

Asset Price Dynamics with Limited Attention

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Abstract

This paper studies the role that limited attention and inefficient risk sharing play in stock price deviations from the efficient prices at horizons from one day to one month. We expand the Duffie (2010) slow-moving capital model to analyze multiple groups of investors who have varying levels of attention. We test the model's implications through an analysis of the joint dynamics of stock price movements and trading by the different types of investors. The model is consistent with contemporaneous, lead, and lag correlations among returns and trading at daily, weekly, biweekly, and monthly frequencies. We quantify limited attention's economic effects on asset prices by estimating a reduced form version of our model on New York Stock Exchange data. A one standard deviation change in market maker inventories is associated with transitory price movements of 65 basis points at a daily frequency and 159 basis points at a monthly frequency. 8% of a stock's daily idiosyncratic return variance and 25% of its monthly idiosyncratic variance are due to transitory price changes (noise) and the trading variables explain 32% of this noise.

Keywords: Transitory Volatility, Limited Attention, Individuals, Market Makers

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1 Introduction

This paper studies the day-to-day, week-to-week, and month-to-month role that investors' limited attention plays in a stock price deviations from the efficient (random walk) price. The paper develops a theoretical model of investors with varying levels of attention, tests the model's implications for price changes and investors' trading with empirical New York Stock Exchange (NYSE) data, and quantifies the economic effects of limited attention on asset prices by estimating a reduced form version of the theoretical model. The economic magnitude of the empirical estimation provides the paper's most significant results: 8% of a stock's daily idiosyncratic return variance and 25% of its monthly idiosyncratic variance are due to transitory price changes (noise) and the trading variables explain 32% of this noise. The consistency between the theoretical model and empirical data of the lead-lag and contemporaneous correlations among investors' trading and price changes provides support for the model's impact of inattention in stocks' return-generating process. Noise in prices of this magnitude has important implications for asset pricing tests (Asparouhova, Bessembinder, and Kalcheva (2010) and Asparouhova, Bessembinder, and Kalcheva (2013)).

Theoretical Framework: The first part of the paper generalizes the Duffie (2010) slow-moving capital framework. Duffie's model contains two types of market participants who trade with each other after an aggregate supply shock. Market clearing in models with two investor types implies that after any initial shock trades of one type are exactly equal and opposite to those from the other type.

Our model differs from Duffie's most significantly by having more types of investors (individuals, institutions, and market makers) and an infinite-date economy. The first two investor types want to trade with each other in order to hedge an exogenous, evolving, and non-tradable endowment. The third investor type, market makers, are not exposed to the endowment shocks, but are willing to provide liquidity to accommodate other investors' trading needs. Market makers act as arbitrageurs and require compensation for assuming inventory risk via a transitory price impact. Investors' inattention and limited arbitrage lead to negative autocorrelation in returns. Three types of investors allow us to describe relations between observed equity returns and trades of the various investor types. We are also able to characterize trading among the different investor types.

Like the Duffie (2010) model, our theoretical framework contains two main frictions. First, all investors are risk-averse. Therefore, there is no risk-neutral agent to take an infinitely large position

to eliminate mispricing. Second, and more importantly, some of the individual investors are not fully attentive. That is, some individuals only periodically consider whether to trade at each possible date. This partially-attentive behavior causes inefficient risk-sharing vis-a-vis the evolving endowment process and the needs of the institutional investors who trade at each date. Specifically, following an endowment shock, there are fewer individuals present than fully attentive institutions. Market makers trade to inter-temporally smooth supply and demand. They subsequently offload their positions to partially-attentive individuals participating in the market with delay.

The link between theory and empirical tests highlights the similarity and key differences between our approach and Duffie's. Price dynamics in our model work in much the same way as in Duffie (2010). A shock initially induces a sharp price movement that is then slowly reversed. The duration of a reversal is closely related to how inattentive investors are. The Duffie (2010) paper provides a number of examples of a price response to a one-time, easily-identified event such as a S&P500 Index deletion. In contrast, we study repeated, not clearly identified shocks on a daily basis. The goal is to apply our model's results to empirical price and trading data. We specifically include individuals and market makers in our investor types to facilitate mapping the model's results to available NYSE data. Duffie (2010) cites papers using observable events to identify shocks to the trading environment. We use empirical trading data to identify repeated shocks that cause investors to trade. Our approach examines whether the limited attention framework is consistent with the data and the impact of inattention in the overall return-generating process.

Tests of the Model's Implications: The second part of this paper describes the NYSE data and tests multiple implications of our model. For a given equity, our model produces implications related to the joint dynamics of three data series at four frequencies (daily, weekly, biweekly, and monthly) for three time-shifts (one-period lag, contemporaneous, and one-period lead.) The three data series consist of equity returns, market makers' inventories, and individuals' aggregate net trades.¹

To test implications regarding dynamic relations among different time series, the second part of the paper focuses on the direction of correlations (positive, zero, or negative). For example, one of our

¹All of our empirical tests exclude institutions' aggregate net trades due to the adding-up constraints from market clearing that aggregate net trading equals zero and aggregate holdings equal supply of the asset. If one knows the aggregate holdings of market makers and individuals, then the aggregate holdings of institutions are also known. Likewise, if one knows the aggregate net trades of market makers and individuals, then aggregate net trades of institutions are known. Note that in our data, there is one dedicated market maker for each stock.

model's predictions is that market maker inventories this period are positively related to individuals' aggregate net trades next period. Another prediction is that individuals' aggregate net trades this period are positively related to returns over the following period. In total, we test predictions related to 12 correlations at four different frequencies. The empirical relations among return and trading data are consistent with all of the predictions.

A calibration exercise examines whether our model can, in addition to correlation signs mentioned above, match the economic magnitudes found in NYSE data. We choose 6 properties (moments) related to risk-sharing trades at daily, weekly, biweekly, and monthly horizons. We find model parameters of the limited attention framework that generate a reasonable quantitative fit of the joint dynamics of stock price movements and trades.

While the results are supportive of a limited attention framework, we note that we cannot directly "test" limited attention per se. Instead, we make predictions that are related to the joint dynamics of equity returns and risk sharing trades.² The strength of our results, and ultimately a contribution of this paper, lies in the breadth of confirmed predictions.

Quantifying Economic Effects with a State-Space (Statistical) Model: The third part of our paper quantifies the economic effects of limited attention on asset prices with a state-space (statistical) model or "SSM". The SSM approach captures the transitory component of prices from the theoretical framework as well as the efficient price. The Kalman filter separates an equity's observed price into these two unobservable components. The SSM allows the NYSE trading data variables to affect the evolution of both the efficient component and the transitory component of prices.³

The final part of our paper pays particular attention to transitory price movements. How large are the deviations from efficient values? How long does a typical deviation last?

Coefficients from the statistical model quantify the magnitude and duration of transitory price movements. Our results apply to a typical NYSE stock over a typical day, week, or month. We find transitory price effects last at least a month and are economically and statistically significant. A

²Our model and empirical focus is on risk-sharing trades. We do, however, allow for information-based trading to affect prices in our empirical work.

³One can also think of the SSM as a linear, reduced form of the non-linear theoretical model. In addition, the NYSE data may also include information-based trading strategies. Therefore, the state-space model to allows for this possibility.

one standard deviation change in individuals' net trades is associated with 12 basis points (bps) of transitory price movement in daily data and 151 bps of transitory price movement in monthly data. Similarly, a one standard deviation change in market makers inventories is associated with 65 bps of transitory price movement in daily data and 159 bps of transitory price movement in monthly data.

An important and final contribution of our paper is the ability to study the “term structure” of transitory price movements. It is well known that most individual stocks experience negative return auto-correlation at a daily horizon. We show how limited attention provides the answers to questions such as: i) Why doesn't return autocorrelation go to zero as one moves from daily, to weekly, to longer (monthly) horizons? ii) Why does return auto-correlation actually become more negative at longer horizons (up to monthly)? iii) Does the ratio of a stock's transitory variance to its idiosyncratic variance decrease, stay the same, or increase as one moves from daily to monthly data? The theoretical model with inattention answers the first two questions as the auto-correlation in returns becomes more negative up to the inattentive horizon. The SSM addresses the third question with estimates of the unobservable efficient and transitory prices. Transitory variance represents 8% of a typical stock's idiosyncratic variance at a daily frequency and 25% of a typical stock's idiosyncratic variance at a monthly frequency. To our knowledge, we are one of the first papers to examine such a term structure of transitory stock price movements.

1.1 Links to Existing Studies

The paper attempts to better understand how investors' limited attention can affect asset prices through a risk-sharing channel, as opposed to through an information (or pseudo-information) channel. While the second channel plays a role in markets, it has been previously studied in papers such as Rashes (2001) and Huberman and Regev (2001). Thus, one goal of our paper is to show how frequent and *i.i.d.* shocks can translate into large and long-lasting movements in a stock's transitory price. The same shocks also lead some types of investors to have highly auto-correlated (aggregate) trades. In these ways, our paper expands an older literature that includes papers such as Poterba and Summers (1988), Roll (1988), and Cochrane (1994). A key difference is that papers such as Poterba and Summers (1988) assume the transitory component of prices evolves as a specific AR process while we use empirical NYSE data to identify the timing and dynamics of transitory price movements.

Because we study the aggregate trades of individual investors, our work is related to attention studies including Barber and Odean (2008). These papers study events that catch individuals' attentions and induce trades. The authors present results that a sudden influx of buy orders (for example) can cause prices to be pushed temporarily higher. Our paper takes a quite different approach. Rather than using liquidity-demanding trades to link attention and transitory prices, we focus on risk sharing issues that arise when some investors have limited attention. In addition, and in NYSE data, individuals sell in aggregate when prices are rising (as in Kaniel, Saar, and Titman (2008) who use shorter sample period of the same NYSE data).

