Delegated Monitoring and Contracting*

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Abstract

We study a continuous-time agency model in which a principal invests in a firm run by a manager and monitored by an intermediary. Both the manager and the intermediary are subject to moral hazard. We analyze two different contracting settings that differ by the type of intermediation. In *delegated monitoring*, the principal can provide the optimal level of incentives to both the intermediary and the manager. In *delegated contracting*, the principal offers a contract only to the intermediary, who in turn designs a contract for the manager. Optimal incentives are qualitatively different across the two cases. Whereas a strong performance shifts incentives from the manager to the intermediary under delegated monitoring, it increases incentives for both agents under delegated contracting. Agency conflicts at the intermediary level lead to an overprovision of managerial incentives under delegated monitoring and an underprovision of managerial incentives under delegated contracting.

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1 Introduction

Monitoring is a key function of financial intermediaries. Hedge funds, private equity funds, and banks all help to contain agency problems for firms by monitoring their managers' activities. Boards of directors play a similar role when they monitor managers on behalf of shareholders. While these intermediaries and institutions may possess unique capacities to monitor, they are subject to agency frictions of their own. Thus, delegated monitoring is a double agency problem. An investor (or a principal) invests in a firm run by a manager (an agent) and is monitored by an intermediary (another agent). The manager and the intermediary are both subject to moral hazard. The goal of this paper is to study dynamic contracting when monitoring is delegated.

Depending on the role played by the intermediary,[1] there are two distinct ways to address these agency conflicts. First, we analyze the *delegated contracting* setting, in which the investor does not directly contract with the manager but delegates this task to the intermediary. Therefore, the investor provides a contract to the intermediary, who in turn contracts with the manager and decides on his own monitoring activity. The contract between the investor and the intermediary depends on observable firm performance but not on the contract between the intermediary and the manager. Second, we analyze a case in which the principal contracts directly with the manager but still has to delegate the monitoring task to the intermediary. As contracting is direct in this case, we refer to it simply as *delegated monitoring* (note that *delegated contracting* also involves monitoring that is delegated but is distinguished by delegated contracting). The main contribution of the paper is its characterization of optimal long-term contracts and the dynamics of the incentives for both intermediary and manager for the two contracting modes.

To study these problems, we formulate a continuous-time contracting model with intermediation. A manager controls firm output via costly but unobservable effort. An intermediary has access to monitoring technology that reduces the potential for moral hazard on the firm level by decreasing the cost of managerial effort. Because the intermediary's monitoring activity is also unobservable to the investor, another agency conflict between the investor and the intermediary exists. We assume that the manager is risk averse (as in Holmstrom and Milgrom (1987)) and that the intermediary has limited liability and is more impatient than the investor (as in DeMarzo and Sannikov (2006)).

The optimal contracts with the intermediary and the manager share a similar structure in

[1]Throughout the paper we refer to institutions between investors and managers that provide monitoring as intermediaries. This notion includes financial intermediaries, but also boards of directors.

either contracting mode. Because the manager's effort and the intermediary's monitoring are costly, it is necessary to provide incentives to both the manager and the intermediary through sufficient exposure to firm performance. As a consequence, both agents are punished (rewarded) if output turns out worse (better) than expected. However, in our model, tying compensation to firm performance is costly for the following reasons. First, the risk-averse manager demands additional compensation for the exposure to output shocks. Second, because the intermediary will not accept negative transfers from the investor, penalties may accumulate and lead to an inefficient termination of the contract. To lower the likelihood of liquidation, it becomes optimal to defer payments to the intermediary, which is again costly because the intermediary is more impatient than the principal. The incentive schemes to the manager and the intermediary interact, but differently depending on the contracting mode. In delegated contracting, the intermediary's performance pay incentivizes both monitoring of and contracting with the manager. Thus, optimal incentives to the intermediary need to account for the fact that they are passed on incentives that the intermediary offers the manager. In delegated monitoring, the manager's performance pay (in this case, set by the investor separately from the intermediary's performance pay) determines managerial effort, which, in turn, affects the intermediary's optimal incentives to monitor.

Methodologically, the model of delegated contracting needs to deal with profitable transfers between the manager and the intermediary beyond those needed to provide incentives. These arise in our model due to the intermediary's limited liability and relative impatience. Because the intermediary cannot commit to payments after his own contract is terminated, the manager must be able to save (and borrow) at the market interest rate to smooth his consumption and obtain his promised contract value. However, because of his impatience, the intermediary has incentives to make the manager raise debt on his behalf while pledging his stake in the firm as collateral. To limit these deals and preclude degenerate solutions, we introduce an exogenous firm liquidation risk that arrives according to a Poisson process.

We now briefly discuss our main findings and implications. First, our model generates sharp predictions about the relation between incentives and firm performance. We find that the intermediary should always receive more incentives after strong past performance, irrespective of the contracting mode. The intermediary's incentives need to be curbed after bad firm performance as a way to limit the threat of inefficient contract termination. By contrast, the sign of the manager's incentives' sensitivity to firm performance differs across the two contracting modes. After strong firm performance under delegated contracting, the intermediary will receive stronger incentives and implement greater managerial effort. Hence, the intermediary passes additional incentives on to the manager and the manager's incentives increase with firm performance. Under delegated monitoring, both the intermediary's incentives and the manage"s incentives exhibit some features of substitutes, in that managerial effort relaxes the intermediary's incentive compatibility constraint. Therefore, the principal finds it optimal to provide high powered incentives to the manager to indirectly incentivize the intermediary. This effect is particularly strong when direct incentives to the intermediary are particularly costly – that is, after poor firm performance and when the likelihood of contract termination is high. After strong performance, direct incentives to the intermediary can substitute indirect incentives such that the manager's incentives decrease.

Second, we analyze how the magnitude of incentives provided to the manager and intermediary changes when contracting is delegated to the intermediary. We also compare the magnitude of incentives to the second-best benchmark in which monitoring is not subject to moral hazard.[2] Because under delegated contracting incentives cannot be provided directly and need to be passed through the intermediary, the manager's incentives and effort are below the second-best levels. By contrast, because under delegated monitoring, the manager's incentives can substitute for the intermediary's incentives, the manager's incentives and effort are above the second-best level. Consequently, the manager receives more incentives under delegated monitoring than under delegated contracting while exactly the opposite is true for the intermediary, who receives more incentives under delegated monitoring.

Third, we analyze when and why the principal should delegate contracting. Our model predicts that intermediation under delegated contracting is particularly valuable for the principal when agency conflicts are severe. This is the case regardless of whether agency conflicts are measured at the firm or intermediary level. It is perhaps most surprising that it is favorable to rely heavily on the intermediary and delegate contracting when the intermediary is plunged by agency conflicts: under delegated contracting, the intermediary optimally has a large stake in the firm and is less affected by increasing moral hazard when compared with the intermediary under delegated monitoring.

We discuss several implications of the above results. A notable implication of the second finding is that investment via an intermediary can result in an over-provision of managerial incentives. To provide more empirical content to our predictions, we interpret private equity investment as an

[2] In the second-best case, the principal can monitor the manager directly without the intermediary or, equivalently, there are no agency frictions at the intermediary's level. In the first-best case, there are no agency frictions at either level.

example of intermediation under delegated monitoring.[3] Empirical evidence shows that private equity investment increases managerial incentives in target firms (see, e.g., Leslie and Oyer (2008), Acharya et al. (2012), and Cronqvist and Fahlenbrach (2013)). The common interpretation of this empirical pattern is that private equity is a superior owner that can improve firm governance. Our model suggests a very different interpretation: increased managerial incentives after private equity investment are an optimal way to deal with moral hazard in monitoring by an intermediary. Conversely, the common interpretation implies that incentives prior to private equity investment are suboptimal, while this is not the case in our model's interpretation.

The model also has implications for the effects of say-on-pay regulations, such as those introduced in the U.S. in the 2010 Dodd-Frank Act. These regulations increase shareholders' participation in determining executive compensation. Thus say-on-pay rules affect the role of boards of directors and, in terms of our model, shift corporate governance from the delegated contracting setting to the delegated monitoring setting. In accordance with the model's predictions discussed above, our theory then suggests that adopting say-on-pay rules will result in increased performance pay and increased sensitivity of pay to poor realizations of performance. Both effects are consistent with empirical evidence (Correa and Lel (2016), Iliev and Vitanova (2018), Ferri and Maber (2013), Alissa (2015)). The proponents and opponents of say-on-pay regulations expected effects, either due to managerial entrenchment or to shareholders' low sophistication (see, e.g., Bebchuk et al. (2007) and Kaplan (2007)). Our explanation is more innocuous and simply relies on changing the contracting environment.

Our theory focuses on the monitoring function of financial intermediaries and complements previous agency-based models of intermediation that consider other functions. Bhattacharya and Pfleiderer (1985) study delegated portfolio management within a one-period model with hidden information, while Ou-Yang (2003) studies portfolio management in a continuous-time model with moral hazard. He and Krishnamurthy (2011, 2013) analyze financial intermediaries that facilitate access to risky assets in general equilibrium models with asset pricing implications.

The fact that the monitoring function of financial intermediaries is limited by their own moral hazard has been studied in the banking literature, starting with Diamond (1984). More closely related to our paper is Holmstrom and Tirole (1997), who consider monitoring by financial inter-

[3] In the private equity investment model, Limited Partners (LPs) act as investors/principals and General Partners (GPs) as intermediaries. The investment horizon, target firm profiles, and governance changes in target firms that GPs implement are largely pre-specified in the contracts between LPs and GPs. Thus private equity investment seems best described by our delegated monitoring model rather than the delegated contracting model.

mediaries in an agency model. Their focus is on intermediaries' financial constraints and their effect on the provision of loans and on equilibrium interest rates. In contrast to these banking theories, our model is set in an infinite-horizon setting and its objective is to examine the provision and dynamics of incentives for both the intermediary and the manager.

Our paper is part of the growing literature on dynamic contracting models. In our model, the impatience and limited liability of the intermediary lead to the same agency-induced inefficiencies as in, among others, DeMarzo and Sannikov (2006), Biais et al. (2007), Sannikov (2008), Biais et al. (2010), DeMarzo et al. (2012), Zhu (2012), and DeMarzo and Sannikov (2016). Furthermore, we render additional tractability to our solution by assuming CARA utility for the manager, building on the dynamic agency models of Holmstrom and Milgrom (1987), He (2011), He et al. (2017), and Gryglewicz and Hartman-Glaser (2017).

Thus far, few models featuring multi-layered moral hazard have been developed, none of which have incorporated dynamic settings. Using a static model with multiple agency conflicts, Scharfstein and Stein (2000) analyze moral hazard that can arise between shareholders, CEOs, and division managers. Holmstrom and Tirole (1997) also features a two-layered moral hazard problem. These models and ours are distinct from two-sided agency problems, which feature two players and the principal is subject to moral hazard, as in the static model of Bhattacharyya and Lafontaine (1995) or in the dynamic model of Hori and Osano (2013). Our theory also relates to papers considering optimal monitoring in two-player agency models in both dynamic (see, e.g., Dilmé et al. (2015), Piskorski and Westerfield (2016), Halac and Prat (2016), Varas et al. (2017), and Malenko (2018)) and static settings (see, e.g., Lazear (2006) and Eeckhout et al. (2010)). Because the investor has to incentivize both monitoring and effort, our model is also related to models of dynamic agency and multitasking (Szydlowski (2016), Hoffmann and Pfeil (2017), Gryglewicz et al. (2018), Marinovic and Varas (2018)).

The paper is organized as follows. Section 1 explains the model and the resulting contracting problems. Sections 2 and 3 provide the solutions under delegated contracting and monitoring, respectively. Section 4 analyzes the model implications and derives empirical predictions. Section 5 concludes. All technical details are deferred to the appendix.

2 Model Setup

In this section, we formulate a continuous-time principal-agent problem in which an investor (principal) commits funds to a firm through an intermediary. A manager is hired to operate the firm and the intermediary possesses unique skills to monitor the management's activities. The firm's cumulative output X_t until liquidation evolves according to

$$dX_t = a_t^M dt + \sigma dZ_t,$$

where $\{Z\}$ is a standard Brownian motion on the complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$, $a_t^M \in [0, \infty)$ is a time-varying drift, and $\sigma > 0$ is a constant volatility. In addition to the Brownian shocks, the firm is exposed to an exogenous failure risk governed by a Poisson point process $\{N\}$ with a constant intensity $\Lambda > 0$ and with $N_0 = 0$. Upon its arrival at time t (i.e., $dN_t = 1$), the firm is no longer able to produce cash flow at all future times $s \geq t$ and is liquidated. Formally, instantaneous output is given by $dX_t = (a_t^M dt + \sigma dZ_t)\mathbf{1}_{\{N_t=0\}}$. Output X_t is observable, in that the public information filtration is given by $\mathbb{F} = \{\mathcal{F}_t : t \geq 0\}$, where $\mathcal{F}_t = \sigma(X_s, N_s : 0 \leq s \leq t)$.

The manager controls the output process via unobservable effort a_t^M at instantaneous monetary cost $g(a_t^M | b_t^I)$. The cost of managerial effort depends on the intermediary's monitoring intensity $b_t^I \in \{b_L, b_H\}$, which is observable to the manager but not to the investor. As in Holmstrom and Tirole (1997), monitoring decreases the cost of managerial effort and generates private costs $h(b_t^I)$ to the intermediary. We need to assume that $g_a(a|b) > 0, g_{aa}(a|b) > 0$, $g_b(a|b) \leq 0$ and $h'(b) > 0$. The strict convexity of g in a guarantees that optimal effort a is bounded. In the following, we work with the functional forms $g(a|b) \equiv \frac{1}{2}\frac{\delta a^2}{b}$ and $h(b) \equiv \lambda(b - b_L)$.

The investor and the intermediary are risk-neutral and maximize their expected (discounted) payoff in contrast to the manager, who is risk averse with CARA preferences and maximizes expected (discounted) utility. The manager's flow utility is given by

$$u(\hat{c}_t^M, a_t^M) = -\frac{1}{\theta}\exp\left[-\theta\left(\hat{c}_t^M - g(a_t^M | b_t^I)\right)\right],$$

where $\hat{c}_t^M$ denotes the manager's consumption flow. The manager and the intermediary possess outside options v_0 and ω_0, measured respectively in their utility units. In case of liquidation, the principal is able to recover a weakly positive value $R \geq 0$. The intermediary has sufficiently deep pockets but is protected by limited liability and cannot commit to any unfavorable payments or to contracts that deliver him a payoff below his outside option at any time with a non-zero probability. We also assume that the intermediary cannot be paid negative wages by the principal. In contrast, the principal is able to commit to any long-term contract. Whereas the investor and the manager discount at the market interest rate r, the intermediary is more impatient and applies a discount rate $\gamma > r$. Finally, we assume that, once employed, the manager can privately save and borrow at the interest rate r, such that his consumption $\hat{c}_t^M$ is not observable. We also set usual regularity conditions to ensure that the problem is solvable and well behaved; these technical conditions are gathered in Appendix A.1.

To close this section, we emphasize that the unobservable Brownian shocks to the output process generate moral hazard, while the observable Poisson liquidation shock does not. As will become clear later, the liquidation risk is required to prevent solutions in which the intermediary can utilize the manager's savings account to raise riskless debt at the fair market price r, while pledging his entire stake in the firm as collateral. In this case, the intermediary would effectively be able to alter the timing of his compensation and remove the impatience friction from the model. This would generate a solution in which the firm is run forever and payouts to the intermediary are indefinitely delayed. In a similar spirit, DeMarzo and Sannikov (2006) assume that the agent cannot borrow at the market interest rate so as to preclude degenerate solutions of the same type.

2.1 First and Second-Best

As a benchmark, we analyze the solutions under first-best (FB) and second-best (SB) cases. Under FB, both monitoring and managerial effort are observable (e.g., it is equivalent to assuming $\sigma = 0$). In SB, monitoring is observable by the principal, whereas managerial effort is not. Alternatively, one could interpret SB as a contracting game between the manager and principal in which only the principal possesses the monitoring technology. In the following, we will refer to third-best as the case in which both the intermediary and the manager are subject to moral hazard. In all scenarios, we assume that it is always optimal to implement full monitoring, $b = b_H$, and provide conditions for this to be the case in the appendix.

Evidently, the time-invariant first-best allocation is achieved by maximizing $a - g(a|b) - h(b)$ over $a \geq 0$ and $b \in \{b_L, b_H\}$, assuming that full monitoring b^{FB} equals b_H, and therefore the optimal effort is $a^{FB} = \frac{b_H}{\delta}$.

The second-best solution is an easy extension of the seminal model of Holmstrom and Milgrom (1987). The optimal contract is linear and therefore implements constant effort and monitoring over time. We gather the results in the following proposition.

Proposition 1. *The following holds:*

a) Under the first-best solution, the principal's value is given by $F^{FB} = f^{FB} - \frac{1}{\theta r}\ln(-\theta r v_0)$. *The constant* f^{FB} *is given by*

$$f^{FB} = \max_{a\geq 0} \frac{1}{r+\Lambda}\left(a - g(a|b_H) - h(b_H) + \Lambda R\right).$$

The optimal effort level is $a^{FB} = \frac{b_H}{\delta}$.

b) Under the second-best solution, the principal's value is given by $F^{SB} = f^{SB} - \frac{1}{\theta r}\ln(-\theta r v_0)$. *The constant* f^{SB} *is given by*

$$f^{SB} = \max_{a\geq 0} \frac{1}{r+\Lambda}\left[a - \frac{1}{2}\frac{\delta a^2}{b_H} - h(b_H) - \frac{1}{2}\theta r\left(\frac{\delta a}{b_H}\sigma\right)^2 + \Lambda R\right].$$

The optimal effort level is $a^{SB} = \bar{a}$ *with* $\bar{a} = \frac{b_H^2}{\delta b_H + \theta r \delta^2 \sigma^2}$.

In the next section, we discuss the contracting problems in the third-best case.

2.2 Delegated Monitoring vs. Delegated Contracting

In this paper, we analyze two different variants of the contracting game. First, we discuss the case in which the investor cannot offer a contract to the manager directly, but only through the intermediary. In this case, the principal contracts *solely* with the intermediary, who in turn offers a contract to the manager. Hence, the intermediary provides a contract Π^M_{DC} to the manager and the principal a contract Π^I_{DC} to the intermediary. The contract Π^M_{DC} specifies recommended savings $\{S^M\}$ and effort $\{a^I\}$, the cash-payments $\{c^M\}$ to the manager, and a termination time τ^M. Similarly, Π^I_{DC} contains the intermediary's compensation $\{c^I\}$ [4], a monitoring recommendation $\{b\}$ for the intermediary, and the termination time τ^I. Formally, we write

$$\Pi^M_{DC} = (\{c^M\},\{a^I\},\{S^M\},\tau^M) \text{ and } \Pi^I_{DC} = \left(\{c^I\},\{b^P\},\tau^I\right),$$

[4] Because the intermediary cannot be paid negative wages, the process $\{c^I\}$ is (in both settings) almost surely non-decreasing, in that $dc^I_t \geq 0$ for all $t \geq 0$ with a probability of one.

where all elements of Π^M_{DC} and Π^I_{DC} are $\mathbb{F}$-progressive. We say that a contract is *terminated*, whenever future transfers equal zero, in that $\tau^K = \inf\{t \geq 0 : dc^K_s = 0 \text{ for all } s \geq t\}$ for $K = I, M$.[5] Because the principal not only has to provide incentives to the intermediary to monitor, but also to select the right menu Π^M_{DC} for the manager, we call this setting *delegated contracting* (DC).

Second, we consider that the investor is able to contract directly with both agents, the manager and the intermediary. That is, the investor offers a contract Π^M_{DM} at $t = 0$ to the manager and Π^I_{DM} to the intermediary. A contract Π^M_{DM} (Π^I_{DM}) specifies the recommended effort (monitoring) process $\{a^P\}$ ($\{b^P\}$), cash transfers to the manager (intermediary) $\{c^M\}$ ($\{c^I\}$), and the termination time of the contract τ^M (τ^I). Further, the contract Π^M_{DM} includes the recommended savings balance $\{S^M\}$. More formally,

$$\Pi^M_{DM} = (\{c^M\}, \{a^P\}, \{S^M\}, \tau^M) \text{ and } \Pi^I_{DM} = \big(\{c^I\}, \{b^P\}, \tau^I\big).$$

All elements of Π^M_{DM} and Π^I_{DM} are $\mathbb{F}$-progressive. Throughout the paper, we call this setting *delegated monitoring* (DM). (We could also interpret this situation as direct instead of delegated contracting or investment.) In both settings, we normalize the initial outlay and the intermediary's outside option to zero.

In summary, we use the following convention regarding effort and monitoring levels. Managerial effort levels a^P and a^I are suggested by the contracts that the principal and the intermediary offer, respectively. The manager's actual effort is a^M. Monitoring level b^P is recommended in a contract, whereas b^I is the actual choice. In the following, we will work with three probability measures $\mathbb{P}^P$, $\mathbb{P}^I$, $\mathbb{P}^M$. As we show in the appendix, each effort path induces a probability measure and we let $\mathbb{P}^K$ be induced by $\{a^K\}$ for $K \in \{P, I, M\}$. Then, $\mathbb{E}^K_t[\cdot]$ denotes the conditional expectation at time t taken under the probability measure $\mathbb{P}^K$. We also write $\mathbb{E}^K[\cdot]$ to denote $\mathbb{E}^K_0[\cdot]$.

3 Solution – Delegated Contracting

3.1 Contracting Problem

The intermediary offers the contract Π^M_{DC} to the manager, which specifies recommended effort and savings levels $\{a^I\}$, $\{S^M\}$ in addition to the payments from the intermediary $\{c^M\}$. Hence, the

[5] The termination times are redundant and implicitly contained in the payment processes $\{c^K\}$. We include them nonetheless for the ease of exposition.

manager's problem reads

$$V_{0^-}(\Pi^M_{DC}) \equiv \max_{\{a^M\},\{\hat{c}^M\}} \mathbb{E}^M \left[\int_0^\infty e^{-rt} u(\hat{c}^M_t, a^M_t) dt \right] \tag{1}$$

$$\text{s.t. } d\hat{S}^M_t = r\hat{S}^M_t dt - \hat{c}^M_t dt + dc^M_t \text{ with } \hat{S}^M_0 = S^M_0. \tag{2}$$

We call Π^M_{DC} *incentive-compatible* if $(a^M_t, \hat{S}^M_t) = (a^I_t, S^M_t)$ for all $t \geq 0$, and *credible* if the intermediary can commit to it. Let the set of incentive-compatible and credible contracts for the manager be IC^M_{DC}. Throughout the remainder of the paper, we focus without loss of generality on incentive-compatible and credible contracts and impose standard regularity conditions, gathered in the Appendix A.1, to ensure that the contracting game is well-behaved.

Because the intermediary faces a contract $\Pi^I_{DC} = (\{c^I\}, \{b^P\}, \tau^I)$, but chooses $\{b^I\}$ and Π^M_{DC}, his problem reads

$$W_{0^-}(\Pi^I_{DC}, \Pi^M_{DC}) \equiv \max_{\{b^I\},\Pi^M \in IC^M_{DC}} \mathbb{E}^I \left[\int_0^{\tau^I} e^{-\gamma t} \left(dc^I_t - h(b^I_t) dt \right) - \int_0^{\tau^M} e^{-\gamma t} dc^M_t \right] \tag{3}$$

$$\text{s.t. } V_0(\Pi^M) \geq v_0 \text{ and } W_t(\Pi^I_{DC}, \Pi^M) = W_t \geq 0 \text{ for all } t \geq 0 \tag{4}$$

$$\text{with } W_t \equiv \mathbb{E}^I_t \left[\int_t^{\tau^I} e^{-\gamma(s-t)} \left(dc^I_s - h(b^I_s) ds \right) - \int_t^{\tau^M} e^{-\gamma(s-t)} dc^M_s \right]. \tag{5}$$

Importantly, the constraint $W_t = W_t(\Pi^I_{DC}, \Pi^M_{DC}) \geq 0$ arises due to the intermediary's limited commitment, in that at each point in time the intermediary's continuation payoff must be at least zero with certainty. If not, the intermediary could not credibly commit to the contract Π^M_{DC}, because he would be better off leaving the contractual relationship at any time t where $W_t < 0$. Put differently, Π^M_{DC} is credible (given Π^I_{DC}), if and only if $W_t(\Pi^I_{DC}, \Pi^M_{DC}) \geq 0$ for all $t \geq 0$.

