CVA in Derivatives Trading

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Talk overview

- Counterparty risk and mitigation
- CVA and DVA: definition, meaning and computation
- XVA in practice: accounting, XVA trading desk
- The future of CVA

Counterparty credit risk

- In a derivatives trade two parties promise to make payments to each other in the future
- We call one of the parties 'the bank'; the other party is 'the counterparty'
- We consider the economics from the point of view of the bank
- Problem: determine adjustment of trade value for the risk of counterparty default (CVA)
- Also: determine adjustment of trade value for the risk of bank default (DVA)

Mechanics of derivatives trading

- Most derivatives counterparties have an (ISDA) Master Agreement specifying
  - close-out (turning derivatives into cash claims)
  - netting (netting derivatives claims against each other)
- A credit support annex (CSA) specifies rules for collateral posting
  - variation margin
  - initial margin

Variation margin specifications

- Collateral calculations are performed in the *base currency*
- Outstanding value will be updated and collateral called at the *margin call frequency*.
  - Typically daily, but weekly, monthly and quarterly calls are also seen.
- Margin can be called only on exposure in excess of the *threshold*.
  - If one of the thresholds is infinite, the agreement is *one-sided*.
- *Minimum transfer amount* is the smallest amount of collateral that will be exchanged.

➢ Both variation and initial margin may be subject to *rating triggers*

Eligible collateral

- Posting of collateral usually involves a choice between eligible assets
- CSA specifies
  - eligible collateral (cash, bonds, shares, gold etc)
  - 'hair-cuts' to apply to different kinds of collateral
- Equivalent cash amount in CSA base currency computed (by hair-cuts and currency exchanges) for all posted collateral
- Collateralisation has important consequences for *funding costs* (different talk. . .)

Margin period of risk (*collateral lag*)

- Since
  - most financial defaults triggered by *liquidity*
  - in derivatives markets collateral is big source/sink of liquidity
- We see that
  - when stressed, a derivatives party can be expected to delay margin payments
  - ➢ This is important for CVA!
- Typical assumption is a lag of 2-4 weeks

Valuation when default is impossible

$$V(t) = N(t) E_t^N \left[\int_t^T \frac{C(s)}{N(s)} ds \right] \qquad (*)$$

- C is the promised cash flow density of the *netting set* with maturity T
- N is the numeraire process, *e.g.*, EUR current account
- $E_t^N[\,\cdot\,] := E^N[\,\cdot\,|\mathcal{F}_t]$ is the expectation operator for the N-induced measure conditioned on the filtration $\mathcal{F}$ to time t.
  - ➢ Must include drift corrections to express all stochastics under single, numeraire-induced measure

Valuation under default risk: no collateral

- Simplest case: no collateral

$$\widehat{V}(t) \;=\; N(t)E_t^N\Bigg[\int_t^{\widehat{T}} \frac{C(s)}{N(s)}ds + \frac{1}{N(\tau_C)}\Big(R_C V(\tau_C)^+ + V(\tau_C)^-\Big)\mathbf{1}_{\tau_C<\tau_B} + \frac{1}{N(\tau_B)}\Big(R_B V(\tau_B)^- + V(\tau_B)^+\Big)\mathbf{1}_{\tau_B<\tau_C}\Bigg] \qquad (\dagger)$$

- τ_B, τ_C default times, R_B, R_C recovery rates

- $\widehat{T}$:= min$\{T, \tau_B, \tau_C\}$

- V (given by $(*)$) at the default time is the *close-out* value

$\Rightarrow$ no CVA of CVA

Adjusted CVA and DVA without collateral

- Rearrange (†) to get

$$\widehat{V}(t) = V(t) + V_{\mathsf{aCVA}}(t) + V_{\mathsf{aDVA}}(t)$$

- "Adjusted" CVA and DVA defined by

$$\begin{aligned} V_{\mathsf{aCVA}}(t) &:= -N(t)E_t^N\left[\frac{1}{N(\tau_C)}(1-R_C)V(\tau_C)^+\mathbf{1}_{\tau_C<\tau_B}\right] \\ V_{\mathsf{aDVA}}(t) &:= -N(t)E_t^N\left[\frac{1}{N(\tau_B)}(1-R_B)V(\tau_B)^-\mathbf{1}_{\tau_B<\tau_C}\right] \end{aligned}$$

  ➢ CVA (DVA) is always non-positive (non-negative)

- Sum of adjusted CVA and DVA known as *bilateral* CVA

Unilateral CVA without collateral

- If we assume $P(\tau_B < \tau_C) = 0$, get

$$V_{\text{aCVA}}(t) = V_{\text{CVA}}(t)$$

- *Unilateral* CVA is

$$V_{\text{CVA}}(t) = -N(t)E_t^N\left[\frac{1}{N(\tau_C)}(1 - R_C)V(\tau_C)^+\right] \tag{‡}$$

- Similarly, unilateral DVA is

$$V_{\text{DVA}}(t) = -N(t)E_t^N\left[\frac{1}{N(\tau_B)}(1 - R_B)V(\tau_B)^-\right]$$