Non-participation in the stock market is an extreme form of limited attention. Such behavior has been shown to affect consumption patterns as well as economists' estimates of quantities such as the equity premium and the inter-temporal marginal rate of substitution. For examples from this literature, see Mankiw and Zeldes (1991), Brav, Constantinides, and Gezcy (2002), and Vissing-Jorgensen (2002). Our paper differentiates itself by considering different levels of limited attention with a focus on transitory price movements.

Finally, we note that our paper provides a single, unified framework tying together a number of existing empirical studies. Financial economics has a long history of studying the relations between an investor type's aggregate net trades and price movements. For example, Lakonishok, Shleifer, and Vishny (1992) find that pension funds tend to buy and sell together. The observed buying and selling behavior is correlated with price movements.⁴ There are two main take-aways from these papers: i) Institutions tend to buy stocks that recently went up and sell stocks that recently went down; ii) Adding up constraints are often assumed such that individual investors' net trades are set equal and opposite to institutions' net trades—see Cohen, Gompers, and Vuolteenaho (2002). Our results confirm that market makers and individuals sell stocks that recently went up (i.e., institutions buy at these times).

Our unified framework provides additional empirical results link institutional and individual trad-

⁴The list of papers that study institutional trades and price movements is too long to include here. Representative papers include Nofsinger and Sias (1999) and Griffin, Harris, and Topaloglu (2003).

ing⁵ to price movements with the literature on market makers.⁶ Our theoretical framework is consistent with existing empirical regularities from these literatures. In addition, our model makes predictions about relations between a stock’s returns, market maker inventories, and individual investors’ net trades. For example, market maker inventories today are positively correlated with individual net buying over the following day, week, and month. We are able to test and confirm the model’s predictions with our NYSE data.

2 Theoretical Model

This paper’s model is inspired by the Duffie (2010) framework. As in Duffie (2010), we focus on a market with frictions—specifically, frictions associated with limited attention and risk aversion. We consider an infinite-date economy with $t = \{0, 1, 2, 3, \dots\}$.

Assets. There are two types of assets in the economy. The first asset is riskless and has a constant per-period gross return of $r \geq 1$. The second asset pays a stochastic dividend of X_t at each date t and is in zero net supply. The dividend is *i.i.d.* with mean zero and variance σ_x^2 .

Participants. The model contains three types of market participants: individual investors, institutional investors, and market makers. Each investor maximizes his or her expected utility of final wealth and has a CARA exponential utility function. Individuals are labeled with a subscript of 1 and institutions with a subscript of 2. Of all investors have a harmonic mean of their absolute risk-aversion coefficients of ϕ .

Institutions: Institutional investors trade at each date t and are a fraction $0 < q_2 < 1$ of all investors.

Market Makers: Market makers trade at each date t . A fraction $1 - q_1 - q_2$ of all investors are market makers.

⁵The literature on individuals’ net trades is also far too large to mention here. Kaniel, Saar, and Titman (2008) is arguably the closest such paper to ours. The authors show that individuals’ net trades this week are positively related to returns the following week. Our model predicts this result and we confirm it empirically.

⁶Papers on market makers date back to Hasbrouck and Sofianos (1993) and Madhavan and Smidt (1993). More recent papers include Hendershott and Seasholes (2007) and Hendershott and Menkveld (2013). Briefly, market makers are found to trade against (contemporaneous) price movements. This strand of literature tends to focus on the relations between market maker inventories and price movements. Our current paper studies these relations at different and longer frequencies as well as relations between market maker aggregate inventories and the net aggregate trades of other investor types.

Individuals: Individual investors in our model are not homogeneous and consist of three subtypes. The first subtype is referred to as the “least-attentive” individuals. A single member of this subtype trades (participates) once every k_1 dates. During the time between two participation dates, the investor does not actively rebalance his/her portfolio. Of all investors in our model, a fraction $0 < q_{11} < 1$ is the least-attentive individuals. The second subtype are “partially-attentive” individuals since they trade (participate) once every k_2 dates with $k_1 > k_2$. There is a fraction q_{12} of these investors. The third subtype are referred to as the “most-attentive” individual investors because they trade at each date t (e.g., retail day traders.) There is a fraction q_{13} of these investors in the economy. We define $q_1 \equiv q_{11} + q_{12} + q_{13}$ to be the total fraction of all individual investors in the economy with $0 < q_1 < 1$.

Individuals’ Life Cycles: Individual investors live a multiple number of their inattentive period. Upon death an individual consumes his or her wealth and is replaced by an arriving investor of the same type. Therefore, in a stationary state the fractions of investor-types at different life stages remain constant. At each date t , we assume $1/k_1$ of the least-attentive individuals participate (trade) and $1/k_2$ of the partially-attentive individuals participate. Thus, analogous to Duffie (2010), the individuals’ “participation frequencies” of $1/k_1$, $1/k_2$, and 1, and not their total life spans, play a role in the aggregate demand functions in our model.

Individuals’ Reinvestment Policies: For simplicity, we assume a least-attentive or partially-attentive individual re-invests his dividends at the riskless rate until his next investment decision date. We use R_{t+k_1} to denote the payout (value) at date $t + k_1$ associated with one unit of investment made at date t . For a least-attentive individual, the gross payout R_{t+k_1} can be written as the sum of the reinvested dividends, plus a possible “extra” or transitory component (S_{t+k_1}) that is associated with the future price of the risky asset: $R_{t+k_1} = \left(\sum_{i=1}^{k_1} r^{k_1-i} X_{t+i} \right) + S_{t+k_1}$. There is a similar expression, $R_{t+k_2} = \left(\sum_{i=1}^{k_2} r^{k_2-i} X_{t+i} \right) + S_{t+k_2}$, for a partially-attentive individual.⁷

Exogenous Endowment Shocks. In the model, individual investors and institutions want to trade with each other in order to hedge exogenous endowment shocks. The payoff of this non-tradable endowment is perfectly correlated with the payoff of the risky asset.⁸ Each institutional investor has

⁷The value of the transitory components (S_{t+k_1} and S_{t+k_2}) result from solving this model and these components are discussed further in Section 2.1.

⁸The use of non-tradeable endowments with payoffs correlated with the payoffs of the risky asset are used in models such as Vayanos and Wang (2012) and Lo, Mamaysky, and Wang (2004). Note that the market makers are not exposed to the endowment shock, but may be willing to trade to help balance supply and demand.

the same exogenous endowment of $-N_t$ at time t , while each individual investor has an endowment $+\frac{q_2}{q_1}N_t$ such that the aggregate endowment is zero. For simplicity at $t = 0$ the per-capital endowment is $N_0 = 0$. For $t > 0$ the per-capital endowment N_t follows a random walk and the per-capital endowment shock is $\Delta N_t \sim N(0, \sigma_{\Delta N})$.

$$N_t = N_{t-1} + \Delta N_t \tag{1}$$

Definitions for variables in our model are given in Appendix A. A detailed description of the model’s setup and solution is in the Internet Appendix.

[Insert Table 1]

Table 1 compares and contrasts our model’s set-up to that in Duffie (2010). Duffie’s paper has two types of investors, while our model has three (individuals, institutions, and market makers). Our individuals can further be classified into three sub-types based attentiveness. The Duffie (2010) model uses an exogenous aggregate supply shock to generate trades, while our model has exogenous (non-tradeable) endowment shocks. Thus, there is a difference in how noise is modeled which leads the Duffie (2010) model to have time-varying aggregate risk while our model does not.

A key difference between the two models is not readily apparent in Table 1. In Duffie’s model, the supply Z_t is constant after $t=1$ such that $\sigma_z=0$ for all $t > 0$. Therefore, prices and investors’ holdings are not stationary. We focus on a stationary solution (directly below) which can be linked to stationary empirical data on returns and trading.

2.1 Stationary Solution

Each investor maximizes the utility of his or her final wealth. Because the risky asset is in zero net supply and its dividend are mean of zero and distributed *i.i.d.*, the expected value of all future dividends is also zero. Therefore, the risky asset’s “price” in this model (denoted “ S_t ”) can be thought of as transitory deviations around the present value of future dividends. As such, S_t may be less than, equal to, or greater than zero based on supply and demand.

Market clearing at a given date t is influenced only by investors who are present. These investors include the institutions, market makers, most-attentive individuals, $1/k_2$ of the partially-attentive

individuals, and $1/k_1$ of the least-attentive individuals. The strategy to identify equilibrium closely follows Duffie (2010). The associated Internet Appendix provides details about each investor-type’s demand function and the market clearing conditions. The risky asset’s transitory price takes the form shown below where c is a $(k_1 + k_2) \times 1$ solution vector:

$$\begin{aligned} S_t &= c^T \cdot Y_t \\ g &= a_1(c) + a_2(c) + b_1(c) + b_2(c) + b_3(c). \end{aligned} \tag{2}$$

Similar to Duffie (2010), a_1 , a_2 , b_1 , b_2 , and b_3 are defined from the demand equations and g is a $(k_1 + k_2) \times 1$ vector of constants. The g vector is derived from market clearing conditions—see equation IA.9 in the Internet Appendix. Y_t is a $(k_1 + k_2) \times 1$ vector of the current endowment process (N_t), the current dividend (X_t), the holdings of the “cohort” of least-attentive individuals who are active, the vector of holdings for the k_1-1 cohorts of least-attentive individuals who are not participating, the holdings of the cohort of partially-attentive individuals who are active, and the vector of holdings for the k_2-1 cohorts of partially-attentive individuals who are not currently participating. As in Duffie (2010), there are no closed-form expressions for the transitory prices, but we can numerically solve for price and holdings given a set of values for the model’s parameters.