In our model, a Poisson shock liquidates the firm and therefore also the intermediary's contract, in that $\tau^I \leq \min\{t \geq 0 : dN_t = 1\}$.[6] As a consequence, payments $\{c^M\}$ must be $\mathbb{F}$-predictable rather than only adapted. This arises because the intermediary cannot fully commit to payments and rationally leaves behind all liabilities to the manager after his contract is terminated at time τ^I. In other words, this optimally sets $dc^M_s = 0$ for all $s \geq \tau^I$, which also implies the end time $\tau \equiv \tau^I = \tau^M$ of the manager's contract Π^M_{DC}.[7] Therefore, all transfers dc^M_t within a credible

[6] Indeed, we verify in the appendix that terminating the contract Π^I_{DC} is optimal for the principal when the output process stops.

[7] In principle, a contract Π^M_{DC} could specify payments from the manager to the intermediary after termination. Such contracts are not optimal, as we show in the Appendix.

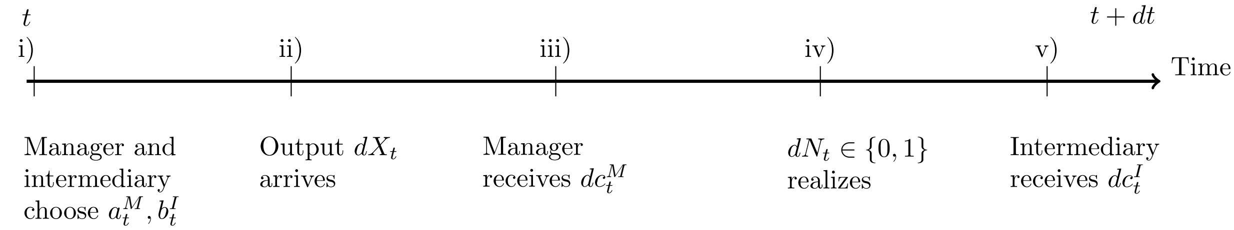

Figure 1: Heuristic timing.

contract Π_{DC}^M have to be made before time τ and, in particular, "just before" a Poisson shock may occur.

Finally, the principal contracts *only* with the intermediary and offers the intermediary a contract Π_{DC}^I, specifying recommended monitoring $\{b^P\}$ and wage $\{c^I\}$. We call Π_{DC}^I incentive-compatible if $b_t^P = b_t^I$ for all t and denote the set of incentive-compatible contracts for the intermediary by IC_{DC}^I.[8] Hence, the principal's problem reads

$$F_{0^-}(\Pi_{DC}^I) \equiv \max_{\Pi^I} \mathbb{E}^P \left[\int_0^\infty e^{-rt} dX_t - \int_0^{\tau^I} e^{-rt} dc_t^I \right] \tag{6}$$

$$\text{s.t. } \Pi^I \in IC_{DC}^I \text{ and } dc_t^I, W_t(\Pi^I, \Pi_{DC}^M) \geq 0 \text{ for all } t \geq 0. \tag{7}$$

Solving her maximization problem, the principal has to respect the intermediary's limited liability constraint $W_t \geq 0$, incentive compatibility and can only pay non-negative wages $dc_t^I \geq 0$.

To complete the description of the contracting problem, we give a heuristic overview of the events happening during one instant $[t, t + dt]$ (also presented in Figure 1):

i) Manager and intermediary choose respectively a_t^M and b_t^I.

ii) Cash flow dX_t is realized.

iii) The manager receives payments dc_t^M from the intermediary and decides on consumption.

iv) With probability Λdt, the firm is liquidated and all contracts are ended, in which case $dN_t = 1$; otherwise $dN_t = 0$.

v) The intermediary collects payments dc_t^I.

[8] Due to full commitment, all contracts that the principal offers are credible.

3.2 The Manager's Problem

Taking contract $\Pi^M_{DC} = (\{a^I\}, \{S^M\}, \{c^M\}, \tau^M) \in IC^M_{DC}$, and monitoring $\{b^I\}$ as given, we are able to define the manager's continuation utility as follows:

$$V_t \equiv \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} u(\hat{c}^M_s, a^M_s) ds \right]. \tag{8}$$

Because the manager's compensation $\{c^M\}$ has to be predictable with respect to $\mathbb{F}$, it is convenient to work with the left-limit of his continuation payoff, V_t, which we denote by $V_{t^-} \equiv \lim_{s \uparrow t} V_s$. Intuitively, while V_t represents the agent's payoff after observing whether a Poisson shock has occurred, V_{t^-} is the respective value before this uncertainty is resolved.

Using standard techniques, we can represent $\{V\}$ by means of a stochastic differential equation.

Lemma 1. *Let $\Pi^M_{DC} \in IC^M_{DC}$. Then, there exist $\mathbb{F}$-predictable processes $\{\alpha^M\}$ and $\{\beta^M\}$, such that $\{V\}$, as defined in (8), solves the stochastic differential equation*

$$dV_t = rV_{t^-} dt - u(\hat{c}^M_t, a^I_t)dt + (-\theta r V_{t^-}) \beta^M_t \big(dX_t - a^I_t dt\big) - (-\theta r V_{t^-}) \alpha^M_t \big(dN_t - \Lambda dt\big). \tag{9}$$

Incentive compatibility, $S^M_t = \hat{S}^M_t$, requires $rV_{t^-} = u(\hat{c}^M_t, a^I_t)$, and incentive compatibility, $a^M_t = a^I_t$, requires $\beta^M_t = \frac{\delta a^I_t}{b^I_t}$.

The rationale behind the two incentive-compatibility conditions is the following. First, to ensure that the manager does not have incentives for additional saving or borrowing (i.e., $S^M_t = \hat{S}^M_t$), the contract Π^M_{DC} has to respect the manager's Euler equation for consumption. This implies that the marginal utility of consumption is a martingale. Because for CARA preferences marginal utility is proportional to flow utility (i.e., there are no wealth effects), it follows that the continuation utility $\{V\}$ is also a martingale, in that $rV_{t^-} = u(\hat{c}^M_t, a^M_t)$.

Second, the sensitivity $\{\beta^M\}$ makes payments contingent on firm performance and thus ensures that the agent possesses sufficient incentives to exert the appropriate amount of effort. At each time t, the manager maximizes the sum of his flow utility and the expected change in his continuation utility, in that he solves

$$\max_{a^M_t \geq 0} u(\hat{c}^M_t, a^M_t)dt + \mathbb{E}^M_t[dV_t] = \max_{a^M_t \geq 0} u(\hat{c}^M_t, a^M_t)dt + (-\theta r V_{t^-}) \beta^M_t (a^M_t - a^I_t) dt.$$

It follows that the incentive compatibility condition is $\beta^M_t = \frac{\delta a^I_t}{b^I_t}$ for interior a^I_t. In this context, it also becomes clear why it is convenient to scale β_t^M by $(-\theta r V_{t-})$. The incentive-compatibility condition for savings $rV_{t^-} = u(\hat{c}_t^M, a_t^M)$ implies that $(-\theta r V_{t-})$ equals the manager's marginal utility; therefore, β_t^M directly measures the *monetary* compensation sensitivity.

Notably, from $rV_{t^-} = u(\hat{c}_t^M, a_t^M)$, we obtain the manager's consumption level implied by an incentive-compatible contract:

$$\hat{c}_t{}^M = -\frac{\ln(-\theta r V_{t-})}{\theta} + g(a_t^M | b_t^I) \Longleftrightarrow \hat{c}_t{}^M - g(a_t^M | b_t^I) = -\frac{\ln(-\theta r V_{t-})}{\theta} \equiv r\mathbb{CE}(V_{t-}). \tag{10}$$

In the following, we will refer to $\mathbb{CE}_{t-} \equiv \frac{\ln(-\theta r V_{t-})}{\theta r}$ as the manager's *certainty equivalent*. Note that at any time t the manager is indifferent to the options of further following a contract with promised value V_t or of receiving an infinite, constant consumption flow $r\mathbb{CE}_{t-}$.

By Itô's Lemma, the certainty equivalent solves

$$d\mathbb{CE}_t = \frac{1}{2}\theta r(\beta_t^M \sigma)^2 dt + \Lambda \alpha_t^M dt + \beta_t^M (dX_t - a_t^I dt) - \frac{\ln(1 + \theta r \alpha_t^M)}{\theta r} dN_t. \tag{11}$$

Note that the certainty equivalent must grow on average to compensate the manager for the two sources of risk that he is exposed to. In the drift term of (11), $\frac{1}{2}\theta r(\beta_t^M \sigma)^2$ captures the compensation for exposure to the Brownian risk in output and $\Lambda \alpha_t^M$ for the exposure to liquidation risk.

Under a credible contract Π_{DC}^M, the manager's savings must be such that $S_{\tau^I}^M = \mathbb{CE}(V_{\tau^I})$.[9] Otherwise, the contract Π_{DC}^M would not guarantee the agent his promised value V_{τ^I} at the termination time of the intermediary's contract τ^I. This dilemma arises because the intermediary cannot commit to payments after termination and would optimally not make any. Therefore, the manager has to derive all of his future consumption $\{\hat{c}_t^M\}_{t \geq \tau^I}$ from the savings pool $S_{\tau^I}^M$ and the subsequently earned interest rate. Thus, if a Poisson shock at time t leads to termination, α_t^M is such that $\mathbb{CE}_t$ and S_t^M are equalized.

To better understand this scenario, imagine that just before time t (i.e., at t^-), the manager's savings fall short of the certainty equivalent, in that $\mathcal{D}_{t^-} \equiv \mathbb{CE}_{t^-} - S_{t^-}^M > 0$, and the intermediary still "owes" the manager the amount $\mathcal{D}_{t-}$. In case a Poisson shock occurs at time t, the contracts Π_{DC}^M and Π_{DC}^I both end at t; therefore, the manager no longer receives cash payments. In particular, the manager is not fully paid the promised amount $\mathbb{CE}_{t-}$, and has a a deficit equal to $\mathcal{D}_{t-}$. As a consequence, his certainty equivalent (or continuation value) is strictly lower after the Poisson

[9]In fact, both contracts end at the same time, i.e. $\tau^M = \tau^I$.

shock while the savings do not change, such that

$$\mathbb{CE}_{t^-} - \mathcal{D}_{t^-} = S_{t^-}^M = S_t^M = \mathbb{CE}_t \Longrightarrow \mathbb{CE}_t - \mathbb{CE}_{t^-} = \mathcal{D}_{t^-} > 0.$$

From there, it follows by means of (11) that $\mathcal{D}_{t^-} = \ln(1 + \theta r \alpha_t^M)/(\theta r) dN_t$ or equivalently

$$\alpha_t^M = \alpha(\mathcal{D}_{t^-}) \equiv \frac{1}{\theta r} \left(\exp(\theta r \mathcal{D}_{t^-}) - 1\right) > 0. \tag{12}$$

The interpretation of (12) is that the promise-keeping constraint determines the sensitivity of the manager's continuation value to liquidation shocks.

3.3 The Intermediary's Problem

As a first step, we analyze the intermediary's continuation value W_t given in (5) and its relation to the manager's value. Combining (2), (10), (11), and $\mathcal{D}_t = \mathbb{CE}_t - S_t^M$, it is straightforward to obtain that the payment process $\{c^M\}$ satisfies the following equation:

$$dc_t^M = dT_t + \beta_t^M (dX_t - a_t^I dt) + r\mathcal{D}_{t^-} dt - d\mathcal{D}_t, \tag{13}$$

where

$$dT_t \equiv g(a_t^I | b_t) dt + \frac{1}{2} \theta r (\beta_t^M \sigma)^2 dt + \Lambda \alpha_t^M dt.$$

The process $\{T\}$ represents the manager's expected compensation for risk and effort costs. As expected, the intermediary's payments to the manager depend on the level and dynamics of the manager's deficit $\mathcal{D}_t$. In general, a higher $\mathcal{D}_t$ means a lower W_t, as more continuation value accrues to the manager. It is thus convenient to introduce a variable that is a sum of the intermediary's continuation value and the manager's deficit, $w_t \equiv W_t + \mathcal{D}_t$. Because the intermediary can at any time appropriate $\mathcal{D}_t$ (and does so in the case that the firm is hit by a liquidation shock), we refer to w as the intermediary's *gross value*. Note that the intermediary's limited liability constraint $W_t \geq 0$ is equivalent to $w_t \geq \mathcal{D}_t$.

Alternatively, one can think of $\mathcal{D}_t$ as the intermediary's liability to the manager. By maintaining a deficit in the manager's continuation value, $\mathcal{D}_t > 0$, the intermediary defers the manager's monetary compensation. Thus, one can interpret $\mathcal{D}_t > 0$ as a debt contract in which the intermediary pledges his stake w_t as collateral and makes the manager "borrow" on his behalf an amount $\mathcal{D}_t \leq w_t$ by drawing on the savings account.

As a next step, we will determine the optimal level of savings S_t^M or, equivalently, the optimal amount of $\mathcal{D}_t$ that the intermediary chooses. Owing to his relative impatience ($\gamma > r$), the intermediary has incentives to delay transfers to the manager, which means setting $\mathcal{D}_t > 0$. However, due to limited commitment, $\mathcal{D}_t > 0$ induces a downward jump of the manager's consumption if a Poisson shock terminates the contract, in which case the intermediary "defaults" on his liabilities, $\mathcal{D}_t$. As a consequence, when $\mathcal{D}_t > 0$, the possibility of "default" requires an additional compensation for the risk-averse manager.

We now illustrate how the optimal contract trades these forces off. Suppose that at time t the intermediary wants to decrease the savings balance, $S_t^M = \mathbb{CE}_t - \mathcal{D}_t$, by a marginal dollar $d\mathcal{D}_t = \varepsilon$, while promising repayment including interest one instant later, at time $t+dt$ via $d\mathcal{D}_{t+dt} = -\varepsilon(1+rdt)$. If this transaction were risk-free, then the intermediary would gain in present value terms at time t

$$\varepsilon - \varepsilon \frac{1+rdt}{1+\gamma dt} = (\gamma - r)\varepsilon dt + o\big((dt)^2\big) \simeq (\gamma - r)\varepsilon dt.$$

Indeed, the risk that Brownian shocks drive the continuation value down to zero during $[t, t+dt)$, given $W_t > 0$, is negligible. However, with probability Λdt, a Poisson shock occurs, thereby triggering termination; in this case, the intermediary does not pay back the promised amount at time $t+dt$, which reduces the certainty equivalent by $\mathcal{D}_t + \varepsilon$. Hence, the intermediary has to pay additional risk compensation $\Lambda\left(\alpha(\mathcal{D}_t + \varepsilon) - \alpha(\mathcal{D}_t)\right)$ during $[t, t+dt)$. Altogether, by increasing $\mathcal{D}_t$ by ε, the intermediary expects a profit of

$$\begin{aligned} &\varepsilon - \varepsilon \frac{1+rdt}{1+\gamma dt}(1 - \Lambda dt) - \Lambda\left(\alpha(\mathcal{D}_t + \varepsilon) - \alpha(\mathcal{D}_t)\right) dt + o\big((dt)^2\big) \\ &\qquad \simeq (\gamma - r + \Lambda)\varepsilon dt - \left(\frac{\partial \alpha(x)}{\partial x}\bigg|_{x=\mathcal{D}_t} \right) \Lambda \varepsilon dt. \qquad (14) \end{aligned}$$

Because the manager is risk averse and therefore $\frac{\partial^2 \alpha(x)}{\partial x^2} > 0$, there exists an upper limit $\mathcal{D}^*$, above which increasing $\mathcal{D}_t$ becomes unprofitable, in that the profit in (14) is zero for $\mathcal{D}_t = \mathcal{D}^*$. We formalize these findings in the following proposition.

Proposition 2. *Let Π_{DC}^M solve problem (3)-(4) given Π_{DC}^I and Π_{DC}^I solve problem (6)-(7). Then,* *the manager's savings satisfy*

$$\mathcal{D}_t = \mathbb{CE}_t - S_t^M = \min\{w_t, \mathcal{D}^*\} \textit{ with } \mathcal{D}^* \equiv \frac{1}{\theta r} \ln\left(\frac{\gamma - r + \Lambda}{\Lambda}\right). \tag{15}$$

Notably, Proposition 2 implies that the continuation value W_t can be directly inferred from knowing w_t via the mapping $\mathcal{D}_t$. The reverse does not necessarily hold true, because the limited commitment constraint is binding only whenever $w_t \in [0, \mathcal{D}^*]$, in which case $W_t = 0$. Interestingly, in this region W_t has neither drift nor volatility and stays constant at zero, which implies that limited liability is never violated and that the intermediary cannot gain from leaving the contractual relationship. In contrast, the gross value $\{w\}$ also has a non-trivial law of motion when $w_t \in [0, \mathcal{D}^*]$, as the following lemma demonstrates.

Lemma 2. *Let $(\Pi_{DC}^M, \Pi_{DC}^I) \in IC_{DC}^M \times IC_{DC}^I$. Then there exists a $\mathbb{F}$-predictable process $\{\beta^I\}$ such that in optimum, $\{w\}$ solves the stochastic differential equation*

$$dw_t = (\gamma + \Lambda) w_{t-} dt + h(b_t^P) dt + dT_t + (r - \gamma - \Lambda)\mathcal{D}_{t-} dt + \beta_t^I (dX_t - a_t^P dt) - w_{t-} dN_t - dc_t^I, \tag{16}$$

where $\mathcal{D}_t$ is given by (15).

From Proposition 2 and equation (16), it becomes clear why the liquidation risk $\Lambda > 0$ is necessary to ensure a well-behaved and interesting solution to the model. When $\Lambda \to 0$, $\mathcal{D}^* \to \infty$, which implies that it is optimal for the intermediary to delay payments to the manager as much as the limited commitment constraint allows. In this case, $\mathcal{D}_t = w_t$ and the intermediary can effectively alter the timing of his compensation, in that he is essentially already able to enjoy all of his promised future payoff w_t at time t. Whence, (16) collapses to

$$dw_t = r w_{t-} dt + h(b_t^P) dt + dT_t + \beta_t^I (dX_t - a_t^P dt) - dc_t^I$$

and the intermediary effectively discounts at rate r. As a consequence, the impatience wedge between principal and intermediary vanishes from the model, as does the corresponding agency friction.[10]

[10] As DeMarzo and Sannikov (2006) also point out in a related setting, unconstrained borrowing at rate r leads to a degenerate solution in which payouts to the intermediary are indefinitely delayed and the firm is run forever.

The interpretation of equation (16) is standard. Incremental payments from the principal $dw_t + dc_t^I$ must grow on average by $(\gamma + \Lambda)w_{t-}dt$ to compensate the intermediary for his time preference, the Poisson risk of termination, and the instantaneous cost $h(b_t^P)dt + dT_t$ that the intermediary incurs.[11] Lastly, the pay-performance sensitivity β_t^I makes rewards contingent on firm performance and incremental output, thereby providing appropriate incentives to the intermediary.

To understand this, we illustrate how β_t^I induces the choice of a_t^I and b_t^I. Observe that the intermediary maximizes the expected change of his continuation value W_t net the instantaneous cost of monitoring and compensating the manager:

$$\max_{b_t^I \in \{b_L, b_H\}, a_t^I \geq 0} \mathbb{E}_t^I \left[dW_t - h(b_t^I)dt - dc_t^M \right] \text{ s.t. } \beta_t^M = \frac{\delta a_t^I}{b_t^I}. \tag{17}$$

To solve problem (17), we first take $b_t^I = b$ as given and determine the optimal effort level a_t^I. Using $\mathbb{E}_t^I[dX_t - a_t^I dt] = 0$, it follows that

$$\mathbb{E}_t^I \left[dW_t - h(b_t)dt - dc_t^M \right] = \mathbb{E}_t^I \left[dw_t - h(b_t)dt - dT_t - r\mathcal{D}_{t-}dt \right].$$

Hence, only the expected change in the intermediary's gross value dw_t is relevant for his incentives, not the expected change in his continuation value dW_t. After rearranging, we obtain

$$a_t^I = a_t^I(b, \beta_t^I) = \arg\max_{a \geq 0} \mathbb{E}_t^I \left[dw_t - dT_t \right] = \arg\max_{a \geq 0} \left(\beta_t^I a - \frac{1}{2} \frac{\delta a^2}{b} - \frac{1}{2} \theta r (\sigma \beta_t^M)^2 \right).$$

If the intermediary were now to deviate and choose $b_s = b_L$ instead of b_H on an interval $[t, t + dt]$, then he would save the cost of monitoring $(h(b_H) - h(b_L))\, dt$ and compensating the manager

$$\mathcal{C}_t dt \equiv \Big[g\big(a_t^I(b_H, \beta_t^I)|b_H\big) - g\big(a_t^I(b_L, \beta_t^I)|b_L\big) \Big] dt + \frac{\theta r \sigma^2 \delta^2}{2} \Big[a_t^I(b_H, \beta_t^I)^2 - a_t^I(b_L, \beta_t^I)^2 \Big] dt$$

but also decrease the output by $(a_t^I(b_H, \beta_t^I) - a_t^I(b_L, \beta_t^I))dt$ on average. Therefore, choosing $b_t^I = b_H$ is optimal if and only if

$$\mathcal{C}_t + h(b_H) - h(b_L) \leq \big(a_t^I(b_H, \beta_t^I) - a_t^I(b_L, \beta_t^I) \big)\, \beta_t^I.$$

[11] This corresponds to the "promise-keeping constraint" in the discrete time formulation of the dynamic agency problem of DeMarzo and Fishman (2007).

This leads to the following proposition.

Proposition 3. *Let $(\Pi^M_{DC}, \Pi^I_{DC}) \in IC^M_{DC} \times IC^I_{DC}$. There then exists a constant $\bar{\beta}$ such that*

a) Optimal effort satisfies $a^I_t = \bar{a}\beta^I_t$.

b) Incentive compatibility, $b^I_t = b^P_t = b_H$, requires $\beta^I_t \geq \bar{\beta}$.

The first result is intuitive. Higher incentives β^I_t push the intermediary to implement greater effort and therefore provide higher incentives β^M_t to the manager. Remarkably, the second-best effort level is $a^{SB} = \bar{a}$ and therefore $a^I_t = \beta^I_t a^{SB}$. Thus β^I_t directly measures the fraction of the second-best effort that is implemented under the third best. The second result states that the intermediary must have sufficient "skin in the game" to ensure diligent monitoring.

3.4 The Principal's Problem

To start with, we argue that w rather than W summarizes the entire contract-relevant history and therefore constitutes the only state variable in the principal's problem.[12] This arises for three reasons. First, the continuation value W can be inferred when w is known and the reverse does not hold true. Second, as the previous section highlighted, the volatility term of dw (i.e., the sensitivity of gross value to incremental output $dw/dX \simeq \beta^I$) is relevant for the intermediary's incentives. Indeed, the volatility of w and W need not coincide, as the latter is zero in the case that w is sufficiently low. Third, and most importantly, the contract Π^I_{DC} must be terminated at time $\tau^I = \min\{t \geq 0 : w_t = 0\}$ – i.e., when w rather than W falls to zero. Because limited liability requires $w_t \geq W_t \geq 0$ for all $t \geq 0$, the process w_t must have zero volatility when w_t hits zero; in this case, the contract Π^I_{DC} must set $\beta^I = 0$ and therefore can no longer provide incentives. Put differently, due to limited liability, $w_t = 0$ implies that future payments dc^I_s are equal to zero, which is equivalent to contract termination. Notably, the contract Π^I_{DC} can still provide incentives to the intermediary (i.e., set $\beta^I > 0$, when $W = 0$, as long as $w > 0$).

Next, we proceed to characterize the optimal payout policy $\{c^I\}$. Observe that it is always possible to compensate the intermediary by any amount $\Delta > 0$. This would reduce promised payments by Δ against a lump-sum transfer of the same magnitude. Hence, the principal's value function $F(w)$ must satisfy $F(w-\Delta)-\Delta \leq F(w)$. Letting Δ go to zero, it follows that $F'(w) \geq -1$.