- CVA (DVA) is mostly taken to mean unilateral CVA (DVA)

CVA formulae

- Can rewrite (‡) as PV of CDS:

$$V_{\mathsf{CVA}}(t) = -\int_t^T \mathcal{E}(t,s)\,dP(\tau \leq s|\tau > t)$$

- *Expected Exposure* (notional of 'roller-coaster CDS' protection leg)

$$\mathcal{E}(t,s) := N(t)E_t^N\left[\frac{1}{N(s)}(1-R)V(s)^+\middle|\,\tau = s\right]$$

- Assuming no explicit dependence on default time, write (‡) as

$$V_{\mathsf{CVA}}(t) = -N(t)E_t^N\left[\int_t^T \frac{1}{N(s)}(1-R(s))V(s)^+ dP(\tau \leq s|\tau > t)\right] \qquad (\S)$$

- This avoids simulating counterparty default—useful in Monte Carlo

CVA with collateral

- Simply replace V by $V - K$ above

- K: equivalent cash amount of collateral held in numerator currency by the the bank

- Variation margin is path-dependent and discontinuous (due to delays and minimum transfer amounts) function of V

- Initial margin may depend on netting set *risk*

Computing CVA

- Netting set value may typically depends on ‘whole market’—interest rates, fx, inflation, credit spreads etc
- Very high-dimensional pricing problem $\Rightarrow$ Monte Carlo
- Monte Carlo setup
  - time line of $\sim$ 200 times out through last maturity ($\lesssim$ 60 yrs)
  - simulation models for all asset classes (calibrated to market *under chosen measure*)
  - valuation techniques for all trades at all times
- Big calculation: using 10,000 paths and 200 times we shall be valuing the netting set two million times! Need fast valuation techniques. . .

Simulation model high-lights

- Four rates factors in each currency to deal with ‘basket option’ nature of CVA

- Stochastic counterparty credit spread for
  - wrong-way risk
  - CSA rating triggers

- Stochastic reference credit spreads for credit derivatives
  - Affine factor model with jumps allows calibration to implied default correlations

More simulation model high-lights

- Consistent discount curves built by basis swapping from base currency
- Many long-dated vols and most correlations unobservable
- Stoch Vol: danger of path decorrelation if done ‘naïvely’
  - must introduce only for global factors
- ‘Backward automatic differentiation’ used for most risk
  - much more efficient than customary ‘bump-and-run’

Valuation...

- Forward pricing sufficiently fast and accurate only for simplest trades (vanilla swaps, CDS's etc)

- Generally price by *backward induction*

- $V_j(x)$: value at time T_j conditioned on $X_j = x$

$$V_j(x) = N(T_j) E^N_{T_j} \left[\frac{1}{N(T_{j+1})} V(T_{j+1}) + \int_{T_j}^{T_{j+1}} \frac{C(t)}{N(t)} dt \,\middle|\, X_j = x \right]$$

- Seek to determine value of *conditional expectation* on rhs from Monte Carlo paths

- Observation: quite generally, conditional expectation *uniquely* solves least-squares regression...

. . . Valuation

- . . . but in practice cannot optimize over global solution set—have to make a choice of X_j—so only approximate

- Path values of operand (k labels paths)

$$\mathcal{V}_{j+1}^{(k)} := N^{(k)}(T_j) \left(\frac{1}{N^{(k)}(T_{j+1})} V^{(k)}(T_{j+1}) + \int_{T_j}^{T_{j+1}} \frac{C^{(k)}(t)}{N^{(k)}(t)} dt \right)$$

- Regress $\mathcal{V}_{j+1}^{(k)}$ against path values of regression variables $X_j^{(k)}$ to determine regression function f_j:

$$V^{(k)}(T_j) \approx f_j\left(X_j^{(k)}\right)$$

- Induction starts at end of time line where $V \equiv 0$

Steps of CVA computation

1. Calibrate simulation models to market

2. Generate simulation paths

3. Compute netting set values along all paths

4. Compute collateral value along each path

5. Estimate CVA from (§) as average over paths

Advantages of this approach

- Support for arbitrary pay-offs, including Bermudans
  - avoids ‘quick-and-dirty’ closed-form pricers required for forward pricing of non-vanilla products
- Matches market pricing in all states
  - can subsequently deform sampling measure, if required

Two slides on FVA. . .

