# The Credit Spread Puzzle - Myth or Reality? *

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**Abstract**

A number of papers find that standard structural models predict spreads that are too low compared to actual spreads, giving rise to the so-called credit spread puzzle. In this paper we examine the existing literature documenting the credit spread puzzle and find that common approaches to testing structural models suffer from strong biases and low statistical power. We then test the Merton model in a bias-free approach using more than half a million transactions in the period 2002-2012. We find almost no evidence of the credit spread puzzle and in particular we find that the Merton model captures both the average level and time series variation of 10-year BBB-AAA spreads.

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# 1. Introduction

Merton (1974) developed structural models, one of the most widely employed frameworks of credit risk. A number of papers find that standard structural models predict spreads that are too low compared to actual spreads, giving rise to the so-called credit spread puzzle[1]. We find that common approaches to testing structural models suffer from strong biases and low statistical power. When we test the Merton model in a bias-free we find almost no evidence of any credit spread puzzle.

One approach to testing structural models is to use average firm variables such as asset volatility and leverage ratio as input in a structural model and compare model-implied spread with average actual spread over a period. This is always done for different rating categories and firm variables are most often averaged over a long historical time period. This introduces a convexity bias because the spread using average variables is typically lower than the average spread. The bias arises both in the cross-section and in the time series dimension. David (2008) points out this bias but there are no empirical tests of the size of the bias.

We empirically examine how important the convexity bias is in our sample period and find the bias to be strong. The bias increases as credit quality increases and as maturity shortens, precisely in the directions where the credit spread puzzle is found to be most severe. For example, the average model-implied 4-year A spread in our sample is 81 basis points while the model-implied 4-year spread of average A financial variables is 10 basis points.

[1] See for example Leland (2006), Cremers, Driessen, and Maenhout (2008), Zhang, Zhou, and Zhu (2009), Chen, Collin-Dufresne, and Goldstein (2009), Chen (2010), Huang and Huang (2012), and McQuade (2013).

This implies that the conclusions in Leland (2006) and McQuade (2013) that standard structural models underpredict spreads are misleading. A similar bias occurs when calculating default probabilities based on average firm variables. As we show this bias is also most severe at short maturities for high-quality issues, so the finding in Leland (2004) and McQuade (2013) that standard structural models underpredict default probabilities at short horizons are not accurate.

Another approach to structural model calibration is to imply out the asset volatility that makes model-implied default probabilities of a representative firm equal to historical default frequencies. This will typically result in a too high asset volatility because the default probabilities of a representative firm are lower than average default probabilities. But since the yield spread of the representative firm is lower than the average yield spread, it is not clear in which direction the model-implied yield spread will be biased relative to the average yield spread. The bias will depend on the parameters and specific structural model. No matter which structural model is calibrated the underlying assumption is that historical default frequencies proxy well for expected default probabilities. To test this assumption, we simulate the defaults of 240,000 identical BBB-rated firms exposed partially to systematic risk over a period of 30 years and repeat the exercise 5,000 times. We find that realized default frequencies are often far from expected default probabilities. In each of our simulations the average 10-year BBB yield spread is 130 basis points and a 95pct confidence interval for the model-implied spread calibrated to the 10-year historical default frequency goes from 23 basis points to 363 basis points. This shows that the statistical power of fitting to historical default frequencies is very low.

Having documented that using average firm variables leads to biased spread predictions and fitting to historical default frequencies to low statistic power, we test the Merton model in a bias-free approach. Specifically, we calculate a Merton spread for each transaction, compute an average, and compare with the average actual spread. To our knowledge Eom, Helwege, and Huang (2004) is the only other paper taking this bias-free approach. Their data set consists of 182 trader quotes in the period 1986-1997. Our data set consists of 534,660 transactions for the period 2002-2012. This allows us to examine in detail the ability of the Merton model to price bonds across maturity, for different ratings, and over a time period that includes both a boom period and a recession.

The most common version of the credit spread puzzle is that the spread between long-term BBB yields and AAA yields is too high to be explained by the Merton model and other standard structural models. We find for bonds with a maturity of more than three years the average difference between the actual and model-implied BBB-AAA spread to be only 4 basis points. Furthermore, the model-implied BBB-AAA spread tracks the time series variation of the actual spread well. We confirm this finding using dealer quotes from Merrill Lynch for the period 1997-2012.

The Merton model predicts very low spreads for high-quality bonds with short maturity and "most researchers view this kind of result as a failure of diffusion-type structural models" according to Huang and Huang (2012). To our knowledge there is no evidence on the size of short-term corporate bond spreads for bonds with a maturity below one year and we fill this gap in the literature. Over the sample period the median yield spread to the LIBOR rate for bonds with a maturity below one year is 3 basis points for AAA/AA and 7 basis points for A. In some periods in 2008-2009 short-term spreads were somewhat higher than zero, but by 2010 spreads were back at a level close to zero. Overall, the low short-maturity spreads in the Merton model is not a failure of the model but aligns well with actual spreads.

For high-quality bonds the Merton model predicts small spreads on long-term bonds. For our sample the median model-implied AAA/AA 10-year spread is only 2 basis points while the median actual spread is 32 basis points. This shows that the Merton model cannot quite match the magnitude of long-term AAA/AA spreads, but a difference of 30 basis points is smaller than previously found. Furthermore, we find the median model-implied 10-year A spread to be 73 basis points and the actual to be 85 basis points, showing that the underprediction of long-term spreads is restricted to bonds of the highest credit quality AAA/AA. So even though we find a puzzle for high-quality long-term spreads, the puzzle is smaller compared to previous findings in terms of spread size and how far down in credit quality the puzzle extents.

The organization of the article is as follows: Section 2 explains the data and how the Merton model is implemented. Section 3 examines common approaches to testing structural models. Although this section uses data described in Section 2, one can easily skip section 2 and read section 3 directly. Section 4 tests the Merton model using transaction data for the period 2002-2012. Section 5 concludes.

# 2. The Merton model: basics and implementation

## 2.1 Data

This section gives an overview the data while a detailed description is relegated to an Appendix.

Since July 1, 2002, members of the Financial Industry Regulatory Authority (FINRA) have been required to report their secondary over-the-counter corporate bond transactions through the TRACE database. Our data comes from two sources, WRDS and FINRA, and covers almost all U.S. Corporate bond transactions for the period July 1, 2002 - June 30, 2012. We limit the sample to senior unsecured fixed rate or zero coupon bonds and exclude bonds that are callable, convertible, putable, perpetual, foreign denominated, Yankee, have sinking fund provisions, or have covenants. Appendix A.1 describes details of the dataset and why it has more transactions than the typical TRACE dataset used in the literature.

To price a bond in the Merton model we need the issuing firm's asset volatility, leverage ratio, and payout ratio. *Leverage ratio* is calculated as the book value of debt divided by firm value (where firm value is calculated as book value of debt plus market value of equity). *Payout ratio* is calculated as the sum of interest payments to debt, dividend payments to equity, and net stock repurchases divided by firm value. *Asset volatility* is not directly observable and we extract it from equity volatility and leverage as we will explain in Section 2.2. Equity volatility is an annualized estimate based on the previous three year's of daily equity returns. All firm variables are obtained from CRSP and Compustat and details are given in Appendix A.2.

## 2.2 Calibration of the Merton model

The asset value in the Merton model follows a Geometric Brownian Motion under the risk-neutral measure,

$$\frac{dV_t}{V_t} = (r - \delta)dt + \sigma_A dW_t \tag{1}$$

where $r$ is the riskfree rate, $\delta$ is the payout rate, and $\sigma_A$ is the volatility of asset value. The firm has financed its assets by issuing equity and a zero-coupon bond with face value $F$ and maturity $T$. If asset value falls below face value at bond maturity, $V_T < F$, the firm cannot repay bond holders and the firm defaults. In the original Merton model bondholders receive 100% of the firm's value in default, but to be consistent with empirical recovery rates, we follow the literature on structural models and assume that bondholders recover only a fraction of firm value in default. According to Moody's (2011) the average recovery rate for senior unsecured bonds for the period 1987-2010 was 49.2% and we follow Eom, Helwege, and Huang (2004) and set the payoff to bondholders to $\min(V_T, 0.492F)$.[2] The bond price at time 0 is calculated as

$$P(0,T) = E^Q[e^{-rT}(1_{\{V_T \geq F\}} + \min(0.492, \frac{V_T}{F})1_{\{V_T < F\}})]$$

and the specific expression is given in Appendix B. The model implies a deadweight cost of bankruptcy; for a 10-year A-rated bond the expected

[2] The average recovery rate in the model is slightly below 49.2% of face value because the recovery is either 49.2% or less. However, the firm value will rarely fall below 49.2% and therefore the expected recovery will be below but close to 49.2%. With an asset risk premium of 4% and using the mean values of leverage, asset volatility, and payout ratio for a A-rated bond in Table 1, the expected recovery on a 10-year bond is 48.3% (see Appendix B.2 for formulas).

deadweight cost of bankruptcy is 32.6%[3]. This is broadly consistent with the empirical estimate of 31.0% in Davydenko, Strebulaev, and Zhao (2012), 36.5% in Alderson and Betker (1995), 45.5% in Gilson (1997), 45% in Glover (2012), and the use of 30% in Leland (2004).

A crucial parameter in any structural model is the volatility of assets and we follow the approach of Schaefer and Strebulaev (2008) in calculating asset volatility. Since firm value as a sum of debt and equity value, asset volatility is given as

$$\sigma_A^2 = (1-L)^2\sigma_E^2 + L^2\sigma_D^2 + 2L(1-L)\sigma_{ED}$$

where $\sigma_A$ is the volatility of assets, $\sigma_D$ volatility of debt, $\sigma_{ED}$ the covariance between the returns on debt and equity, and $L$ is leverage ratio. If we assume that debt volatility is zero, asset volatility reduces to $\sigma_A = (1-L)\sigma_E$. This is a lower bound on asset volatility. Schaefer and Strebulaev (2008) (SS) compute this lower bound along with an estimate of asset volatility based on volatility of equity, debt, and the correlation between debt and equity returns. They find that for investment grade companies with low leverage the lower bound and their estimate of asset volatility are very similar while for junk bonds with high leverage there is a substantial difference. We compute the lower bound of asset volatility, $(1-L)\sigma_E$, and multiply this lower bound with SS's estimate of the fraction between asset volatility and the lower bound. Specifically, we estimate $(1-L)\sigma_E$ and multiply this with 1 if $L < 0.25$, 1.05 if $0.25 < L \leq 0.35$, 1.10 if $0.35 < L \leq 0.45$, 1.20 if $0.45 < L \leq 0.55$, 1.40 if

[3]This assumes an asset risk premium of 4% and uses the mean values of leverage, asset volatility, and payout ratio for a A-rated bond in Table 1. The formula is given in Appendix B.2. An asset premium of 3% respectively 5% gives an expected deadweight cost of bankruptcy of 31.8% respectively 33.4%.

$0.55 < L \leq 0.75$, and 1.80 if $L > 0.75$.[4] [5] This estimation approach has the advantage of being transparent and easy to replicate.