[12] To avoid clutter, we now drop time subscripts and refer only to w or W. These can be thought of as the realizations of the random variable w_{t^-} or W_{t^-}, respectively.

Thus, the marginal cost of delaying the manager's payout can never exceed the cost of an immediate transfer. Because delaying payments is costly (owing to the intermediary's relative impatience), there exists a barrier $\overline{w}$ above which payouts become optimal, in that

$$F'(\overline{w}) = 1 \text{ and } dc^I = \max\{w - \overline{w}, 0\}.$$

The location of the boundary $\overline{w}$ is uniquely determined by the smooth pasting condition $F''(\overline{w}) = 0$. Finally, termination at $w = 0$ implies that $F(0) = R$.[13] Because in optimum for $w \in [0, \overline{w}]$, the principal must earn instantaneous return $rF(w)dt$ equal to $\mathbb{E}^P[(dX + dF(w)]$, the value function solves the following HJB-equation subject to the previously derived boundary conditions:

$$\begin{aligned}(r+\Lambda)F(w) = \max_{\beta^I \geq \bar{\beta}} \Bigg\{ a + \Bigg[&(\gamma+\Lambda)w + h(b_H) + g(a|b_H) + \Lambda\alpha \\ &- \mathcal{D}(\gamma+\Lambda-r) + \frac{1}{2}\theta r(\sigma\beta^M)^2 \Bigg] F'(w) + \frac{1}{2}(\beta^I\sigma)^2 F''(w) \Bigg\} + \Lambda R, \quad (18)\end{aligned}$$

where $a = \bar{a}\beta^I$ and $\beta^M = \frac{a\delta}{b_H}$. It can be shown that $F(\cdot)$ is strictly concave on the interval $[0, \overline{w})$. The concavity of the value function captures the risk of inefficient contract termination when w falls down to zero.

Eventually, we are able to characterize the optimal incentives $\{\beta^I\}$ that the principal can provide. To get a sense of this, we look at the marginal benefit of increasing sensitivity β^I

$$\frac{\partial F(w)}{\partial \beta^I} = \underbrace{\bar{a}}_{\substack{\text{Increase}\\ \text{in output}}} + \underbrace{F'(w)\frac{\delta \bar{a}^2 \beta^I}{b_H}}_{\substack{\text{Direct}\\ \text{effort cost}}} + \underbrace{r\theta\sigma^2 F'(w)\frac{\delta^2 \bar{a}^2 \beta^I}{b_H^2}}_{\substack{\text{Risk}\\ \text{compensation}}} + \underbrace{F''(w)\sigma^2\beta^I}_{\substack{\text{Additional}\\ \text{volatility}}}.$$

Notably, by choosing β^I, the investor not only maintains incentive compatibility, but also induces the intermediary to implement greater managerial effort. However, this requires the intermediary to compensate the manager for the direct costs of effort and the additional risk that the manager carries. Furthermore, providing higher incentives, β^I is costly to the principal, as it increase the volatility of w and thus the risk of inefficient termination.

To close this section, we summarize our findings.

Proposition 4. *Let* Π^M_{DC} *solve problem* (3)-(4) *given* Π^I_{DC} *and* Π^I_{DC} *solve problem* (6)-(7). *Then*

[13] We collectively refer to $F'(\overline{w}) - 1 = F''(\overline{w}) = F(0) - R = 0$ as "the boundary conditions," if no confusion is likely to arise.

the following holds:

a) *The principal's value function $F(w)$ is the unique solution to equation (18) on the interval $[0, \overline{w}]$. For $w > \overline{w}$ the function $F(w)$ satisfies $F(w) = F(\overline{w}) - (w - \overline{w})$ and cash-payments $dc^I = \max\{w - \overline{w}, 0\}$ reflect w back to $\overline{w}$.*

b) *The function $F(\cdot)$ is strictly concave on $[0, \overline{w})$.*

c) *For all $w \in [0, \overline{w}]$ there exists $\beta^* = \beta^*(w)$ solving the first-order condition $\frac{\partial F(w)}{\partial \beta^I} = 0$. The principal optimally chooses*

$$\beta^I = \beta^I(w) = \max\left\{\bar{\beta}, \beta^*(w)\right\}.$$

4 Solution – Delegated Monitoring

4.1 Contracting Problem

We now discuss the setting in which the principal can contract with both the intermediary and manager directly. If no confusion is likely to arise, we use the same notation as the previous section.

Provided a contract Π^M_{DM}, the manager's problem remains unchanged and is given by (1)-(2). The intermediary then chooses the optimal monitoring $\{b^I\}$ problem, taking the menu Π^I_{DM} as a given, i.e.,

$$W_0(\Pi^I_{DM}) \equiv \max_{\{b^I\}} \mathbb{E}^I \left[\int_0^{\tau^I} e^{-\gamma t} \left(dc^I_t - h(b^I_t) dt \right) \right].$$

As usual, we will call Π^M_{DM} incentive-compatible if $(S^M_t, a^M_t) = (\hat{S}^M_t, a^P_t)$ and Π^I_{DM} incentive-compatible if $b^I_t = b^P_t = b_H$ for any $t \geq 0$.[14]

Eventually, the principal's problem is given by

$$F_{0^-}(\Pi^M_{DM}, \Pi^I_{DM}) \equiv \max_{\Pi^M, \Pi^I} \mathbb{E}^P \left[\int_0^{\infty} e^{-rt} dX_t - \int_0^{\tau^I} e^{-rt} dc^I_t - \int_0^{\tau^M} e^{-rt} dc^M_t \right] \tag{19}$$

$$\text{s.t. } (\Pi^M, \Pi^I) \in IC^M_{DM} \times IC^I_{DM}; \tag{20}$$

$$V_0(\Pi^M_{DM}) \geq v_0, dc^I_t \geq 0 \text{ and } W_t(\Pi^I_{DM}) \geq 0 \text{ for all } t \geq 0. \tag{21}$$

[14]Further, let IC^K_{DM} be the respective sets of incentive-compatible contracts for $K \in \{I, M\}$. Also note that credibility is not an issue under delegated monitoring, because the principal can commit to any long-term contract.

4.2 The Manager's Problem

Let the manager's continuation value under a contract Π_{DM}^M be defined by equation (8). The following lemma establishes the law of motion of $\{V\}$.

Lemma 3. *Let $\Pi_{DM}^M \in IC_{DM}^M$. There then exists a $\mathbb{F}$-predictable process $\{\beta^M\}$ such that the manager's continuation value $\{V\}$ solves the stochastic differential equation*

$$dV_t = rV_{t-}dt - u(\hat{c}_t^M, a_t^P)dt + (-\theta r V_{t-})\beta_t^M\big(dX_t - a_t^P dt\big). \tag{22}$$

Incentive compatibility $S_t^M = \hat{S}_t^M$ requires $rV_{t^-} = u(\hat{c}_t^M, a_t^P)$ and incentive compatibility $a_t^M = a_t^P$ requires $\beta_t^M = \frac{\delta a_t^P}{b_H}$. The manager's implied consumption is given by (10).

The results of Lemma 3 are analogous to those in Lemma 1, albeit with one important difference. The continuation value $\{V\}$ is now independent of the Poisson risk $\{N\}$. Because the principal can credibly commit to any payments, it is optimal not to expose the risk-averse manager to Poisson shocks beyond his influence. As a consequence, the manager's certainty equivalent evolves according to

$$d\mathbb{CE}_t = \frac{1}{2}\theta r(\beta_t^M\sigma)^2 dt + \beta_t^M(dX_t - a_t^P dt). \tag{23}$$

In addition, due to full commitment, we may assume without loss of generality that $S_t^M = \mathbb{CE}_t$ is implemented in equilibrium.[15] Therefore, payments $\{c^M\}$ need to satisfy $dc_t^M = d\mathbb{CE}_t + g(a_t^M|b_H)dt$.

4.3 The Intermediary's Problem

Given an incentive-compatible contract $\Pi_{DI}^I = (\{c^I\}, \{b\}, \tau^I)$, the intermediary's continuation value reads

$$W_t \equiv \mathbb{E}_t^I\left[\int_t^{\tau^I} e^{-\gamma(s-t)}\left(dc_s^I - h(b_s)ds\right)\right].$$

Using standard techniques, we obtain the following result.

[15] Because the principal can fully commit and discounts at rate r equal to the market interest rate, this assumption is without loss of generality. We allow the manager to save so as to better highlight the differences and similarities across our two contracting modes. Other dynamic contracting papers with a CARA-agent usually assume zero savings (see He (2011), He et al. (2017), Marinovic and Varas (2018), Gryglewicz and Hartman-Glaser (2017), or Hackbarth et al. (2018)).

Lemma 4. *Let $\Pi^I_{DM} \in IC^I_{DM}$. Then, there exists a $\mathbb{F}$-predictable process $\{\beta^I\}$ such that in optimum*

$$dW_t = (\gamma + \Lambda)W_{t^-}dt + h(b_H)dt + \beta^I_t\left(dX_t - a^P_t dt\right) - W_{t^-}dN_t - dc^I_t. \tag{24}$$

Incentive compatibility $b^I_t = b^P_t = b_H$ requires $\beta^I_t \geq \frac{\lambda b_H}{a^P_t}$.

Equation (24) is the natural analogue to equation (16), the delegated contracting problem. To better understand the incentive compatibility condition, imagine that the intermediary deviates by choosing $b_s = b_L$ during one instant $[t, t + dt)$. Because the manager observes $b_s = b_L$, he will exert effort a^M_s equal to $b_L\beta^M_t/\delta$, instead of $b_H\beta^M_t/\delta$, which reduces the expected output. However, the intermediary also saves monitoring cost $\lambda(b_H - b_L)dt$. Hence, the intermediary's deviation is not profitable if

$$\lambda(b_H - b_L)dt \leq \frac{(b_H - b_L)\beta^M_t}{\delta}\beta^I_t dt \Longrightarrow \frac{\lambda b_H}{a^M_t} \leq \beta^I_t. \tag{25}$$

Interestingly, the constraint on β^I_t decreases in a^M_t. This means that all else being equal, it is easier to incentivize monitoring when managerial effort is high.

4.4 The Principal's Problem

The principal's problem can be simplified be considering the gross value $f(W_0) \equiv F_{0^-} + \mathbb{CE}_0$ instead of F. The gross value $f(W)$ is a function of only W due to the absence of wealth effects with the manager's CARA utility.

Similarly to the delegated contracting model, the optimal payout policy follows a barrier strategy. That is, there exists $\overline{W}$ which satisfies

$$dc^I = \max\{0, W - \overline{W}\}, f'(\overline{W}) = -1 \text{ and } f''(\overline{W}) = 0.$$

Furthermore, the intermediary's limited liability mandates a termination of the contract Π^I_{DM} when W falls to zero, i.e., $\tau^I = \min\{t \geq 0 : W_t = 0\}$ and $f(0) = R$.

By the dynamic programming principle, when $dc^I = 0$, the principal's required return $rf(W)dt$ must be equal to the expected net payoff plus the expected change in value $\mathbb{E}^P[dX - dc^M + df(W)]$.

This implies the following HJB equation:

$$(r+\Lambda)f(W) = \max_{a\geq 0, \beta^I \geq \frac{\lambda b_H}{a}} \Big[a - g(a|b_H) + f'(W)\left((\gamma+\Lambda)W + h(b_H)\right) + \frac{1}{2}f''(W)(\beta^I\sigma)^2 - \frac{1}{2}\theta r(\beta^M\sigma)^2 \Big] + \Lambda R, \quad (26)$$

where $\beta^M = \frac{\delta a}{b_H}$. Because of agency-induced risk aversion, $f(\cdot)$ is strictly concave on $[0, \overline{W})$ and therefore $\beta^I = \frac{\lambda b_H}{a}$. This means that the intermediary is exposed to as little cash-flow risk as possible and the intermediary's incentive-compatibility condition must bind in optimum.

Optimal managerial effort is then pinned down by the first-order condition of maximization, $\frac{\partial f(W)}{\partial a} = 0$, which is equivalent to

$$\underbrace{1 - \frac{\delta a}{b_H}}_{\text{Net benefits of effort}} \underbrace{- r\theta\sigma^2\frac{\delta^2 a}{b_H^2}}_{\text{Risk compensation}} \underbrace{- f''(W)\frac{(\lambda b_H \sigma)^2}{a^3}}_{\text{Incentives to intermediary}} = 0. \quad (27)$$

Remarkably, along its direct effect on output, managerial effort has a beneficial effect on the intermediary's incentives (as $f''(W) < 0$, the last term on the left-hand side of (27) is positive). Higher effort a motivates the intermediary to monitor, in that a lower level of direct incentives β^I are needed to induce monitoring b_H. As this makes the continuation payoff W less volatile, the threat of termination becomes less severe. However, while decreasing the intermediary's risk exposure is beneficial, a higher level of a increases the volatility of the manager's continuation value V and therefore also the risk that the manager carries, which is costly due to his risk-aversion. We summarize our findings in the proposition below.

Proposition 5. *Let* (Π^M_{DM}, Π^I_{DM}) *solve problem* (19)-(21). *Then the following holds true:*

a) The principal's value function $f(W)$ *is the unique solution to equation (26) on the interval* $[0, \overline{W}]$. *For* $W > \overline{W}$ *the function* $f(W)$ *satisfies* $f(W) = f(\overline{W}) - (W - \overline{W})$ *and cash-payments* $dc^I = \max\{W - \overline{W}, 0\}$ *reflect* W *back to* $\overline{W}$.

b) The function $f(\cdot)$ *is strictly concave on* $[0, \overline{W})$.

5 Model Analysis

5.1 Incentives, Effort, and Performance

We now investigate the consequences for each of the two contracting settings, delegated contracting and delegated monitoring, in terms of the levels and dynamics of their respective effort and incentives. In particular, we examine whether financially sound firms that performed well in the past should employ incentive schemes different from firms in financial distress. To facilitate comparisons, in this section we introduce subscripts for effort levels $\{a_K\}$ and incentives $\{\beta_K^I\}$, $\{\beta_K^M\}$ to correspond with the optimal contracts Π_K^I, Π_K^M for $K \in \{DC, DM\}$.

5.1.1 Delegated Contracting

We first analyze the delegated contracting settings. In this case, the intermediary is provided incentives for two reasons. First, the intermediary should have enough "skin in the game" to sufficiently monitor the manager's activities. Second, the intermediary's compensation scheme determines the contract that the intermediary offers the manager. In this regard, more incentive pay, reflected by a higher value β_{DC}^I, motivates the intermediary to contract for more managerial effort – that is, to provide more incentives β_{DC}^M to the manager. Loosely speaking, the intermediary passes the incentives that he receives from the principal to the manager, for whom he acts as a principal. Formally, this can be seen from the result $a_{DC}^I = \bar{a}\beta_{DC}^I$ (as established in Lemma 3) and $\beta_{DC}^M = a_{DC}^I\delta/b_H$ (as established in Lemma 1), such that the manager's incentives β_{DC}^M and effort a_{DC}^I are both linear and increasing in the intermediary's incentives β_{DC}^I. This also means that the sensitivities β_{DC}^M and β_{DC}^I co-move over time in the same direction.

To analyze the dynamics of incentives, we consider their level along the state variable w. As w moves with output shocks dX_t, a high w signifies financial strength after good performance and a low w arises after a series of low output realizations, which bring the firm close to liquidation. Recall that incentive compatibility for monitoring requires that $\beta_{DC}^I(w)$ is at least $\bar{\beta}$, but that the cost-benefit trade-off of the intermediary's incentives calls for $\beta_{DC}^I(w) = \beta^*(w)$. Thus, optimal $\beta_{DC}^I(w)$ equals $\max\{\bar{\beta}, \beta^*(w)\}$. As pointed out previously, a higher value of β_{DC}^I creates additional volatility in the intermediary's compensation, as it leads the intermediary's gross value w to react more strongly to output realizations. Due to limited liability, the intermediary's employment has to be terminated if weak firm performance drives w to zero. With everything kept constant, increasing β_{DC}^I makes liquidation more likely. As a consequence, when the threat of termination is severe and

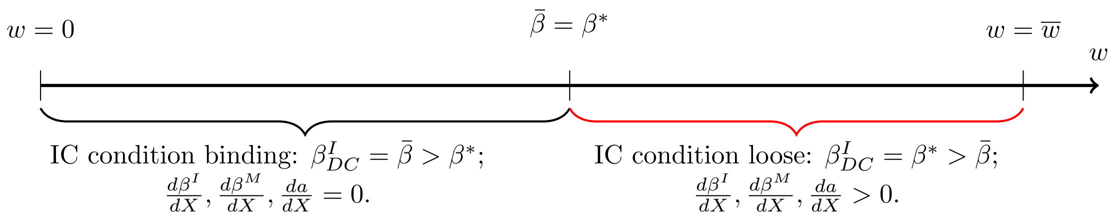

Figure 2: High-powered incentives for the intermediary for large w.

the firm undergoes distress – i.e., w is close to zero – the investor will provide just enough incentives $\beta^I_{DC}(w) = \bar{\beta} > \beta^*(w)$ to ensure sufficiently high monitoring $b_t = b_H$. Under these circumstances, β^I_{DC}, β^M_{DC} remain constant and are not sensitive to small changes in cash flow dX.

By contrast, after sufficiently strong past performance, when w is high termination is less of a concern and it instead becomes optimal to rely on incentive pay. The intermediary is then provided with high-powered incentives $\beta^I_{DC}(w) = \beta^*(w) > \bar{\beta}$ to stimulate managerial effort. The incentive-compatibility condition for monitoring is then non-binding. Increasing w in this region further reduces any agency-induced frictions, leading to higher incentives for both the intermediary and the manager, in that $d\beta^I_{DC}/dX > 0$ and $d\beta^M_{DC}/dX > 0$. This means that both the manager's and the intermediary's incentives have option-like features. Figure 2 depicts the two regions in w with binding and loose incentive compatibility conditions.[16] Figure 3 provides a numerical illustration of the dynamics of the manager's effort and the intermediary's incentives. Importantly, moral hazard leads to average output levels being strictly below the first- and second-best benchmarks (i.e., $a_{DC}(w) < a^{SB}$ for $w < \overline{w}$). This also means that introducing intermediation moral hazard generates an underprovision of managerial incentives – i.e., the manager's incentives are lower in the third-best than in the second-best. Only when the agency conflict between the principal and the intermediary becomes locally resolved at the payout boundary $w = \overline{w}$ is the intermediary provided the right amount of incentives to implement the second-best effort (i.e., $a_{DC}(\overline{w}) = a^{SB}$). The following corollary gathers the formal results for the preceding discussion.

Corollary 1. *Let Π^M_{DC} solve problem (3)-(4) given Π^I_{DC}, and Π^I_{DC} solve problem (6)-(7). Further let the corresponding the value function $p(w)$ solve the HJB equation (18). Then the following holds true:*

a) Effort satisfies $a_{DC}(\overline{w}) = a^{SB}$.

[16] To ensure that the region where $\beta^I_{DC}(w) > \bar{\beta}$ is non-empty, we have to assume that exogenous parameters are such that $\beta^*(\overline{w}) = 1 > \bar{\beta}$. Corollary 1 makes this assumption explicit.

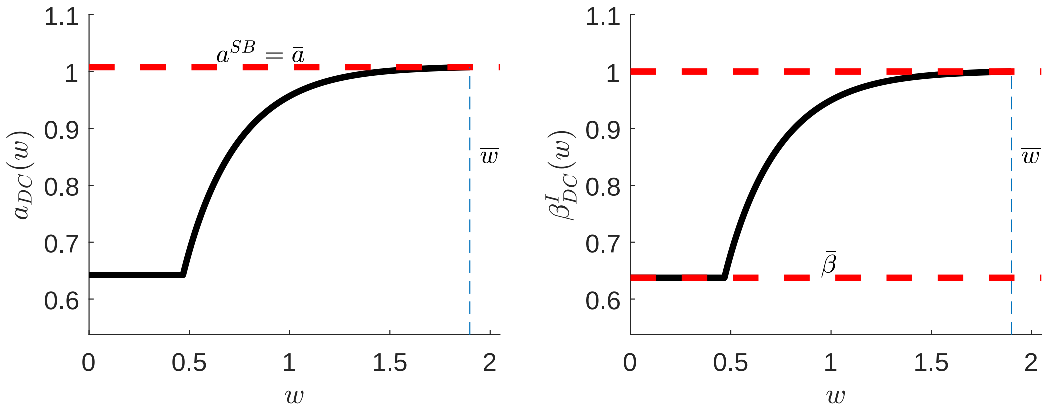

Figure 3: Effort and incentives under delegated contracting. Effort a_{DC} is below the second-best benchmark and increases after good performance. The intermediary's incentives are also increasing in w. The parameters are such that $a^{SB} = 1$: $r = 0.046$, $\gamma = 0.05$, $\Lambda = 0.2$ $\sigma = 0.7$, $\delta = 1$, $\lambda = 0.2$, $b_H = 1.11$, $b_L = b_H/2$, $\theta = 5$, $A = 2$, $R = 0.2$.

b) The optimal sensitivities $\beta^I_{DC}, \beta^M_{DC}$ increase in w on an interval $[w', \overline{w}]$, in that

$$\frac{\partial \beta^I_{DC}(w)}{\partial w} \geq 0 \text{ and } \frac{\partial \beta^M_{DC}(w)}{\partial w} \geq 0.$$

c) There exists a unique value $w'' \in (0, \overline{w})$ such that $\beta^*(w)$ increases in w and $\beta^I_{DC}(w) > \bar{\beta}$ on $(w'', \overline{w}]$ if and only if $\beta^*(\overline{w}) = 1 > \bar{\beta}$. Further, it holds on this interval that both sensitivities increase strictly and that $a_{DC}(w) < a^{SB}$.

5.1.2 Delegated Monitoring

We now turn to analyze implications of the optimal contracts under delegated monitoring. From the previous section, we recall that under delegated contracting, the principal has to provide the intermediary's incentives β^I_{DC} to stimulate both monitoring and contracting with the manager. The principal sets the manager's incentives β^M_{DC} only indirectly via β^I_{DC}. By contrast, under delegated monitoring, she can directly set both β^I_{DM} and β^M_{DM}. This means that the principal selects β^I_{DM} solely to stimulate monitoring. As before, providing incentives to the intermediary is costly due to the agency-induced risk of termination. The principal also chooses the manager's incentives β^M_{DM}, which determine the effort level a_{DM}. Incentive-provision to the manager is costly, as this requires exposing him to risk. However, in contrast to raising β^I_{DM}, increases to managerial incentives β^M_{DM} do not also increase the likelihood of termination, but instead require additional risk compensations to be paid to the risk-averse manager.

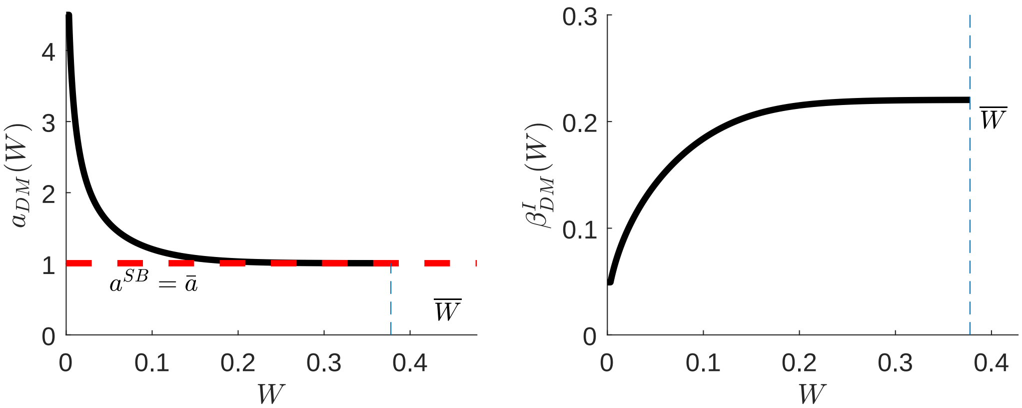

Figure 4: Effort and incentives under delegated monitoring. Effort a_{DM} always exceeds the second-best benchmark and decreases after good performance. The intermediary's incentives increase but are less than the incentives under delegated contracting. The parameters are such that $a^{SB} = 1$: $r = 0.046$, $\gamma = 0.05$, $\Lambda = 0.2$ $\sigma = 0.7$, $\delta = 1$, $\lambda = 0.2$, $b_H = 1.11$, $b_L = b_H/2$, $\theta = 5$, $A = 2$, $R = 0.2$.