2.2 Numerical Example

A numerical example illustrates the trading and price dynamics in our model and compares and contrasts those with Duffie (2010). The example is based on an exogenous endowment shock of size $\sigma_{\Delta N}/q_2$ at $t=1$. The parameters are chosen based on NYSE data and further discussed in Section 3.6. In our model, we set the fractions of least-attentive, partially-attentive, and most-attentive individuals to: $q_{11} = 0.24$; $q_{12} = 0.24$; and $q_{13} = 0.12$ respectively.⁹ The fraction of institutional investors is set to $q_2 = 0.20$ and the remaining fraction of market makers is 0.20. We set the inattentive frequencies to $k_1 = 42$ days and to $k_2 = 10$ days, respectively. The gross riskless rate is $r = 1.0001$, which equals the daily average 1-month T-bill rate from 1999 to 2005. Also, $\sigma_{\Delta N}^2 = 2.00$ and $\sigma_x^2 = 0.0005$. For the Duffie (2010) results we set $k = 42$, $\sigma_x^2 = 0.0005$, $r = 1.0001$, and $q = 0.48$ in his model to generate results that we can compare to our model. In both models the the absolute risk-aversion coefficients

⁹In this example, the exogenous endowment shock occurs at $t=1$ and is equal to $\Delta N_1 = \sigma_{\Delta N}/q_2$ with $N_1 = N_2 = \dots = N_t = \sigma_{\Delta N}/q_2$. Therefore, the aggregate shift for institutional investors is $-\sigma_{\Delta N}$ and the aggregate demand shift of individual investors is $+\sigma_{\Delta N}$. This formulation is equivalent to a standardized impulse response function.

ϕ equal 1.00.

[Insert Figure 1]

The top graphs in Figure 1 (Panel A and Panel B) show that our stock price's impulse response function is qualitatively similar to the one produced by the Duffie (2010) model. In our example, equilibrium (transitory) prices initially decline sharply (at $t=1$) before rising back to zero by $t \geq 40$.

Figure 1's middle-left and bottom-left graphs show the aggregate asset holdings and net trades of our three investor types. Clearly, the price deviation is negatively associated with the market makers' inventory levels. In addition, the price deviation is also negatively associated with the individuals' net trades. Conversely, price deviation is positively related with the net trades of the institutional investors.¹⁰ Notice that in the Duffie framework there is an aggregate supply shock of size $\sqrt{2}$ (see the graphs in Panel B). The middle-right and bottom-right panels show that, in the Duffie framework, aggregate holdings increase by 1.41 units (for all dates), net trades sum to 1.41 at date $t=1$, and net trades sum to zero for $t > 1$.

Much of our model's economic intuition can be seen in Figure 1's middle-left and bottom-left graphs. Market maker inventories spike up at $t=1$ and have a half life of approximately six days. The inventories are negatively correlated with contemporaneous and lagged returns (changes in log prices), but positively correlated with future returns. In this panel, institutions sell and market makers buy at $t=1$. Individuals with limited attention show up and want to buy at future dates ($t+1, t+2, \dots$) Market makers liquidate their positions to these individuals. Interestingly, institutions also sell to these individuals causing $Corr(\Delta Inst_t, \Delta Indv_{t+1} < 0)$.

To conclude this section: The top row of Figure 1 shows the Duffie (2010) framework is sufficient to describe the shape of transitory price movements. Rows 2 and 3 highlight our paper's contributions. Specifically, our paper makes numerous predictions about the joint dynamics of returns and trading variables that can be tested with NYSE data.

¹⁰We focus more on individuals' and institutions' net trades instead of their holding levels due to non-stationarity of the level series. In all of our empirical work, we focus on three data series: i) Stock returns; ii) Market makers' inventory levels; and iii) Individuals' net trades. Institutional investors' trading activity is omitted due to the adding-up constraints from market clearing requires that aggregate net trading equals zero and aggregate holdings equal supply of the asset.

3 Data and Tests of the Model’s Predictions

3.1 Data Description

Our data start in January 1999 and end in December 2005. Throughout this section, all tests are carried out at four frequencies (daily, weekly, biweekly, and monthly). Thus, our sample consists of 1,760 days, 365 weeks, 182 two-week periods, and/or 84 months of data. Four sources provide the data used in this paper.

- An internal New York Stock Exchange (“NYSE”) database called the Specialist Summary File (or “SPETS”) contains specialists’ closing inventory positions for each stock at the end of each month. The NYSE assigns one specialist per stock and a given specialist is responsible for making a market in approximately ten stocks. See Hasbrouck and Sofianos (1993) for further discussion of the SPETS database.
- An internal NYSE database called the Consolidated Equity Audit Trail Data (or “CAUD”) contains the number of shares bought and sold by individual investors, for each stock, over each month. In addition, the CAUD database provides trading volume. See Kaniel, Saar, and Titman (2008) for further discussion of the CAUD database.
- The Trades and Quotes (“TAQ”) database provides closing midquotes prices. Prices and returns in this paper are measured at the midquote to avoid bid-ask bounce. All prices are adjusted to account for stock splits and dividends.
- The Center for Research in Security Prices (“CRSP”) provides the number of shares outstanding (used to calculate market capitalizations) and information necessary to adjust prices for stock splits/dividends.

We start with the 2,357 common stocks that can be matched across the NYSE, TAQ, and CRSP databases. We construct a balanced panel of data to ensure results are comparable throughout time. There are 1,037 stocks that exist for the entire sample period. Stocks with an average share price of less than \$5 or larger than \$1,000 are removed from the sample. The final sample consists of 1,019 stocks.

The trading variables are for market-makers and individuals. We convert market makers' inventory positions and individual net trades to US dollars (both variables are originally in number of shares.) For each stock, we multiply the number of shares by the stock's sample average price so as not to introduce price changes directly into the trading variables.

To ensure stationarity, our empirical work uses market makers' inventories (levels) and individuals' net trades (changes in levels). The Internet Appendix provides formal Dickey-Fuller tests to support the choice of market maker inventory levels and individuals' net trades (changes in levels) in our empirical work.

3.2 Summary Statistics

Table 2, Panel A presents summary statistics for seven "raw" variables. Over our sample period, the average company market capitalization is \$8.10 billion, the average daily trading volume is 0.71 million shares, and the average closing midquote price is \$34.56.

Panel A also shows trading variables including market makers' inventories (in both thousands of shares and in dollars) and individuals' net trades. On average, market makers hold \$168,590 dollars of inventory per stock.

[Insert Table 2]

Individuals' average net trades are negative across all frequencies indicating that individuals' direct holdings decline over our sample period. Individual investors, in aggregate and on average, sell \$177,990 per stock-day, \$857,770 per stock-week, \$1.7 million per stock-two weeks, and \$3.75 per stock-month.

3.3 Idiosyncratic Variables

Risks associated with market-wide return shocks can be hedged with highly-liquid index products. Therefore, our empirical analysis focuses on the idiosyncratic components of our variables. For each return and trading variable, we construct a common factor equal to the market capitalization weighted average of the underlying variable. We regress each variable on its common factor and save the residual as the corresponding idiosyncratic variable. This procedure is detailed in Appendix A. For notational

simplicity, we omit any subscripts or superscripts referring to “idiosyncratic,” and use $MM_{i,t}$ (for example) to denote the idiosyncratic portion of the market maker’s dollar inventory in stock i .

Table 2, Panel B provides summary statistics for idiosyncratic trading variables used in this paper. Since the idiosyncratic variables are defined as residuals from a market model regression, means are zero. The panel focuses on standard deviations at each of four frequencies. For market makers, the standard deviations of their positions range from \$565,700 (daily) to \$618,500 (monthly). For individuals, the standard deviations of their net trades range from \$872,200 (daily) to \$7.79 million (monthly).

3.4 Correlations of Price and Trading Variables (NYSE Data)

From our NYSE data we calculate the correlations of the market maker inventories, individuals’ aggregate net trades, and the idiosyncratic part of returns. The correlations are calculated at daily, weekly, biweekly, and monthly frequencies contemporaneously as well as for one lag and one lead at each of the four frequencies. Calculations are first done on a stock-by-stock basis and then averaged across stocks. Table 3 presents the correlations with t -statistics based on double-clustered standard errors (stock and time). See Petersen (2009) for a discussion of clustered standard errors.

[Insert Table 3]

The autocorrelation of returns is -0.01 at a daily frequency and -0.06 at a monthly frequency, indicating that deviations from the efficient price may be long-lived. Each autocorrelation appears twice in the table as $Corr(X_{t-1}, X_t) = Corr(X_t, X_{t+1})$ for any variable X_t . The -0.01 daily return autocorrelation, therefore, appears in the top-left box (rows=“Daily” and columns=“Lag”) and in the top-right box (rows=“Daily” and columns=“Lead”).

Market maker inventory levels are positively autocorrelated, though relations decrease monotonically as the horizon gets longer. The values declines from 0.60 (daily) to 0.17 (monthly). Individual investors’ net trades are also positively autocorrelated (daily = 0.27 and monthly = 0.32). As one can see, these values do not go to zero at longer horizons.

Persistent relations can also be seen in some cross-autocorrelations. For example $Corr(MM_{t-1}, \Delta Indv_t)$ goes from 0.06 (daily) to 0.08 (monthly) as can be seen in the lag columns. Also, the $Corr(MM_{t-1}, r_t)$ goes from 0.01 (daily) to 0.05 (monthly) also in the “Lag” column group. Both of these persistent cross-autocorrelations are consistent with our model of limited attention. Market makers take long (short) positions at date $t-1$ when they can be compensated via future price rises (falls). Market makers unwind positions from date $t-1$ by trading with individuals at date t .

3.5 Model’s Predicted Relations Between Returns and Trading Variables

We assess the model’s ability to predict relations between returns and trading variables by comparing model-generated and empirical (NYSE) correlations’ signs, i.e., positive or negative. The first step is to choose parameter values. For most of the parameters, we consider a range of values. The coefficient of absolute risk aversion is set to one of four values $\phi \in \{0.001, 0.010, 0.100, 1.000\}$. The fraction of individual investors in the market is $q_1 \in \{1/5, 2/5, 3/5\}$. The fraction of institutional investors in the market is $q_2 \in \{1/5, 2/5, 3/5\}$ with the restriction that q_1 and q_2 cannot both be $3/5$. The least-attentive individuals’ inattentive frequency is set to $k_1 \in \{42, 63\}$ and expressed in days, while the partially-attentive individuals’ inattentive frequency is set to one of four values $k_2 \in \{2, 5, 10, 21\}$ and expressed in days⁻¹. Finally, the variance of the exogenous endowment shock is $\sigma_{\Delta N}^2 \in \{1, 2, 10, 100, 1000\}$. In total, there are 2,052 parameter combinations and numerically solving the model for these combinations takes approximately 5 days of computation time.

