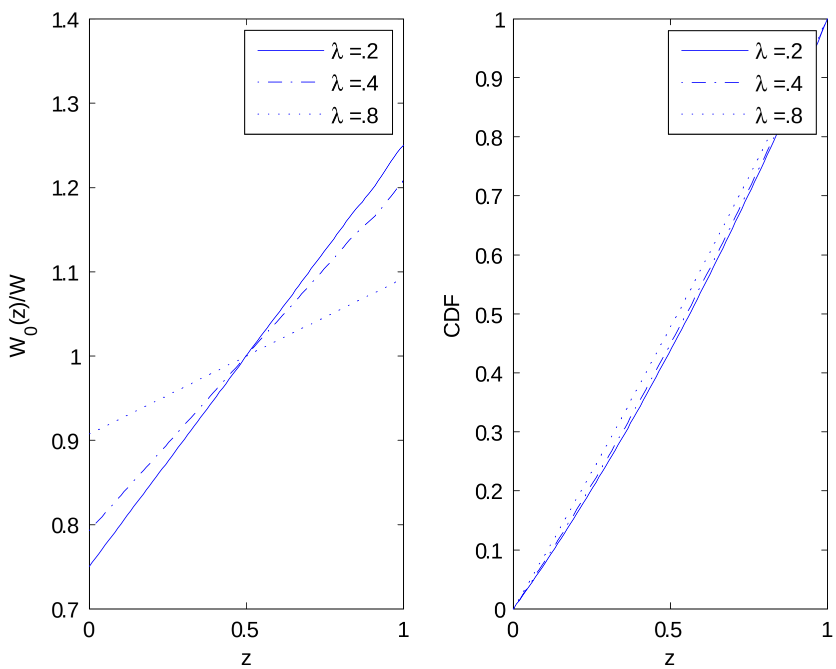


Figure 2: Cross sectional wealth. This picture shows the PDF (left panel) and the CDF (right panel) of the wealth distribution across loss-averse agents.

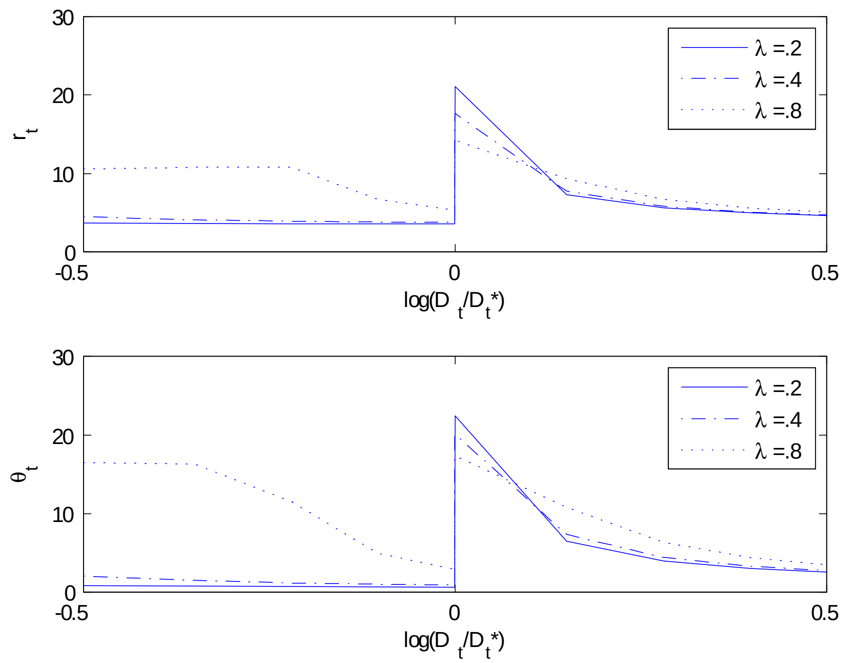


Figure 3: Market price of risk and risk free rate. This figure shows the market price of risk and the risk free rate as a function of the aggregate andowment. Preference and consumption parameters from Table 1, $t = 1$.