The sensitivities β^M_{DM} and β^I_{DM} are interrelated but not in the same way as in the delegated contracting environment. The principal can now freely set the manager's incentives and – taking the desired level a_{DM} as given – optimally sets $\beta^I_{DM} = b_H\lambda/a_{DM}$ such that the intermediary's incentive-compatibility constraint binds. Notably, an increase in managerial effort a_{DM} decreases β^I_{DM} and thus relaxes the intermediary's incentive compatibility constraint. This owes to a higher output rate, which makes a deviation in monitoring more costly for the intermediary. As a consequence, the choice of managerial effort has opposite effects on the incentives and risk that both the manager and intermediary face. High managerial effort increases incentives and risk for the manager, but decreases incentives and risk for the intermediary. The optimal effort choice is thus also determined by risk-sharing considerations between the manager and the intermediary. Whereas the cost of exposing the manager to risk is stationary, additional volatility β^I_{DM} for the intermediary's continuation value is particularly costly when W is low. Therefore, under distress, it is optimal for the principal to provide high-powered incentives to the manager and low-powered incentives to the intermediary. The opposite occurs after good performance, when W is large. Figure 4 illustrates this point in a numerical example. Positive performance leads to a gradual shift of risk exposure from the manager to the intermediary – that is, $d\beta^M_{DM}/dX < 0$ and $d\beta^I_{DM}/dX > 0$. Notably, the manager's compensation under DM is particularity sensitive to negative performance, which is a stark contrast to the option-like characteristics of optimal compensation under DC.

Given that higher effort alleviates moral hazard frictions by relaxing the intermediary's incentive compatibility constraint, the principal finds it optimal to implement effort *above* at the second-best level — i.e. to set $a_{DM}(W) > a^{SB}$. Agency frictions between the intermediary and principal resolve once W approaches $\overline{W}$; implemented effort $a_{DM}(W)$ then decreases in W such that eventually $a_{DM}(\overline{W}) = a^{SB}$. Remarkably, managerial incentives behave differently under the two contracting environments. While strong firm performance leads to decreased managerial incentives under delegated monitoring, it leads to increased managerial incentives under delegated contracting.

Because risk can be shifted from the intermediary to the manager under delegated monitoring, the intermediary generally receives *stronger incentives* when contracting is delegated – that is, $\beta^I_{DM} < \beta^I_{DC}$.[17] By contrast, the manager generally receives *weaker incentives* when contracting is delegated – that is, $\beta^M_{DM} > \beta^M_{DC}$. The following corollary gathers the formal results related to the discussion in this section.

Corollary 2. *Let* (Π^M_{DM}, Π^I_{DM}) *solve problems* (19)-(21), *and let the corresponding value function* $f(\cdot)$ *solve the HJB equation (26). The following then holds true:*

a) Effort satisfies $a_{DM}(W) \geq a^{SB}$ *with equality, if and only if* $W = \overline{W}$.

b) If $\bar{\beta} \geq \frac{\lambda b_H}{\bar{a}}$, *then* $\beta^I_{DM}(W) \leq \beta^I_{DC}(w)$ *for all* $(W, w) \in [0, \overline{W}] \times [0, \overline{w}]$, *where the inequality is strict if* $W < \overline{W}$.

c) If $\bar{\beta} < 1$, *then there exists a value* $w' \in [0, \overline{w})$ *such that* $\beta^M_{DM}(W) \geq \beta^M_{DC}(w)$ *for all* $(W, w) \in [0, \overline{W}] \times [w', \overline{w}]$, *where the inequality is strict if* $W < \overline{W}$.

d) The optimal sensitivities β^I_{DM} *and* β^M_{DM} *move in different directions, i.e.,*

$$\frac{\partial \beta^I_{DM}(W)}{\partial W} \times \frac{\partial \beta^M_{DM}(W)}{\partial W} < 0.$$

Further, there is a unique value $W' \in [0, \overline{W}]$ *such that* $\beta^I_{DM}(W)$ *strictly increases and* $\beta^M_{DM}(W)$ *strictly decreases to the right of* W'. *If* $\gamma - r$ *is sufficiently small, the claim holds true everywhere, in that* $W' = 0$.

[17] This holds under some weak assumptions on exogenous parameters, which we found to be easily satisfied in our numerical analysis. See Corollary 2.

5.2 Modes of Contracting and Implications for Managerial Incentives

5.2.1 Financial Intermediation

To map the two modes of contracting to actual types of financial intermediation, we would ideally like to know the explicit and implicit components of the investor-intermediary relationship. While there are valuable systematic studies of contracts between intermediaries and portfolio firms (see Kaplan and Strömberg (2003) for the case of venture capital), there is little empirical research on contracting between investors and financial intermediaries, such as venture capital funds, private equity funds, or hedge funds. With this caveat, we posit that one can interpret private equity investment as an example of intermediation under delegated monitoring. Limited partners (investors) provide funds and contract with general partners (intermediaries). Contracting with managers of portfolio firms is, to a large degree, standardized; the conditions of such contracts are (possibly implicit) parts of contracts between limited partners and general partners. Kaplan and Strömberg (2009) term the set of standard incentives and governance practices applied to portfolio firms as the "governance engineering" of private equity. Other aspects of investment such as time horizon, financing structure, target firms characteristics are also pre-specified. Altogether, these features align private equity intermediation with our model's DM mode.

With this interpretation of private equity investment, the predictions of the previous section prompt a reevaluation of some empirical evidence of private equity performance. Numerous studies, such as Leslie and Oyer (2008), Acharya et al. (2012), and Cronqvist and Fahlenbrach (2013), show that private equity investment increases managerial incentives in target firms. The common interpretation of this empirical pattern is that private equity is a superior owner who can improve firm governance. By contrast, our model of intermediation under delegated monitoring suggests that increased managerial incentives after private equity investment are an optimal way to deal with moral hazard in monitoring by an intermediary. To put it differently, the common interpretation implies that incentives before private equity investment are suboptimal and "governance engineering" fixes them, but this is not the case in our model's interpretation.[18]

More generally, there are two possible cases for how the mode of intermediation is determined in practice. First, under some circumstances, contracting directly with the manager is not feasible, such that intermediated investment in the delegated contracting mode is the only possibility. This

[18]Firm ownership and monitoring by other types of financial intermediaries has been shown to increase managerial incentives in, e.g., Hartzell and Starks (2003) and Brav et al. (2008). Our model suggests a need to pay attention to the different types of intermediaries and to exercise caution when interpreting higher managerial incentives as a marker of the intermediary's performance or superior monitoring skills.

occurs when the principal has no direct contact with the manager and when the contract that the intermediary offers to the manager is not contractible for the relation between the principal and the intermediary. If this is the case, the principal *has* to involve a specialized intermediary.

Second, the mode of contracting can be determined by a cost-benefit analysis. As expected, agency conflicts can be easier to address when the principal is able to contract directly with the manager rather than via an intermediary who is also subject to moral hazard. Specifically, as the principal can commit to any feasible contract, she can therefore replicate the DC outcome in the DM environment. In practice, contracting with ultimate agents can be complex and time consuming, and the DM mode can become more costly than the DC mode.[19] Given the level of this cost, it can be optimal to delegate the contracting task to an intermediary. We consider situations in which the mode of contracting can be chosen optimally in Section 5.3.

5.2.2 Boards of Directors and Say-on-Pay

In another interpretation of our model, the intermediary represents a board of directors. In this case, both modes of contracting can arise depending of the regulatory environment.[20] In their traditional roles, shareholders delegate to boards both monitoring of and contracting with managers. This environment corresponds to the DC setting of our model. Adoption of various say-on-pay regulations[21] changed these traditional roles and brought about an increase in shareholders' direct participation in arranging executive compensation. These regulations shifted the shareholders-board-manager relationship toward the DM setting of our model, in which shareholders determine the manager's compensation directly.

As much as adoption of say-on-pay regulations switches the contracting setting from DC to DM, our model predicts two main effects related to executive compensation. First, the manager's performance pay would increase. Second, compensation contracts would lose their option-like features and instead incentives would increase after poor performance. There is clear empirical evidence for

[19]Papers such as Aghion and Tirole (1997) and Burkart et al. (1997) suggest endogenous reasons for delegation in principal-agent models, building on asymmetric information, communication, and dispersed ownership. Such far-reaching extensions are beyond the scope of this paper.

[20]To further support the interpretation in which the intermediary represents a board of directors, we solve a variant of our model in which the intermediary and the manager have similar preferences and are subject to similar frictions. Section S.3 of the Supplementary Appendix shows that our model's quantitative predictions for incentive levels carry over into a setting in which the intermediary's frictions to contracting come from CARA preferences rather than from limited liability. As the problem is stationary, the optimal contract does not feature option-like characteristics, in that incentives do not change with performance.

[21]The first say-on-pay regulation was introduced in the U.K. in 2002, mandating advisory shareholder votes on executive compensation. In the U.S., a related regulation was included in the Dodd-Frank Act of 2010. Some European countries introduced binding say-on-pay votes.

both effects. Pay-for-performance increases after adoption of pay-on-say regulation (Correa and Lel (2016), Iliev and Vitanova (2018)). Shareholders' say-on-pay votes result in increasing sensitivity of pay to poor realizations of performance (Ferri and Maber (2013), Alissa (2015)). Notably, our explanation of these empirical facts simply accounts for changes in the contracting environment with say-on-pay adoption. It thus differs from the two most common explanations of say-on-pay regulations' effects, which postulate frictions due either to managerial power (e.g., Coates (2009), Bebchuk et al. (2007)) or to unsophisticated shareholders (e.g., Bainbridge (2008), Kaplan (2007)).

5.3 How to Deal with Severe Agency Conflicts?

Are more severe agency conflicts at the firm level better addressed by intermediation with delegated contracting or delegated monitoring? Do more severe agency conflicts at the intermedairy level promote delegated contracting or delegated monitoring? In this section, we analyze these questions and study the investor's and the intermediary's values under optimal contracts with varying levels of agency conflicts. In our model, moral hazard at the intermediary level is more pronounced when λ or σ is high. Whereas a higher cost of monitoring (measured by λ) increases the intermediary's benefit of deviating and not monitoring, a more noisy output process (measured by σ) makes it more difficult for the principal to detect deviations. Similarly, moral hazard at the firm level is severe whenever δ or σ are high.

To study the investor's point of view, we analyze differences in the initial values for $F^{DM} - F^{DC}$ at the time that the contract is initiated. Similarly, the intermediary's initial difference in values is given by $W^{DC} - W^{DM}$. For a clear comparison, we assume that the principal has all of the bargaining power and that the manager's outside option is zero.[22] As discussed above, because of some of the fixed costs of delegated monitoring, delegated contracting can be more attractive for the investor. Thus, the effects of changes in agency cost parameters on $F^{DM} - F^{DC}$ represent a shifting preference for the investor for this form of intermediation – even if $F^{DM} - F^{DC}$ remains positive under our assumptions. In the subsequent analysis, we use the same parameter values as in Section 5.1. While we cannot prove the presented relations analytically, they are robust to extensive numerical simulations under the various parametrizations we have tried.

The top panels of Figure 5 show that more severe agency problems at the intermediary level, or a higher λ, make delegated contracting relatively more attractive for the principal. This may be

[22] Under the principal's full bargaining power, $W^{DC} \equiv W^{DC}_{0^-} \in \arg\max_{W \geq 0} F^{DC}(W)$ and $F^{DC} \equiv F^{DC}(W^{DC})$. Similarly, $W^{DM} \equiv W^{DM}_{0^-} = \arg\max_{W \geq 0} F^{DM}(W)$ and $F^{DM} \equiv F^{DM}(W^{DM})$. Note that with our assumption that $\mathbb{CE}_{0^-} = 0$, W^{DC} equals w^{DC} and F^{DM} equals f^{DM} at time zero.

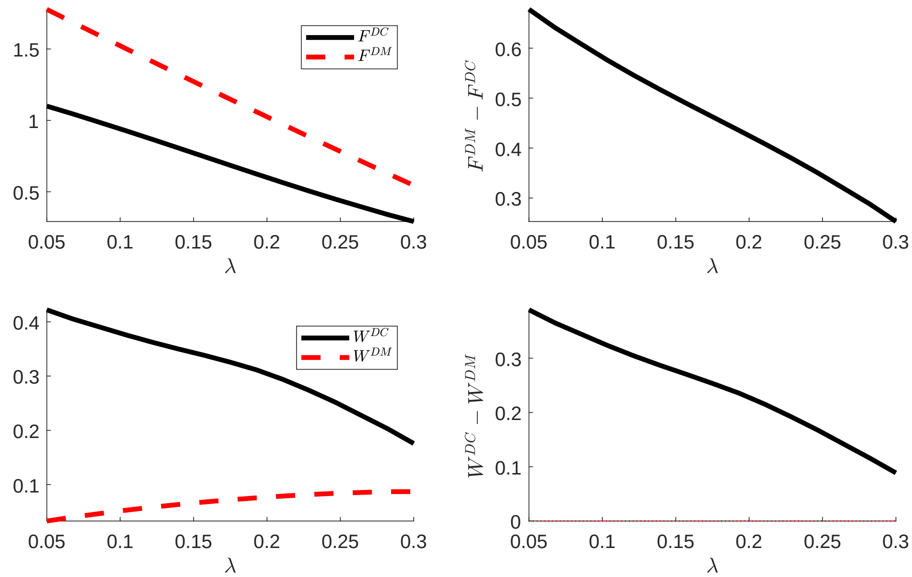

Figure 5: Delegated contracting becomes more suitable when monitoring costs increase. The parameters are: $r = 0.046$, $\gamma = 0.05$, $\Lambda = 0.2$, $\sigma = 0.7$, $\delta = 1$, $b_H = 1.11$, $b_L = b_H/2$, $\theta = 5$, $A = 2$, $R = 0.2$.

surprising, as it implies that the more difficult it is for the intermediary to monitor the manager, the more the principal relies on the intermediary by delegating him the responsibility to contract with the manager. The rationale for our result is that in the optimal DC contract, the intermediary has more "skin in the game" than under the optimal DM contract and is thus less sensitive to the increasing costs of monitoring. More specifically, note that an increase in λ tightens the intermediary's incentive compatibility constraints in both cases – that is, $\beta^I_{DM} \geq \lambda b_H/a$ under DM and $\beta^I_{DC} \geq \bar{\beta}$ (as $\partial\bar{\beta}/\partial\lambda > 0$) under DC. While under DM, the incentive compatibility constraint is always binding, under DC this is only the case when w is close to zero; otherwise, incentives are high-powered and $\beta^I_{DC} > \bar{\beta}$. Consequently, an increase in λ is particularly costly for the principal under DM when he directly contracts with the manager. Our results thus suggest that a difficult monitoring task can be efficiently incentivized by delegating the responsibility of contracting with the manager to the intermediary, which provides the intermediary additional skin in the game and thus the necessary incentives.

In a similar spirit, the top panels of Figure 6 show that severe moral hazard at the firm level, or a high δ, is also best dealt with by delegating the contracting task. According to our model, this occurs because higher effort costs, ceteris paribus, lead to lower managerial effort throughout the whole lifetime of the firm. This reduction in effort again tightens the intermediary's incentive

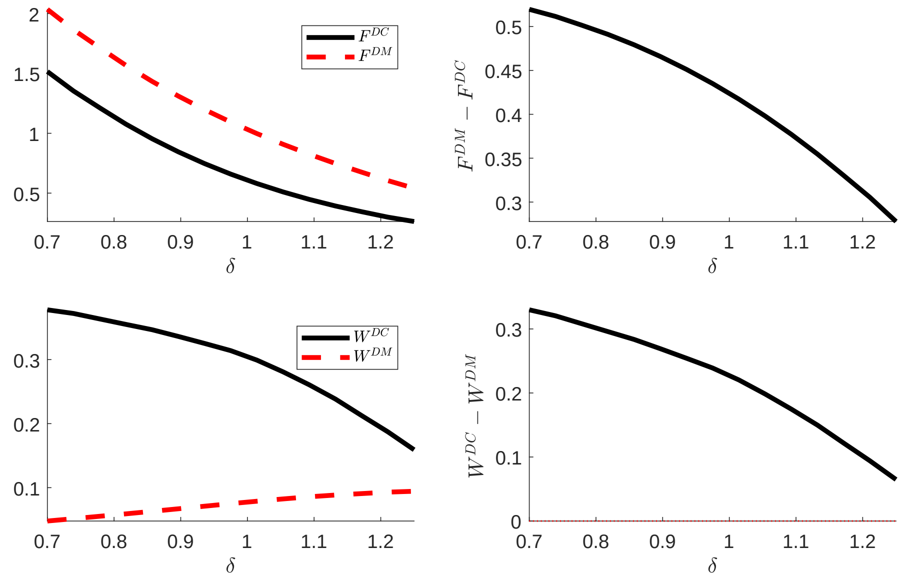

Figure 6: Delegated contracting becomes more suitable to address severe moral hazard at the firm level. The parameters are: $r = 0.046$, $\gamma = 0.05$, $\Lambda = 0.2$, $\sigma = 0.7$, $\lambda = 0.2$, $b_H = 1.11$, $b_L = b_H/2$, $\theta = 5$, $A = 2$, $R = 0.2$.

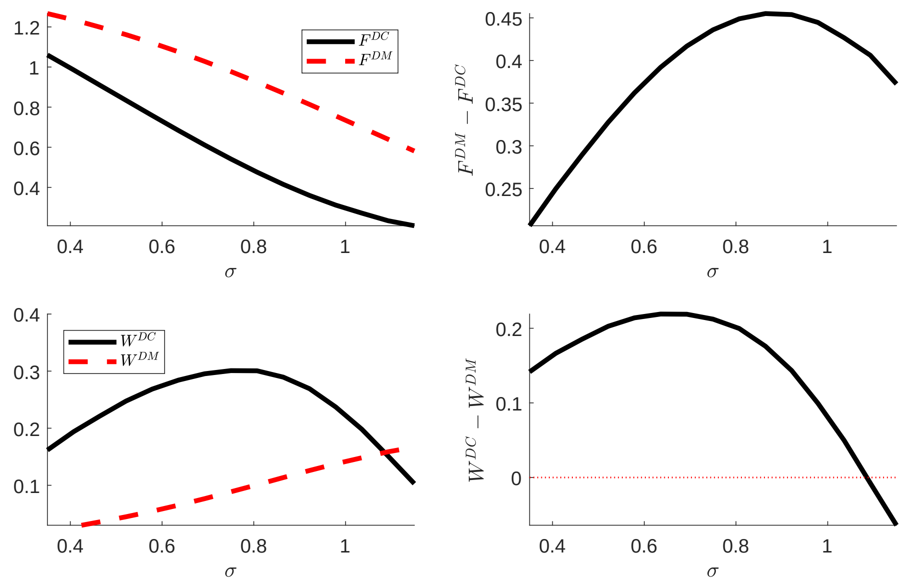

Figure 7: Cash-flow volatility σ has an ambiguous effect. The parameters are: $r = 0.046$, $\gamma = 0.05$, $\Lambda = 0.2$, $\delta = 1$, $\lambda = 0.2$, $b_H = 1.11$, $b_L = b_H/2$, $\theta = 5$, $A = 2$, $R = 0.2$.

compatibility constraints in both contracting modes and is therefore more harmful under delegated monitoring than under delegated contracting. The bottom line is that whenever moral hazard at the firm or intermediary level is severe, contracting with the manager becomes difficult and the investor may find it more attractive to delegate this task.

The top panels of Figure 7 present the effects of output risk σ on the investor's value. Remarkably, increasing σ has an ambiguous effect on $F^{DM} - F^{DC}$. The difference between the working of agency costs measured by σ versus λ and δ is driven by two mechanisms. While under DC, λ and δ had direct effects on the the intermediary's risk only when w is large, a higher σ increases the intermediary's risk exposure at any time. Because the intermediary receives higher powered incentives when he contracts with the manager (see Corollary 2), the adverse effects of additional volatility σ are stronger under DC. Furthermore, an increase in volatility reduces implemented effort and triggers a tighter incentive compatibility constraint under DM. As shown in Figure 7, the first effect outlined above dominates for small values of σ and so an increase in σ makes direct and then delegated contracting more attractive.

The effect of agency conflicts' severity on the intermediary's preference for DC versus DM tends to be the opposite of the investor's; see the bottom panels of Figures 5, 6, and 7. Specifically, with increasing λ and δ, the difference between the intermediary's initial value under DC and DM decreases. The primary effect is that severe agency conflicts decrease managerial effort and incentives. Weak managerial incentives require stronger incentives for monitoring under DM but not under DC. Thus, the intermediary's expected rent decreases in λ and δ under DC and increases under DM. By contrast, higher output volatility σ makes it optimal to limit termination risk by initially promising a higher stake to the intermediary. This effect is stronger under DC than under DM – i.e., when the intermediary is highly incentivized – and dominates for lower σ values. This results in the largest difference between the intermediary's stake across the two contracting modes for intermediate σ values.

6 Discussion of Assumptions and Robustness

Our model entails a number of assumptions that are mainly designed to enhance simplicity and to facilitate a clear analysis of the main forces in a tractable model. Below, we discuss these assumptions and the robustness of the results.

The manager's patience. In our exposition, we assumed throughout that the manager discounts at the market interest rate r and is therefore more patient than the intermediary. This assumption is without loss of generality: our results remain qualitatively unchanged if the manager discounted at some rate $\rho \neq r$ (e.g., $\rho \geq \gamma$), as becomes apparent from the model solution with a general discount rate in the Supplementary Appendix S.1.

The manager's CARA preference. With CARA preferences, the state variable V_t – the agent's continuation utility – separates out due to the absence of wealth effects. This means that one does not have to explicitly keep track of V_t. As a consequence, we are able to characterize the optimal contract by means of an ordinary differential equation. If we assumed a different utility function, the agent's continuation value would become a relevant state and, in general, we would have to solve a partial differential equation. Furthermore, non-CARA preferences would add further complications to the analysis of incentive provision for the agent with hidden savings (see, e.g., Di Tella and Sannikov (2016) or He (2012)).

Unobservable savings and consumption. Because the intermediary is protected by limited liability and cannot fully commit, we assume that the manager can maintain a savings account that can be consumed after firm termination. When savings are not observable, incentive compatibility requires that (discounted) marginal utility follows a martingale, which pins down the manager's consumption. If, by contrast, the manager's savings were observable, then the principal could in effect control the manager's consumption, adding another control variable to the principal's maximization problem. While under these circumstances the optimal contract is still characterized by an ODE, the problem would be far less tractable, as illustrated in, e.g., He et al. (2017). The model outcomes are unlikely to hinge on the assumption of unobservable savings.

The intermediary's risk neutrality. In the Supplementary Appendix S.3, we solve a model where the intermediary also has CARA preferences. In this case, the problem becomes fully stationary, the provision incentives become independent of past firm performance, and, in particular, the intermediary's contract loses its option-like features. However, some of our main findings on incentives remain unchanged. In particular, the intermediary (manager) receives more (less) incentives under delegated contracting.

7 Conclusions

In this paper, we study the effect of intermediation and monitoring on optimal long-term contracting. In a dynamic model that features an investor, an intermediary, and a manager, we focus on the provision and dynamics of incentives. The investor contracts with the intermediary and separately with the manager in the delegated monitoring mode. The investor contracts only with the intermediary and the intermediary contracts with the manager in the delegated contracting mode. Optimal contracts provide incentives to both the intermediary and the manager as an exposure to firm performance. In delegated contracting, the manager incentives need to be passed over through the intermediary's incentives. This has an effect that the manager's incentives and effort are below the second-best levels (i.e., those without the intermediary's moral hazard), but increase with firm performance. In delegated monitoring, high managerial incentives and effort can alleviate the intermediary's incentive compatibility constraint. As a consequence, the manager's incentives and effort are above the second-best levels and decrease with firm performance. In our analysis, we discuss scenarios in which either of the two contracting modes may be optimal.