Finally, the riskfree rate $r$ is chosen to be the swap rate at the same maturity as the bond. For maturities shorter than one year we use LIBOR rates.[6]

## 2.3 Summary statistics

Table 1 shows summary statistics for the firms in the investment grade sample while statistics for the speculative grade sample is in Table 2. We have joined AAA and AA into one rating group, because there are only four AAA-rated firms in our sample.

Focusing on investment grade bonds, we see in Table 1 that the median leverage ratio in the sample period is 0.17 for AAA/AA, 0.21 for A, and 0.50 for BBB. These figures are broadly consistent with the ratios used in Huang and Huang (2012) (hereafter called HH), namely 0.13/0.21 for AAA/AA, 0.32 for A, and 0.43 for BBB. Payout ratio is similar across rating with median payout ratio between 3.9% and 5.3%. Asset volatility is slightly increasing as

[4]These fractions are based on Table 7 in SS apart from 1.80 which we have set somewhat ad hoc. Results are insensitive to other choices of values for $L > 0.75$.

[5]We rely on estimates from SS instead of applying their procedure because the nature of our dataset is different from theirs. Their dataset consists of daily quotes and therefore they have daily return observations for every bond. Our dataset consist of actual transactions which are unevenly spaced in time and constructing a return series is considerably more difficult.

[6]Previous literature uses Treasury yields as riskfree rates, but recent evidence shows that swap rates are more appropriate to use than Treasury yields. The reason is that Treasury bonds enjoy a convenience yield that pushes their yields below riskfree rates. Hull, Predescu, and White (2004) find that the riskfree rate is 63bps higher than Treasury yields and 7bps lower than swap rates, Feldhütter and Lando (2008) find riskfree rates to be approximately 53bps higher than Treasury yields and 8bps lower than swap rates, while Krishnamurthy and Vissing-Jorgensen (2012) find that Treasury yields are on average 73bps lower than riskfree rates.

rating decreases. Interestingly, asset volatility remains fairly constant from the first half of the sample to the second half. This is the case even for BBB that sees a dramatic increase in equity volatility from 40% to 100%. This suggests that the high equity volatility during the subprime crisis was caused primarily through increased leverage and not because of increased asset volatility.

We see in Table 2 that speculative grade firms are significantly more leveraged. However, their asset volatilities are close to those for investment grade bonds with median asset volatility at 22% for BB, 23% for B, and 19% for C. Median payout ratios between 3.8% and 5.1% are in the same range as those for investment grade bonds, so the higher coupons speculative grade firms pay are countered by lower dividends to equity holders.

Turning to bond statistics in Table 3 we see that the number of speculative grade bonds is small relative to the number of investment grade bonds. For example, the number of bonds maturing within one year during the sample period is between 84 and 114 for investment grade bonds while it is between 0 and 25 for speculative grade bonds. The reason is that speculative grade firms are more likely to issue bonds with call and covenant features. Therefore, many of the speculative grade observations we have in the dataset are from fallen angels.

Since the number of speculative grade firm and bond observations in the sample is small relative to the number of observations in the investment grade segment, we focus on investment grade bonds in the empirical section. This does not substantially limit our examination of the credit spread puzzle since the puzzle is mostly confined to investment grade bonds. For example, HH report that between 16 and 29% of the 10-year spread of investment grade bonds can be explained by credit risk while the corresponding range for speculative grade bonds is 60-83%.

# 3 Existing tests of structural models

There is a large number of papers finding that standard structural models - often the Merton model - cannot match the level of credit spreads, particularly for short maturities and high credit quality issuers (Huang and Huang (2012), McQuade (2013), Chen, Collin-Dufresne, and Goldstein (2008), Cremers, Driessen, and Maenhout (2008), Leland (2006), Chen (2010), and Ericsson, Jiang, Elkhamhi, and Du (2013) among others). This finding has been coined the credit spread puzzle and the standard reference for the puzzle is Huang and Huang (2012) (while the paper is published in 2012 the latest working paper version of the paper is from 2003). A recent review of the inability of the Merton model to capture the level of credit spreads and extensions of the model is Sundaresan (2013). The only paper we are aware of that does not find a credit spread puzzle is Eom, Helwege, and Huang (2004). While credit spreads might be influenced by illiquidity and other factors, default probabilities are not. Leland (2004) and McQuade (2013) find that the Merton model underpredicts default probabilities at short horizons consistent with the underprediction evidence for spreads.

In this Section we discuss how the above mentioned papers test structural models and show that common tests are biased or suffer from low statistical power. Furthermore, we reconcile the conclusion in Eom, Helwege, and Huang (2004) with the findings in the rest of the literature.

### 3.1 Notation

Before we begin the discussion of structural model tests we define some notation. For a given corporate bond transaction at time $t$ in a bond with maturity $T$ issued by firm $i$ we call the actual spread $s_T^A(i,t)$. To calculate the theoretical credit spread in the Merton model we need firm leverage $L_{it}$, asset volatility $\sigma_{Ait}$, payout ratio $\delta_{it}$, and the riskfree rate at maturity $T$ $r_{tT}$. We denote the vector of parameters for firm $i$ at time $t$ for $\theta_{it} = (L_{it}, \sigma_{Ait}, \delta_{it}, r_{tT})$[7]. We define the parameter vector *without* asset volatility as $\theta_{it}^{\setminus \sigma_A} = (L_{it}, \delta_{it}, r_{tT})$. We denote the model-implied Merton credit spread as $s_T^M(\theta_{it})$ and the explicit formula is given in equation (3) and (6). Given an asset risk premium $\pi_A$ Appendix B.2 reviews the calculation of the cumulative default probability over the next $T$ years and we call this $PD_T(\theta_{it}, \pi_A)$.

### 3.2 Convexity bias in spreads

Leland (2006) and McQuade (2013) use historical averages of leverage ratio, payout rate, the riskfree rate, and asset volatility (obtained from Schaefer and Strebulaev (2008)) to calculate model-implied credit spreads from a standard structural model and compare with historical averages of actual credit spreads. Following the tradition of the literature they do this for individual rating classes and in the rest of the paper we assume that all such comparisons are done within a rating class without explicitly mentioning this.

[7]Payout ratio, asset volatility, and the risk free rate are assumed to be constant in the Merton model, but in the implementation we estimate them day by day and therefore they might vary over time for a given firm. This approach is standard in the literature (see for example Eom, Helwege, and Huang (2004), Schaefer and Strebulaev (2008), and Bao and Pan (2013)) and since we explicitly study previous results in the literature we follow this tradition.

In their approach

$$s_T^M(\frac{1}{T_{i,t}}\sum_{i,t}\theta_{it})$$

is compared with

$$\frac{1}{T_{i,t}}\sum_{i,t}s_T^A(i,t)$$

where $T_{i,t}$ is the number of spread observations for bonds with maturity $T$ over the corresponding time period. The approach suffers from a Jensen's inequality bias because spreads are typically convex in firm variables. David (2008) discusses this bias but there are no empirical results on the severity of the bias. We therefore examine the bias by computing the average Merton spread and compare it with the Merton spread of the average firm variables in our sample. Specifically, we compute the ratio

$$\frac{s_T^M(\frac{1}{T_{i,t}}\sum_{i,t}\theta_{it})}{\frac{1}{T_{i,t}}\sum_{i,t}s_T^M(\theta_{it})}$$

in Table 4. If the ratio is below one calculating model spreads based on average firm variables underestimates the size of spreads the Merton model generates.

We see in the table that the bias is big and increases with credit quality and as maturity shortens. Spreads based on average firm variables are only 39% of the correct average model-implied spread for a 10-year AAA/AA rated bond. For 10-year BBB-rated bonds it is 77% and for BB-rated bonds it is 107% showing that for long-maturity speculative grade bonds spreads can be biased upwards. For bonds with a maturity of less than one year the spread based on average firm variables is zero for all investment grade bonds while the correct average spread is up to 601 basis points. We also see that the bias is similar in the first (2002-2007) and second half (2007-2012) of the sample period showing that the bias is a robust phenomenon and not due to the large variation in spreads in the latter period.

Overall, the results show that calculating model-implied spreads based on average leverage ratio, payout ratio, and asset volatility causes a downward bias in spreads for investment grade bonds and this bias becomes larger where the credit spread puzzle has been found to be most severe, for short maturities and high-quality firms. Thus, the findings in Leland (2006) and McQuade (2013) that the Merton model underpredict spreads are not accurate.

## 3.3 Convexity bias in default probabilities

Leland (2004) and McQuade (2013) examine default probabilities implied by structural models. Default probabilites are not contaminated by liquidity, recovery rates, and other potential factors influencing spreads. They choose leverage ratio, payout rate, and the riskfree rate based on historical averages, an asset risk premium of 4% and compute model-implied default probabilities. Leland finds that default frequencies at horizons below five years are underestimated and McQuade finds similar results.

Default probabilities are subject to a convexity bias similar to the bias spreads are exposed to. Table 5 shows that a similar bias for default probabilities as for spreads occurs when fitting the Merton model to average firm variables; the bias increases in credit quality and as maturity shortens. Once average default probabilities are calculated correctly by doing it on a firm-by-firm basis short-term default probabilities are very different than those obtained by calculating default probabilities for the average firm. In short, calculating default probabilities based on average firm variables and comparing with realized default probabilities is not very meaningful.

### 3.4 Calibrating to historical default frequencies

The most common approach when calibrating structural models is to leave asset volatility as a free parameter, set leverage ratio and payout ratio to historical averages, and imply out asset volatility by matching the expected default probabilities to historical default probabilities (see for example Huang and Huang (2012), Cremers, Driessen, and Maenhout (2008), Chen, Collin-Dufresne, and Goldstein (2009), Zhang, Zhou, and Zhu (2009), and Chen (2010)). Specifically, for a given bond maturity $T$ (and rating), the asset volatility $\sigma_A$ is implied out by the equation

$$PD_T(\frac{1}{T_{i,t}}\sum_{i,t}\theta_{it}^{\setminus\sigma_A},\sigma_A) = RD_T$$

where $RD_T$ is the historical average default frequency over a long period, in Huang and Huang (2012)'s case 28 years. The backed-out asset volatility $\hat{\sigma}_A$ is then used to calculate the model-implied spread

$$s_T^M(\frac{1}{T_{i,t}}\sum_{i,t}\theta_{it}^{\setminus\sigma_A},\hat{\sigma}_A)$$

and compared with average actual spreads $\frac{1}{T_{i,t}}\sum_{i,t} s_T^A(i,t)$. Given the documented bias in Table 5, this approach generates a biased calibration. Since predicted default probability using average firm variables is lower than average default probability, $\hat{\sigma}_A$ is higher than average $\sigma_A$ to compensate for this. In turn, model-implied spreads using average firm variables are too low as Table 4 documents, so a too high asset volatility counters this bias. Overall, it is not clear in which direction the sum of the biases go, but in untabulated results we find that predicted spreads are mostly lower than actual average spreads.