For a given parameter combination, the model is solved numerically to create moving average (MA) representations of prices and quantities. Correlations are generated from the impulse response functions (IRF) following an endowment shock at $t=0$. For an IRF in the form $y_t = \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-1} + \dots$, the autocorrelation of y_t can be derived as a function of the θ ’s. As in Table 3 results are shown for four frequencies (daily, weekly, biweekly, and monthly) as well as three time shifts (one period lagged, contemporaneous, and one period lead).

[Insert Table 4]

Table 4 presents the predominant correlation signs across the 2,052 parameter combinations. Below each “+” or “-” sign is the fraction of parameter combinations with the same sign as the predominant

sign. For example, in the top left of the table (rows='Daily' and columns='Lag'), our model predicts that the autocorrelation of market-makers' daily inventories (holdings) is positive with 99.7% of the 2,052 parameter combinations produce positively autocorrelated market maker inventories at a daily frequency.

In the rows='Daily' and columns='Lag', we see that individuals net purchases ($\Delta Indv_t$) are positively correlated with market maker inventories from yesterday (MM_{t-1}). This prediction is new and unique to our model and 100% of the parameter combinations produce this positive correlation. Economically, when institutions want to sell at the time of a shock, not enough individuals are "attentive" and market makers step in to buy. As shown in Figure 1, as the partially-attentive and least-attentive individuals eventually come to trade, market makers are able to unwind their positions. Market makers sell, individuals buy, and our model predicts there is a positive correlation between lagged market-maker inventories and individuals' net trades. Panel A, shows that this positive correlation exists at daily, weekly, biweekly, and monthly frequencies.

There are two main take-aways from Table 4. First, the model-predicted correlation signs in Table 4 match the empirically-calculated correlation signs in Table 3 for all relations and at all frequencies. Second, Table 4 shows that for 96.5% to 100% of the 2,052 parameter combinations, the correlation between two model-generated data series produces a consistent sign. For example, the model predicts individuals' net trades are positively autocorrelated at a weekly frequency for 99.3% of the 2,052 parameter combinations. The similarly signed correlations (empirical data and model-generated data) provide evidence that the limited-attention framework can describe the joint dynamics of prices and trading variables.

3.6 Model Calibration

Beyond the qualitative comparison of Tables 3 and 4 we next examine whether our model can replicate the economic magnitudes observed in the empirical data. In other words, we grid-search through the 2,052 parameter combinations to identify a "best fit" set of parameters. The calibration exercise is based on 24 properties (i.e., moments) related to risk-sharing and the joint dynamics of returns and trading. In models of informed trading with risk neutral market makers, e.g., Glosten and Milgrom (1985) and Kyle (1985), informed order flows cause contemporaneous relations between themselves and

returns. Informed trading induces no autocorrelation in returns as lagged total order flow and lagged returns are uncorrelated with subsequent returns. If individuals are adversely selected by informed traders, then their trading is negatively correlated with future returns. Therefore, we exclude the moments related to this from the calibration exercise: the contemporaneous correlations between returns and trading variables, the correlations between past trading variables and subsequent returns, and the cross correlations between the trading variables. Tables 3 and 4 shows that the model produces correlations of the same sign as in the data. However, informed trading induces variation in the trading variables different from inattention. This makes it difficult for the model to produce correlations for the omitted moments that quantitatively match the data.

[Insert Table 5]

Table 5 lists the 24 properties used to calibrate our model. There is one group of six properties for each of four time frequencies. 20 of the 24 properties involve correlations in Tables 4 and 3. 4 of the 24 properties are the volatility of individuals’ trading divided by the volatility of market-maker inventories (also at daily, weekly, biweekly, and monthly frequencies). These link the magnitude of trading by the different groups in the data to the model. The column of “Target Values” is calculated with the observable NYSE data described in Table 3 in Section 3. The column of “Best fit” values comes from model-generated data. The best-fit model’s parameters are chosen from a grid-search over the same set of parameter values as in Table 4 with the goal of minimizing the sum of (percentage) squared error.

$$Criterion\ Function = \sum_i \left(\frac{Model_i - Target_i}{Target_i} \right)^2. \quad (3)$$

Table 5 reports target and best-fit properties of our data (empirical and model-generated respectively). As previously noted, we match the sign of all 20 target correlations, while the four standard deviation ratios are positive by definition. For the most part, our model comes close to matching the magnitudes of the 20 target correlations. For example, the autocorrelation of daily returns is -0.01 in both our sample of NYSE data and in our model-generated data. The daily net trades of individuals ($\Delta Indv_t$) has a positive autocorrelation of 0.27 in the NYSE data and 0.20 in our model-generated data. Individuals buy after prices fall at all frequencies in both the NYSE data and the model-generated data (see cross autocorrelations of -0.06 and -0.02 at a daily frequency). We are less successful at matching the ratio of standard deviations. Best fits are about half the magnitudes of

their target values.

Tables 3, 4, and 5 provide support that our model produces results that are consistent with NYSE price and trading dynamics. This spans both correlations signs and economic magnitudes. Next we turn to quantifying the economic effects. In particular, the correlations provide insight into how investors trade and how these relate to price changes. There is no straightforward way to use the correlations to estimate the importance of the trading variables, and, therefore, limited attention, in the overall return-generating process.

4 State-Space Model

In this section, we estimate the economic effects of limited attention on asset prices based on a state space (statistical) model that includes measures of market makers' inventories and individuals' net trades. The statistical model follows from equations in the Internet Appendix (see Eq.IA.4 to IA.9 and Eq.IA.12 for example) and can be thought of a reduced-form (linear) version of the non-linear model described in Section 2.

Following directly from Eq.IA.12, the state space model defines a stock's observed price ($p_{i,t}$) as the sum of two unobservable components. The first component is its efficient price ($m_{i,t}$) and the second component is the transitory price ($s_{i,t}$).

$$p_{i,t} = m_{i,t} + s_{i,t} \tag{4}$$

$$m_{i,t} = m_{i,t-1} + \delta_{i,t} + \beta_i f_t + w_{i,t} \tag{5}$$

$$w_{i,t} = \kappa_i^{MM} \tilde{M}M_{i,t} + \kappa_i^{indv} \Delta \tilde{I}ndv_{i,t} + u_{i,t} \tag{6}$$

$$s_{i,t} = \alpha_i^{MM} MM_{i,t} + \alpha_i^{indv} \Delta I ndv_{i,t} + \epsilon_{i,t} \tag{7}$$

We use $p_{i,t}$ to denote stock i 's log price. The efficient price ($m_{i,t}$ in logs) is modeled as a process of uncorrelated increments with a nonzero drift equal to the stock's required return. This characterization is appropriate for a process with information arrival. The required return ($\delta_{i,t}$) is assumed to be equal to the monthly risk free rate plus the stock's beta times a market risk premium of 6%. The increments consist of the market factor ($\beta_i f_t$) and an idiosyncratic increment ($w_{i,t}$). We note that β_i is a coefficient to be estimated while f_t represents the demeaned market return. Appendix A.2 provides details related

to calculating both $\delta_{i,t}$ and f_t .

The idiosyncratic innovation of stock i 's efficient price is denoted $w_{i,t}$ and is one focus of this section since it represents undiversifiable risk to those who temporarily hold inefficient positions (e.g., the market makers). Equation 6 defines $w_{i,t}$ for stock i over period t . The inclusion of the trading variables $\tilde{M}M_{i,t}$ and $\Delta\tilde{I}ndv_{i,t}$ in the equation is important for identification if we believe these variables include trading based on private information. A tilde over the trading variable's name indicates autocorrelation has been removed using an autoregressive model. Appendix A.2 provides details related to calculating both $\tilde{M}M_{i,t}$ and $\Delta\tilde{I}ndv_{i,t}$.

The transitory component of price ($s_{i,t}$ in logs) is assumed to be stationary. In Eq.7, we allow for both market makers' inventories ($MM_{i,t}$) and individuals' net trades ($\Delta I ndv_{i,t}$) to affect the transitory component of price.

4.1 Estimation Procedure

The state-space model is estimated on a stock-by-stock basis using maximum likelihood. The procedure exploits a Kalman filter. The estimation is implemented in `Ox` using standard optimization techniques. The Kalman filter routines are from an add-on package called `ssfpack`. See Koopman, Shephard, and Doornik (1999) for additional information about related estimation procedures. The optimization procedure follows steps designed to avoid getting stuck in local maxima. The associated Internet Appendix has additional details. There are at least three advantages with using a state space model in our setting.

1. A state space model explicitly separates transitory (short-term) effects from permanent (long-term) effects. This separation allows for parsimonious modeling of how an observed variable might affect different horizons.
2. Maximum likelihood estimation is asymptotically unbiased and efficient.
3. The state space statistical model helps identify effects that would otherwise be unobserved. After estimation, the Kalman filter offers an in-sample decomposition of an observed price (time series) into its efficient and transitory components. The decomposition is available at any point

in the sample period using past and current prices.¹¹

The t -statistics reported in this section assume that residuals are uncorrelated across stocks as the state-space model is estimated on a stock-by-stock basis. Estimating all stocks together is not computationally feasible. Therefore, we test the robustness of our results with an alternative (ARIMA) statistical model. We estimate the ARIMA model with OLS and t -statistics are based on standard errors clustered by month.¹² A more detailed discussion of the ARIMA model, along with its results that are quantitatively similar to those reported in this paper, can be found in the Internet Appendix.