Appendix A: Preliminaries

A.1 Regularity Conditions

Throughout the paper and for all problems, we impose finite utility

$$\mathbb{E}^M\left[\int_0^{\tau^M} e^{-rs}|u(\hat{c}_s^M, a_s^M)|ds\right] < \infty$$

and the usual square integrability conditions of transfers:

$$\mathbb{E}^K\left[\int_0^{\tau^I} e^{-rt}dc_s^I\right]^2 < \infty \text{ and } \mathbb{E}^K\left[\int_0^{\tau^M} e^{-rt}dc_s^M\right]^2 < \infty \tag{A.1}$$

for all $K \in \{P, I, M\}$. Next, note that

$$\hat{S}_t^M = \int_0^t e^{r(t-s)}dc_s^M - \int_0^t e^{r(t-s)}\tilde{c}_s^M ds + \hat{S}_0^M e^{rt}$$

for a consumption process $\{\tilde{c}^M\}$. Define $\{\hat{c}^M\}$ as the (up to a null set) unique process, implying $S_t^M = \hat{S}_t^M$ for all t with probability one, given $S_0^M = \hat{S}_0^M$.

Furthermore, we impose the transversality condition

$$\lim_{t\to\infty}|S_t^M - \hat{S}_t^M| = 0 \text{ a.s. } \implies \lim_{t\to\infty}|\hat{c}_t^M - \tilde{c}_t^M| = 0 \text{ a.s. },$$

where $\{\tilde{c}^M\}$ is the almost surely unique consumption process, which implies a certain savings process $\{\hat{S}^M\}$.

For technical reasons, we postulate that the processes $\{\beta^K\}$, $\{\alpha^K\}$ are almost surely bounded, so that $|\beta_t^K|, |\alpha_t^K| < M$ almost surely, i.e. $\mathbb{P}\big(|\psi_t^K| < M\big) = 1$ for $\psi \in \{\alpha, \beta\}$, for any t and $K = I, M$. The equivalence of the measures $\{\mathbb{P}, \mathbb{P}^K : K = P, I, M\}$ (to be discussed in the next paragraph) ensures that the sensitivities are almost surely bounded under each probability measure used throughout the paper. We assume $M \in \mathbb{R}_+$ to be sufficiently large, so that this imposed constraint actually never binds in optimum.

A.2 Change of Measure

To start with, fix a probability measure $\mathbb{P}^0$, such that $dX_t = \sigma dZ_t^0$ with a $\mathbb{F}$-progressive standard Brownian Motion $\{Z^0\}$ under the measure $\mathbb{P}^0$. Take a progressive process $\{a_t\}_{0\le t\le\tau^M}$ of bounded variation and define the process $\{\chi\}$ via $\chi_t = a_t/\sigma$ for all $t \ge 0$, almost surely. Further, let

$$\Gamma_t = \Gamma_t(a) = \exp\Big(\int_0^t \chi_u dZ_u^0 - \frac{1}{2}\int_0^t \chi_u^2 du\Big).$$

Assuming that the so-called Novikov condition is satisfied, i.e.,

$$\mathbb{E}^0\left[\exp\left(\frac{1}{2}\int_0^{\tau^M} \chi_t^2 dt\right)\right] < \infty,$$

it follows that $\{\Gamma_t\}_{0\le t\le\tau^M}$ follows a martingale. Given our restriction of bounded sensitivities, the Novikov-Condition is evidently met. Due to $\mathbb{E}^0[\Gamma_0] = 1$, it is evident that $\{\Gamma_t\}_{0\le t\le\tau^M}$ is a progressive density process and defines a probability measure $\mathbb{P}^a$ via the Radon-Nikodyn derivative

$$\left(\frac{d\mathbb{P}^a}{d\mathbb{P}}\right)\bigg|_{\mathcal{F}_t} = \Gamma_t.$$

Under the probability measure $\mathbb{P}^a$, the process $\{Z\}$ with

$$Z_t = Z_t^0 - \int_0^t \chi_u du = \frac{X_t - \int_0^t a_u du}{\sigma}$$

follows a standard Brownian Motion up to the stopping time τ^M. In the following, given a process $\{a^K\}$ that satisfies the above stated conditions, we adopt the notation $\mathbb{P}^K \equiv \mathbb{P}^{a^K}$ for all $K \in \{P, I, M\}$. All measures $\{\mathbb{P}^0, \mathbb{P}^a : \{a\}\}$ are equivalent for suitable processes $\{a\}$, that satisfy the above stated conditions, such that the measures share the same null sets.

Appendix B: The Manager's Problem (DC) - Proof of Lemma 1

We split up the proof in two parts. First, we establish the representation of $\{V\}$ by means of a stochastic differential equation, given a contract Π_{DC}^M. From there, we proceed to show the claim regarding incentive compatibility.

B.1 Martingale Representation

Proof. Let in the following $\Pi_{DC}^M = (\{a^I\}, \{c^M\}, \{S^M\}, \tau^M)$ represent the manager's contract. We denote the manager's continuation value by

$$V_t = V_t(\Pi_{DI}^M) = \mathbb{E}_t^I \left[\int_t^\infty e^{-\rho(s-t)} u(\hat{c}_s^M, a_s^I) ds\right] = \mathbb{E}_t^I \left[\int_t^\infty e^{-\rho(s-t)} u(\hat{c}_s^M, a_s^I) ds\right],$$

because the measures $\mathbb{P}^I$ and $\mathbb{P}^M$ agree in equilibrium and $\{a^M\} = \{a^I\}$, $\{\tilde{c}^M\} = \{\hat{c}^M\}$, where $\{\hat{c}^M\}$, is the "recommended consumption" and $\{\tilde{c}^M\}$ the "actual consumption" process. Define

$$A_t \equiv \mathbb{E}_t^I \left[\int_0^\infty e^{-rt} u(\hat{c}_s^M, a_s^I) ds\right] = \int_0^t e^{-rs} u(\hat{c}_s^M, a_s^I) ds + e^{-rt} V_t(\Pi_{DI}^M) \tag{B.1}$$

By construction, $\{A_t : 0 \leq t \leq \infty\}$ is a square integrable martingale, progressive with respect to $\mathbb{F}$ under $\mathbb{P}^I = \mathbb{P}^M$. By the martingale representation theorem, there exist now $\mathbb{F}$-predictable processes $\{\alpha^M\}, \{\beta^M\}$ such that

$$e^{rt} dA_t = (-\theta r V_{t-}) \beta_t^M \big(dX_t - a_t^I dt\big) - (-\theta r V_{t-}) \alpha_t^M \big(dN_t - \Lambda dt\big).$$

and therefore

$$dV_t = r V_{t-} dt - u(\hat{c}_t^M, a_t^I) dt + (-\theta r V_{t-}) \beta_t^M \big(dX_t - a_t^I dt\big) - (-\theta r V_{t-}) \alpha_t^M \big(dN_t - \Lambda dt\big).$$

□

B.2 Incentive Compatibility

Proof. We prove first the following auxiliary Lemma

Lemma 5. *Fix $\mathbb{F}$-predictable processes $\{a^M\}$, $\{\hat{c}^M\}$ and let $\mathcal{S} \in \mathbb{R}$. Consider the problem*

$$V_t = \max_{\{\tilde{c}_s^M\}_{s\geq t}} \mathbb{E}_t^M \left[\int_t^\infty e^{-r(s-t)} u(\tilde{c}_s^M, a_s^M) ds \right]$$
$$\text{subject to } d\Delta_s^M = r\Delta_s^M ds + \hat{c}_s^M ds - \tilde{c}_s^M ds, \Delta_t^M = 0 \text{ and } \lim_{s\to\infty} |\Delta_t^M - \Delta_s^M| = 0 \text{ a.s.}$$

Next consider the problem

$$V_t' = \max_{\{c_s\}_{s\geq t}} \mathbb{E}_t^M \left[\int_t^\infty e^{-r(s-t)} u(c_s, a_s^M) ds \right]$$
$$\text{subject to } d\Delta_s^M = r\Delta_s^M ds + \hat{c}_s^M ds - c_s ds, \Delta_t^M = \mathcal{S} \text{ and } \lim_{s\to\infty} |\Delta_t^M - \Delta_s^M| = 0 \text{ a.s.}$$

Then, $c_t + r\mathcal{S} = \tilde{c}_t^M$ and $V_t' = e^{-\theta r \mathcal{S}} V_t$.

Proof. Suppose that there exists a process $\{c'\} \neq \{c\}$, which satisfies the transversality condition, such that

$$V_t'(\{c'\}) > V_t'(\{c\}) = e^{-\theta r \mathcal{S}} V_t.$$

Define the process $\{c''\}$ via $\tilde{c}_t^M = c_t'' - r\mathcal{S}$. Then $\{c''\}$ satisfies the transversality condition and

$$\mathbb{E}_t^M \left[\int_t^\infty e^{-r(s-t)} u(c_s'', a_s^M) ds \right] = e^{\theta r \mathcal{S}} V_t'(\{c'\}) > V_t,$$

a contradiction. □

Next, we provide necessary and sufficient conditions for Π_{DC}^M to be incentive-compatible, in that $\hat{S}_t^M = S_t^M$ and $a_t^M = a_t^I$ for all $t \geq 0$ holds almost surely. Define $\Delta_t \equiv \hat{S}_t^M - S_t^M$ the deviation state with $\Delta_0 = 0$ and note that

$$d\Delta_t = r\Delta_t dt + \hat{c}_t^M dt - \tilde{c}_t^M dt,$$

where $\{\hat{c}^M\}$ is such that $S_t^M = \hat{S}_t^M$, i.e. $\Delta_t = 0$ for all t. Note that $dZ_t^M \equiv (dX_t - a_t^M dt)/\sigma$ is the increment of a standard Brownian Motion under the measure $\mathbb{P}^M$. We rewrite

$$dV_t = rV_{t^-} dt - u(\hat{c}_t^M, a_t^I)dt + (-\theta r V_{t^-})\beta_t^M \big(dZ_t^M + (a_t^M - a_t^I)dt\big) - (-\theta r V_{t^-})\alpha_t^M \big(dN_t - \Lambda dt\big).$$

Let $t > 0$ and suppose the manager follows the recommended policy from time t onwards, in that $a_s^M = a_s^I$ and $\tilde{c}_s^M = \hat{c}_s^M + r\Delta_t$ for all $s \geq t$. The payoff from following this strategy is represented by the auxiliary gain process

$$\begin{aligned} G_t^M \equiv G_t^M(\tilde{c}^M, a^M) &= \int_0^t e^{-rs} u(\tilde{c}_s^M, a_s^M) ds + e^{-\theta r \Delta_t} e^{-rt} V_t \\ &= \int_0^\infty e^{-rs} u(\tilde{c}_s, a_s^M) ds + \int_t^\infty e^{-rs} \Big(e^{-\theta r \Delta_t} u(\hat{c}_s^M, a_s^I) - u(\tilde{c}_s^M, a_s^M) \Big) ds \end{aligned}$$

and by means of Lemma 5, it suffices to consider deviations of this type, which yield weakly higher payoff than deviation of any other type.

Next, note that the transversality condition $\lim_{t\to\infty} \Delta_t = 0$ a.s. implies that $\lim_{t\to\infty} |\hat{c}_t^M - \tilde{c}_t^M| =$

0 a.s. for any possible strategy of the manager. Therefore,

$$\lim_{t\to\infty}\int_t^\infty e^{-rs}\Big(e^{-\theta r\Delta_t}u(\hat{c}_s^M,a_s^I)-u(\tilde{c}_s^M,a_s^M)\Big)ds=0\text{ a.s.,}$$

which implies that the manager's actual payoff equals

$$V'_{0^-}=\max_{\{\tilde{c}\},\{\hat{a}\}}\mathbb{E}^M\int_0^\infty e^{-rs}u(\tilde{c}_s,\hat{a}_s)ds=\max_{\{\tilde{c}\},\{\hat{a}\}}\mathbb{E}^M G_\infty^M=\max_{\{\tilde{c}\},\{\hat{a}\}}\mathbb{E}^M\lim_{t\to\infty}G_t^M.$$

By Itô's Lemma:

$$\begin{aligned}
&e^{\theta r\Delta_t}e^{rt}dG_t^M\\
&=\Big(u(\tilde{c}_t^M,a_t^M)e^{\theta r\Delta_t}-u(\hat{c}_t^M,a_t^I)-\theta rV_{t-}(r\Delta_t+\hat{c}_t^M-\tilde{c}_t^M)+(-\theta rV_{t-})\beta_t^M(a_t^I-a_t^M)\Big)dt\\
&+(-\theta rV_{t-})\beta_t^M dZ_t^M-(-\theta rV_{t-})\alpha_t^M(dN_t-\Lambda dt)\\
&\equiv\mu_{tG}^M(\cdot)dt+(-\theta rV_{t-})\beta_t^M dZ_t^M-(-\theta rV_{t-})\alpha_t^M(dN_t-\Lambda dt)
\end{aligned}$$

Observe that, because $\{\alpha^M\},\{\beta^M\}$ are bounded and finite utility is imposed, we have

$$\mathbb{E}^K\bigg(\int_0^t e^{-rs}\beta_s^M(-\theta rV_{s-})dZ_s\bigg)=\mathbb{E}^K\bigg(\int_0^t e^{-rs}\alpha_s^M(-\theta rV_{s-})(dN_s-\Lambda ds)\bigg)=0,$$

for any $K\in\{P,I,M\}$. It is then evident that by choosing $a_t^M=a_t^I$, $\tilde{c}_t^M=\hat{c}_t^M$, the manager can ensure that $\Delta_t=\mu_{tG}^M(\cdot)=0$ for all $t\geq 0$, in which case $\{G^M(\hat{c}^M,a^I)\}$ follows a martingale under $\mathbb{P}^M$ with last element $G_\infty^M(\cdot)$, such that $\mathbb{E}^M|G_\infty^M(\hat{c}^M,a)|<\infty$ due to the regularity conditions we impose. Hence, by optional sampling

$$V'_{0^-}=\max_{\{\tilde{c}^M\},\{\hat{a}\}}\mathbb{E}^M G_\infty^M(\tilde{c}^M,a^M)\geq\mathbb{E}^M G_\infty^M(\hat{c}^M,a^I)=\lim_{t\to\infty}\mathbb{E}^M G_t^M(\hat{c}^M,a^I)=V_{0^-}.$$

Next, observe that the highest value that $\mu_{tG}^M(\cdot)$ can obtain given Δ_t is given by the maximization over $\tilde{c}_t^M$ and a_t^M, where the solution evidently satisfies the FOC:

$$\begin{aligned}
&u_c(\tilde{c}_t^M,a_t^M)e^{r\theta\Delta_t}=-\theta rV_{t-}\iff u_c(\tilde{c}_t^M+r\Delta_t,a_t^M)=-\theta rV_{t-}\\
&\iff u(\tilde{c}_t^M+r\Delta_t,a_t^M)=rV_{t-};\\
&u_a(\tilde{c}_t^M,a_t^M)e^{\theta r\Delta_t}=(\theta rV_{t-})\beta_t^M\iff -u_c(\tilde{c}_t^M+r\Delta_t,a_t^M)\frac{\delta a_t^M}{b_t^I}=(\theta rV_{t-})\beta_t^M\\
&\iff u(\tilde{c}_t^M+r\Delta_t,a_t^M)\frac{\delta a_t^M}{b_t^I}=rV_{t-}\beta_t^M.
\end{aligned}$$

If Π_{DC}^M is such that $rV_{t-}e^{-\theta r\Delta_t}=u(\hat{c}_t^M,a_t^I)$ and $\beta_t^M=a_t^I\delta/b_t^I$ hold for all $t\geq 0$, it follows that the FOC are satisfied by $\tilde{c}_t^M=\hat{c}_t^M$ $a_t^M=a_t^I$ for all $t\geq 0$, in which case $\Delta_t=\mu_{tG}^M(\cdot)=0$. Indeed, because the deviation gains are concave in the state Δ, the first order conditions are sufficient.

Hence, any other strategy $\{\tilde{c}^M\},\{a^M\}$ makes the process $\{G^M(\tilde{c}^M,a^M)\}$ a supermartingale under the measure $\mathbb{P}^M$, i.e.

$$V_{0^-}=G_0^M(\hat{c}^M,a^I)\geq\mathbb{E}^M G_t^M(\tilde{c}^M,a^M)$$

Because our regularity conditions ensure that $\{G^M(\tilde{c}^M, a^M)\}$ is bounded from below, we can thus take limits on both sides and apply optional sampling to obtain

$$V_{0^-} \geq \lim_{t\to\infty} \mathbb{E}^M G_t^M(\tilde{c}^M, a^M) = \mathbb{E}^M \lim_{t\to\infty} G_t^M(\tilde{c}^M, a^M) = \mathbb{E}^M G_\infty^M(\tilde{c}^M, a^M)$$

and in particular

$$V_{0^-} \geq \max_{\{\tilde{c}^M\},\{\hat{a}\}} \mathbb{E}^M G_\infty^M(\tilde{c}^M, a^M) = V'_{0^-}.$$

Therefore, $V_{0^-} = V'_{0^-}$ and $(\tilde{c}_t^M, a_t^M) = (\hat{c}_t^M, a_t^I)$ for all $t \geq 0$ is the optimal strategy for the agent, in that $\Pi_{DC}^M \in IC_{DC}^M$.

If there exists a $\mathbb{F}$ stopping time τ' such that $\mathbb{P}^M(\tau' < \tau^M) > 0$ and either $rV_{(\tau')^-}e^{-\theta r\Delta_{\tau'}} = u(\hat{c}_{\tau'}^M, a_t^I)$ or $\beta_{\tau'}^M = a_{\tau'}^I\delta/b_{\tau'}^I$ fail to hold, then there are also processes $\{\tilde{c}^M\}, \{a^M\}$, and a set $\mathcal{A} \in [0,\tau) \times \Omega$ with

$$\mu_{tG}^M > 0 \text{ for all } (t,\omega) \in \mathcal{A} \text{ and } \hat{\mathcal{L}} \otimes \mathbb{P}^M(\mathcal{A}) > 0,$$

where $\hat{\mathcal{L}}$ is the Lebesgue measure on the Lebesgue sigma algebra in $\mathbb{R}$. Then,

$$V'_{0^-} \geq V_{0^-} + \int_{\mathcal{A}} e^{-rz}\mu_{zG}^M d(\hat{\mathcal{L}}(z) \otimes \mathbb{P}^M(\omega)) > V_{0^-}.$$

Consequently, $\{a^I\}, \{\hat{c}^M\}$ is not the optimal strategy for the agent, such that there exists with positive probability a time t where $(\tilde{c}_t^M a_t^M) = (\hat{c}_t^M, a_t^I)$ fails. Whence, $\Pi_{DC}^M \notin IC_{DC}^M$, which concludes the proof. □

Appendix C: The Intermediary's Problem (DC) - Proof of Lemma 2 and Propositions 2, 3

In this section, we prove the claims of Lemma 2 (see Step I) and Propositions 2, 3 (see Step III). In step II, we establish an auxiliary result, which is then utilized in step III to conclude the proof.

In the following let $\Pi_{DC}^I = (\{b^P\}, \{c^I\}, \tau^I)$ be the intermediary's contract. We may assume that the intermediary's contract is terminated at the latest when a Poisson shock terminates the firm, i.e. $\tau^I \leq \min\{t \geq 0 : N_t = 1\}$; we verify this claim when discussing the principal's problem. For notational convenience, we write $\tau = \tau^I$, if no confusion is likely to arise.

C.1 Step I - Proof of Lemma 2

Proof. Utilizing the result, i.e. (S.2), from supplementary appendix S.2 (integration by parts) we obtain an integral expression for w_t. Because the measures $\mathbb{P}^P$ and $\mathbb{P}^I$ agree in equilibrium/optimum, it follows that for $t < \tau$:

$$w_t \equiv \mathbb{E}_t^P\Big[\int_t^\tau e^{-\gamma(s-t)}dc_s^I - \int_t^\tau e^{-\gamma(s-t)}\big[h(b_s^P) + (\gamma + \Lambda - r)\mathcal{D}_{s^-}\big]ds - \int_t^\tau e^{-\gamma(s-t)}dT_s\Big]$$

and $\{b^I\} = \{b^P\}$ and $\{a^I\} = \{a^P\}$. Next, define for $t < \tau^I$

$$A_t^P \equiv \mathbb{E}_t^P\bigg[\int_0^\tau e^{-\gamma s}dc_s^I - \int_0^{\tau^I} e^{-\gamma s}\big[h(b_s^P) + (\gamma + \Lambda - r)\mathcal{D}_{s-}\big]ds - \int_0^\tau e^{-\gamma s}dT_s\bigg]$$
$$= \bigg[\int_0^t e^{-\gamma s}dc_s^I - \int_0^t e^{-\gamma s}\big[h(b_s^P) + (\gamma + \Lambda - r)\mathcal{D}_{s-}\big]ds - \int_0^t e^{-\gamma s}dT_s\bigg] + e^{-\gamma t}w_t.$$

By construction $\{A^P\}$ is a $\mathbb{F}$-progressive martingale under $\mathbb{P}^P = \mathbb{P}^I$. By the martingale representation theorem, there are now $\mathbb{F}$-predictable processes $\{\alpha^I\}$ and $\{\beta^I\}$ such that

$$e^{\gamma t}dA_t^P = \beta_t^I(dX_t - a_t^P dt) - \alpha_t^I(dN_t - \Lambda dt),$$

from which (24) follows, provided that $\alpha_t^I = w_{t-}$, which is directly implied by $\tau^I \leq \inf\{t \geq 0 : N_t = 1\}$. □

C.2 Step II - Auxiliary Results

We first prove the following auxiliary Lemma

Lemma 6. *Let $\tau' \equiv \inf\{t \geq 0 : dN_t = 1\}$. Fix a process $\{\mathcal{D}_t\}_{t<\tau'}$ and assume $\alpha_t^I = w_{t-}$ for all $t \geq 0$, i.e. $\tau \leq \tau'$. Then, $\Pi_{DC}^M \in IC_{DC}^M$ implies*

$$\alpha_t^M \geq \alpha(\mathcal{D}_{t-}) \equiv \frac{1}{\theta r}\Big(\exp(\mathcal{D}_{t-}\theta r) - 1\Big) > 0 \text{ and } dc_s^M = 0 \text{ for } s > \tau.$$

Hence, both contracts Π_{DC}^I and Π_{DC}^M optimally end at the same time, in that $\tau = \tau^I = \tau^M$.

Proof. To start with, let us show the second part of the claim. First, suppose that there is a stopping time τ'' with $\mathbb{P}^I\big(\tau'' < \infty \wedge \tau'' > \tau\big) > 0$, such that $dc_{\tau''}^M < 0$. Because after time τ payments dc_s^I are no longer made and $w_s = 0$ for $s \geq \tau$, the intermediary does not gain from producing output, if possible at all, and therefore sets $\beta_s^M = 0$ for all $s \geq \tau$. Hence, the payment $dc_{\tau''}^M < 0$ is deterministic, conditional on Π_{DC}^I being terminated. But then specifying at time τ a payment $d\tilde{c}_{\tau''-\varepsilon}e^{-r\varepsilon}$ rather than $dc_{\tau''}$ for arbitrary $\tau'' - \tau \geq \varepsilon > 0$ yields the same payoff V_τ for the manager, but a strictly higher payoff W_τ for the intermediary, in that $dc_{\tau''}^M < 0$ cannot be optimal. Repeating this reasoning implies that in optimum there cannot be any payment $dc_s^M < 0$ for $s > \tau$, so $dc_s^M \geq 0$

Second, suppose there exists a stopping time τ'' with $\mathbb{P}^I\big(\tau'' < \infty \wedge \tau'' > \tau\big) > 0$, such that $dc_{\tau''}^M > 0$. However, by definition $dc_s^I = 0$ for all $s \geq \tau$. But from there it follows that

$$W_\tau' \leq w_\tau - \mathbb{E}_\tau^I e^{-\gamma(\tau''-\tau)}dc_{\tau''} = -\mathbb{E}_\tau^I e^{-\gamma(\tau''-\tau)}dc_{\tau''} < 0,$$

which violates limited liability. Hence, $dc_s^M \leq 0$

By the previous arguments, it must be $\mathcal{D}_\tau = 0$, as $\mathcal{D}_\tau > 0$ induces a payment $dc_s^I > 0$ for $s > \tau$, which violates limited liability, and $\mathcal{D}_\tau < 0$ induces a payment $dc_s^I < 0$ for $s > \tau$, which is sub-optimal for the intermediary.