While it is not clear whether the model-implied spread when fitting to average default probabilities is higher or lower than average model-implied spreads, it is clear that asset volatility is biased upwards. This bias is more severe for shorter maturities and higher credit quality. Consequently, other model implications are likely to be biased. And even though the biases *might* largely cancel out when fitting to 10-year default rates and predicting 10-year spreads, they are unlikely to cancel out when default rates and spreads have different time horizons. For example, Cremers, Driessen, and Maenhout (2008) fit to historical 10-year historical default frequencies and look at implied spreads and default probabilities for maturities 1-10 year while Zhang, Zhou, and Zhu (2009) fit to historical 5-year historical default frequencies and look at implied spreads and default probabilities for maturities 1-5 year. These exercises are likely strongly biased. Finally, the spread and default probability biases might not be of similar magnitude in more advanced structural models.

## 3.5 Statistical uncertainty when fitting to historical default frequencies

Besides the biases when fitting to historical default frequencies documented in the previous section, there is another problem with this approach, namely low statistical power. The underlying assumption is that historical default frequencies over a given period proxy well for average expected default frequencies over the same period. For example, Huang and Huang (2012) (HH) use average spreads over the period 1973-1993 and realized default frequencies over the period 1970-1998.

To test how accurate it is to use realized default frequencies as a proxy for average expected default probabilities we run simulation experiment similar in spirit to Strebulaev (2007). In the simulation we populate the economy with BBB firms who are exposed to both idiosyncratic and systematic risk and then compare expected 10-year default frequencies with realized 10-year default frequencies.

The simulation is done as follows. Assume we have an index of BBB-rated bonds. In month 1 we have 1,000 BBB firms where firm $i$'s value under the risk-neutral measure follows the process

$$\frac{dV_t^i}{V_t^i} = (r - \delta)dt + \sqrt{1-\rho}\sigma_A dW_{it}^Q + \sqrt{\rho}\sigma_A dW_{st}^Q$$

where $W_i$ is a Wiener process specific for firm $i$ and $W_s$ is a Wiener process common to all firms. All Wiener processes are independent. When pricing bonds the relative contribution of systematic and idiosyncratic risk is not relevant and we can write the dynamics as

$$\frac{dV_t^i}{V_t^i} = (r - \delta)dt + \sigma_A dW_t^Q.$$

Under the actual measure the firm value follows the process

$$\frac{dV_t^i}{V_t^i} = (\pi_A + r - \delta)dt + \sqrt{1-\rho}\sigma_A dW_{it}^P + \sqrt{\rho}\sigma_A dW_{st}^P \qquad (2)$$

where $\pi_A$ is the asset risk premium. The realized 10-year default frequency for BBB-rated firms in month 1 is found by simulating the idiosyncratic and systematic processes 10 years ahead. In month 2 the BBB-rated firms from month 1 exit the index and 1,000 new BBB firms enter the index. The firms in month 2 are identical to the previous firms as they entered the index in month 1. We calculate the realized 10-year default frequency of firms entering the index in month 2 as before. Note that in month i their common shock $W_{si}^P$ is the same as the common shock in month $i$ for firms born in month 1, etc. We do this for 240 months and calculate the overall realized default frequency in the economy by taking an average of the default frequencies in each month. We repeat this simulation 5,000 times.

We use the parameters in HH for BBB firms in our simulation: leverage ratio is 43.28%, recovery rate is 51.31%, payout rate is $\delta = 6\%$, riskfree rate is $r = 8\%$, and asset risk premium is $\pi = 5\%$. We attribute half of firm volatility to systematic volatility ($\rho = 0.5$) consistent with evidence in Choi and Richardson (2012)[8]. We set asset volatility to be the average asset volatility for BBB-rated firms in our sample, $\sigma_A = 28\%$. We simulate 20 years of firms (in total 12*1,000*20=240,000 firms) and since we look at default frequencies up to 10 years, we simulate 30 years of data. These time spans are almost the same as the 20 years of spreads and 28 years of default frequencies HH base their results on.

Figure 2 shows the results of the simulation study. The top graph shows the distribution of realized default frequencies. The black line shows the expected default probability in the economy.[9] Given that we simulate 240,000 firms over a period of 30 years, it might be a surprise that realized default fre-

[8]See Table 5 in Choi and Richardson (2012). $\rho = 0.5$ is also roughly consistent with the fact that during our sample period the annualized volatility of daily returns of the S&P500 index was 21.5% while the median equity volatility for A-rated forms respectively BBB-rated firms was 31% respectively 47%.

[9]The expected default probability when a firm enters the index is 9.8% and since all firms are identical and firms stay in the index only one month, the average expected default probability is also 9.8%.

quencies can be so far away from expected default probabilities. The reason is that there is systematic risk in the economy and this induces correlation in defaults among firms. If there is no systematic risk in the economy a 95% confidence interval for the realized default frequency is (9.67%; 9.91%).

The implications for the 10-year BBB spread using the approach in HH is shown in the bottom graph in Figure 2. For each simulation we back out the asset volatility that in the Merton model would imply an expected default probability equal to the realized default probabilty and then calculate the 10-year spread using this asset volatility[10]. The graph shows the distribution of model-implied spreads across the 5,000 simulations. The black line shows the spread using the true expected default probability.

The observed spread in the simulated economy is 130 basis points; close to the reported 10-year BBB-spread to the swap rate of 142 in HH[11]. We see that the variation in model-implied spreads is huge and a 95% confidence interval for the model-implied spread is (23bp; 363bp). Moreover, we often see quite small model-implied spreads. HH fit their benchmark structural model to a realized default probability of 4.39% and calculate a model-implied spread of 56.5 basis points. In our simulated economy 20% of the simulations lead to a smaller model-implied spread than reported by HH. This shows that the statistical power when comparing actual spreads to model spreads implied

[10]Note that since all firms are identical in the cross-section and over time, there is no convexity bias at play, and the reported results are entirely due to statistical uncertainty.

[11]HH report a 10-year BBB-spread to Treasury of 194 basis points and a spread between the swap rate and Treasury rate of 52 basis points, giving a 142 basis points spread to the swap rate. Since HH 10-year BBB-spreads include yields from callable bonds, a more acurate estimate is to use Duffee (1998)'s estimates based on a sample of non-callable bonds (HH use Duffee (1998)'s 4-year spreads but not his 10-year spreads). Duffee (1998)'s estimate of the 10-year BBB-spread to Treasury is 148 basis points, giving a 96 basis points spread to the swap rate.

by fitting to historical default frequencies is low.

These results are obtained under benign assumptions. In practice 1) there is a range of default rates within a rating category, 2) rating agencies rate "through the cycle" meaning that the expected default probability for a given rating varies over time, and 3) BBB-rated companies stay BBB-rated for a while creating dependence over time between the idiosyncratic risk of BBB-rated firms in the simulations. If we included these features in the simulations the variations in default frequencies would likely be even larger.

HH fit structural models to 10-year default frequencies of 4.39% and 0.77% for BBB- and AAA-rated bonds based on the period 1970-1998. They report an average actual long-term BBB-AAA spread of 131 basis points. According to Ou, Chiu, Wen, and Metz (2013) the 10-year default frequencies over the period 1970-2012 were 4.74% and 0.50% for BBB- and AAA-rated bonds. The long-term BBB-AAA spread was 112 basis points during the period 1970-2012[12]. These updated numbers are broadly in line with those in HH. For the period 1920-1970 the 10-year default frequencies were 9.10% and 1.17% for BBB- and AAA-rated bonds while the long BBB-AAA spread was 126 basis points. That is, roughly twice the default frequencies in 1920-1970 compared to 1970-2012 and similar BBB-AAA spreads. To the extent that differences in the BBB-AAA spread reflect differences in expected default frequencies, this example further illustrates that realized default frequencies are poor proxies for expected default probabilities even over periods of 40-50 years.

[12] According to Moody's seasoned Aaa and Baa data downloaded from the Federal Reserve's webpage.

## 4. The credit spread puzzle

We showed in the previous section that tests of structural models that use average firm variables as input and/or fit to historical default frequencies are unreliable. A correct approach to testing structural models is to compare model-implied and actual spreads on a transaction-by-transaction basis. To our knowledge Eom, Helwege, and Huang (2004) is the only paper taking this approach. Interestingly, they find that structural models do not systematically underpredict spreads. Due to the limited data availability before TRACE their data set consists of 182 trader quotes in the period 1986-1997. With the availability of TRACE we can conduct a large scale examination of the Merton model using 534,660 transactions for the period 2002-2012. This allows us to examine in detail the ability of the Merton model to price bonds across maturity, for different ratings, and over a time period that includes both a boom period and a recession.

There are broadly three versions of the puzzle:

- Puzzle I: *Yield spreads between BBB- and AAA-rated bonds are too high to be explained by standard structural models of credit risk.* The yield spread is typically for bonds with a maturity of 4 or 10 years. If potential non-default components of yield spreads like taxes or liquidity are the same for AAA- and BBB-rated bonds, this version of the credit spread puzzle offers a "clean" spread uncontaminated by non-credit effects.

- Puzzle II: *Yield spreads on high-quality bonds with short maturity are too high to be explained by standard structural models of credit risk.*

Short maturity is 1 year or less and high-quality refers to bonds with a rating of AAA, AA, or A. Since standard structural models typically predict yield spreads close to zero for high-quality bonds at short maturities, this version of the puzzle is not very sensitive to model specification; if there is a significantly positive short-term spread there is a puzzle.

- Puzzle III: *Yield spreads on high-quality bonds with long maturity are too high to be explained by standard structural models of credit risk.* Long maturity is typically 10 years but sometimes also 4 years and high-quality refers to bonds with a rating of AAA.

Puzzle I has received most attention in the literature and we will focus on this puzzle first before examining Puzzle II and III.

When we calculate spreads we take the median across all bond transactions and weight by the volume of each transaction.[13] The use of medians is robust to the presence of potential outliers and in most cases we also report volume-weighted 10 and 90pct quantiles which are informative about the distribution of spreads. We weight by volume following the recommendation by Bessembinder, Kahle, Maxwell, and Xu (2009).

## 4.1 Puzzle I: BBB-AAA yield spreads

Table 6 shows the actual and model-implied bond spreads for our sample. The actual median 10-year BBB spread is 95bps while the model-implied

[13] To calculate the volume-weighted median, we sort spreads in increasing size $s_1 < s_2 < ... < s_T$ with corresponding normalized volumes $\tilde{v}_1, \tilde{v}_2, ..., \tilde{v}_T$, where $\tilde{v}_i = \frac{v_i}{\sum_{j=1}^{T} v_j}$ and then find $t$ such that $\sum_{i=1}^{t} \tilde{v}_i \geq 0.5$ and $\sum_{i=1}^{t-1} \tilde{v}_i < 0.5$. The volume-weighted median spread is then $s_t$.

spread is 110bps. Thus, on average the Merton model does not underpredict long-term BBB-AAA spreads in contrast to what most of the previous literature has found. For 4-year BBB bonds the median actual spread is 278bps while the model-implied spread is 126bps. In both cases the distance between the 10pct and 90pct quantile is big. This reflects that the sample period includes periods with low spreads such as the 2005-2006 period and periods with very high spreads like the 2008-2009 period. Ignoring differences in spreads over time can easily lead to incorrect conclusions so Figure 3 shows the time series variation in long-term BBB-AAA spreads along with 10pct and 90pct quantiles.[14] Model-implied and actual 10-year BBB-AAA spreads are in the top panel. The graph suggests that the model-implied spread cannot quite match the level and time series variation of the actual spread during 2005-2007, but apart from this period the model-implied spread tracks the actual spread well. Although the 4-year BBB-AAA model spread tracks the actual spread less convincingly in the bottom graph than for 10-year spread the conclusion is broadly similar; Apart from the period 2005-2007, the actual spread is matched fairly well.