Ideally, the model would be estimated at the highest frequency, e.g., daily, and the longer horizon results could be calculated from those results. However, the linearity of the empirical model may prevent it from identifying longer horizon transitory price dynamics in higher frequency data. To examine and confirm this we generate data with our theoretical model. We then estimate a state-space (statistical) model on these model-generated data. The Internet Appendix provides notes on the model generated data, as well as results of the state-space estimation.

4.2 Estimation Results

Table 6 shows that both market makers' and individuals' trading variables play an important role in our state space model. Initially we focus on results at a monthly frequency. Results in the 'Efficient Price' section show both κ_i^{MM} and κ_i^{indv} are negative with values of -0.94 and -0.10 respectively.¹³ Both investor types buy as prices are falling and both types tend to sell as prices are rising. The coefficient estimates show the slopes of efficient price changes per unit of trading (in this case, the units are bps per \$1,000 of trading).

[Insert Table 6]

¹¹Although not applicable to our study, a fourth advantage of the Kalman filter is that it deals with missing observations in the most informationally efficient way.

¹²Beyond the previously discussed advantages of the state space model, another drawback of the ARIMA approximation is that it requires one to add, in theory, infinite lagged polynomials in order to disentangle short- and long-term effects.

¹³The negative values, together with the market clearing constraints, imply that estimating the model with institutional net trades would yield $\kappa_i^{inst} > 0$. This is consistent with institutional traders having value-relevant information.

To better capture economic magnitudes, we multiply the absolute value of our coefficient estimates by the relevant variable’s standard deviation. A one standard deviation movement in market makers’ inventory is associated with a 235 bp movement in a stock’s efficient price. For individuals’ net trades, the associated quantity is 260 bp. By comparison the average total permanent volatility of 932 basis points per month is shown in the fifth column of the efficient price section of Table 6.

The ‘Transitory Price’ section of Table 6 also provides a number of results. One of this paper’s goals is to answer questions such as: *i*) Does one investor type’s trading variables “drive out” the other types’s variables? *ii*) Or, do trading variables from both market makers and individuals combine to help explain transitory volatility? Estimates from the transitory equation answer these questions. Both α_i^{MM} and α_i^{indv} are negative. Trades/holdings from one investor type do not “drive out” trades/holdings from the other type. At a monthly frequency, we see α_i^{MM} is -0.33 which multiplied by a one standard deviation of market market inventories yields 159 bps in the transitory price equation. For individuals’ net trades α_i^{indv} is -0.05 which multiplied by a one standard deviation yields 151 bps in the transitory price equation.¹⁴

4.3 Term Structure of Transitory Price Movements

By separately estimating the state space model on daily, weekly, biweekly, and monthly data we are able to study the term structure of transitory price movements. The middle section of Table 6 (under ‘Transitory Equation’) shows that trading related shocks do not disappear at longer horizons. For example, a one standard deviation movement of market maker inventories is associated with transitory volatility of 12, 35, 76, and 151 basis points respectively. A similar pattern can be seen when looking at a one standard deviation movement of individuals’ net trading over different horizons ranging from daily to monthly.

¹⁴Table 3 shows both investor types’ trading variables are positively autocorrelated at a monthly frequency (0.17 for market makers and 0.32 for individuals). These autocorrelations lead to positive autocorrelation of price pressures and the total transitory component of prices. The average monthly autocorrelation for the transitory component of price ($s_{i,t}$) is 0.15. Ultimately, one might like to study horizons greater than one month. Doing so requires much longer time series than we currently have.

4.4 Variance Decomposition

Our state space model also allows us to decompose stock price variance. We are particularly interested in two questions asked at the start of this paper: How “noisy” are daily, weekly, biweekly, and monthly price and return data? How economically large are transitory price deviations? Given the state space model defined by Eq.4 to 7, the idiosyncratic return of stock i over period t can be written as: $r_{i,t} \equiv w_{i,t} + s_{i,t} - s_{i,t-1} = w_{i,t} + \Delta s_{i,t}$. The variances of our price variables are shown below and one can think of $\sigma(w)$ as the size of permanent price changes while $\sigma(\Delta s)$ is the size of changes to the transitory component.

$$\begin{aligned}\sigma^2(w) &= \text{var}[\kappa_i^{MM} \tilde{M}M_{i,t} + \kappa_i^{indv} \Delta \tilde{I}ndv_{i,t}] + \sigma^2(u) \\ \sigma^2(s) &= \text{var}[\alpha_i^{MM} MM_{i,t} + \alpha_i^{indv} \Delta Indv_{i,t}] + \sigma^2(\epsilon) \\ \sigma^2(\Delta s) &= \text{var}[\alpha_i^{MM} (MM_{i,t} - MM_{i,t-1}) + \alpha_i^{indv} (\Delta Indv_{i,t} - \Delta Indv_{i,t-1})] + 2\sigma^2(\epsilon) \\ \sigma^2(r) &= \sigma^2(w) + \sigma^2(\Delta s) + 2\text{cov}(w, \Delta s)\end{aligned}$$

where,

$$\begin{aligned}\text{cov}(w, \Delta s) &= \text{cov}[\kappa_i^{MM} \tilde{M}M_{i,t} + \kappa_i^{indv} \Delta \tilde{I}ndv_{i,t}, \\ &\quad \alpha_i^{MM} (MM_{i,t} - MM_{i,t-1}) + \alpha_i^{indv} (\Delta Indv_{i,t} - \Delta Indv_{i,t-1})]\end{aligned}$$

The last two columns of Table 6 show the results of the variance decomposition. The ratio $\frac{\sigma^2(\Delta s)}{\sigma^2(r)}$ reflects the size of transitory variance relative to idiosyncratic return variance and we find a 0.25 ratio at a monthly frequency.¹⁵ The ratio in the last column is $\frac{\sigma^2(\Delta s) - 2\sigma^2(\epsilon)}{\sigma^2(r)}$ and represents the size of transitory variance that is explained by our trading variables relative to the idiosyncratic return variance. One can think of the numerator as the size of price pressure variance and we see daily and monthly values of 0.03 and 0.08, respectively.

We compare the degree of price pressure explained by our trading variables to the total movements of the transitory component. We estimate that market makers’ inventories and individuals’ net trades account for $\frac{0.08}{0.25}$ or 32% of transitory variance.

Finally, we revisit the term structure of transitory volatility. Taking first differences of Eq.4, we see a stock’s observed return can be written as the sum of its efficient return plus its transitory return

¹⁵The Roll (1988, p.564, Table IV) decomposition of idiosyncratic volatility yields a noise component that is roughly 25% of idiosyncratic variance. See Foucault, Sraer, and Thesmar (2011) for further discussion.

($\Delta p_{i,t} = \Delta m_{i,t} + \Delta s_{i,t}$). The results in Table 6 allow us to consider returns over horizons of one to twenty days.

The second to last column in Table 6 shows the size of transitory variance relative to idiosyncratic returns. Our daily results are in line with those suggested by French and Roll (1986) and Roll (1988). The variance ratio grows moving from daily data (ratio is 0.08) to monthly data (ratio is 0.25). This result is consistent with recent momentum studies such as Gutierrez and Kelley (2008). Their Figure 1 shows that a reversal portfolio increases in profitability out to a month and the overall reversal effect lasts about a quarter. At horizons longer than a quarter, momentum becomes more prominent than reversals.

Also consistent with our results are the expressions for the return-generating process given above. The autocorrelation of $\Delta s_{i,t}$ is positive (roughly) at lead/lags up to the longest inattentive period. Hence, the variance of $\Delta s_{i,t}$ increases with frequency from daily to monthly data. Given that the calibration exercise indicates the longest inattentive period is on the order of 42 days, it would make sense to estimate the SSM with bimonthly and quarterly data. Unfortunately, there are only 84 months of data in our sample. Cutting the sample length to 42 or 28 observations is not feasible given the requirements of the SSM. For now, we have to wait until longer datasets become available.

The last column in Table 6 shows the size of the price pressure caused by trading variables relative to the idiosyncratic return variance. At first glance it may be surprising that the transitory variance does not “die out” after a few days. In fact, when compared with idiosyncratic price movements, the importance of transitory movements grows as the horizon becomes longer. Our model leads us to believe that trading-related price pressure should last approximately as long as the least attentive investors’ horizons.

In fact, the last two columns of Table 6 point to interesting avenues for future research. At what horizons do transitory movements cease to be important in modern financial markets? Can we link long horizon effects to extremely limited investor attention? Answering such questions will require much longer time series than the NYSE data used in this paper. We leave these questions for future work.

5 Conclusions

This paper studies the joint dynamics of stock price movements and the trading of individuals, institutions, and market makers. We present a dynamic model in which individuals and institutions trade a risky asset in order to hedge an exogenous endowment shock. Some individuals have limited attention and do not participate in trading at each possible date. Their limited participation creates supply and demand imbalances as well as opportunities for market makers to step in and trade.

The model produces a wide variety of testable predictions regarding the joint dynamics of stock price movements and trading variables. The theoretical framework assumes that some individuals have limited attention and do not trade at each possible date. Their lack of participation creates an opportunity for market makers to step in and balance supply and demand. NYSE data confirms that both individuals and market makers tend to buy as prices are falling. The model shows that only one investor type in aggregate acts as a market maker in the traditional sense—that is, they trade and then the unwind their positions relatively quickly. While the most attentive individuals follow a related strategy, individuals in aggregate do not unwind their positions in our model nor do they in the NYSE data. There are other predictions of the limited attention framework that are consistent with the empirical results using NYSE price and trading data. For example, our framework predicts new relations such as market maker inventories today are positively correlated with individual net buying over the following day, week, and month.

Our paper highlights a well-known pitfall of studying aggregated trading data. Groups of investors that financial economists would like to study may be quite heterogenous. For example, some individuals in both our model and in the real world are quite attentive (day traders). Some are not. General statements about an investor type (in this case, individuals) require nuance. We specifically model three subtypes of individual investors, solve for their optimal trading strategies, aggregate their model-generated data, and compare the aggregate data with empirical data. These steps produce quantitatively consistent results between the model-generated data and NYSE data for numerous moments related to the correlations among returns and the different investor types' trading.