Next, the process $\{\mathcal{D}\}$ associated with strategy $\{\alpha(\mathcal{D}_{t-})\}$ evolves according to

$$d\mathcal{D}_t = d\mathcal{D}_{t^-} - \mathcal{D}_{t^-}dN_t = d\mathcal{D}_{t^-} - \mathcal{D}_{t^-}\mathbf{1}_{t=\tau'},$$

where we denote the continuous component of the stochastic integral in differential form by $d\mathcal{D}_{t^-}$. Consider any other strategy $\{\hat{\alpha}\}$. If it were $\hat{\alpha}_t < \alpha(\mathcal{D}_{t^-})$, then there is a process $\{\varepsilon\}$ with $\varepsilon_t > 0$ such that the induced process $\{\mathcal{D}''\}$ follows

$$d\mathcal{D}'_t = d\mathcal{D}_{t^-} - \big(\mathcal{D}_{t^-} - \varepsilon_t\big)dN_t = d\mathcal{D}_{t^-} - \big(\mathcal{D}_{t^-} - \varepsilon_t\big)\mathbf{1}_{t=\tau'}.$$

Hence, it occurs with positive probability that for the continuation value $\{W'\}$ under the alternative

$$W'_{\tau'} = w'_{\tau'} - \mathcal{D}'_{\tau'} = w_{\tau'} - \mathcal{D}_{\tau'} - \varepsilon_{\tau'} < 0,$$

which violates limited liability. Therefore, the contract Π^I_{DC} is not credible as it induces a deviation motive from the recommended savings such that $\Pi^I_{DC} \notin IC^I_{DC}$. It follows that $\alpha^M_t \geq \alpha(\mathcal{D}_{t^-})$, which concludes the proof. □

To the end of this part, we would like to emphasize that even though the previous result implies $dc^M_s = \mathcal{D}_\tau = 0$ for $s > \tau$ and $\alpha^M_t \geq \alpha(\mathcal{D}_{t^-})$, it does not imply yet that $\alpha^M_t = \alpha(\mathcal{D}_{t^-})$. This is because we did not rule out potential payments $dc^M_\tau < 0$ yet (while we have shown $dc^M_\tau \leq 0$).

C.3 Step III - Proof of Proposition 2 and Lemma 3

Proof. In the following, let the intermediary's strategy from the optimal contract $\Pi^M_{DC} \in IC^M_{DC}$ given Π^I_{DC} be represented by

$$\mathbb{S}^* = \big(\{b^P\}, \{a^P\}, \{\mathcal{D}\}, \{\alpha^M\}\big),$$

where

$$\mathcal{D}_t = \min\{w_t, \mathcal{D}^*\}, \alpha^M_t = \alpha(\mathcal{D}_{t^-}), b^P_t = b_H,$$
$$\text{and } a^P_t = \frac{\beta^I_t b^P_t}{\delta + \theta r \sigma^2 \delta^2} \equiv \bar{a}\beta^I_t.$$

Additionally, consider an alternative strategy

$$\mathbb{S} = \big(\{b^I\}, \{a^I\}, \{\hat{\mathcal{D}}\}, \{\hat{\alpha}^M\}\big)$$

under any incentive-compatible contract $\Pi^M \in IC^M_{DC}$ given Π^I_{DC}. We verify the optimality of $\Pi^M_{DC} \in IC^M_{DC}$ and derive conditions for $\Pi^I_{DC} \in IC^I_{DC}$. In particular, we show that the strategy $\mathbb{S}^*$ is indeed optimal if and only if $\Pi^I_{DC} \in IC^I_{DC}$.

Note that $dZ^I_t = \big(dX_t - a^I_t dt\big)/\sigma$ is the increment of a standard Brownian Motion under the measure $\mathbb{P}^I = \mathbb{P}^{a^I}$. The profit from following the alternative strategy up to time t and then switching to the proposed strategy is represented by the auxiliary gain process

$$G^I_t \equiv G^I_t(\mathbb{S}) = \hat{\mathcal{D}}_{0^-} + \int_0^t e^{-\gamma s} dc^I_s - \int_0^t e^{-\gamma s}\big[h(b^I_s) - r\hat{\mathcal{D}}_{s^-}\big]ds$$
$$- \int_0^t e^{-\gamma s} d\hat{\mathcal{D}}_s - \int_0^t e^{-\gamma s} d\hat{T}_s + e^{-\gamma t}(w_t - \hat{\mathcal{D}}_{t^-}),$$

where $G^I_{0^-} = w_{0^-} = w_0$. By Itô's Lemma:

$$
\begin{aligned}
e^{\gamma t} dG^I_t = & \Bigg[h(b^P_t) - h(b^I_t) + g(a^P_t | b^P_t) - g(a^I_t | b^I_t) + \frac{\gamma r \delta \sigma^2}{2} \Big(\frac{(a^P_t)^2}{b_t} - \frac{(a^I_t)^2}{b^I_t} \Big) \\
& + \Lambda(\alpha^M_t - \hat{\alpha}^M_t) + (\gamma + \Lambda - r)\big(\hat{\mathcal{D}}_{t^-} - \mathcal{D}_{t^-}\big) \\
& + \Lambda \big[\ln(1 + \theta r \hat{\alpha}^M_t)/(\theta r) - \hat{\mathcal{D}}_{t^-} \big] + \beta^I_t \big(a^I_t - a^P_t \big) \Bigg] dt \\
& + \beta^I_t \sigma dZ^I_t + \Big[\ln(1 + \theta r \hat{\alpha}^M_t)/(\theta r) - \alpha^I_t \Big] \big(dN_t - \Lambda dt \big) \\
& \equiv \mu^G_t(\mathbb{S}) dt + \beta^I_t dZ^I_t + \Big[\ln(1 + \theta r \hat{\alpha}^M_t)/(\theta r) - \alpha^I_t \Big] \big(dN_t - \Lambda dt \big)
\end{aligned}
$$

Next, note that our regularity conditions, that $\{\beta^I\}, \{\alpha^I\}, \{\hat{\alpha}^M\}$ are bounded, ensure that

$$
\mathbb{E}^P \bigg(\int_0^t e^{-\gamma s} \beta^I_s dZ_s \bigg) = \mathbb{E}^P \bigg(\int_0^t e^{-\gamma s} \Big[\ln(1 + \theta r \hat{\alpha}^M_t)/(\theta r) - \alpha^I_s \Big] (dN_s - \Lambda ds) \bigg) = 0
$$

It is then clear that by choosing $a^I_t = a^P_t$, $b^I_t = b^P_t$, $\hat{\mathcal{D}}_t = \mathcal{D}_t$ and $\hat{\alpha}^M_t = \alpha^M_t = \alpha(\mathcal{D}_{t^-})$ for all $t \geq 0$, the intermediary achieves that $\mu^G_t(\cdot) \equiv 0$, in which case $\{G^I\}$ follows a martingale under $\mathbb{P}^I$. Optional sampling implies that the stopped process $\{G^I_{t \wedge \tau}\}$ also follows a martingale. Whence,

$$
\max_{\mathbb{S}} \mathbb{E}^I G^I_{t \wedge \tau}(\mathbb{S}) \geq \mathbb{E}^I G^I_{t \wedge \tau}(\mathbb{S}^*) = w_{0^-}.
$$

Next, observe that the mapping

$$
\max_{\mathbb{S}} \mathbb{E}^I G_{t \wedge \tau} : [0, \infty) \to \mathbb{R} \text{ with } t \mapsto \max_{\mathbb{S}} \mathbb{E}^I G_{t \wedge \tau}
$$

is monotonically increasing in t and that our regularity conditions ensure that $\{G^I\}$ is bounded in expectation for all strategies $\mathbb{S}$ and therefore also the stopped process. Consequently, we can take limits to obtain that the intermediary's actual payoff w'_{0^-} is given by:

$$
w'_{0^-} \equiv \max_{\mathbb{S}} \mathbb{E}^I G^I_\tau(\mathbb{S}) \geq \lim_{t \to \infty} \max_{\mathbb{S}} \mathbb{E}^I G^I_{t \wedge \tau}(\mathbb{S}) \geq \mathbb{E}^I G^I_\tau(\mathbb{S}^*) = w_{0^-}.
$$

The maximal value $\mu^G_t(\cdot)$ can take, is obtained by solving

$$
\begin{aligned}
& \max_{\mathbb{S}_t} \mu^G_t(\mathbb{S}_t) \text{ with } \mathbb{S}_t = \big(a^I_t, b^I_t, \hat{\alpha}^M_t, \hat{\mathcal{D}}_t \big) \\
& \text{s.t. } \hat{\alpha}^M_t \geq \alpha(\hat{\mathcal{D}}_{t^-}); \mathcal{D}_t \leq w_t,
\end{aligned}
$$

where the first constraint follows from Step II of the proof and the second constraint arises due to limited liability.[23]

First, due to

$$
\frac{\partial \mu^G_t(\mathbb{S}_t)}{\partial \hat{\alpha}^M_t} = \Lambda \Big(-1 + \frac{1}{1 + \theta r \hat{\alpha}^M_t} \Big) < 0,
$$

[23] Adopting the previously used notation, we can write $\mathbb{S} = \{\mathbb{S}_t\}$.

the drift term is maximized $\mu_t^G(\mathbb{S}_t)$ for $\hat{\alpha}_t^M = \alpha(\hat{\mathcal{D}}_{t^-})$.

After plugging the optimal $\hat{\alpha}_t^M$ into the expression for $\mu_t^G(\mathbb{S}_t)$, one can verify that $\hat{\mathcal{D}}_t = \mathcal{D}_t$ solves the First Order Condition $\frac{\partial \mu_t^G(\mathbb{S}_t)}{\partial \hat{\mathcal{D}}_{t^-}} = 0$, so that $\hat{\mathcal{D}}_{t^-} = \mathcal{D}_{t^-}$ maximizes $\mu_t^G(\mathbb{S}_t)$, which proves Proposition 2. It can be seen that the objective is concave in all control variables, so that the First-Order Conditions are indeed a sufficient optimality criterion.

Next, going through the maximization, we see that

$$a_t^I = \frac{\beta_t^I b_t^I}{\delta + \theta r \sigma^2 \delta^2}$$

maximizes the drift $\mu_t^G(\mathbb{S}_t)$, given β_t^I, b_t^I. Furthermore, straightforward but tedious calculations yield that $b_t^I = b_t^P = b_H$ maximizes the drift if and only if $\beta_t^I \geq \bar{\beta}$, where

$$\bar{\beta} \equiv \left(\frac{\lambda(b_H - b_L)}{\bar{a} - \bar{a}_L - \frac{\delta}{2}\left(\frac{\bar{a}^2}{b_H} - \frac{\bar{a}_L^2}{b_L}\right) - \frac{\theta r \sigma^2 \delta^2}{2}\left(\frac{\bar{a}^2}{b_H^2} - \frac{\bar{a}_L^2}{b_L^2}\right)} \right)^{\frac{1}{2}} = \left(\frac{\lambda(b_H - b_L)}{\bar{a} - \bar{a}_L - \frac{\delta}{2}\left(\frac{\bar{a}^2}{b_H} - \frac{\bar{a}_L^2}{b_L}\right)} \right)^{\frac{1}{2}}; \bar{a}_L = \frac{b_L}{\delta + \theta r \sigma^2 \delta^2}.$$

Consequently, if and only if $\beta_t^I \geq \bar{\beta}$, each strategy $\mathbb{S}$ will make the process $\{G^I(\mathbb{S})\}$ a supermartingale under $\mathbb{P}^I$ and therefore also the stopped process $\{G_{t\wedge\tau}^I(\mathbb{S})\}$, which implies

$$w_{0^-} = \mathbb{E}^I \mathbb{G}_0^I(\mathbb{S}) \geq \mathbb{E}^I \mathbb{G}_{t\wedge\tau}^I(\mathbb{S})$$

for any $t \geq 0$. Because our regularity conditions ensure that $\mathbb{E}|\mathbb{G}_\tau^I| < \infty$, we can take limits on both sides, such that by the optional sampling theorem

$$w_{0^-} = \mathbb{E}^I G_0^I(\mathbb{S}) \geq \lim_{t\to\infty} \mathbb{E}^I G_{t\wedge\tau}^I(\mathbb{S}) = \mathbb{E}^I \lim_{t\to\infty} G_{t\wedge\tau}^I(\mathbb{S}) = \mathbb{E}^I G_\tau^I(\mathbb{S})$$

and in particular

$$w_{0^-} = \mathbb{E} G_0^I \geq \max_{\mathbb{S}} \mathbb{E} G_\tau^I(\mathbb{S}) = w'_{0^-}.$$

Hence, if and only if $\beta_t^I \geq \bar{\beta}$ for all $t \geq 0$, it follows that $w_{0^-} = w'_{0^-}$ and $\mathbb{S} = \mathbb{S}^*$ is the intermediary's optimal choice – i.e., the contract Π_{DC}^M is indeed optimal and $\Pi_{DC}^I \in IC_{DC}^I$, which concludes the proof. $\square$

Appendix D: Appendix D: The Principal's Problem (DC) - Proof of Proposition 4

We proceed in three steps. In step I, we guess the optimal contract, the associated principal's value function, and the implied policies under the optimal contract. In step II, we show that the value function is (strictly) concave. Finally, in step III, we verify that our guess in step I is indeed the optimal contract, in that the value function represents the maximal payoff that the principal can obtain.

D.1 Step I - Value Function

The principal's value function solves the following HJB equation when $dc_t^I = 0$:

$$(r+\Lambda)F(w_{t^-}) = \max_{\beta_t^I, \alpha_t^I} \Big\{ \Lambda F^T(w_{t^-} - \alpha_t^I) + a_t^I + F'(w_{t^-}) \Big[\big(\gamma w_{t^-} + \Lambda \alpha_t^I\big) + h(b_H) + g(a_t^I | b_H) + \Lambda \alpha(\mathcal{D}_{t^-}) - \mathcal{D}_{t^-}(\gamma + \Lambda - r) + \frac{1}{2}\theta r(\sigma \beta_t^M)^2 \Big] + \frac{1}{2} F''(w_{t^-}) \big(\beta_t^I \sigma\big)^2 \Big\}$$

subject to

$$\alpha_t^I \le w_{t^-}, \beta_t^I \ge \bar{\beta}, a_t^I = a_t^I(\beta_t^I), F(0) - R = F'(\overline{w}) - 1 = F''(\overline{w}) = 0.$$

In the following, we assume that the above HJB equation admits a unique, twice continuously differentiable solution $F(\cdot) \in C^2$. A formal existence proof is beyond the scope of the paper and therefore omitted.

Here, $F^T(\cdot)$ denotes the principal's value after a Poisson shock. It is evident that $F^T(w_{t^-}) = R - w_{t^-}$. Because $F'(w_{t^-}) \ge -1$, it follows that

$$\frac{\partial F(w_{t^-})}{\partial \alpha_t^I} \propto (F^T)'(w_{t^-} - \alpha_t^I) + F'(w_{t^-}) = -1 + F'(w_{t^-}) \ge 0$$

and $\alpha_t^I = w_{t^-}$ is indeed optimal.

Further, one can verify that optimal β_t^I is given by

$$\beta_t^I = \max\{\bar{\beta}, \beta^*(w_{t^-})\},$$

where

$$\beta^*(w_{t^-}) = \frac{\bar{a}}{-F'(w_{t^-})\Big(\frac{\delta \bar{a}^2}{b_H} + r\theta\sigma^2 \frac{\delta^2 \bar{a}^2}{b_H^2}\Big) - F''(w_{t^-})\sigma^2} = \frac{1}{-F'(w_{t^-}) - \frac{F''(w_{t^-})}{\bar{a}}\sigma^2},$$

assuming that $F(\cdot)$ is concave.

D.2 Concavity of the Value Function

Proof. To avoid clutter, we omit in this section time subscripts, if no confusion is likely to arise. Using the envelope theorem and differentiating both sides of (18), we obtain

$$F'''(w) = \frac{2}{(\beta^I \sigma)^2} \times \Big\{ F'(w) \Big[r - \gamma - \Lambda \frac{\partial \alpha(\mathcal{D})}{\partial w} + (\gamma - r + \Lambda) \frac{\partial \mathcal{D}}{\partial w} \Big] \tag{D.1}$$

$$- F''(w) \Big[(\gamma + \Lambda) w + h(b_H) + g(a^I | b_H) + \Lambda \alpha(\mathcal{D}) - \mathcal{D}(\gamma + \Lambda - r) + \frac{1}{2}\theta r(\sigma \beta^M)^2 \Big] \Big\}. \tag{D.2}$$

Next, note that at the boundary $F'(\overline{w}) - 1 = F''(\overline{w}) = 0$ implies that

$$F'''(\overline{w}) \propto \gamma - r + \Lambda \frac{\partial \alpha(\mathcal{D})}{\partial w} - (\gamma - r + \Lambda) \frac{\partial \mathcal{D}}{\partial w}.$$

If $\overline{w} > \mathcal{D}^*$, it readily follows that $\mathcal{D}$ is constant at $\overline{w}$ and therefore $F'''(\overline{w}) \propto \gamma - r > 0$.

If $\overline{w} \leq \mathcal{D}^*$, then $\mathcal{D} = w$ and $\alpha(\mathcal{D}) = \frac{1}{\theta r}(\exp(w\theta r) - 1)$. Thus,

$$F'''(\overline{w}) \propto \big(\exp(\overline{w}\theta r) - 1\big)\Lambda > 0,$$

as $\overline{w} > 0$. Continuity and $F'''(\overline{w}) > 0$ imply now that, there exists $\varepsilon > 0$ such that $F''(w) < 0$ for all $w \in (\overline{w} - \varepsilon, \overline{w})$.

Next, suppose $w^0 \in [0, \overline{w})$ with $F''(w^0) \geq 0$ and define $w' = \sup\{w \in [0, \overline{w}) : F''(w) \geq 0\}$. By continuity of $F''(\cdot)$, it follows that $F''(w') = 0$ and by the previous step $w' < \overline{w}$.

We further show that $F'(w') < 0$. Suppose to the contrary $F'(w') \geq 0$. Then, by the HJB equation

$$\begin{aligned}(r + \Lambda)F(w') \geq \max_{\beta^I \geq \bar{\beta}} \Bigg\{ & a^I(\beta^I) + F'(w')\Big[(\gamma + \Lambda)w' + h(b_H) \\ & + g(a^I|b_H) + \Lambda\alpha(\mathcal{D}) - \mathcal{D}(\gamma + \Lambda - r) + \frac{1}{2}\theta r(\sigma\beta^M)^2\Big]\Bigg\} \\ \geq & F^{FB},\end{aligned}$$

because $F'(w') \geq 0$ and the drift of $\{w\}$ is evidently positive due to $w \geq \mathcal{D}$. This yields the desired contradiction. Thus, $F'(w') < 0$.

But $F'(w') < 0$ also implies that $F'''(w') > 0$, and hence there is a value of $w'' > w'$ with $F''(w'') > 0$, which contradicts the definition of w'. This completes the argument. □

D.3 Verification

Proof. Let $\Pi^I_{DC} \in IC^I_{DC}$ the optimal contract and consider any other incentive-compatible contract $\Pi^I \in IC^I_{DC}$.

We show now that the value function $F(\cdot)$ solving (18) represents the principal's optimal profit, in that the contract Π^I_{DC} outlined in the Proposition is indeed optimal.

Let $\Pi^I_{DC} \in IC^I_{DC}$ the optimal contract and consider any other incentive-compatible contract $\Pi^I \in IC^I_{DC}$. It boils down to show that

$$F_0(\Pi^I) \leq F_0(\Pi^I_{DC}) = F(w_{0-})$$

with equality if and only if $\Pi^I = \Pi^I_{DC}$.

Under any incentive-compatible contract $\Pi^I = (\{c^I\}, \{b\}, \tau^I)$ the process $\{w\}$ solves (16) for some predictable process $\{\beta^I\}$. Define for $t < \tau^I$ the auxiliary gain process

$$G^P_t = G^P_t(\Pi^I) = \int_0^t e^{-rs}\big(dX_s - dc^I_s\big) + e^{-rt}F(w_t).$$

By Itô's Lemma:

$$
\begin{aligned}
e^{rt}dG_t^P = \Bigg\{ & -(r+\Lambda)F(w_{t^-}) + \Lambda F^T(w_{t^-} - \alpha_t^I) + a_t^I(\beta_t^I) + F'(w_{t^-})\Big[\gamma w_{t^-} + \Lambda\alpha_t^I + h(b_H) \\
& + g(a_t^I|b_H) + \Lambda\alpha(\mathcal{D}_{t^-}) - \mathcal{D}_{t^-}(\gamma+\Lambda-r) + \frac{1}{2}\theta r(\sigma\beta_t^M)^2\Big] + \frac{1}{2}F''(w_{t^-})\big(\beta_t^I\sigma\big)^2\Bigg\}dt \\
& + \big[F^T(w_{s^-} - \alpha_s^I) - F(w_{t^-})\big](dN_s - \Lambda ds) \\
& - \big(F'(w_{t^-}) + 1\big)dc_t^I + \sigma\big(1 + \beta_t^I F'(w_{t^-})\big)dZ_t \\
& \equiv \mu_t^G(\Pi^I)dt - \big(F'(w_{t^-}) + 1\big)dc_t^I \\
& + \sigma\big(1 + \beta_t^I F'(w_{t^-})\big)dZ_t + \big[F^T(w_{s^-} - \alpha_s^I) - F(w_{t^-})\big](dN_s - \Lambda ds).
\end{aligned}
$$

By the HJB equation (18), the drift term is zero under the optimal contract Π_{DC}^I, i.e. $\mu_t^G(\Pi_{DC}^I) = 0$, while each other strategy/contract will make this term (weakly) negative, i.e $\mu_t^G(\Pi^I) \leq 0$. Because the process $\{c^I\}$ is almost surely increasing and the fact that $F'(w_{t^-}) \geq -1$, the term $\big(F'(w_{t^-}) - 1\big)dc_t^I$ is (weakly) negative under any contract Π^I and zero under the optimal contract Π_{DC}^I. Next, our regularity conditions ensure that $\{\alpha^I\}$ and $\{\beta^I\}$ are almost surely bounded. Therefore,

$$
\begin{aligned}
& \mathbb{E}^P\left(\int_0^t e^{-rs}(1 + \beta_s^I F'(w_{s^-})dZ_s\right) \\
& = \mathbb{E}^P\left(\int_0^t e^{-rs}\big[F^T(W_{s^-} - \alpha_s^I) - F(w_{s^-})\big](dN_s - \Lambda ds)\right) = 0
\end{aligned}
$$

for all $t < \tau$. Therefore, $\{G^P(\Pi^I)\}$ follows a supermartingale, while $\{G^P(\Pi_{DC}^I)\}$ follows a martingale under the measure $\mathbb{P}^P$ and so do the stopped processes $\{G^P(\Pi^I)_{t\wedge\tau^I}\}$ and $\{G^P(\Pi_{DC}^I)_{t\wedge\tau^I}\}$. Hence,

$$
F(w_{0^-}) = G_0^P(\Pi^I) \geq \mathbb{E}^P G_{t\wedge\tau^I}^P(\Pi^I).
$$

It then follows for any t that

$$
\begin{aligned}
F_0(\Pi^I) &= \mathbb{E}^P\left(\int_0^{\tau^I} e^{-rs}\big(dX_s - dc_s^I\big)\right) = \mathbb{E}^P G_{\tau^I}^P(\Pi^I) \\
&= \mathbb{E}^P\left(G_{t\wedge\tau^I}^P(\Pi^I) + \mathbf{1}_{t\leq\tau}\Big[\int_t^{\tau^I} e^{-rs}(dX_s - dc_s^I) + e^{-r\tau^I}R - e^{-rt}F(w_{t^-})\Big]\right) \\
&= \mathbb{E}^P G_{t\wedge\tau^I}^P(\Pi^I) + e^{-rt}\mathbb{E}_t^P\mathbf{1}_{t\leq\tau}\left(\int_t^{\tau^I} e^{-r(s-t)}(dX_s - dc_s^I) + e^{-r(\tau-t)}R + F(w_{t^-})\right) \\
&\leq F(w_{0^-}) + e^{-rt}\big(F^{FB} - R\big),
\end{aligned}
$$

where we used the supermartingale property and the fact that

$$
\mathbb{E}_t^P\left(\int_t^{\tau^I} e^{-r(s-t)}(dX_s - dc_s^I) + e^{-r(\tau-t)}R \leq F^{FB} - w_{t^-}\right.
$$

and

$$F(w_{t^-}) + w_{t^-} \geq R \Longleftrightarrow F^{FB} - F(w_{t^-}) - w_{t^-} \leq F^{FB} - R.$$

From the above arguments, we readily obtain $F_0(\Pi^I) \leq F(w_{0^-})$ for all contracts $\Pi^I \in IC^I_{DC}$. However, under the optimal contract Π^I_{DC} the principal's payoff $F_0(\Pi^I_{DC})$ achieves $F(w_{0^-})$, as the above weak inequality holds in equality when $t \to \infty$. This concludes the proof. □

Appendix E: The Intermediary's and Manager's Problems (DM)

E.1 The Manager's Problem - Proof of Lemma 3

Proof. The proof is more or less identical to the proof of its counterpart under delegated contracting, i.e. Lemma 1, albeit with slight notational adaption and is therefore omitted to conserve space. □

E.2 The Intermediary's Problem (DM) - Proof of Lemma 24

Proof. The martingale representation can be shown, replicating the arguments of the proof of Lemma 2, and is therefore not given here.