In Figure 4 we plot the median quarterly pricing residual - actual BBB-AAA/AA spread minus model-implied BBB-AAA/AA spread for all bonds with a maturity of three years of more.[15] This graph provides more broad

[14]We calculate the quantiles in each quarter by simulation: we draw a transaction in a BBB bond from the pool of actual BBB transactions where each transaction is weighted by transaction volume. In the same way we draw a AAA/AA transaction. We calculate from the two transactions a BBB-AAA/AA spread. We repeat this procedure 5,000 times and calculate the 10pct and 90pct quantile in the 5,000 simulated BBB-AAA/AA spreads.

[15]We calculate the quantiles in each quarter by simulation: we draw a transaction in a BBB bond from the pool of actual BBB transactions where each transaction is weighted by transaction volume. In the same way we draw a AAA/AA transaction. We calculate from the two transactions the actual BBB-AAA/AA spread minus the model-implied BBB-AAA/AA spread. We repeat this procedure 5,000 times and calculate the 10pct and

evidence on the BBB-AAA/AA spread across bonds with longer maturities. Consistent with the evidence in the previous graph, we do not see a consistent pattern of actual spreads being a higher than model-implied spreads as the credit spread puzzle suggests. The average difference between actual and model-implied BBB-AAA/AA in the graph is only 4bps.

Overall, we find no evidence that actual long-term BBB-AAA/AA spreads are consistently higher than model-implied long-term BBB-AAA/AA spreads.

## 4.2 Puzzle II: Short-term yield spreads on high-quality bonds

Predicted spreads in standard models of credit risk for short-maturity high-quality bonds are very low and this has been viewed as a failure of structural models. To the best of our knowledge there is no empirical evidence on the size of corporate bond credit spreads for maturities shorter than one year. The reason is that previous research had to rely on quotes and typically only bonds part of an index, like the Lehman or Merrill Lynch index, were carefully quoted. Bonds drop out of indices when the maturity falls below one year and so the bonds typically stop being quoted. Since we use transactions data, we observe transactions of bonds with any maturity, and our results on short-term bonds provide new evidence on the size of short-term corporate bond spreads.

In Table 6 we see that the median yield spread for AAA/AA-rated bonds is 3bps and 7bps for A-rated bonds. This is close to zero even considering that the spread is with respect to maturity-matched LIBOR rates and the median LIBOR spread to the riskfree rate is 5-10bps (Feldhütter and Lando (2008)).

90pct quantile in the 5,000 simulated pricing residuals.

Figure 5 shows the time variation of short-term spreads for ratings AAA/AA, A, and BBB. The three graphs on the left-hand side and right-hand side are identical apart from the y-axis. For AAA/AA-rated bonds model-implied spreads are zero throughout the sample period. Actual spreads are also close to zero and the 10pct quantile is below zero for the whole sample period except during the volatility 2008 where we seee a jump in short-term spreads of around 50bps. For A-rated bonds we see a similar pattern. Short-term spreads for BBB-rated bonds in first half of the sample period are in the range of 20-100bps and confidence bands do no contain zero. For the first half of the sample period model-implied spreads are zero. Proposed explanations in the literature for the range of spread of 20-100bps are incomplete accounting information (Duffie and Lando (2001)) or jumps in firm value (Zhou (2001)). However, any explanation raising short-term BBB credit spreads must have a modest effect on AAA/AA/A short-term spreads since they are close to zero.

In the second half of the sample period we see that actual BBB short-term credit spreads are high but model-implied spreads are even higher. When calculating the credit spread, we assume that all of the firm's debt is due when the bond matures. In practice, most firms have debt maturing at different maturities. For long-term bonds most of the debt outstanding today has been repaid when the bond matures and the assumption - although an approximation - is not unreasonable. For short-maturity bonds, the assumption is that the firm defaults if it cannot repay all of its debt outstanding in the near future, and intuitively this leads to an overestimation of the credit risk.[16] It is interesting to incorporate the maturity structure of debt in the Merton model, but we leave this for future research.

Overall, we find that short-term corporate yield spreads for high-quality bonds are close to zero consistent with predictions from the Merton model.

### 4.3 Puzzle III: Long-term yield spreads on high-quality bonds

Table 6 shows that the median actual AAA/AA 10-year spread over the sample period is 32bps while the model-implied is a mere 2bps. Figure 6 shows the time variation of the 10-year AAA/AA spread. In 2002-2005 the model-implied spread tracks the actual spread fairly well, while from 2006 and onwards the actual spread is almost always higher than the model spread. To examine in more detail the spread underprediction Figure 7 shows the pricing residuals across subsamples and maturity. The pricing residual is the actual spread minus model-implied spread. Since the patterns of pricing residuals are similar for A, AA, and AAA, we have grouped the three rating categories into one category.

In 2002Q3-2004Q4 the underprediction is small at around 10bps and the the size is similar across maturity. The 10-90pct quantiles contain 0 for all maturities. This shows that the Merton model prices high-quality corporate bonds fairly well in this period.

In 2005Q1-2007Q2 the spread underprediction increases with maturity. At short maturities it is close to zero while at long maturities it is around 30bps and the 10-90pct quantiles do not contain 0. This shows that the Merton model cannot capture the size of spreads in this period. One potential

[16] We thank Darrell Duffie for pointing this out.

explanation is that during this period leveraged buyout activity was high. Highly-rated companies have low leverage ratios and the Merton model suggests a low yield spread. If the market assigns a positive probability of a sudden increase in leverage in association with a leveraged buyout, spreads will be higher than the Merton model suggests. An extension of the Merton model where leverage ratio can jump as a consequence of an LBO transaction would have a small effect on short-maturity spreads but an increasing effect at longer maturities. The reason is that for short maturities even a jump in leverage to say 70pct would not produce a significant probability of the firm defaulting within a short time interval. This is an interesting extension of the Merton model but outside the scope of this paper.

In the 2007Q3-2009Q4 period the Merton model underpredicts spreads quite strongly with the underprediction being around 20bps for short-maturity spreads and 70bps for long maturity spreads. Dick-Nielsen, Feldhütter, and Lando (2012) find a significant illiquidity component to corporate bond spreads during this period. Their estimates of 24bps for short-maturity AA-rated bonds and 65bps for long-maturity AA-rated bonds suggests are of similar magnitude suggesting that the underprediction can be attributed to illiquidity of corporate bonds during this period.

The last part of the sample period, 2010Q1-2012Q2, has an upward-sloping term structure of pricing errors. Although the graph of pricing errors looks similar to that during 2005Q1-2007Q2, leveraged buyout activity was low in this period and therefore an unlikely explanation for this pattern. A plausible reason is that the riskfree rate is underestimated during this period. As mentioned in Footnote 6 Treasury yields are downward biased measures of riskfree rates because Treasury bonds enjoy a convenience yield and therefore we use swap rates. However, swap rates are occasionally pushed down due to market imbalances as shown in Feldhütter and Lando (2008). The default of Lehman left market participants with unhedged risks causing a demand in interest rate swaps. According to Bloomberg "Pension funds need to hedge long-term liabilities by receiving fixed on long-maturity swap rates...When Lehman dissolved, pension funds found themselves with unmatched hedging needs and then needed to cover these positions in the market with other counterparties. This demand for receiving fixed in the long end drove swap spreads tighter."[17] In the 10 years before the Lehman default the average 30-year swap spread to Treasury was 61bps with a minimum of 24bps, while the average swap spread following the Lehman default and until the end of the sample period has been -23bps. This effect is stronger at longer maturities consistent with the increased mispricing as maturity lengthens.

Overall, we do find that the Merton model underpredicts long-term high-quality spreads at times, but apart from the crisis period 2008-2009 the underprediction is no more than 30-40bps.

## 4.4 Further evidence using Merrill Lynch quotes

The evidence so far has relied on the TRACE data set which contains almost all corporate bond transactions in the US since 2002. In contrast, previous literature has had to rely on dealer quotes instead of transactions data. To test the extent to which our conclusions are influenced by the use of transactions data instead of dealer quotes, we repeat our analysis using daily quotes

[17] http://www.bloomberg.com/apps/news?pid=newsarchive&sid=aUq.d1dYuhEA

provided by Merrill Lynch on all corporate bonds part of the Merrill Lynch investment grade and high-yield indices.

A concern when using dealer quotes is that they are prices at which dealers are willing to buy and therefore they represent bid prices and are sensitive to time variation in the bid-ask spread. Using a database of transactions data for the period 1995-1997, Schultz (2001) finds that for investment grade bonds dealer buy prices on average exceeds Lehman quotes by 6 cent and dealer sell prices on average exceeds Lehman quotes by 34 cent (per notional $100). Thus, he finds that Lehman quotes used by HH, Duffee (1998) and many others are lower than actual transactions even at the bid.

To examine the severity of this bias we search for every TRACE inter-dealer transaction a corresponding Merrill Lynch quote on the same day and record the difference. We only use interdealer transactions so that we look at midprices. Figure 8 shows the volume-weighted average difference.[18] We see that in the first half of the sample period the bias negligible, but in the second half the bias is quite large with a peak of 70 basis points for BBB in 2008.[19] We also see that the bias in the second sample half is larger as we move down in rating. The bias is of a magnitude that can in certain periods lead to misleading conclusions. It is also conceivable that the bias is smaller in our sample period than before TRACE, because all market participants know at which price current transactions occur, while before TRACE the market was opaque with no post-trade transparency.

With the documented bias in mind, we repeat the analysis of the previous

[18]The difference is winsorized at -500bps and 500bps.

[19]In results not reported we find the bias to be largest for short-maturity bonds. The shortest maturity is one year in the Merrill Lynch data.

sections using Merrill Lynch data instead of TRACE data. Since the Merrill Lynch data does not have transaction volume, we do not volume-weight, but an advantage of the data is that it extends back to January 2, 1997.

Figure 9 shows the 10- and 4-year actual and model-implied BBB-AAA/AA yield spreads. The figure confirms our earlier results that the actual BBB-AAA/AA spread is matched well by the model-implied spread and that there is no credit spread puzzle. The extra five years of data does hint to a pattern of slight overprediction of spreads in periods with high spreads (2000-2003 and 2008-2012) and slight underprediction in low-spread periods (1998-1999 and 2004-2007), but further evidence is needed to confirm this observation.

Figure 10 shows the 10-year AAA/AA spread. This graph shows that the Merton model underpredicts spreads consistent with evidence using TRACE data. We see that in the period 1997-2007 the spread underprediction is seldomnly more than 30bps and in the first five years of the sample quite close to zero. Thus, the conclusion that the Merton model underpredicts spreads but up until the subprime crisis the underprediction was no more than 30-40bps is confirmed.