We estimate a reduced-form version of our theory model to demonstrate that transitory price movements are not merely a short-lived microstructure phenomenon. Deviations from efficient prices are common, large in magnitude, and last at least a month. For example, a one standard deviation

change in market makers' monthly positions is associated with transitory volatility of 1.59% (monthly). We estimate that between 8% and 25% of stock's idiosyncratic return variance is due to transitory price changes (noise). Interestingly, the fraction does not fall at longer horizons. Instead, we find the 8% applies to daily data while the 25% applies to monthly data. Asparouhova, Bessembinder, and Kalcheva (2010) and Asparouhova, Bessembinder, and Kalcheva (2013) demonstrate the importance and implications of noise in prices for asset pricing tests. Understanding at what horizon transitory price changes cease to be important is an open question which requires a longer time series of data.

The results in our paper provide insights into dynamic relations in markets and point to unanswered questions and future possible research directions. Can we identify and better understand the shocks that induce investors to trade? Are there shocks that tend to affect some types of investors, but not others? Put differently, can we estimate different types' loadings on shocks?

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Table 1: Differences and Similarities Between Our Model and the Duffie (2010) Model

This table compares and contrasts aspects of our theory model with aspects of the Duffie (2010) model.

<i>Investor Types</i> \longrightarrow	Our Model				Duffie (2010)		
	“Least-attentive” Individuals	“Partially-attentive” Individuals	“Most-attentive” Individuals	Institutional Investors	Market Makers	Inattentive Investors	Frequent Investors
Panel A: Differences							
Time Between Trading Activity ^(a)	k_1	k_2	1	1	1	k	1
Proportion of Investor Types ^(b)	q_{11}	q_{12}	q_{13}	q_2	$1 - q_1 - q_2$	q	$1 - q$
Exogenous Per-Capita Endow ^(c)	$\frac{q_2 N_t}{q_1}$	$\frac{q_2 N_t}{q_1}$	$\frac{q_2 N_t}{q_1}$	$-N_t$	0	0	0
Exogenous Agg Supply Shocks	\longleftarrow	\longleftarrow	\longleftarrow	\longleftarrow	\longrightarrow	\longleftarrow	$Z_t \longrightarrow$
Panel B: Similarities							
Risk Aversion Coefficients	\longleftarrow	\longleftarrow	ϕ	\longrightarrow	\longrightarrow	\longleftarrow	$\phi \longrightarrow$
Dividend Payments	\longleftarrow	\longleftarrow	X_t	\longrightarrow	\longrightarrow	\longleftarrow	$X_t \longrightarrow$

^(a) $k_1 > k_2 > 1$

^(b) $q_1 \equiv q_{11} + q_{12} + q_{13}$

^(c) Per unit of investor

Table 2: Summary Statistics (Empirical Data)

This table presents summary statistics of our empirical data. The data come from two NYSE databases (called SPETS and CAUD) and two public databases (TAQ and CRSP). We construct a balanced panel that contains 1,019 NYSE common stocks starting January 1999 and ending December 2005. Panel A provides averages over the sample. Panel B provides the cross-sectional averages of stocks' time series standard deviations of the idiosyncratic variables. We adjust all price series to account for stock splits and dividends. Appendix A.2 defines all empirical variables used in this paper.

Panel A: Raw Variables

Variable	Description	Units	Source	Daily	Weekly	Biweekly	Monthly
MarCap $_{i,t}$	Market Capitalization	\$ billion	CRSP	←	←	←	←
Volume $_{i,t}^{sh}$	Average Daily Share Volume	Millions	TAQ	←	←	←	←
P $_{i,t}$	Closing Midquote Price	\$	NYSE	←	←	←	←
MM $_{i,t}^{sh}$	Market Makers'	1,000 shares	NYSE	←	←	←	←
MM $_{i,t}^s$	Closing Inventories	\$ 1,000 ^b	NYSE	←	←	←	←
Δ Indo $_{i,t}^{sh}$	Individual's Net Trades	1,000 shares	NYSE	-4.58	-22.08	-44.09	-96.43
Δ Indo $_{i,t}^s$	"	\$ 1,000 ^b	NYSE	-177.99	-857.77	-1,713.99	-3,746.11

Panel B: Stdev of Idiosyncratic Variables

Variable	Description	Units	Source	Avg of Daily		Avg of Weekly		Avg of Biweekly		Avg of Monthly	
				Stdevs	Stdevs	Stdevs	Stdevs	Stdevs	Stdevs	Stdevs	Stdevs
$MM_{i,t}$	Market Makers' Inventory	\$ 1,000	NYSE	565.7	563.8	569.9	618.5				
$\Delta Indv_{i,t}$	Individual's Net Trades	\$ 1,000	NYSE	872.2	2,752.5	4,437.4	7,788.7				
$r_{i,t}$	Idiosyncratic Returns	%	CRSP	2.61	5.32	7.27	10.22				

Table 3: **Correlations of Trading and Price Variables (Empirical Data)**

This table presents correlations of our main three empirical variables: NYSE market maker inventories ($MM_{i,t}$) and individuals' net trades ($\Delta Indv_{i,t}$), and the idiosyncratic part of returns ($r_{i,t}$). Appendix A describes all variables and shows the number of observations. Correlations are first calculated on a stock-by-stock basis. The table shows average correlations (across stocks). t -statistics are reported in parentheses and based on double-clustered standard errors (stocks and time).

		Lag (t-1)			Contemporaneous			Lead (t+1)		
		MM_{t-1}	$\Delta Indv_{t-1}$	r_{t-1}	MM_t	$\Delta Indv_t$	r_t	MM_{t+1}	$\Delta Indv_{t+1}$	r_{t+1}
Daily	MM_t	0.60 (73.6)			1.0			0.60 (73.6)		
	$\Delta Indv_t$	0.06 (28.6)	0.27 (74.6)		0.06 (23.1)	1.0		0.04 (17.5)	0.27 (74.6)	
	r_t	0.01 (5.7)	0.01 (9.4)	-0.01 (-3.7)	-0.25 (-68.3)	-0.09 (-22.8)	1.0	-0.12 (-43.4)	-0.06 (-26.2)	-0.01 (-3.7)
Weekly	MM_t	0.37 (41.2)			1.0			0.37 (41.2)		
	$\Delta Indv_t$	0.07 (18.8)	0.28 (54.4)		0.06 (14.5)	1.0		0.04 (11.1)	0.28 (54.4)	
	r_t	0.02 (4.2)	0.02 (6.5)	-0.06 (-7.3)	-0.27 (-37.4)	-0.13 (-20.6)	1.0	-0.09 (-19.1)	-0.11 (-28.0)	-0.06 (-7.3)
Biweekly	MM_t	0.27 (28.3)			1.0			0.27 (28.3)		
	$\Delta Indv_t$	0.08 (17.7)	0.33 (52.3)		0.06 (12.8)	1.0		0.03 (7.5)	0.33 (52.3)	
	r_t	0.02 (2.6)	0.03 (5.6)	-0.05 (-4.4)	-0.26 (-39.9)	-0.16 (-19.3)	1.0	-0.08 (-11.8)	-0.16 (-28.1)	-0.05 (-4.4)
Monthly	MM_t	0.17 (12.8)			1.0			0.17 (12.8)		
	$\Delta Indv_t$	0.08 (12.0)	0.32 (39.2)		0.06 (8.7)	1.0		0.02 (3.0)	0.32 (39.2)	
	r_t	0.05 (4.8)	0.03 (4.3)	-0.06 (-4.5)	-0.23 (-26.6)	-0.20 (-21.6)	1.0	-0.05 (-6.5)	-0.20 (-21.3)	-0.06 (-4.5)

Table 4: Correlation Signs of Trading and Price Variables (Model-Generated Data)

This table presents the correlation signs of market makers inventories (MM_t), individuals' net trades ($\Delta Indv_t$), and returns (r_t). We study 2,052 different combinations of our model's parameters (grid points). For each combination, we generate data from our model and calculate the correlations of the three variables shown above. The sign shown (“+” or “-”) represents the predominant correlation sign across all 2,052 parameter combinations. The proportion of grid points with the same sign (as the most predominant correlation sign) is reported below in parentheses.

		Lag (t-1)			Contemporaneous			Lead (t+1)		
		MM_{t-1}	$\Delta Indv_{t-1}$	r_{t-1}	MM_t	$\Delta Indv_t$	r_t	MM_{t+1}	$\Delta Indv_{t+1}$	r_{t+1}
Daily	MM_t	+			1.0			+		
		(99.7%)						(99.7%)		
	$\Delta Indv_t$	+	+		+	1.0		+	+	
	(100%)	(99.6%)		(99.2%)			(99.4%)	(99.6%)		
	r_t	+	+	-	-	-	1.0	-	-	-
		(100%)	(99.2%)	(99.6%)	(99.6%)	(99.3%)		(99.8%)	(99.5%)	(99.6%)
Weekly	MM_t	+			1.0			+		
		(99.4%)						(99.4%)		
	$\Delta Indv_t$	+	+		+	1.0		+	+	
	(100%)	(99.3%)		(99.2%)			(99.7%)	(99.3%)		
	r_t	+	+	-	-	-	1.0	-	-	-
		(99.9%)	(99.3%)	(100%)	(99.9%)	(99.9%)		(99.6%)	(99.8%)	(100%)
Biweekly	MM_t	+			1.0			+		
		(99.7%)						(99.7%)		
	$\Delta Indv_t$	+	+		+	1.0		+	+	
	(100%)	(99.2%)		(99.2%)			(99.5%)	(99.2%)		
	r_t	+	+	-	-	-	1.0	-	-	-
		(99.9%)	(99.3%)	(100%)	(99.8%)	(99.9%)		(99.6%)	(99.6%)	(100%)
Monthly	MM_t	+			1.0			+		
		(99.5%)						(99.5%)		
	$\Delta Indv_t$	+	+		+	1.0		+	+	
	(100%)	(99.2%)		(99.2%)			(97.9%)	(99.2%)		
	r_t	+	+	-	-	-	1.0	-	-	-
		(99.9%)	(99.4%)	(100%)	(99.8%)	(96.5%)		(99.3%)	(99.7%)	(100%)

Table 5: **Model Calibration**

This table compares 24 different properties related to trading variables and returns. Entries in the “Target Values” column are calculated from our empirical data. Entries in the “Best-Fit Values” column are from a numerical calibration of our theoretical model using the following parameters: $q_{11}=0.24$; $q_{12}=0.24$; $q_{13}=0.12$; $q_2=0.20$; $1/k_1=1/42$ days; $1/k_2=1/10$ days; $\sigma_{\Delta N}^2=2.00$; $\sigma_x^2=0.0005$; $\phi=1.00$; and $r=1.0001$.