We now prove the claim regarding incentive compatibility. Let $b^P_t = b_H$ and $b^P \equiv \{b^P\}$ be the proposed monitoring process by the optimal contract Π^I_{DM}, taking the manager's contract Π^M_{DM} as given, and $b^I \equiv \{b^I\}$ be any other monitoring strategy. Further, let the associated managerial effort under the proposed strategy be $\{a^P\}$ and under the alternative $\{a^I\}$. We derive conditions for $\Pi^I_{DM} \in IC^I_{DM}$, i.e., for b^P representing the optimal strategy.

The profit from following the alternative strategy b^I up to time t and then switching to the proposed strategy b^P is given by the auxiliary gain process

$$G^I_t \equiv G^I_t(b^I) = \int_0^t e^{-\gamma s}\big(dc^I_s - h(b^I_s)\big)ds + e^{-\gamma t}W_t,$$

where $G^I_{0^-} = W_{0^-} = W_0$. By Itô's Lemma:

$$\begin{aligned} e^{\gamma t}dG^I_t &= \Big[h(b_H) - h(b^I_t) + \beta^I_t\big(a^I_t - a^P_t\big)\Big]dt + \beta^I_t \sigma dZ^I_t - \alpha^I_t\big(dN_t - \Lambda dt\big) \\ &\equiv \mu^G_t(b^I)dt + \beta^I_t dZ^I_t - \alpha^I_t\big(dN_t - \Lambda dt\big) \end{aligned}$$

Next, note that our regularity conditions, i.e., $\{\beta^I\}, \{\alpha^I\}$ are bounded, ensuring that

$$\mathbb{E}^P\bigg(\int_0^t e^{-\gamma s}\beta^I_s dZ_s\bigg) = \mathbb{E}^P\bigg(\int_0^t e^{-\gamma s}\alpha^I_s(dN_s - \Lambda ds)\bigg) = 0$$

It is then clear that by choosing $b^I_t = b^P_t$ for all $t \geq 0$, the intermediary achieves $\mu^G_t(\cdot) \equiv 0$, in which case $\{G^I\}$ follows a martingale under $\mathbb{P}^I$. Optional sampling implies that also the "stopped process" $\{G^I_{t\wedge\tau}\}$ follows a martingale. Whence,

$$\max_{b^I} \mathbb{E}^I G^I_{t\wedge\tau}(b^I) \geq \mathbb{E}^I G^I_{t\wedge\tau}(b^P) = W_{0^-}.$$

Next, observe that the mapping

$$\max_{b^I} \mathbb{E}^I G_{t\wedge\tau} : [0,\infty) \to \mathbb{R} \text{ with } t \mapsto \max_{b^I} \mathbb{E}^I G_{t\wedge\tau}$$

is monotonically increasing in t and that our regularity conditions ensure that $\{G^I\}$ is bounded in expectation for all strategies b^I and therefore also the stopped process. Consequently, we can take limits to obtain that the intermediary's actual payoff W'_{0^-} is given by:

$$W'_{0^-} \equiv \max_{b^I} \mathbb{E}^I G^I_\tau(b^I) \geq \lim_{t\to\infty} \max_{b^I} \mathbb{E}^I G^I_{t\wedge\tau}(b^I) \geq \mathbb{E}^I G^I_\tau(b^P) = w_{0^-}.$$

The maximal value $\mu^G_t(\cdot)$ can take, is obtained through solving

$$\max_{b^I_t\in\{b_L,b_H\}} \mu^G_t(b^I_t) \text{ with } a^I_t = \frac{\beta^M_t b^I_t}{\delta}, a^I_t = \frac{\beta^M_t b^P_t}{\delta}, \beta^M_t = \frac{\delta a^P_t}{b_H}.$$

As can be verified, $b^I_t = b^P_t = b_H$ maximizes $\mu^G_t(\cdot)$ if and only if $\beta^I_t \geq \frac{\lambda b^P_t}{a^P_t}$.

Consequently, if and only if $\beta^I_t \geq \frac{\lambda b^P_t}{a^P_t}$, each strategy b^I will make the process $\{G^I(b^I)\}$ a supermartingale under the measure $\mathbb{P}^I$ and therefore also the stopped process $\{G^I_{t\wedge\tau}(b^I)\}$, which implies

$$W_{0^-} = \mathbb{E}^I \mathbb{G}^I_0(b^I) \geq \mathbb{E}^I \mathbb{G}^I_{t\wedge\tau}(b^I)$$

for any $t \geq 0$. Because our regularity conditions ensure that $\mathbb{E}|\mathbb{G}^I_\tau| < \infty$, we can take limits on both sides, such that by the optional sampling theorem

$$w_{0^-} = \mathbb{E}^I G^I_0(b^I) \geq \lim_{t\to\infty} \mathbb{E}^I G^I_{t\wedge\tau}(b^I) = \mathbb{E}^I \lim_{t\to\infty} G^I_{t\wedge\tau}(b^I) = \mathbb{E}^I G^I_\tau(b^I)$$

and in particular

$$W_{0^-} = \mathbb{E}G^I_0 \geq \max_{b^I} \mathbb{E}G^I_\tau(b^I) = W'_{0^-}.$$

Hence, if and only if $\beta^I_t \geq \frac{\lambda b^P_t}{a^P_t}$ for all $t \geq 0$, it follows that $W_{0^-} = W'_{0^-}$ and $b^I = b^P$ is the intermediary's optimal choice, and $\Pi^I_{DM} \in IC^I_{DM}$, which concludes the proof. □

Appendix F: The Principal's Problem (DM) - Proof of Proposition 5

Proof. A formal existence proof of the solution is beyond the scope of the paper and concavity can be established, utilizing essentially the same arguments as in the proof of Proposition 4. Both proofs are therefore omitted.

Let $(\Pi^I_{DM}, \Pi^M_{DM}) \in IC^I_{DM} \times IC^M_{DM}$ be the optimal contracts, solving the principal's problem (19)-(21), and consider any other incentive-compatible contracts $(\Pi^I, \Pi^M) \in IC^I_{DM} \times IC^M_{DM}$.

We show now that the value function $F(\cdot) = f(\cdot) + \mathbb{CE}(v_0)$ with $f(\cdot)$ solving (26) represents the principal's optimal profit, in that the contracts (Π^I_{DM}, Π^M_{DM}) outlined in the Proposition are indeed optimal. For the sake of exposition, we normalize without loss of generality $\mathbb{CE}(v_0)$ to zero, so that $F(\cdot) = f(\cdot)$.

Consequently, we need to show that

$$F_0(\Pi^I, \Pi^M) = f_0(\Pi^I, \Pi^M) \leq F_0(\Pi^I_{DM}, \Pi^M_{DM}) = f_0(\Pi^I_{DM}, \Pi^M_{DM}) = f(W_{0^-})$$

with equality if and only if $(\Pi^I, \Pi^M) = (\Pi^I_{DM}, \Pi^M_{DM})$.

Under any incentive-compatible contract $\Pi^I = (\{c^I\}, \{b\}, \tau^I)$ the process $\{W\}$ solves (16) for some predictable process $\{\beta^I\}$. Further, let the contract Π^M be represented by the tuple $(\{c^M\}, \{a^P\}, \{S^M\}, \tau^M)$. Define for $t < \tau^I$ the auxiliary gain process

$$G_t^P = G_t^P(\Pi^I, \Pi^M) = \mathcal{D}_{0^-} + \int_0^t e^{-rs}\big(dX_s - dc_s^I - dc_s^M\big) + e^{-rt}(f(w_t) - \mathcal{D}_{t^-}),$$

where $\{\mathcal{D}\}$ is implied by the savings under Π^M and savings under Π^M_{DM} imply $\mathcal{D}_t = 0$. By Itô's Lemma:

$$\begin{aligned}
e^{rt} dG_t^P = \Bigg\{ &- (r+\Lambda) f(W_{t^-}) + \Lambda f^T(W_{t^-} - \alpha_t^I) + a_t^M + f'(W_{t^-})\big(\gamma W_{t^-} + \Lambda \alpha_t^I\big) + h(b_H)\big) \\
&- g(a_t^I | b_H) - \Lambda \alpha_t^M + \Lambda \ln(1 + \theta r \alpha_t^M)/(\theta r) - \frac{1}{2}\theta r (\sigma \beta_t^M)^2 + \frac{1}{2} F''(W_{t^-})\big(\beta_t^I \sigma\big)^2 \Bigg\} dt \\
&- \big(f'(W_{t^-}) + 1\big) dc_t^I + \sigma\big(1 + \beta_t^I f'(W_{t^-}) - \beta_t^M\big) dZ_t \\
&+ \big(f^T(W_{t^-} - \alpha_t^I) - f(W_{t^-}) + \ln(1 + \theta r \alpha_t^M)/(\theta r)\big)(dN_t - \Lambda dt) \\
&\equiv \mu_t^G(\Pi^I, \Pi^M) dt - \big(f'(w_{t^-}) + 1\big) dc_t^I \\
&+ \sigma\big(1 + \beta_t^I f'(w_{t^-}) - \beta_t^M\big) dZ_t \\
&+ \big[f^T(W_{t^-} - \alpha_t^I) - f(W_{t^-}) + \ln(1 + \theta r \alpha_t^M)/(\theta r)\big](dN_t - \Lambda dt).
\end{aligned}$$

Note that all terms involving $\mathcal{D}_t$ cancel out, which implies that the choice of the savings does not affect the principal's payoff and is therefore without loss of generality.

Further, $f^T(W_{t^-}) = R - W_{t^-}$ is the principal's value after Poisson termination and the optimal contract sets $\alpha_t^I = -W_{t^-}$ and $\alpha_t^M = 0$.

By the HJB equation (26), the drift term is zero under the optimal contracts Π^I_{DM}, Π^M_{DM}, i.e. $\mu_t^G(\Pi^I_{DM}, \Pi^M_{DM}) = 0$, while each other strategy/contract will make this term (weakly) negative, i.e $\mu_t^G(\Pi^I, \Pi^M) \leq 0$. Because the process $\{c^I\}$ is almost surely increasing and the fact that $f'(w_{t^-}) \geq -1$, the term $\big(f'(w_{t^-}) - 1\big) dc_t^I$ is (weakly) negative under any contract Π^I and zero due under the contract Π^I_{DM}.

Next, our regularity conditions ensure that $\{\alpha^M\}$, $\{\alpha^I\}$, $\{\beta^I\}$, $\{\beta^M\}$ are bounded and therefore

$$\begin{aligned}
&\mathbb{E}^P\bigg(\int_0^t e^{-rs}(1 + \beta_s^I f'(w_{s^-}) - \beta_s^M\big) dZ_s\bigg) \\
&= \mathbb{E}^P\bigg(\int_0^t e^{-rs}\big[f^T(W_{s^-} + \alpha_s^I) - f(W_{s^-}) + \ln(1 + \theta r \alpha_t^M)/(\theta r)\big](dN_s - \Lambda ds)\bigg) = 0
\end{aligned}$$

for all $t < \tau$. Therefore, $\{G^P(\Pi^I, \Pi^M)\}$ follows a supermartingale, while $\{G^P(\Pi^I_{DM}, \Pi^M_{DM})\}$ follows a martingale under the measure $\mathbb{P}^P$ and so do the stopped processes $\{G^P(\Pi^I)_{t \wedge \tau^I}\}$ and $\{G^P(\Pi^I_{DM}, \Pi^M_{DM})_{t \wedge \tau^I}\}$. Hence,

$$f(W_{0^-}) = G_{0^-}^P(\Pi^I, \Pi^M) \geq \mathbb{E}^P G^P_{t \wedge \tau^I}(\Pi^I, \Pi^M).$$

Then it follows for any t:

$$
\begin{aligned}
f_0(\Pi^I, \Pi^M) &= \mathbb{E}^P\bigg(\int_0^{\tau^I} e^{-rs}\big(dX_s - dc_s^I - dc_s^M\big)\bigg) = \mathbb{E}^P G^P_{\tau^I}(\Pi^I, \Pi^M) \\
&= \mathbb{E}^P\bigg(G^P_{t\wedge\tau^I}(\Pi^I, \Pi^M) + \mathbf{1}_{t\le\tau}\Big[\int_t^{\tau^I} e^{-rs}(dX_s - dc_s^I - dc_s^M) + e^{-r\tau^I}R - e^{-rt}f(W_{t^-})\Big]\bigg) \\
&= \mathbb{E}^P G^P_{t\wedge\tau^I}(\Pi^I, \Pi^M) \\
&\quad + e^{-rt}\mathbb{E}_t^P \mathbf{1}_{t\le\tau}\bigg(\int_t^{\tau^I} e^{-r(s-t)}(dX_s - dc_s^I - dc_s^M) + e^{-r(\tau-t)}R + f(W_{t^-})\bigg) \\
&\le f(W_{0^-}) + e^{-rt}\big(F^{FB} - R\big),
\end{aligned}
$$

where we used the supermartingale property and the fact that

$$
\mathbb{E}_t^P\bigg(\int_t^{\tau^I} e^{-r(s-t)}(dX_s - dc_s^I - dc_s^M) + e^{-r(\tau-t)}R \le F^{FB} - W_{t^-}
$$

and

$$
f(W_{t^-}) + W_{t^-} \ge R \Longleftrightarrow F^{FB} - f(W_{t^-}) - W_{t^-} \le F^{FB} - R.
$$

From the above arguments, we readily obtain $f_0(\Pi^I, \Pi^M) \le f(W_{0^-})$ for all contracts $(\Pi^I, \Pi^M) \in IC^I_{DM} \times IC^M_{DM}$. On the other hand, under the optimal contract (Π^I_{DM}, Π^M_{DM}) the principal's payoff $f_0(\Pi^I_{DM}, \Pi^M_{DM}))$ achieves $f(W_{0^-})$, as the above weak inequality holds in equality when $t \to \infty$. This concludes the proof. □

Appendix G: Further Results

G.1 Proof of Corollary 1

In this proof, we abbreviate the optimal senstivities by $\beta^K(\cdot) = \beta^K_{DC}(\cdot)$ for $K \in \{I, M\}$, if no confusion is likely to arise.

a) *Proof.* At the boundary, $F'(\overline{w}) - 1 = F''(\overline{w}) = 0$ and therefore

$$
a^I(\overline{w}) = \bar{a}\beta^I(\overline{w}) = \frac{\bar{a}}{\frac{\delta\bar{a}}{b_H} + r\theta\sigma^2\frac{\delta^2\bar{a}}{b_H^2}} = \bar{a} = a^{SB}
$$

and $\beta^I(\overline{w}) = 1$, provided that $\beta^*(\overline{w}) = 1 > \bar{\beta}$. □

b) *Proof.* First, we observe that due to $F''(\cdot) \le 0$ and $F'(w^*) = 0$, it follows by (D.2) that $F'''(w) > 0$ on $[w^*, \overline{w}]$. Hence, $w' \le w^*$, such that $F'''(w) > 0$ for all $w \ge w'''$. This implies that $F''(\cdot)$ is an increasing function to the right of w'''. Because $F''(\overline{w}) = 0$, $\beta^*(w)$ increases, whenever $F''(w) + F'''(w)\sigma^2/\bar{a} \ge 0$, and $\beta^I(w), \beta^M(w)$ are proportional to each other, there must be w' with $\overline{w} > w' \ge w'''$, so that $\beta^I(w), \beta^M(w)$ increase on $[w', \overline{w}]$. □

c) *Proof.* Note that $\beta^*(w)$ is increasing whenever $F''(w) + F'''(w)\sigma^2/\bar{a} \ge 0$. As $F''(\overline{w}) = 0 < F'''(\overline{w})$, there must be a value $w' \in [0, \overline{w}]$, such that $\beta^*(\cdot)$ increases on $[0, \overline{w})$ with $\beta^*(\overline{w}) = 1$. Provided that $\bar{\beta} < 1$, there must be also a value w'' with $\overline{w} > w'' \ge w'$, such that

$\beta^I(w) = \beta^*(w) > \bar{\beta}$ on $(w'', \overline{w}]$, from which it follows that both sensitivities increase strictly on $(w'', \overline{w}]$. Because $\beta^I(\overline{w}) = 1$ and $\beta^I(\cdot)$ increases on $(w'', \overline{w}]$, it follows that $\beta^I(w) \leq 1$ for $w \in (w'', \overline{w}]$, where the inequality is strict for $w \neq \overline{w}$. Thus, $a(w) = a_{DC}(w) < \bar{a} = a^{SB}$ for all $w \in (w'', \overline{w})$. □

G.2 Proof of Corollary 2

In this proof, we abbreviate $a_x(\cdot) = a(\cdot)$ and $\beta_x^K(\cdot) = \beta^K$ for $K \in \{I, M\}$ and $x \in \{DC, DM\}$, if no confusion is likely to arise.

a) *Proof.* From (27) and the concavity of $f(\cdot)$, it follows that

$$\begin{aligned}\frac{\partial (r+\Lambda) f(W)}{\partial a} &= 1 - \frac{a\delta}{b_H} - r\theta\sigma^2 \frac{\delta^2 a}{b_H^2} - f''(W) \frac{\left(\lambda b_H \sigma\right)^2}{a^3} \\ &= \frac{\partial (r+\Lambda) f^{SB}(a)}{\partial a} - f''(W) \frac{\left(\lambda b_H \sigma\right)^2}{a^3} \\ &\geq \frac{\partial (r+\Lambda) f^{SB}(a)}{\partial a},\end{aligned}$$

where the inequality is strict if and only if $W < \overline{W}$ as $f''(\overline{W}) = 0$. Hence, $a(W)$ solving $\frac{\partial f(W)}{\partial a} = 0$ satisfies

$$a(W) \geq a^{SB},$$

where the above inequality holds in equality if and only if $W = \overline{W}$. □

b) *Proof.* The IC-condition requires that $\beta_{DC}^I(w) \geq \bar{\beta}$ for all $w \in [0, \overline{w}]$. Further, by part a), we know that $a(W) \geq a^{SB} = \bar{a}$, from where it follows by means of the intermediary's IC-constraint under "DM," which binds in optimum, that

$$\beta_{DM}^I(W) = \frac{\lambda b_H}{a(W)} \leq \frac{\lambda b_H}{\bar{a}} \leq \bar{\beta} \leq \beta_{DC}^I(w)$$

for any pair $(W, w) \in [0, \overline{W}] \times [0, \overline{w}]$. The inequality is strict if $W < \overline{W}$. □

c) *Proof.* By Corollary 1), there is $w'' \in (0, \overline{w})$, such that $a_{DC}(w) < a^{SB} = \bar{a}$ for all $w \in (w'', \overline{w})$. Hence,

$$\beta_{DM}^M(W) = \frac{\delta a_{DM}(W)}{b_H} \geq \beta_{DC}^M(w) = \frac{\delta a_{DC}(w)}{b_H}$$

for all $(W, w) \in [0, \overline{W}] \times (w'', \overline{w}]$, where the inequality is strict if $W \neq \overline{W}$ or $w \neq \overline{w}$. □

d) *Proof.* The first part of the claim readily follows from the IC-conditions $\beta_t^M = \delta a_t / b_H$ and

$$\beta_t^M = \frac{\delta a_t}{b_H} \Longleftrightarrow a_t = \frac{\beta_t^M b_H}{\delta} \text{ and } \beta_t^I = \frac{\lambda b_H}{a_t} = \frac{\lambda \delta}{\beta_t^M},$$

that is β_t^I, β^M are indirectly proportional to each other, utilizing the fact the intermediary's IC-condition is active in equilibrium.

For the second part of the claim, we first use the envelope theorem and differentiate both sides of (26), to obtain

$$f'''(W) = \frac{2}{(\beta^I \sigma)^2} \times \Big\{ f'(W)(r-\gamma) - f''(W)\Big[(\gamma+\Lambda)W + h(b_H)\Big]\Big\}. \tag{G.1}$$

It is evident that due to concavity, $f'(W) \geq 0$ implies $f'''(W) > 0$. Hence, there exists $W' \leq W^*$ with $f'(W^*) = 0$, such that $f'''(W) > 0$ for all $W \geq W'$.

By the implicit function theorem, we can differentiate both sides of the First-Order condition $\frac{\partial f(W)}{\partial a} = 0$, to obtain

$$\begin{aligned} 0 &= \frac{da(W)}{dW}\Big(-\frac{\delta}{b_H} - r\theta\sigma^2\frac{\delta^2}{b_H^2}\Big) - f'''(W)\frac{\big(\lambda b_H \sigma\big)^2}{a(W)^3} + 3f''(W)\frac{\big(\lambda b_H \sigma\big)^2}{a(W)^4}\frac{da(W)}{dW} \\ \Longrightarrow \frac{da(W)}{dW} &= -\frac{1}{-\frac{\delta}{b_H} - r\theta\sigma^2\frac{\delta^2}{b_H^2} + 3f''(W)\frac{(\lambda b_H\sigma)^2}{a(W)^4}} \times f'''(W)\frac{\big(\lambda b_H \sigma\big)^2}{a(W)^3} < 0 \end{aligned}$$

provided that $f'''(W) > 0$ and, in particular, for all $W \geq W'$. Because β_t^M is directly proportional to a_t, the second part of the claim follows.

For the third part, it suffices to observe that (G.1) implies $\lim_{\gamma \downarrow r} f'''(W) > 0$ for any W and hence, for $\gamma - r$ sufficiently small, it holds that $f'''(W) \geq 0$ for all $W \geq 0$. □

G.3 Optimality of Full Monitoring

Here, we provide necessary and sufficient conditions for full monitoring, $b_t = b_H$, to be part of the optimal contract. We do so for all our different scenarios.