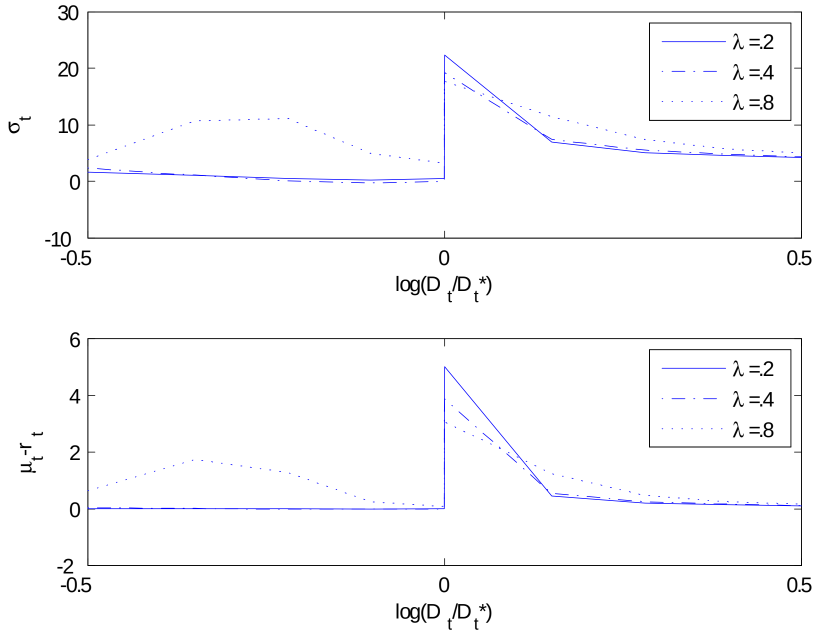


Figure 4: Return volatility and equity premium. This Figure shows the return volatility and the equity premium (both in percentage) as a function of the aggregate endowment. Preference and consumption parameters from Table 1, $t = 1$.

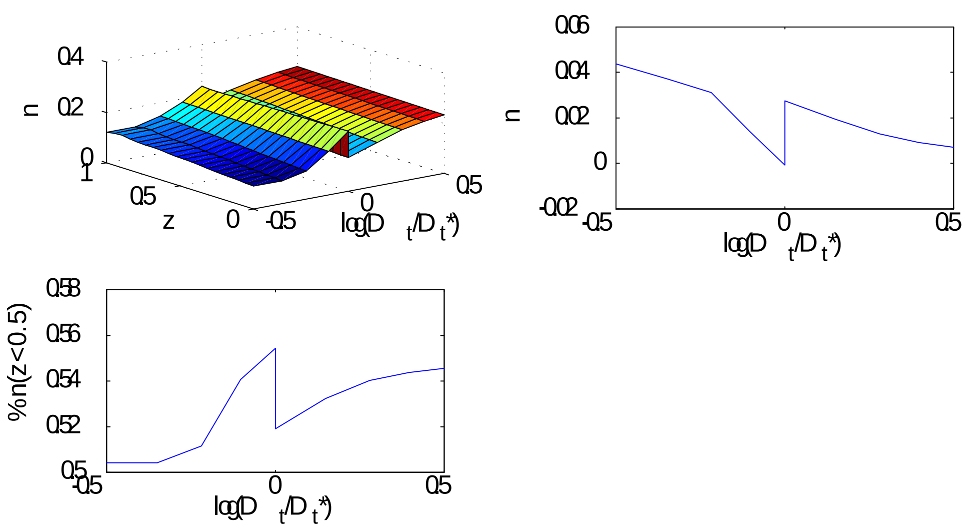


Figure 5: Portfolios. This figure shows the number of stock held by the loss-averse (first column) and risk-averse (second column). The last row shows the fraction of stock held by the loss-averse agents with reference $z \leq 0.5$.