# 5. Conclusion

Often the credit spread puzzle is documenting by calibrating structural models to average firm variables and/or historical default frequencies and comparing model-implied spreads to average actual spreads. We find that there are two problems with this approach. The first problem is that spreads are typically convex in firm variables, so average spreads are higher than spreads of average firm variables. A similar bias occurs when looking at default probabilities. We examine these biases empirically and find them to be significant. The second problem is that when fitting to historical default frequencies, the implicit assumption is that historical default frequencies proxy well for expected default probabilities. In a simulation study we find that even over a period of 30 years historical default frequency can differ dramatically from expected default frequency. This shows that the statistical power of fitting to historical default frequencies is low.

We then test the Merton in a bias-free approach and find that the Merton model captures the level and time series variation of long-term BBB-AAA US corporate bond spreads during 1997-2012. We document that short-term yield spreads of AAA-A-rated bonds are close to zero in normal times. We find that the Merton can explain the size of long-term A spreads, but undershoots long-term AAA/AA spreads by around 30 basis points in normal times. Overall, this shows that there is very little evidence for a credit spread puzzle.

Our results show that when testing structural models it is important take into account the cross-sectional variation of firms and the time series variation of firm leverage and other firm variables. While it is useful to assess the default probabilities implied by models, using historical default frequencies results in large statistical uncertainty. An alternative approach is to compare model-implied default probabilities with default probabilities implied from a statistical model such as the model in Duffie, Saita, and Wang (2007). This comparison can be done for any firm at any time.

# A. Data

This Appendix gives details on the corporate bond transactions dataset and how firm variables, leverage, payout rate, and equity volatility are calculated using CRSP/Compustat.

## A.1 Bond data

Since July 1, 2002, members of the Financial Industry Regulatory Authority have been required to report their secondary over-the-counter corporate bond transactions through the Trade Reporting and Compliance Engine (TRACE) and the transactions are disseminated to the public within 15 minutes.[20]

Initially, the collected trade information was publicly disseminated only for investment grade bonds with issue sizes greater than $1 billion. Gradually, the set of bonds subject to transaction dissemination increased and since January 9, 2006 transactions in all non-144A bonds transactions have been immediately disseminated.[21] Goldstein and Hotchkiss (2008) provide a detailed account of the dissemination stages. In the publicly disseminated data the trade size is capped at $5 million in investment grade transactions and $1 million in speculative grade transactions. Since November 3, 2008, the publicly available TRACE data indicate whether a transaction is an interdealer transaction or a transaction with a customer and, if a customer transaction,

[20] In the initial phase of TRACE the disseminating times were longer than 15 minutes. Since July 1, 2005 the reporting and dissemination is required to occur within 15 minutes after the trade.

[21] Rule 144a allows for private resale of certain restricted securities to qualified institutional buyers. According to TRACE Fact Book 2011, the percent of rule 144A transactions relative to all transactions was 2.0% in investment grade bonds and 8.4% in speculative grade bonds. Also, transactions reported on or through an exchange are not included in TRACE.

whether the broker-dealer is on the buy or the sell side. This publicly disseminated data is available through Wharton Research Data Services (WRDS) and is used in for example Dick-Nielsen, Feldhütter, and Lando (2012) and Bao, Pan, and Wang (2011). We use this data for the period September 15, 2011- June 30, 2012.

Through FINRA we have access to historical transactions information not previously disseminated. The historical data is richer than the WRDS data in three aspects. First, the data contains all transactions in non-144A bonds since July 2002, so the data set for the first years of TRACE is significantly larger than the WRDS data set. Second, the data has buy/sell indicators for all transactions, not just after October 2008 as in the WRDS data set. Third, trade volumes are not capped. FINRA provide access to the enhanced historical data with a lag of 18 months. We use this data for the period July 1, 2002-September 14, 2011.

We obtain bond information from the Mergent Fixed Income Securities Database (FISD) and limit the sample to senior unsecured fixed rate or zero coupon bonds. We exclude bonds that are callable, convertible, putable, perpetual, foreign denominated, Yankee, have sinking fund provisions, or have covenants. For bond rating, we use the lower of Moody's rating and S&P's rating and discard any transactions that do not have a Moody's or S&P rating on transaction day. We track rating changes on a bond, so the same bond can appear in several rating categories over time. Bonds for which FISD do not provide information are dropped from the sample. Erroneous trades are filtering out as described in Dick-Nielsen (2009). We exclude transactions with a yield of 99999.9999% or 99999.99%.

## A.2 Firm data

To compute bond prices in the Merton model we need the issuing firm's leverage ratio, payout ratio, and asset volatility. Firm variables are collected in CRSP and Compustat. To do so we match a bond's CUSIP with CRSP's CUSIP. In theory the first 6 digits of the bond cusip plus the digits '10' corresponds to CRSP's CUSIP, but in practice only a small fraction of firms is matched this way. Even if there is a match we check if the issuing firm has experienced M&A activity during the life of the bond. If there is no match, we hand-match a bond cusip with firm variables in CRSP/Compustat.

Leverage ratio: Equity value is calculated on a daily basis by multiplying the number of shares outstanding with the price of shares. Debt value is calculated in Compustat as the latest quarter observation of long-term debt (DLTTQ) plus debt in current liabilities (DLCQ). Leverage ratio is calculated as $\frac{\text{Debt value}}{\text{Debt value+Equity value}}$.

Payout ratio: The total outflow to stake holders in the firm is interest payments to debt holders, dividend payments to equity holders, and net stock repurchases. Interest payments to debt holders is calculated as the previous year's total interest payments (previous fourth quarter's INTPNY). Dividend payments to equity holders is the indicated annual dividend (DVI) multiplied with the number of shares. The indicated annual dividend is updated on a daily basis and is adjusted for stock splits etc. Net stock repurchases is the previous year's total purchase of common and preferred stock (previous fourth quarter's PRSTKCY). Payout ratio is the total outflow to stake holders divided by firm value, where firm value is equity value plus debt value.

Equity volatility: We calculate the standard deviation of daily returns (RET in CRSP) in the past three years to estimate daily volatility. We multiply the daily standard deviation with $\sqrt{255}$ to calculate annualized equity volatility. If there are no return observations on more than half the days in the three year historical window, we do not calculate equity volatility and discard any bond transactions on that day.

# B. Pricing formulas

For completeness we include in this Appendix formulas for bond prices, default probabilities, and deadweight losses.

## B.1 Bond price

Equation (1) states firm value as a Geometric Brownian motion under the pricing measure,

$$\frac{dV_t}{V_t} = (r - \delta)dt + \sigma_A dW_t,$$

and the firm defaults if firm value is below face value of the bond at bond maturity, $V_T < F$. The bond price at time 0 is calculated as

$$P(0,T) = E^Q[e^{-rT}(1_{\{V_T \geq F\}} + \min(R, \frac{V_T}{F})1_{\{V_T<F\}})] \tag{3}$$

where $R$ is the recovery rate. From Eom, Helwege, and Huang (2004) Appendix A.1 we have that

$$E^Q[1_{\{V_T \geq F\}}] = N(d_2(F,T)) \tag{4}$$

and

$$E^Q[1_{\{V_T<F\}} \min(\psi, V_T)] = \frac{V_0}{D(0,T)} e^{-\delta T} N(-d_1(\psi,T)) + \psi[N(d_2(\psi,T)) - N(d_2(F,T))], \tag{5}$$

where $\psi \in [0, K]$, N represents the cumulative standard normal function,

$$\begin{aligned} d_1(x,T) &= \frac{\log(\frac{V_0}{xD(0,T)}) + (-\delta + \sigma_A^2/2)T}{\sigma_A\sqrt{T}}, \\ d_2(x,T) &= d_1(x,T) - \sigma_A\sqrt{T}, \end{aligned}$$

and $D(0,T) = \exp(-rT)$. Using equation (4) and (5) in (3) gives the solution to the bond price. The yield spread is calculated as

$$s(0,T) = -\frac{\log(P(0,T))}{T} - r. \tag{6}$$

## B.2 Default probabilities and deadweight loss

Under the physical measure firm value is given as the Geometric Brownian motion

$$\frac{dV_t}{V_t} = (\pi_A + r - \delta)dt + \sigma_A dW_t,$$

where $\pi_A$ is the asset risk premium.

Default probabilities are given by (4) where the expectation is taken under the physical measure.

The expected deadweight loss in bankruptcy as a fraction of asset value is given as

$$1 - E^P[\frac{\min(V_T, RF)}{V_T}|V_T < F]$$

and this expression can be solved numerically using that $V_T$ is log-normally distributed.

# References

Alderson, M. and B. Betker (1995). Liquidation costs and capital structure. *Journal of Financial Economics 39*, 45–69.

Bao, J. and J. Pan (2013). Bond Illiquidity and Excess Volatility. *Review of Financial Studies, forthcoming*.

Bao, J., J. Pan, and J. Wang (2011). The illiquidity of Corporate Bonds. *Journal of Finance 66*, 911–946.

Bessembinder, H., K. M. Kahle, W. F. Maxwell, and D. Xu (2009). Measuring abnormal bond performance. *Review of Financial Studies 22*(10), 4219–4258.

Chen, H. (2010). Macroeconomic conditions and the puzzles of credit spreads and capital structure. *Journal of Finance 65*(6), 2171–2212.

Chen, L., P. Collin-Dufresne, and R. Goldstein (2008). On the relation between the credit spread puzzle and the equity premium puzzle. *Forthcoming, Review of Financial Studies*.

Chen, L., P. Collin-Dufresne, and R. S. Goldstein (2009). On the relation between the credit spread puzzle and the equity premium puzzle. *Review of Financial Studies 22*, 3367–3409.

Choi, J. and M. Richardson (2012). The volatility of firm's assets and the leverage effect. *Working Paper*.

Cremers, M., J. Driessen, and P. Maenhout (2008). Explaining the level of credit spreads: Option-implied jump risk premia in a firm value model. *Review of Financial Studies 21*, 2209–2242.

David, A. (2008). Inflation Uncertainty, Asset Valuations, and the Credit Spread Puzzle. *Review of Financial Studies 21*(6), 2487–2534.

Davydenko, S. A., I. A. Strebulaev, and X. Zhao (2012). A Market-Based Study of the Cost of Default. *Review of Financial Studies 25*(10), 2959–2999.

Dick-Nielsen, J. (2009). Liquidity biases in TRACE. *Journal of Fixed Income 19*(2), 43–55.

Dick-Nielsen, J., P. Feldhütter, and D. Lando (2012). Corporate bond liquidity before and after the onset of the subprime crisis. *Journal of Financial Economics 103*, 471–492.

Duffee, G. R. (1998). The Relation Between Treasury Yields and Corporate Bond Yield Spreads. *Journal of Finance 53*(6), 2225–2241.

Duffie, D. and D. Lando (2001, May). Term Structures of Credit Spreads with Incomplete Accounting Information. *Econometrica*, 633–664.