Property	Target Values from NYSE Data	Best-Fit Values from Model- Generated Data
Panel A: Daily Data		
1. Corr of r_t and r_{t-1}	-0.01	-0.01
2. Corr of $\Delta Indv_t$ and $\Delta Indv_{t-1}$	+0.27	+0.20
3. Corr of MM_t and MM_{t-1}	+0.60	+0.89
4. Corr of MM_t and r_{t-1}	-0.12	-0.09
5. Corr of $\Delta Indv_t$ and r_{t-1}	-0.06	-0.02
6. $\sigma(\Delta Indv)/\sigma(MM)$	+1.54	+0.75
Panel B: Weekly Data		
7. Corr of r_t and r_{t-1}	-0.06	-0.03
8. Corr of $\Delta Indv_t$ and $\Delta Indv_{t-1}$	+0.28	+0.42
9. Corr of MM_t and MM_{t-1}	+0.37	+0.54
10. Corr of MM_t and r_{t-1}	-0.09	-0.08
11. Corr of $\Delta Indv_t$ and r_{t-1}	-0.11	-0.10
12. $\sigma(\Delta Indv)/\sigma(MM)$	+4.88	+2.19
Panel C: Biweekly Data		
13. Corr of r_t and r_{t-1}	-0.05	-0.03
14. Corr of $\Delta Indv_t$ and $\Delta Indv_{t-1}$	+0.33	+0.27
15. Corr of MM_t and MM_{t-1}	+0.27	+0.34
16. Corr of MM_t and r_{t-1}	-0.08	-0.05
17. Corr of $\Delta Indv_t$ and r_{t-1}	-0.16	-0.08
18. $\sigma(\Delta Indv)/\sigma(MM)$	+7.79	+3.69
Panel D: Monthly Data		
19. Corr of r_t and r_{t-1}	-0.06	-0.04
20. Corr of $\Delta Indv_t$ and $\Delta Indv_{t-1}$	+0.32	+0.20
21. Corr of MM_t and MM_{t-1}	+0.17	+0.17
22. Corr of MM_t and r_{t-1}	-0.05	-0.04
23. Corr of $\Delta Indv_t$ and r_{t-1}	-0.20	-0.04
24. $\sigma(\Delta Indv)/\sigma(MM)$	12.59	+6.06

Table 6: State Space Model Estimates

This table presents estimates from a state space model. The model is estimated on a stock-by-stock basis using maximum likelihood estimates. $p_{i,t}$ is the observable log price of stock i at the end of period t (daily, weekly, biweekly, and monthly). $m_{i,t}$ is the unobservable efficient price. $s_{i,t}$ is the unobservable transitory component of prices. $\delta_{i,t}$ is the required rate of return. f_t is a market factor. Δs is the change in the transitory component. $r_{i,t} = w_{i,t} + \Delta s_{i,t}$ is the idiosyncratic return implied by the state space model. The ratio $\sigma^2(\Delta s)/\sigma^2(r)$ reflects the size of transitory variance relative to the idiosyncratic return variance. $(\sigma^2(\Delta s) - 2\sigma^2(\epsilon)) / \sigma^2(r)$ represents the size of price pressure caused by trading variables relative to the idiosyncratic return variance. The error terms $u_{i,t}$ and $\epsilon_{i,t}$ are assumed to be normally and independently distributed. Full descriptions and definitions of variables are given in Appendix A. The table reports t -values in parentheses assuming zero correlations across stocks.

$$\begin{aligned}
 p_{i,t} &= m_{i,t} + s_{i,t} \\
 m_{i,t} &= m_{i,t-1} + \delta_{i,t} + \beta_i f_t + w_{i,t} \\
 w_{i,t} &= \kappa_i^{MM} \bar{M} M_{i,t} + \kappa_i^{indv} \Delta \bar{I} n d v_{i,t} + u_{i,t} \\
 s_{i,t} &= \alpha_i^{MM} \bar{M} M_{i,t} + \alpha_i^{indv} \Delta \bar{I} n d v_{i,t} + \epsilon_{i,t}
 \end{aligned}$$

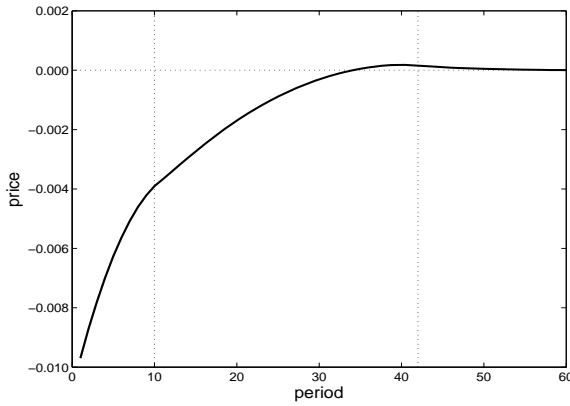
	Efficient Price Equation			Transitory Equation			Variance Decomposition					
	κ_i^{MM}	$ \kappa_i^{MM} \times \sigma(\bar{M}M)$	$\kappa_i^{indv} \kappa_i^{indv} \times \sigma(\Delta \bar{I} n d v)$	$\sigma(w)$	α_i^{MM}	$ \alpha_i^{MM} \times \sigma(MM)$	$\alpha_i^{indv} \alpha_i^{indv} \times \sigma(\Delta \bar{I} n d v)$	$\frac{\sigma^2(\Delta s)}{\sigma^2(r)}$	$\frac{\sigma^2(\Delta s) - 2\sigma^2(\epsilon)}{\sigma^2(r)}$			
Daily	-0.50 (-68.7)	75	-0.03 (-1.9)	25	228	-0.26 (-46.1)	65	-0.05 (-4.3)	12	81	0.08	0.03
Weekly	-0.83 (-48.4)	145	-0.06 (-9.2)	82	495	-0.12 (-7.7)	77	-0.03 (-7.3)	35	191	0.16	0.03
Biweekly	-0.94 (-39.6)	189	-0.07 (-6.9)	141	671	-0.13 (-6.1)	103	-0.05 (-7.4)	76	295	0.19	0.05
Monthly	-0.94 (-25.0)	235	-0.10 (-8.0)	260	932	-0.33 (-10.6)	159	-0.05 (-5.9)	151	500	0.25	0.08

Figure 1: Price and Trading Paths Given a 1σ Exogenous Shock

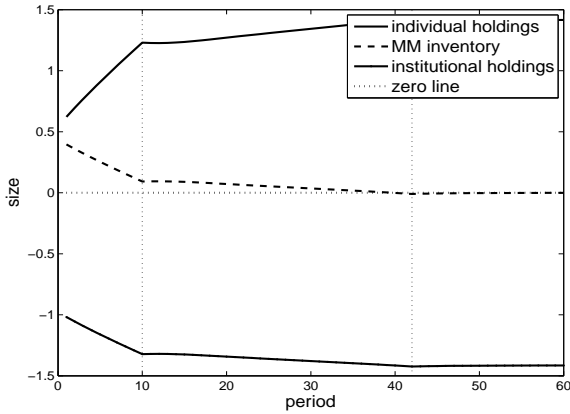
This figure shows price and trading paths given a one standard deviation exogenous shock in the endowment process and the aggregate supply, in our model and the Duffie (2010) model, respectively. For both models $\sigma_x^2 = 0.0005$, $\phi=1.00$, and $r = 1.0001$. For our model the parameters are: $q_{11} = 0.24$; $q_{12} = 0.24$; $q_{13} = 0.12$; $q_2 = 0.20$; $k_1 = 42$ days; $k_2 = 10$ days; $\sigma_{\Delta N}^2 = 2.00$. For the Duffie model the parameters are: $q = 0.48$ and $k = 42$.