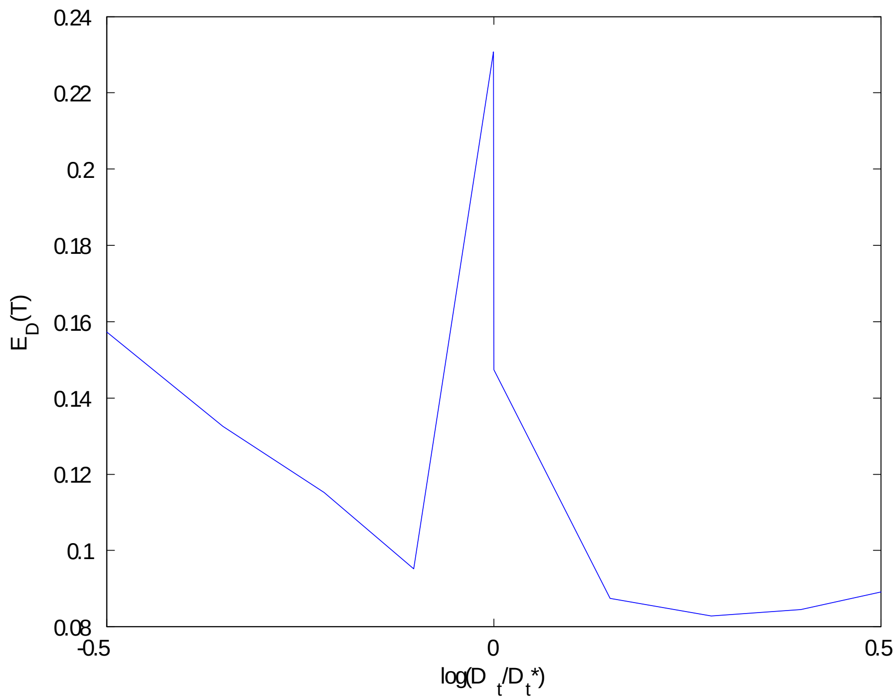


Figure 6: Trading volume. This Figure shows the expected trading volume conditional on the aggregate endowment at the first trading period (D_{t_1}).

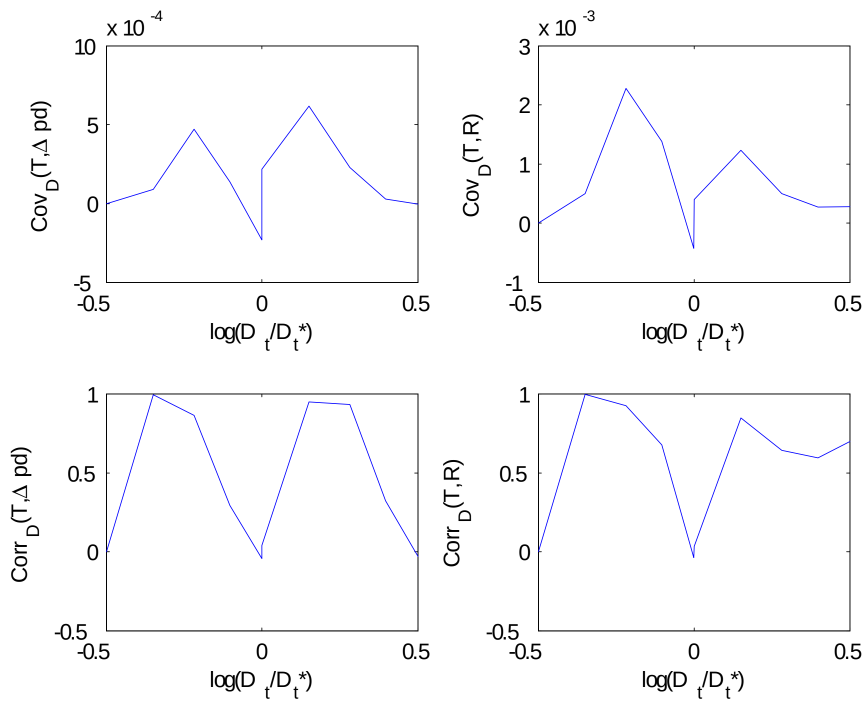


Figure 7: Covariances and correlations. This Figure shows the covariances (upper row) correlation between trading volume and price changes (first column) and between trading volume and stock returns conditional on the aggregate endowment at the first trading period (D_{t_1}).

Symbol	Value	Description
μ_D	.0184	Expected dividend growth rate
σ_D	.033	Volatility of dividend growth rate
λ	.2,.4,.8	Fraction of risk-averse agents
γ_1	5	Risk aversion of power-utility agents
γ_2	.3	Curvature of loss-averse agents utility
B	2.25	Loss-aversion parameter
α	.0085	Growth rate of the reference level
$\bar{z}$	1	Maximum reference level
ρ	.03	Subjective discount rate

Table 1: Model parameters: preferences and aggregate consumption process.