Duffie, D., L. Saita, and K. Wang (2007). Multi-period corporate default prediction with stochastic covariates. *Journal of Financial Economics 83*, 635–665.

Eom, Y. H., J. Helwege, and J.-Z. Huang (2004). Structural models of corporate bond pricing: An empirical analysis. *Review of Financial Studies 17*(2), 499–544.

Ericsson, J., M. Jiang, R. Elkhamhi, and D. Du (2013). Time-varying asset volatility and the credit spread puzzle. *Working Paper*.

Feldhütter, P. and D. Lando (2008). Decomposing Swap Spreads. *Journal of Financial Economics 88*, 375–405.

Gilson, S. (1997). Transactions costs and capital structure choice: Evidence from financially distressed firms. *Journal of Finance 52*(1), 161–196.

Glover, B. (2012). The Expected Cost of Default. *Working Paper*.

Goldstein, M. A. and E. Hotchkiss (2008). Dealer Behavior and the Trading of Newly Issued Corporate Bonds. *Working Paper*.

Huang, J. and M. Huang (2012). How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk? *Review of Asset Pricing Studies 2*(2), 153–202.

Hull, J., M. Predescu, and A. White (2004). The Relationship Between Credit Default Swap Spreads, Bond Yields, and Credit Rating Announcements. *Journal of Banking and Finance 28*, 2789–2811.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012). The Aggregate Demand for Treasury Debt. *Journal of Political Economy 120*(2), 233–267.

Leland, H. (2004). Predictions of Default Probabilities in Structural Models of Debt. *Journal of Investment Management 2*.

Leland, H. (2006). Structural Models of Corporate Financial Choice. *Princeton Lectures in Finance, Lecture 1*.

McQuade, T. J. (2013). Stochastic Volatility and Asset Pricing Puzzles. *Working Paper, Harvard University*.

Merton, R. (1974). On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance 29*, 449–470.

Moody's (2011). Corporate Default and Recovery Rates, 1920-2010. *Moody's Investors Service*, 1–66.

Ou, S., D. Chiu, B. Wen, and A. Metz (2013). Annual Default Study: Corporate Default and Recovery Rates, 1920-2012. *Special Comment. New York: Moody's Investors Services*, 1–64.

Schaefer, S. and I. Strebulaev (2008). Structural models of credit risk are useful: Evidence from hedge ratios on corporate bonds. *Journal of Financial Economics 90*, 1–19.

Schultz, P. (2001). Bond trading costs: a peak behind the curtain. *Journal of Finance 56*(2), 677–698.

Strebulaev, I. (2007). Do Tests of Capital Structure Theory Mean What They Say? *Journal of Finance 62*(4), 1747–1787.

Sundaresan, S. (2013). A Review of Merton's Model of the Firm's Capital Structure with its Wide Applications. *Annual Review of Financial Economics 5*.

Zhang, B. Y., H. Zhou, and H. Zhu (2009). Explaining Credit Default Swap Spreads with the Equity Volatility and Jump Risks of Individual Firms. *Review of Financial Studies 22*(12), 5101–5131.

Zhou, C. (2001). The term structure of credit spreads with jump risk. *Journal of Banking and Finance 25*, 2015–2040.

| | Firms | Mean | 10th | 25th | Median | 75th | 90th |
|---|---|---|---|---|---|---|---|
| | | Full sample period (2002Q3-2012Q4) | | | | | |
| **AAA/AA** | 13 | | | | | | |
| Leverage ratio | | 0.26 | 0.05 | 0.09 | 0.17 | 0.45 | 0.52 |
| Equity volatility | | 0.25 | 0.17 | 0.20 | 0.25 | 0.30 | 0.34 |
| Asset volatility | | 0.19 | 0.12 | 0.14 | 0.20 | 0.23 | 0.26 |
| Payout ratio | | 0.041 | 0.021 | 0.023 | 0.039 | 0.051 | 0.068 |
| **A** | 34 | | | | | | |
| Leverage ratio | | 0.29 | 0.14 | 0.16 | 0.21 | 0.32 | 0.68 |
| Equity volatility | | 0.32 | 0.19 | 0.23 | 0.31 | 0.39 | 0.44 |
| Asset volatility | | 0.23 | 0.13 | 0.16 | 0.23 | 0.30 | 0.33 |
| Payout ratio | | 0.060 | 0.023 | 0.041 | 0.053 | 0.072 | 0.107 |
| **BBB** | 53 | | | | | | |
| Leverage ratio | | 0.53 | 0.20 | 0.27 | 0.50 | 0.78 | 0.97 |
| Equity volatility | | 0.66 | 0.32 | 0.39 | 0.47 | 0.97 | 1.30 |
| Asset volatility | | 0.28 | 0.06 | 0.15 | 0.29 | 0.34 | 0.52 |
| Payout ratio | | 0.052 | 0.018 | 0.029 | 0.041 | 0.064 | 0.076 |
| | | 2002Q3-2007Q2 | | | | | |
| **AAA/AA** | 12 | | | | | | |
| Leverage ratio | | 0.28 | 0.04 | 0.10 | 0.36 | 0.45 | 0.49 |
| Equity volatility | | 0.26 | 0.16 | 0.20 | 0.24 | 0.32 | 0.35 |
| Asset volatility | | 0.19 | 0.12 | 0.14 | 0.20 | 0.22 | 0.25 |
| Payout ratio | | 0.033 | 0.019 | 0.022 | 0.025 | 0.046 | 0.052 |
| **A** | 29 | | | | | | |
| Leverage ratio | | 0.22 | 0.13 | 0.15 | 0.20 | 0.30 | 0.35 |
| Equity volatility | | 0.30 | 0.18 | 0.22 | 0.30 | 0.40 | 0.44 |
| Asset volatility | | 0.24 | 0.15 | 0.18 | 0.23 | 0.30 | 0.34 |
| Payout ratio | | 0.045 | 0.023 | 0.031 | 0.047 | 0.054 | 0.061 |
| **BBB** | 46 | | | | | | |
| Leverage ratio | | 0.35 | 0.19 | 0.22 | 0.28 | 0.41 | 0.72 |
| Equity volatility | | 0.40 | 0.27 | 0.35 | 0.40 | 0.45 | 0.51 |
| Asset volatility | | 0.27 | 0.11 | 0.22 | 0.29 | 0.33 | 0.34 |
| Payout ratio | | 0.043 | 0.017 | 0.020 | 0.044 | 0.059 | 0.069 |
| | | 2007Q3-2012Q4 | | | | | |
| **AAA/AA** | 10 | | | | | | |
| Leverage ratio | | 0.24 | 0.08 | 0.09 | 0.13 | 0.52 | 0.64 |
| Equity volatility | | 0.24 | 0.18 | 0.20 | 0.25 | 0.28 | 0.30 |
| Asset volatility | | 0.19 | 0.11 | 0.13 | 0.21 | 0.25 | 0.27 |
| Payout ratio | | 0.051 | 0.026 | 0.030 | 0.050 | 0.061 | 0.085 |
| **A** | 22 | | | | | | |
| Leverage ratio | | 0.36 | 0.15 | 0.17 | 0.22 | 0.52 | 0.75 |
| Equity volatility | | 0.34 | 0.19 | 0.26 | 0.33 | 0.39 | 0.46 |
| Asset volatility | | 0.21 | 0.11 | 0.15 | 0.21 | 0.29 | 0.33 |
| Payout ratio | | 0.076 | 0.027 | 0.053 | 0.068 | 0.101 | 0.138 |
| **BBB** | 24 | | | | | | |
| Leverage ratio | | 0.72 | 0.38 | 0.60 | 0.68 | 0.97 | 0.98 |
| Equity volatility | | 0.93 | 0.47 | 0.72 | 1.00 | 1.27 | 1.32 |
| Asset volatility | | 0.29 | 0.05 | 0.07 | 0.27 | 0.48 | 0.58 |
| Payout ratio | | 0.062 | 0.029 | 0.031 | 0.041 | 0.067 | 0.078 |



**Table 1** *Firm summary statistics, investment grade bonds.* For each bond transaction, the leverage ratio, equity volatility, asset volatility, and payout ratio is calculated for the issuing firm on the day of the transaction. This table shows the distribution of firm values across transactions in the sample. Leverage ratio is the ratio of the book value of debt to the market value of equity plus the book value of debt. Equity volatility is the annualized volatility of daily equity returns from the last three years. Asset volatility is the unlevered equity volatility as explained in the text. Payout ratio is yearly interest payments plus dividends plus share repurchases divided by firm value. Bond transactions cover the period 2002Q3-2012Q4 and are from TRACE while firm variables are based on data from CRSP and Compustat.

| | Firms | Mean | 10th | 25th | Median | 75th | 90th |
|---|---|---|---|---|---|---|---|
| Full sample period (2002Q3-2012Q2) | | | | | | | |
| **BB** | 22 | | | | | | |
| Leverage ratio | | 0.71 | 0.30 | 0.58 | 0.72 | 0.97 | 0.97 |
| Equity volatility | | 0.93 | 0.39 | 0.53 | 0.75 | 1.35 | 1.35 |
| Asset volatility | | 0.25 | 0.06 | 0.08 | 0.22 | 0.34 | 0.52 |
| Payout ratio | | 0.042 | 0.029 | 0.035 | 0.038 | 0.042 | 0.053 |
| **B** | 9 | | | | | | |
| Leverage ratio | | 0.66 | 0.31 | 0.45 | 0.74 | 0.83 | 0.93 |
| Equity volatility | | 0.60 | 0.30 | 0.44 | 0.70 | 0.74 | 0.74 |
| Asset volatility | | 0.23 | 0.08 | 0.15 | 0.23 | 0.29 | 0.33 |
| Payout ratio | | 0.042 | 0.030 | 0.038 | 0.040 | 0.048 | 0.054 |
| **C** | 4 | | | | | | |
| Leverage ratio | | 0.87 | 0.78 | 0.82 | 0.88 | 0.93 | 0.96 |
| Equity volatility | | 0.73 | 0.34 | 0.70 | 0.73 | 0.92 | 0.98 |
| Asset volatility | | 0.17 | 0.04 | 0.05 | 0.19 | 0.25 | 0.29 |
| Payout ratio | | 0.050 | 0.040 | 0.045 | 0.051 | 0.056 | 0.061 |
| 2002Q3-2007Q2 | | | | | | | |
| **BB** | 18 | | | | | | |
| Leverage ratio | | 0.51 | 0.22 | 0.31 | 0.46 | 0.73 | 0.91 |
| Equity volatility | | 0.45 | 0.31 | 0.33 | 0.46 | 0.54 | 0.62 |
| Asset volatility | | 0.25 | 0.05 | 0.19 | 0.28 | 0.33 | 0.34 |
| Payout ratio | | 0.054 | 0.032 | 0.039 | 0.046 | 0.056 | 0.072 |
| **B** | 6 | | | | | | |
| Leverage ratio | | 0.64 | 0.31 | 0.34 | 0.55 | 0.92 | 0.95 |
| Equity volatility | | 0.52 | 0.29 | 0.31 | 0.65 | 0.70 | 0.73 |
| Asset volatility | | 0.18 | 0.06 | 0.09 | 0.20 | 0.22 | 0.37 |
| Payout ratio | | 0.040 | 0.029 | 0.031 | 0.039 | 0.048 | 0.054 |
| **C** | 3 | | | | | | |
| Leverage ratio | | 0.89 | 0.76 | 0.87 | 0.92 | 0.93 | 0.96 |
| Equity volatility | | 0.73 | 0.31 | 0.53 | 0.80 | 0.98 | 0.99 |
| Asset volatility | | 0.14 | 0.04 | 0.05 | 0.13 | 0.21 | 0.26 |
| Payout ratio | | 0.045 | 0.038 | 0.041 | 0.044 | 0.047 | 0.054 |
| 2007Q3-2012Q2 | | | | | | | |
| **BB** | 11 | | | | | | |
| Leverage ratio | | 0.76 | 0.35 | 0.64 | 0.77 | 0.97 | 0.97 |
| Equity volatility | | 1.05 | 0.48 | 0.70 | 1.33 | 1.35 | 1.35 |
| Asset volatility | | 0.25 | 0.07 | 0.08 | 0.18 | 0.36 | 0.57 |
| Payout ratio | | 0.040 | 0.028 | 0.035 | 0.038 | 0.040 | 0.042 |
| **B** | 7 | | | | | | |
| Leverage ratio | | 0.68 | 0.34 | 0.73 | 0.75 | 0.77 | 0.80 |
| Equity volatility | | 0.66 | 0.43 | 0.68 | 0.73 | 0.74 | 0.74 |
| Asset volatility | | 0.28 | 0.17 | 0.26 | 0.27 | 0.31 | 0.33 |
| Payout ratio | | 0.044 | 0.038 | 0.039 | 0.040 | 0.048 | 0.065 |
| **C** | 3 | | | | | | |
| Leverage ratio | | 0.86 | 0.78 | 0.80 | 0.85 | 0.92 | 0.97 |
| Equity volatility | | 0.73 | 0.53 | 0.72 | 0.73 | 0.77 | 0.94 |
| Asset volatility | | 0.19 | 0.04 | 0.11 | 0.20 | 0.28 | 0.30 |
| Payout ratio | | 0.055 | 0.046 | 0.051 | 0.055 | 0.059 | 0.063 |