- Master equation (FB-SDE) for valuation with funding adjustment in risk-neutral measure

$$
\begin{aligned}
V(t) &= N_R(t)E_t\left[\int_t^T \frac{1}{N_R(u)}\Big[C(u)+(r_R(u)-r_F(u))(V(u)-K(u))\right. \\
&\qquad \left. +\,(r_R(u)-r_K(u))K(u)\Big]\,du\right] \qquad (\P)
\end{aligned}
$$

$$N_R(t) := e^{\int_0^t r_R(u)du}$$

  - second term: funding of non-collateralised value
  - third term: funding of collateral

- Can show solution V to (¶) independent of arbitrary discount rate r_R

- “Risk-neutral”: value of perfectly collateralised claim grows at collateral rate

. . . two slides on FVA

- Special case: perfect collateralisation ($K \equiv V$) at reference rate r_0

$$V(t) = V_0(t) := N_0(t) E \left[\int_t^T \frac{C(u)}{N_0(u)} du \right] \qquad (\|)$$

- FVA defined as difference between (‖) and solution to (¶)

- Important special case: perfect one-sided collateralisation in counterparty favour: $K \equiv V^-$

$$\begin{aligned} V_{\mathrm{FVA}}(t) &= N_0(t) E_t \left[\int_t^T \frac{1}{N_0(u)} \left[s_F(u) V(u)^+ + s_K(u) V(u)^- \right] du \right] \\ &\approx N_0(t) E_t \left[\int_t^T \frac{1}{N_0(u)} \left[s_F(u) V_0(u)^+ + s_K(u) V_0(u)^- \right] du \right], \end{aligned}$$

$s_F := r_F - r_0$, $s_K := r_K - r_0$

➢ Computationally similar to CVA

A somewhat academic point

- From an economic point of view, close-out value includes FVA
- So should use

$$V_{\mathsf{CVA}}(t) = -N(t)E_t^N\left[\frac{1}{N(\tau_C)}(1-R_C)V(\tau_C)^+\right],$$

where V now solves 'master equation' (¶), rather than $(*)$

- Value including of FVA and CVA and cross-effects (CVA of FVA etc)

$$V_{\mathsf{net}}(t) = V(t) + V_{\mathsf{CVA}}(t)$$

- Note: FVA of FVA is included in (¶) and there is no CVA of CVA
- Fortunately: can ignore cross-effects to first order in all spreads in sight

The rise of the ‘XVA’ desk

- XVA additive at netting set level, but non-additive at trade level
  $\Rightarrow$ cannot be handled at trading desk level

- Must have dedicated XVA desk

- XVA desk involved in all “netting set” risk
  - charges trading desk for *incremental* XVA
  - XVA trading and risk management
  - CSA renegotiations and collateral management
  - CVA capital

Why nobody likes DVA

- Impossible to hedge/monetise risk to own credit curve
  - creates P/L volatility
  - can partially hedge/monetise by selling protection on peers, but plenty of basis risk
- ‘Funny money’: we make money when our credit deteriorates
  - DVA major contribution to official P/L of dealer banks in bad quarters, *e.g.*, Q3 ’11
  - banks emphasize results *without* DVA
- DVA quite volatile
  - dealer CDS spreads quite volatile

Why nobody likes CVA

- Charging unilateral CVA creates market friction

- Difficult and expensive to hedge
  - risk in all assets (and vols)
  - significant cross-gammas—mostly unhedgeable

- Quite volatile
  - CDS spreads quite volatile and quite highly correlated

- Attracts Basle III ‘CVA Risk Charge’ regulatory capital

IFRS 13 accounting standard (effective Jan 2013)

- Fair value
  - "is the price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date"
  - "reflects the effect of non-performance risk, which is the risk that an entity will not fulfil an obligation. Non-performance risk includes an entitys own credit risk"
- So XVA *must* be included in accounting
  - ➢ To the extent they are priced by the market

Initial margin to the rescue

- If both parties post sufficient initial margin, CVA and DVA disappear
- IM works best if
  - it is segregated
  - it increases with netting set market risk
- Clearing houses (CCP's) require one-sided IM
- Some global dealers have introduced IM bilaterally
  - static IM
  - illiquid assets used ('dead' balance sheet)

Regulatory IM initiative

- New BIS regulation (EMIR in EU) requires IM in CSA's from end 2015
- Designed to 'take out' CVA and DVA
- IM must be segregated and cannot be rehypothecated
- IM must be high quality assets (or subject to severe hair-cuts)
- IM threshold $\leq$ M€50; IM minimum transfer amount $\leq$ €500,000
- IM amount must be consistent with 10-day 99% stressed VaR of netting set

Market impact of using IM

- Parties have to *fund* the assets pledged as collateral

- Ban on rehypothecation means funding at own funding rate

- So: increased market friction (bid/ask spreads) from increased costs

- Use of liquid assets for segregated IM reduces market liquidity
  - estimated extent: T€1-7

- Funding costs non-additive at trade level, so handled by XVA desk

What about the clients?

- IM will apply only to financials and very large non-financials
  - many clients post no collateral at all
- For most clients, CVA will still be around
- Problem: no (liquid) CDS's
  - how to find default intensity?
  - how to hedge default risk?

Two ways out

- Use synthetic CDS curve generated from proxies and/or indexes
  i.e., estimate market hedge/replication cost

- Use physical default intensities
  i.e., estimate physical expected cost

- This is the basic choice between *hedging* and *diversifying* (event risk)

- Market—and accounting—practice has not quite settled yet

The end

Thank you!