G.3.1 Delegated Contracting

It follows from the HJB equation (18) that $b_t = b_H$ is optimal for all $t \geq 0$ if and only if for all $w \in [0, \overline{w}]$:

$$\begin{aligned} (r+\Lambda)F(w) \geq \max_{\tilde{\beta}^I < \bar{\beta}} \Big\{ a + \Big[&(\gamma+\Lambda)w + g(a|b_L) + \Lambda\alpha \\ &-\mathcal{D}(\gamma+\Lambda-r) + \frac{1}{2}\theta r(\sigma\tilde{\beta}^M)^2\Big] F'(w) + \frac{1}{2}(\tilde{\beta}^I\sigma)^2 F''(w)\Big\} + \Lambda R \\ \text{s.t. } a = \tilde{\beta}^I a_L = \frac{\tilde{\beta}^I b_L}{\delta + \theta r \sigma^2 \delta^2} &\text{ and } \tilde{\beta}^M = \frac{\delta a}{b_L}. \end{aligned}$$

G.3.2 Delegated Contracting

It follows from the HJB equation (18) that $b_t = b_H$ is optimal for all $t \geq 0$ if and only if for all $W \in [0, \overline{W}]$:

$$(r+\Lambda)f(W) \geq \max_{a\geq 0}\left[a - g(a|b_L) + f'(W)\left((\gamma+\Lambda)W\right) \; - \frac{1}{2}\theta r(\beta^M\sigma)^2\right] + \Lambda R$$

$$\text{s.t. } \tilde{\beta}^M = \frac{\delta a}{b_L} \Longleftrightarrow a = \frac{\tilde{\beta}^M b_L}{\delta}.$$

G.3.3 First- and Second-Best

Under first-best $b = b_H$ is optimal if and only if

$$\max_{a\geq 0}\left(a - \frac{1}{2}\frac{\delta a^2}{b_H} - h(b_H)\right) \geq \max_{a\geq 0}\left(a - \frac{1}{2}\frac{\delta a^2}{b_L} - h(b_L)\right).$$

Under second-best $b = b_H$ is optimal if and only if

$$\max_{a\geq 0}\left(a - \frac{1}{2}\frac{\delta a^2}{b_H} - h(b_H) - \frac{1}{2}\theta r\left(\frac{\delta a}{b_H}\sigma\right)^2\right) \geq \max_{a\geq 0}\left(a - \frac{1}{2}\frac{\delta a^2}{b_L} - h(b_L) - \frac{1}{2}\theta r\left(\frac{\delta a}{b_L}\sigma\right)^2\right)$$

Supplementary Appendix

S.1 Solution for general discount rate ρ

In this section, we solve the problem in which the manager possesses a discount rate ρ not necessarily equal to r.

We first prove the following auxiliary Lemma:

Lemma 7. *Fix $\mathbb{F}$-predictable processes $\{a^M\}$ and $\{b^I\}$ and let the probability measure induced by $\{a^M\}$ be $\mathbb{P}^M$. Consider the problem*

$$V_t = V_t(\{\tilde{c}^M\}) = \max_{\{\tilde{c}^M_s\}_{s\geq t}} \mathbb{E}^M_t\left(\int_t^\infty e^{-\rho(s-t)}u(\tilde{c}^M_s, a^M_s)ds\right)$$
$$\text{subject to } d\Delta^M_s = r\Delta^M_s ds + \hat{c}^M_s ds - \tilde{c}^M_s ds, \Delta^M_t = 0 \text{ and } \lim_{s\to\infty}\Delta^M_s = 0 \text{ a.s.}$$

The solution $\{\tilde{c}^M\}$ then satisfies

$$rV_t = u(\tilde{c}^M_t, a^M_t)$$

for all $t \geq 0$ with a probability of one.

Proof. By standard arguments, the Euler equation reads

$$u_c(\tilde{c}^M_{s'}, a^M_{s'}) = \mathbb{E}^M_{s'} u_c(\tilde{c}^M_s, a^M_s)e^{-(\rho-r)(s-s')}$$

for any $t \leq s' < s$, which implies that marginal utility $u_c(\tilde{c}^M_s, a^M_s)e^{-(\rho-r)s}$ follows a martingale under $\mathbb{P}^M$. Due to CARA-preferences it follows that marginal utility $u_c(\cdot)$ is proportional to flow utility $u(\cdot)$, so that $u(\tilde{c}^M_s, a^M_s)e^{-(\rho-r)s}$ is also a martingale. Hence,

$$\begin{aligned}
V_t &= \mathbb{E}^M_t\left(\int_t^\infty e^{-\rho(s-t)}u(\tilde{c}^M_s, a^M_s)ds\right)\\
&= \mathbb{E}^M_t\left(\int_t^\infty e^{-\rho(s-t)}e^{(\rho-r)s}e^{-(\rho-r)s}u(\tilde{c}^M_s, a^M_s)ds\right)\\
&= \int_t^\infty e^{-\rho(s-t)}e^{(\rho-r)s}\mathbb{E}^M_t\left(e^{-(\rho-r)s}u(\tilde{c}^M_s, a^M_s)\right)ds\\
&= \int_t^\infty e^{-\rho(s-t)}e^{(\rho-r)s}e^{-(\rho-r)t}u(\tilde{c}^M_t, a^M_t)ds = \frac{u(\tilde{c}^M_t, a^M_t)}{\rho-(\rho-r)}\\
&\Longrightarrow rV_t = u(\tilde{c}^M_t, a^M_t),
\end{aligned}$$

which was to show.

□

S.1.1 Sketch of the Solution under DC

As is usual, we introduce the left limit and note that the statement of Lemma 7 is valid when replacing V_t by V_{t-} and we use this for $\hat{c}^M_t = \tilde{c}^M_t$.

Note that marginal utility is given by $u_c(\hat{c}^M_t, a^M_t) = -\theta rV_{t-}$ and marginal cost by $u_a(\hat{c}^M_t, a^M_t) = g_a(a^M_t|b^I_t)\theta rV_{t-}$. Inverting the relation from the previous Lemma yields

$$\hat{c}_t{}^M = -\frac{\ln(-\theta rV_{t-})}{\theta} + g(a^M_t|b^I_t) \Longleftrightarrow \hat{c}_t{}^M - g(a^M_t|b^I_t) = -\frac{\ln(-\theta rV_{t-})}{\theta} \equiv r\mathbb{CE}(V_{t-}).$$

Importantly, we still assume that the agent consumes at each point in time the interest rate of his certainty equivalent, in that we define $\mathbb{CE}(\cdot)$ such that this holds.

Further, $\{V\}$ solves the SDE

$$\begin{aligned} dV_t &= \rho V_{t^-}dt - u(\hat{c}_t^M, a_t^M)dt + (-\theta r V_{t^-})\beta_t^M\big(dX_t - a_t^I dt\big) - (-\theta r V_{t^-})\alpha_t^M\big(dN_t - \Lambda dt\big) \\ &= (\rho - r)V_{t^-}dt + (-\theta r V_{t^-})\beta_t^M\big(dX_t - a_t^I dt\big) - (-\theta r V_{t^-})\alpha_t^M\big(dN_t - \Lambda dt\big), \end{aligned}$$

where we used the incentive condition with respect to savings, namely $rV_{t^-} = u(\hat{c}_t^M, a_t^M)$. Because we scale the volatility β_t^M by marginal utility $-\theta r V_{t^-}$, the incentive condition remains unchanged compared to the baseline case $\rho = r$.

Next, Itô's Lemma implies

$$d\mathbb{CE}_t = \frac{\theta r}{2}(\beta_t^M\sigma)^2 dt + \frac{r-\rho}{\theta r}dt + \Lambda\alpha_t^M dt + \beta_t^M(dX_t - a_t^I dt) - \frac{\ln(1+\theta r\alpha_t^M)}{\theta r}dN_t.$$

From there, we obtain for $\mathbb{CE}_t = S_t^M + \mathcal{D}_t$ due to limited commitment of the intermediary that

$$\alpha_t^M = \alpha(\mathcal{D}_{t^-}) \equiv \frac{1}{\theta r}\left(\exp(\theta r\mathcal{D}_{t^-}) - 1\right) > 0.$$

The intermediary chooses $\mathcal{D}_t = \min\{w_t, \mathcal{D}^*\}$ with $\mathcal{D}^*$ solving

$$(\gamma - r + \Lambda) - \Lambda\alpha'(\mathcal{D}) = 0,$$

such that $\mathcal{D}^*$ and the process $\{\mathcal{D}\}$ remain "unaffected" (in a qualitative sense) from the discount rate change.

We can now write down the HJB-equation:

$$\begin{aligned} (r+\Lambda)F(w) = \max_{\beta^I \geq \bar{\beta}} \Bigg\{ a^I + F'(w)\Bigg[\big(\gamma+\Lambda\big)w + h(b_H) + g(a^I|b_H) + \Lambda\alpha(\mathcal{D}) \\ - \mathcal{D}(\gamma+\Lambda-r) + \frac{\theta r^2}{2r}(\beta^M\sigma)^2 + \frac{r-\rho}{\theta r}\Bigg] + \frac{1}{2}F''(w)\big(\beta^I\sigma\big)^2\Bigg\} + \Lambda R, \end{aligned}$$

subject to $F(0) - R = F'(\overline{w}) - 1 = F''(\overline{w}) = 0$ and $a^I = \beta^I\bar{a}$.

Compared to the case where $\rho = r$, one additional term shows up in the HJB-equation, which – importantly – does not qualitatively affect incentives and effort, i.e., it does not enter the FOC wrt. to β^I. The factor $\frac{r-\rho}{\theta r}$ stems from the manager's intertemporal consumption smoothing, in that consumption tends to be frontloaded (backloaded) when $\rho > r$ ($\rho < r$). It captures the effect of shifting consumption (without any risk) with the interest rate r, the discount rate ρ, and the elasticity of intertemporal substitution $\frac{1}{\theta r}$. Evidently, this effect is positive if and only if $r > \rho$ and greater if the agent is less risk averse, i.e., for smaller θ. The manager's certainty equivalent must increase by this factor to encourage the agent to comply to the prescribed savings path.

We emphasize here that the discount rate differential $\rho - r$ does not enter incentive compatibility or optimality conditions, so that our key findings do not depend on the assumption of equal discount rates.

S.1.2 Sketch of the Solution under DM

We omit all further derivations and immediately give the HJB-equation:

$$(r+\Lambda)f(W) = \max_{a\geq 0,\beta^I\geq\frac{\lambda b_H}{a}}\Big[a - g(a|b_H) - \frac{r-\rho}{\theta r} + f'(W)\left((\gamma+\Lambda)W + h(b_H)\right) + \frac{1}{2}f''(W)(\beta^I\sigma)^2 - \frac{\theta r}{2}(\beta_t^M\sigma)^2\Big] + \Lambda R,$$

subject to $f(0) - R = f'(\overline{w}) - 1 = f''(\overline{w}) = 0$ and $\beta^M = \delta a/b_H$. Again, compared to the case in which $\rho = r$, only one additional term shows up in the HJB equation, which does not affect the FOC with respect to a and therefore does not qualitatively affect incentives and effort.

Looking at the above HJB-equation, it becomes clear that the principal's profit increases in ρ, everything else remaining unchanged. The rationale for this result is as follows. When ρ is high, the agent cares more about the present and current consumption and thus has strong incentives to borrow. To maintain incentive compatibility with respect to savings, the principal therefore promises high consumption today and low consumption at any future time, which allows the principal to provide the promised value in a cheap way.

As before, it becomes evident that the discount rate differential does not qualitatively affect findings about incentives and/or effort.

S.2 Integration by parts

We provide the details for rewriting the continuation value $W_t = w_t - \mathcal{D}_t$ by means of integration by parts, so that

$$W_t = \mathbb{E}_t^I\left[\int_t^\tau e^{-\gamma(s-t)}dc_s^I - \int_t^\tau e^{-\gamma(s-t)}\left[h(b_s^P) + (\gamma+\Lambda-r)\mathcal{D}_{s-}\right]ds - \int_t^\tau e^{-\gamma(s-t)}dT_s\right] - \mathcal{D}_t. \quad \text{(S.2)}$$

Proof. Assume that $\alpha_t^M = \alpha(\mathcal{D}_{t-})$ and that the intermediary's contract is terminated upon $dN_t = 1$, that is $\tau \leq \inf\{t\geq 0 : dN_t = 1\}$.

In general, the process $\{\mathcal{D}\}$ can we written as the sum of an almost-surely continuous, predictable process $\{\mathcal{L}\}$ and a point process $\{\mathcal{P}\}$, in that

$$\mathcal{D}_t = \mathcal{L}_0 + \int_0^t d\mathcal{L}_s + \mathcal{P}_0 + \int_0^t d\mathcal{P}_s = \mathcal{L}_t + \mathcal{P}_t.$$

Or in our specific case, we may write

$$\mathcal{D}_t = \int_0^t d\mathcal{D}_{s^-} - \int_0^t D_{s^-}dN_s = \mathcal{D}_{t^-} - \mathcal{D}_{t^-}\mathbf{1}_{t=\tau} = \mathcal{D}_{t^-}\mathbf{1}_{t<\tau} \Longrightarrow \mathcal{D}_t\mathbf{1}_{t<\tau\wedge N_\tau=0} = \mathcal{D}_{t^-},$$

because a Poisson shock brings $\{\mathcal{D}\}$ down to zero and in particular $\mathcal{D}_\tau = 0$. Integration by parts for $t < \tau$ now yields

$$\begin{aligned}
\int_t^\tau e^{-\gamma(s-t)} d\mathcal{D}_s &= e^{-\gamma(\tau-t)}\mathcal{D}_{\tau^-} - \mathcal{D}_{t^-} + \gamma \int_t^\tau e^{-\gamma(s-t)}\mathcal{D}_{s^-} ds - \int_t^\tau e^{-\gamma(s-t)} D_{s^-} dN_s \\
&= e^{-\gamma(\tau-t)}\mathcal{D}_{\tau^-} - \mathcal{D}_t + \gamma \int_t^\tau e^{-\gamma(s-t)}\mathcal{D}_{s^-} ds \\
&- \int_t^{\tau^-} e^{-\gamma(s-t)}\mathcal{D}_{s^-} dN_s - e^{-\gamma(\tau-t)}\mathcal{D}_{\tau^-}\mathbf{1}_{N_\tau=1} \\
&= \gamma \int_t^\tau e^{-\gamma(s-t)}\mathcal{D}_{s^-} ds - \mathcal{D}_t - \int_t^{\tau^-} e^{-\gamma(s-t)}\mathcal{D}_{s^-} dN_s,
\end{aligned}$$

due to $\mathcal{D}_{\tau^-} = \mathcal{D}_\tau = 0$ in case $N_\tau = 0$. Note that

$$\int_t^{\tau^-} e^{-\gamma(s-t)}\mathcal{D}_{s^-} ds = \int_t^\tau e^{-\gamma(s-t)}\mathcal{D}_{s^-} ds = \int_t^\tau e^{-\gamma(s-t)}\mathcal{D}_s ds$$

because the processes $\{\mathcal{D}_{t^-}\}$ and $\{\mathcal{D}_t\}$ do not coincide on at most finitely many points, which is a Lebesgue null set and therefore does not change the value of the integral.[24]

The claim follows now from the so-called "smoothing formula." Define for appropriate Δ and a given pair $t < \tau$ the set $\Pi \equiv \{s + n\Delta : n \in \mathbb{N} \wedge t \leq s + n\Delta < \tau\}$.

$$\begin{aligned}
&\mathbb{E}_t^I\left(\int_t^{\tau^-} e^{-\gamma(s-t)}\mathcal{D}_{s^-} dN_s\right) = \mathbb{E}_t^I \sum_{t\leq s<\tau}\left[e^{-\gamma(s-t)}\mathcal{D}_{s^-}(N_s - N_{s^-})\right] \\
&= \lim_{\Delta\downarrow 0}\sum_{s\in\Pi}\mathbb{E}_t^I\mathbb{E}_{s-\Delta}^I\left[e^{-\gamma(s-t)}\mathcal{D}_{s-\Delta}(N_s - N_{s-\Delta})\right] \\
&= \lim_{\Delta\downarrow 0}\sum_{s\in\Pi}\mathbb{E}_t^I\left[e^{-\gamma(s-t)}\mathcal{D}_{s-\Delta}\mathbb{E}_{s-\Delta}^I(N_s - N_{s-\Delta})\right] \\
&= \mathbb{E}_t^I \lim_{\Delta\downarrow 0}\sum_{s\in\Pi}\left[e^{-\gamma(s-t)}\mathcal{D}_{s-\Delta}\Lambda\Delta\right] \\
&= \mathbb{E}_t^I\left(\int_s^{\tau^-} e^{-\gamma(s-t)}\Lambda\mathcal{D}_{s^-} ds\right),
\end{aligned}$$

where we used the towering property of conditional expectation, $\mathcal{F}_t \subseteq \mathcal{F}_{s-\Delta}$ and the fact that $\mathbb{E}_s^I(N_{s+\Delta} - N_s) = \Lambda\Delta$. We can readily combine our results to obtain

$$\mathbb{E}_t^I \int_t^\tau e^{-\gamma(s-t)} d\mathcal{D}_s = (\gamma + \Lambda)\mathbb{E}_t^I\left(\int_t^\tau e^{-\gamma(s-t)}\mathcal{D}_{s^-} ds\right) - \mathcal{D}_t,$$

which was to show.

Alternatively, define for $s \geq t$ the probability measure $\tilde{\mathbb{P}}^I$ via the Radon Nikodyn derivative

$$\frac{d\widetilde{\mathbb{P}}^I}{d\mathbb{P}^I}|_{\mathcal{F}_t} = e^{\Lambda(s-t)}$$

[24]Here, all integrals are to be understood in the Lebesgue-Stieltjes sense. For convenience we write $\int_t^s d\mathcal{X}_s$ for $\int_{[t,s]} d\mathcal{X}_s$ and $\int_t^{s^-} d\mathcal{X}_s$ for $\int_{[t,s)} d\mathcal{X}_s$ for any process integrable process $\{\mathcal{X}\}$.

and apply Girsanov's theorem to write

$$
\begin{aligned}
W_t &\equiv \mathbb{E}_t^I\bigg(\int_t^\tau e^{-\gamma(s-t)}dc_s^I - \int_t^\tau e^{-\gamma(s-t)}h(\hat{b}_s)ds - \int_t^\tau e^{-\gamma(s-t)}dc_s^M\bigg) \\
&= \widetilde{\mathbb{E}}_t^I\bigg(\int_t^\tau e^{-(\gamma+\Lambda)(s-t)}dc_s^I - \int_t^\tau e^{-(\gamma+\Lambda)(s-t)}h(\hat{b}_s)ds - \int_t^\tau e^{-(\gamma+\Lambda)(s-t)}dc_s^M\bigg),
\end{aligned}
$$

where the operator $\widetilde{\mathbb{E}}_t^I(\cdot)$ denotes the expectation conditional on $\mathcal{F}_t$, taken under the measure $\widetilde{\mathbb{P}}^I$. In particular, integration by parts and the change of measure collectively yield

$$
\begin{aligned}
&\mathbb{E}_t^I\bigg(\int_t^\tau e^{-\gamma(s-t)}d\mathcal{D}_s\bigg) = \widetilde{\mathbb{E}}_t^I\bigg(\int_t^\tau e^{-(\gamma+\Lambda)(s-t)}d\mathcal{D}_s\bigg) \\
&= \widetilde{\mathbb{E}}_t^I\bigg((\gamma+\Lambda)\int_t^\tau e^{-(\gamma+\Lambda)(s-t)}\mathcal{D}_{s^-}ds\bigg) - \mathcal{D}_t = \mathbb{E}_t^I\bigg((\gamma+\Lambda)\int_t^\tau e^{-\gamma(s-t)}\mathcal{D}_{s^-}ds\bigg) - \mathcal{D}_t,
\end{aligned}
$$

because of $\mathcal{D}_\tau = 0$. The result follows upon noticing that

$$
dc_t^M = dT_t + r\mathcal{D}_{t^-}dt - d\mathcal{D}_t.
$$

□

S.3 A Model with Risk-Averse Intermediary

In this section, we solve the model for the case in which both agents, the manager and the intermediary, have CARA preferences and are not protected by limited liability. For simplicity, we assume that all players discount at the market interest rate r and that the intermediary and the manager can privately save and borrow. Further, we denote by $\theta^I > 0$ the risk aversion coefficient of the intermediary and by $\theta^M > 0$ the risk-aversion coefficient of the manager. Let us focus on incentive-compatible contracts and for simplicity normalize the manager's and intermediary's outside option in monetary terms to zero.

By standard arguments, one can see that the problem is stationary and that the principal's value under delegated contracting is given by

$$
\begin{aligned}
\bar{f}^{DC} &= \max_{\beta^I} \frac{1}{r+\Lambda}\left[a - \frac{1}{2}\frac{\delta a^2}{b_H} - h(b_H) - \frac{1}{2}\theta^M r\left(\sigma\beta^M)^2\right) - \frac{1}{2}\theta^I r\left(\sigma\beta^I\right)^2\right] \\
&\quad \text{s.t. } \beta_{DM}^M = \beta^M = \frac{\delta a}{b_H}, a = \bar{a}\beta^I = a_{DC} \text{ and } \beta_{DC}^I = \beta^I \geq \bar{\beta}.
\end{aligned}
$$

The first order conditions with respect to β^I read

$$
\frac{\partial \bar{f}^{DC}}{\partial \beta^I} = 0 \Longleftrightarrow \bar{a} - \delta\bar{a}^2/b_H\beta^I - \theta^M r\sigma^2\delta^2\bar{a}^2/b_H^2\beta^I - \theta^I r\sigma^2\beta^I = 0,
$$

so that $\beta_{DC}^I = \beta^I = \max\{\bar{\beta}, \beta^*\}$ where

$$
\bar{\beta} \equiv \left(\frac{\lambda(b_H - b_L)}{\bar{a} - \bar{a}_L - \frac{\delta}{2}\left(\frac{\bar{a}^2}{b_H} - \frac{\bar{a}_L^2}{b_L}\right) - \frac{\theta r\sigma^2\delta^2}{2}\left(\frac{\bar{a}^2}{b_H^2} - \frac{\bar{a}_L^2}{b_L^2}\right)}\right)^{\frac{1}{2}} = \left(\frac{\lambda(b_H - b_L)}{\bar{a} - \bar{a}_L - \frac{\delta}{2}\left(\frac{\bar{a}^2}{b_H} - \frac{\bar{a}_L^2}{b_L}\right)}\right)^{\frac{1}{2}}; \bar{a}_L = \frac{b_L}{\delta + \theta r\sigma^2\delta^2}.
$$

and

$$\beta^* = \frac{\bar{a}}{\delta\bar{a}^2/b_H + \theta^M r\sigma^2\delta^2\bar{a}^2/b_H^2\beta^I + \theta^I r\sigma^2} = \frac{1}{1+\frac{\theta^I r\sigma^2}{\bar{a}}} < 1,$$

so that $a_{DC} = \beta^I \bar{a} < \bar{a}$.

Likewise, under delegated monitoring, the principal's value under delegated contracting is given by

$$\bar{f}^{DM} = \max_{\beta^I,\beta^M} \frac{1}{r+\Lambda}\left[a - \frac{1}{2}\frac{\delta a^2}{b_H} - h(b_H) - \frac{1}{2}\theta^M r\left(\sigma\beta^M)^2\right) - \frac{1}{2}\theta^I r\left(\sigma\beta^I)^2\right)\right]$$
$$\text{s.t. } \beta^M_{DM} = \beta^M = \frac{\delta a}{b_H} \text{ and } \beta^I_{DM} = \beta^I \geq \frac{\lambda b_H}{a}.$$

It is obvious that the IC-constraint for the intermediary is just binding, in that $\beta^I = \frac{\lambda b_H}{a}$, which yields the FOC wrt. to effort a:

$$\frac{\partial \bar{f}^{DM}}{\partial a} = \frac{\partial \bar{f}^{SB}}{\partial a} + \frac{1}{r+\Lambda}\theta^I r(\sigma\lambda b_H)^2/a^3 = 0.$$

Optimal effort $a_{DM} = a$ solves the above equation. Because $\bar{a}$ solves $\frac{\partial \bar{f}^{SB}}{\partial a} = 0$, it follows that $a_{DM} > \bar{a} > a_{DC}$ and therefore $\beta^M_{DM} > \beta^M_{DC}$. That is, the manager receives higher incentives under delegated monitoring.

Furthermore, under the assumption (on exogeneous model parameters) $\bar{\beta} \geq \frac{\lambda b_H}{\bar{a}}$, we have that

$$\beta^I_{DM} = \frac{\lambda b_H}{a_{DM}} < \frac{\lambda b_H}{\bar{a}} \leq \bar{\beta} \leq \beta^I_{DC}.$$

Hence, the intermediary receives higher incentives under delegated contracting.

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