**Table 2** *Firm summary statistics, speculative grade bonds.* For each bond transaction, the leverage ratio, equity volatility, asset volatility, and payout ratio is calculated for the issuing firm on the day of the transaction. This table shows the distribution of firm values across transactions in the sample. Leverage ratio is the ratio of the book value of debt to the market value of equity plus the book value of debt. Equity volatility is the annualized volatility of daily equity returns from the last three years. Asset volatility is the unlevered equity volatility as explained in the text. Payout ratio is yearly interest payments plus dividends plus share repurchases divided by firm value. Bond transactions cover the period 2002Q3-2012Q4 and are from TRACE while firm variables are based on data from CRSP and Compustat.

| | Bond maturity | | | |
|---|---|---|---|---|
| | 0-1y | 1-3y | 3-5y | 5-30y |
| **AAA/AA** | | | | |
| Number of bonds | 84 | 76 | 48 | 40 |
| Mean number of bonds pr quarter | 6.96 | 9.75 | 5.73 | 5.84 |
| Mean number of transactions pr quarter | 300.2 | 798.9 | 703.2 | 472.6 |
| Age | 4.57 | 3.16 | 1.74 | 7.77 |
| Coupon | 4.90 | 4.69 | 4.72 | 6.28 |
| Amount outstanding ($mm) | 371 | 433 | 619 | 512 |
| Trade size (in 1,000) | 421 | 380 | 361 | 330 |
| Time-to-maturity | 0.56 | 1.98 | 4.10 | 8.07 |
| **A** | | | | |
| Number of bonds | 115 | 125 | 87 | 70 |
| Mean number of bonds pr quarter | 9.1 | 14.8 | 10 | 9.96 |
| Mean number of transactions pr quarter | 404.7 | 1014 | 672.7 | 809.4 |
| Age | 7.18 | 5.75 | 5.39 | 7.65 |
| Coupon | 5.58 | 5.63 | 5.94 | 6.92 |
| Amount outstanding ($mm) | 461 | 578 | 674 | 640 |
| Trade size (in 1,000) | 245 | 204 | 229 | 275 |
| Time-to-maturity | 0.54 | 1.99 | 3.95 | 11.02 |
| **BBB** | | | | |
| Number of bonds | 114 | 120 | 86 | 84 |
| Mean number of bonds pr quarter | 7.9 | 12.9 | 8.98 | 17.9 |
| Mean number of transactions pr quarter | 472.4 | 1225 | 715.3 | 923.1 |
| Age | 5.60 | 4.62 | 3.51 | 6.20 |
| Coupon | 5.56 | 6.06 | 5.77 | 7.10 |
| Amount outstanding ($mm) | 405 | 578 | 437 | 491 |
| Trade size (in 1,000) | 223 | 265 | 371 | 384 |
| Time-to-maturity | 0.58 | 1.91 | 3.87 | 13.66 |
| **BB** | | | | |
| Number of bonds | 25 | 30 | 16 | 10 |
| Mean number of bonds pr quarter | 2.73 | 3.5 | 2.28 | 2.45 |
| Mean number of transactions pr quarter | 285.2 | 494.9 | 328.9 | 431.8 |
| Age | 4.52 | 4.16 | 6.62 | 11.88 |
| Coupon | 5.46 | 5.67 | 6.27 | 8.18 |
| Amount outstanding ($mm) | 580 | 591 | 625 | 255 |
| Trade size (in 1,000) | 299 | 448 | 476 | 190 |
| Time-to-maturity | 0.66 | 1.92 | 3.64 | 12.83 |
| **B** | | | | |
| Number of bonds | 3 | 3 | 1 | 9 |
| Mean number of bonds pr quarter | 1.13 | 1.08 | 1 | 1.8 |
| Mean number of transactions pr quarter | 106.2 | 117.3 | 2 | 173.6 |
| Age | 4.74 | 3.73 | 11.52 | 14.08 |
| Coupon | 5.43 | 3.95 | 7.77 | 8.56 |
| Amount outstanding ($mm) | 395 | 232 | 1 | 179 |
| Trade size (in 1,000) | 179 | 192 | 2500 | 123 |
| Time-to-maturity | 0.47 | 1.81 | 3.51 | 13.37 |
| **C** | | | | |
| Number of bonds | 0 | 0 | 1 | 5 |
| Mean number of bonds pr quarter | NaN | NaN | 1 | 1.79 |
| Mean number of transactions pr quarter | NaN | NaN | 330.7 | 441.6 |
| Age | NaN | NaN | 25.19 | 16.47 |
| Coupon | NaN | NaN | 9.00 | 9.03 |
| Amount outstanding ($mm) | NaN | NaN | 100 | 190 |
| Trade size (in 1,000) | NaN | NaN | 33 | 106 |
| Time-to-maturity | NaN | NaN | 4.83 | 12.78 |



**Table 3** *Bond summary statistics.* Only unsecured senior industrial bonds with a fixed coupon that are not callable, putable, perpetual, asset-backed, convertable, Yankee, foreign currency, and do not contain sinking fund provisions or covenants are used. All transactions in those bonds for which there are also firm variables in Crisp/Compustat are used. This table shows summary statistics for the bond transactions. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

| | Bond maturity | | | |
|---|---|---|---|---|
| | 6m | 2y | 4y | 10y |
| Full sample period (2002Q3-2012Q2) | | | | |
| **AAA/AA** | | | | |
| $E[s(\theta_{it})]$ | 0 | 2 | 11 | 20 |
| $s(E[\theta_{it}])$ | 0 | 0 | 2 | 8 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.00 | 0.00 | 0.15 | 0.39 |
| **A** | | | | |
| $E[s(\theta_{it})]$ | 103 | 127 | 81 | 100 |
| $s(E[\theta_{it}])$ | 0 | 1 | 10 | 56 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.00 | 0.01 | 0.13 | 0.56 |
| **BBB** | | | | |
| $E[s(\theta_{it})]$ | 601 | 565 | 279 | 210 |
| $s(E[\theta_{it}])$ | 0 | 77 | 94 | 162 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.00 | 0.14 | 0.34 | 0.77 |
| **BB** | | | | |
| $E[s(\theta_{it})]$ | 972 | 483 | 500 | 281 |
| $s(E[\theta_{it}])$ | 4 | 46 | 321 | 302 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.00 | 0.10 | 0.64 | 1.07 |
| 2002Q3-2007Q2 | | | | |
| **AAA/AA** | | | | |
| $E[s(\theta_{it})]$ | 0 | 4 | 16 | 13 |
| $s(E[\theta_{it}])$ | 0 | 0 | 3 | 5 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.00 | 0.01 | 0.20 | 0.41 |
| **A** | | | | |
| $E[s(\theta_{it})]$ | 3 | 14 | 18 | 36 |
| $s(E[\theta_{it}])$ | 0 | 0 | 3 | 26 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.00 | 0.00 | 0.18 | 0.71 |
| **BBB** | | | | |
| $E[s(\theta_{it})]$ | 1 | 20 | 41 | 82 |
| $s(E[\theta_{it}])$ | 0 | 1 | 13 | 84 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.00 | 0.06 | 0.31 | 1.03 |
| **BB** | | | | |
| $E[s(\theta_{it})]$ | 170 | 189 | 313 | 230 |
| $s(E[\theta_{it}])$ | 0 | 17 | 168 | 254 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.00 | 0.09 | 0.54 | 1.10 |
| 2007Q3-2012Q2 | | | | |
| **AAA/AA** | | | | |
| $E[s(\theta_{it})]$ | 0 | 1 | 6 | 27 |
| $s(E[\theta_{it}])$ | 0 | 0 | 1 | 11 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.00 | 0.00 | 0.09 | 0.41 |
| **A** | | | | |
| $E[s(\theta_{it})]$ | 207 | 240 | 149 | 163 |
| $s(E[\theta_{it}])$ | 0 | 10 | 28 | 107 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.00 | 0.04 | 0.19 | 0.65 |
| **BBB** | | | | |
| $E[s(\theta_{it})]$ | 1200 | 1111 | 539 | 338 |
| $s(E[\theta_{it}])$ | 14 | 430 | 314 | 275 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.01 | 0.39 | 0.58 | 0.81 |
| **BB** | | | | |
| $E[s(\theta_{it})]$ | 2308 | 924 | 755 | 336 |
| $s(E[\theta_{it}])$ | 268 | 138 | 558 | 354 |
| $s(E[\theta_{it}])/E[s(\theta_{it})]$ | 0.12 | 0.15 | 0.74 | 1.05 |