Panel A: Our Model

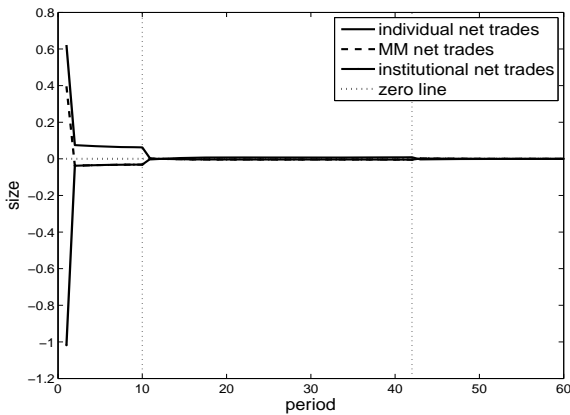
Price Path Following a Shock



Holdings in Levels

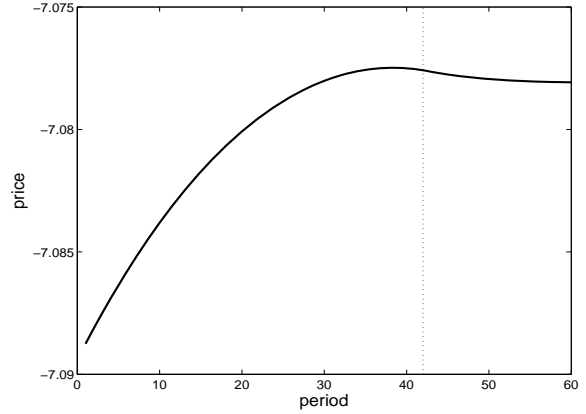


Net Trades (Changes in Holdings)

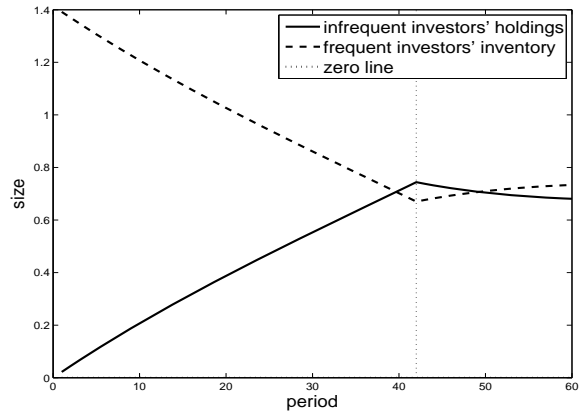


Panel B: Duffie (2010)

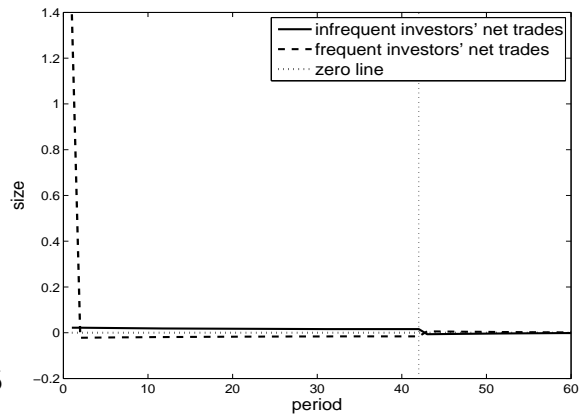
Price Path Following a Shock



Holdings in Levels



Net Trades (Changes in Holdings)



A Variable Definitions

A.1 Variables in the Theoretical Model

$D_{1,t}$	Demand of “least-attentive” individual investors
$D_{2,t}$	Demand of “partially-attentive” individual investors
$D_{3,t}$	Demand of “most-attentive” individual investors
H_t	Vector of dimension $(k_1 - 1)$ of quantities held by least-attentive individuals when these investors are not participating in the market $H_t = (D_{1,t-1}, D_{1,t-2}, \dots, D_{1,t-k_1+1})$
G_t	Vector of dimension $(k_2 - 1)$ of quantities held by partially-attentive individuals when these investors are not participating in the market $G_t = (D_{2,t-1}, D_{2,t-2}, \dots, D_{2,t-k_2+1})$
k_1	Participation frequency for least-attentive individuals
k_2	Participation frequency for partially-attentive individuals
$K_{1,t}$	Demand of institutional investors
$K_{2,t}$	Demand of market makers
N_t	Exogenous endowment
ΔN_t	Exogenous endowment shock
q_1	Fraction of all investors who are individuals (any subtype)
q_{11}	Fraction of “least-attentive” individual investors
q_{12}	Fraction of “partially-attentive” individual investors
q_{13}	Fraction of “most-attentive” individual investors
q_2	Fraction of all investors who are institutions

r	Gross riskless interest
R_{t+k_1}	Value (payoff) at date $t + k_1$ of one unit invested at date t by the least-attentive individuals $R_{t+k_1} = \left(\sum_{i=1}^{k_1} r^{k_1-i} X_{t+i} \right) + S_{t+k_1}$
R_{t+k_2}	Value (payoff) at date $t + k_2$ of one unit invested at date t by the partially-attentive individuals $R_{t+k_2} = \left(\sum_{i=1}^{k_2} r^{k_2-i} X_{t+i} \right) + S_{t+k_2}$
S_t	Equilibrium price of the risky asset after taking out the effect of dividends
X_t	Dividend payment of the risky asset
Y_t	State vector with $Y_t = [N_t, X_t, H_t, G_t]^T$
ϕ	Harmonic mean of the all investors
σ_x^2	Variance of the dividend payment
$\sigma_{\Delta N}^2$	Variance of exogenous endowment shock

A.2 Variables in the Empirical Analysis

β_i	Stock i 's beta coefficient from a standard CAPM regression.
$\delta_{i,t}$	Required return of stock i 's over period t . Defined as: $\delta_{i,t} = r_{f,t} + \beta_i \left(1.06^{\frac{1}{12}} - 1\right)$.
f_t	Demeaned series of market-wide returns. Defined as: $f_t = r_{m,t} - \bar{r}_m$.
γ_t^{Indv}	Common (market-wide) cumulative net trading factor at the end of period t . Defined as: $\gamma_t^{Indv} = \sum_i \omega_i \times Indv_{i,t}^{std}$. Where ω_i is the weight of stock i in our "market" of 1,019 stocks.
γ_t^{MM}	Common (market-wide) inventory factor at the end of period t . Defined as: $\gamma_t^{MM} = \sum_i \omega_i \times MM_{i,t}^{std}$.
$\Delta\gamma_t^{Indv}$	Net trading of common factor over period t : $\Delta\gamma_t^{Indv} = \gamma_t^{Indv} - \gamma_{t-1}^{Indv}$.
$Indv_{i,t}^{sh}$	Individuals' cumulative net trading (in shares) of stock i at the end of period t .
$Indv_{i,t}^{\$}$	Individuals' cumulative net trading (in dollars) of stock i at the end of period t . Defined as: $Indv_{i,t}^{\$} = Indv_{i,t}^{sh} \times \bar{P}_i$.
$Indv_{i,t}^{std}$	Standardized value of individuals' cumulative net trading of stock i at the end of period t . Defined as: $Indv_{i,t}^{std} = \frac{Indv_{i,t}^{\$} - \overline{Indv_{i,t}^{\$}}}{std(Indv_{i,t}^{\$})}$.
$Indv_{i,t}$	Idiosyncratic part of individuals' cumulative net trading. Defined as: $Indv_{i,t} = \varepsilon_{i,t}$ from the regression: $Indv_{i,t}^{\$} = \alpha + \beta \cdot \gamma_t^{Indv} + \varepsilon_{i,t}$.
$\Delta Indv_{i,t}^{\$}$	Individuals' net trading (in dollars) of stock i 's at the end of period t . Defined as: $\Delta Indv_{i,t}^{\$} = Indv_{i,t}^{\$} - Indv_{i,t-1}^{\$}$.
$\Delta Indv_{i,t}$	Idiosyncratic part of net trading. Defined as: $\Delta Indv_{i,t} = \varepsilon_{i,t}$ from the regression $\Delta Indv_{i,t}^{\$} = \alpha + \beta \cdot \Delta\gamma_t^{Indv} + \varepsilon_{i,t}$.
$\Delta \tilde{Indv}_{i,t}$	Defined as the residual from an AR(1): $\Delta \tilde{Indv}_{i,t} = \varepsilon_{i,t}$ from the regression: $\Delta Indv_{i,t} = \phi_0 + \phi_1 \Delta Indv_{i,t-1} + \varepsilon_{i,t}$.
$MktCap_{i,t}$	Market capitalization of stock i , in dollars, at the end of period t .
\overline{MktCap}_i	Average market capitalization of stock i , in dollars, over the sample period.

$MM_{i,t}^{sh}$	Market Maker's inventory (in shares) of stock i at the end of period t .
$MM_{i,t}^{\$}$	Market Maker's inventory (in dollars) of stock i at the end of period t . Defined as: $MM_{i,t}^{\$} = MM_{i,t}^{sh} \times \bar{P}_i$.
$MM_{i,t}^{std}$	Standardized value of market maker's inventory of stock i 's at the end of period t . Defined as: $MM_{i,t}^{std} = \frac{MM_{i,t}^{\$} - \overline{MM_{i,t}^{\$}}}{std(MM_{i,t}^{\$})}$.
$MM_{i,t}$	Idiosyncratic part of market maker's inventory. Defined as: $MM_{i,t} = \varepsilon_{i,t}$ from the regression: $MM_{i,t}^{\$} = \alpha + \beta \cdot \gamma_t^{MM} + \varepsilon_{i,t}$.
$\tilde{M}M_{i,t}$	Defined as the residual from an AR(1): $\tilde{M}M_{i,t} = \varepsilon_{i,t}$ from the regression $MM_{i,t} = \phi_0 + \phi_1 MM_{i,t-1} + \varepsilon_{i,t}$.
$\Delta MM_{i,t}$	Defined as: $MM_{i,t} - MM_{i,t-1}$.
$P_{i,t}$	Price of stock i , in dollars, at the end of period t .
\bar{P}_i	Average price of stock i , in dollars, over the sample period.
$p_{i,t}$	Natural log of stock i 's price at the end of period t .
$r_{f,t}$	Return of riskfree rate over period t from Ken French's website.
$r_{i,t}^{total}$	Return of stock i 's over period t : $r_{i,t}^{total} = p_{i,t} - p_{i,t-1}$.
$r_{i,t}$	Idiosyncratic portion of stock i 's return. Defined as: $r_{i,t} = \xi_{i,t}$.
$r_{m,t}$	Value-weighted market return from CRSP.
\bar{r}_m	Average of the market wide return over the sample period: $\bar{r}_m = \frac{1}{84} \sum_{t=1}^{84} r_{m,t}$. from the regression: $r_{i,t}^{total} = \alpha + \beta_i r_{m,t} + \xi_{i,t}$.

A.3 Number of Observations in the Empirical Analysis

Daily	1,019 stocks	×	1,760 days	=	1,793,440 observations
Weekly	1,019 stocks	×	365 weeks	=	371,935 observations
Biweekly	1,019 stocks	×	182 2 weeks	=	185,458 observations
Monthly	1,019 stocks	×	84 months	=	85,596 observations