**Table 4** *Convexity bias when calculating yield spreads in the Merton model using average firm variables.* It is a common approach to compare average actual spreads to model-implied spreads, where the model-implied spreads are calculated by using average firm variables. This introduces a bias because the spread in structural models is a non-linear function of firm variables. This table shows the magnitude of this bias. To calculate $E[s(\theta_{it})]$ we compute for every transaction in the sample the volume-weighted average Merton spread on a monthly basis and then average the monthly average spreads over the sample period. To calculate $s(E[\theta_{it}])$ we compute on a monthly basis the volume-weighted average firm variables (leverage ratio, asset volatility, payout ratio, maturity, bond maturity, riskfree rate at same as bond), average the monthly firm variables over the sample period, and then use the averaged firm variables to calculate the spread in the Merton spread. Bond transactions are grouped into groups where the issued bond has remaining maturity 0-1y, 1-3y, 3-5y, and 5-30y.

| | Bond maturity | | | |
|---|---|---|---|---|
| | 6m | 2y | 4y | 10y |
| Full sample period (2002Q3-2012Q2) | | | | |
| **AAA/AA** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 0.00 | 0.05 | 0.33 | 1.22 |
| $\text{PD}(E[\theta_{it}])$ | 0.00 | 0.00 | 0.03 | 0.24 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.00 | 0.00 | 0.10 | 0.20 |
| **A** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 0.61 | 2.17 | 2.29 | 7.43 |
| $\text{PD}(E[\theta_{it}])$ | 0.00 | 0.02 | 0.31 | 4.05 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.00 | 0.01 | 0.14 | 0.54 |
| **BBB** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 4.83 | 11.28 | 10.13 | 14.97 |
| $\text{PD}(E[\theta_{it}])$ | 0.00 | 1.81 | 4.00 | 14.05 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.00 | 0.16 | 0.40 | 0.94 |
| **BB** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 7.64 | 10.15 | 18.58 | 26.93 |
| $\text{PD}(E[\theta_{it}])$ | 0.03 | 1.01 | 14.46 | 32.86 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.00 | 0.10 | 0.78 | 1.22 |
| 2002Q3-2007Q2 | | | | |
| **AAA/AA** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 0.00 | 0.08 | 0.55 | 0.68 |
| $\text{PD}(E[\theta_{it}])$ | 0.00 | 0.00 | 0.07 | 0.14 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.00 | 0.01 | 0.12 | 0.21 |
| **A** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 0.04 | 0.34 | 0.77 | 2.90 |
| $\text{PD}(E[\theta_{it}])$ | 0.00 | 0.00 | 0.08 | 1.53 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.00 | 0.00 | 0.11 | 0.53 |
| **BBB** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 0.01 | 0.55 | 1.89 | 6.91 |
| $\text{PD}(E[\theta_{it}])$ | 0.00 | 0.02 | 0.46 | 7.41 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.00 | 0.03 | 0.25 | 1.07 |
| **BB** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 1.95 | 4.75 | 14.13 | 21.02 |
| $\text{PD}(E[\theta_{it}])$ | 0.00 | 0.35 | 6.52 | 27.18 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.00 | 0.07 | 0.46 | 1.29 |
| 2007Q3-2012Q2 | | | | |
| **AAA/AA** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 0.00 | 0.01 | 0.11 | 1.75 |
| $\text{PD}(E[\theta_{it}])$ | 0.00 | 0.00 | 0.01 | 0.37 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.00 | 0.00 | 0.09 | 0.21 |
| **A** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 1.22 | 4.00 | 3.94 | 11.97 |
| $\text{PD}(E[\theta_{it}])$ | 0.00 | 0.19 | 0.98 | 9.34 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.00 | 0.05 | 0.25 | 0.78 |
| **BBB** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 9.66 | 22.01 | 19.13 | 23.02 |
| $\text{PD}(E[\theta_{it}])$ | 0.10 | 12.12 | 14.50 | 22.26 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.01 | 0.55 | 0.76 | 0.97 |
| **BB** | | | | |
| $E[\text{PD}(\theta_{it})]$ | 17.14 | 18.24 | 24.67 | 33.27 |
| $\text{PD}(E[\theta_{it}])$ | 2.62 | 3.22 | 27.12 | 38.66 |
| $\text{PD}(E[\theta_{it}])/E[\text{PD}(\theta_{it})]$ | 0.15 | 0.18 | 1.10 | 1.16 |

**Table 5** *Conexity bias when calculating model-implied default probabilities in the Merton model using average firm parameters.* It is a common approach to compare historical default frequencies to model-implied default probabilities, where model-implied default probabilities are calculated by using average firm variables. This introduces a bias because the default probability in structural models is a non-linear function of firm value parameters. This table shows the magnitude of this bias. To calculate $E[\text{PD}_T(\theta_{it})]$ we compute the volume-weighted average default probability implied by the Merton on a monthly basis and then average the monthly default probabilities over the sample period. To calculate $\text{PD}_T(E[\theta_{it}])$ we compute the volume-weighted average firm variables (leverage ratio, asset volatility, payout ratio, bond maturity, riskfree rate at same as bond), take the simple average of the monthly volume-weighted averages, and use the averaged firm variables to calculate the implied default probability in the Merton model.

| | Bond maturity | | | |
|---|---|---|---|---|
| | 6m | 2y | 4y | 10y |
| **AAA/AA** | | | | |
| Actual spread (bp) | 3<br>(−13;35) | 9<br>(−16;111) | 24<br>(−3;142) | 32<br>(3;112) |
| Model spread (bp) | 0<br>(0;0) | 0<br>(0;15) | 1<br>(0;41) | 2<br>(0;46) |
| **A** | | | | |
| Actual spread (bp) | 7<br>(−7;114) | 30<br>(−0;575) | 79<br>(7;1325) | 85<br>(24;297) |
| Model spread (bp) | 0<br>(0;0) | 1<br>(0;614) | 13<br>(0;1568) | 73<br>(0;311) |
| **BBB** | | | | |
| Actual spread (bp) | 104<br>(16;1249) | 375<br>(30;1635) | 277<br>(40;1262) | 95<br>(42;364) |
| Model spread (bp) | 0<br>(0;3247) | 1099<br>(0;2511) | 126<br>(1;1255) | 110<br>(23;272) |
| **BB** | | | | |
| Actual spread (bp) | 362<br>(68;689) | 512<br>(182;1028) | 604<br>(111;1000) | 173<br>(38;611) |
| Model spread (bp) | 2573<br>(0;4570) | 1809<br>(0;2866) | 1117<br>(0;1503) | 302<br>(65;370) |
| **B** | | | | |
| Actual spread (bp) | 128<br>(66;228) | 145<br>(35;250) | 463<br>(463;479) | 501<br>(264;1427) |
| Model spread (bp) | 0<br>(0;0) | 1<br>(0;484) | 856<br>(856;856) | 317<br>(62;484) |
| **C** | | | | |
| Actual spread (bp) | 0<br>(0;0) | 0<br>(0;0) | 6009<br>(2164;6811) | 843<br>(391;4581) |
| Model spread (bp) | 0<br>(0;0) | 0<br>(0;0) | 1483<br>(1031;1501) | 327<br>(147;486) |

**Table 6** *Actual and Merton-model yield spreads.* This table shows the actual and model-implied industrial bond spreads across maturity and rating. Bond transactions are grouped into groups where the issued bond has remaining maturity 0-1y, 1-3y, 3-5y, and 5-30y. 'Actual spread' is the volume-weighted median actual spread to the swap rate while 'Model spread' is the volume-weighted median spread implied by the Merton model. Below the spreads in parantheses are the 10% and 90% volume-weighted quantiles. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

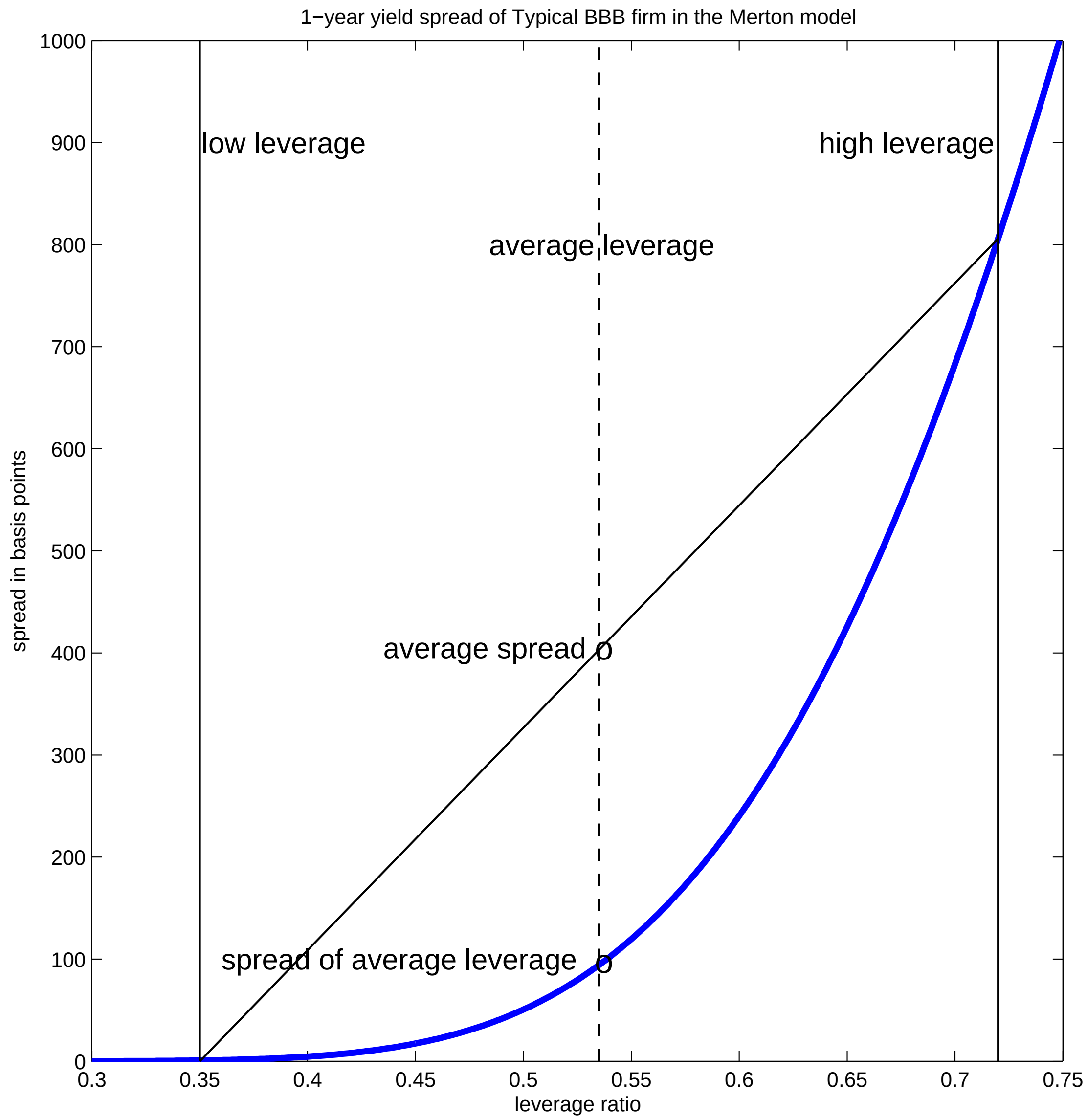
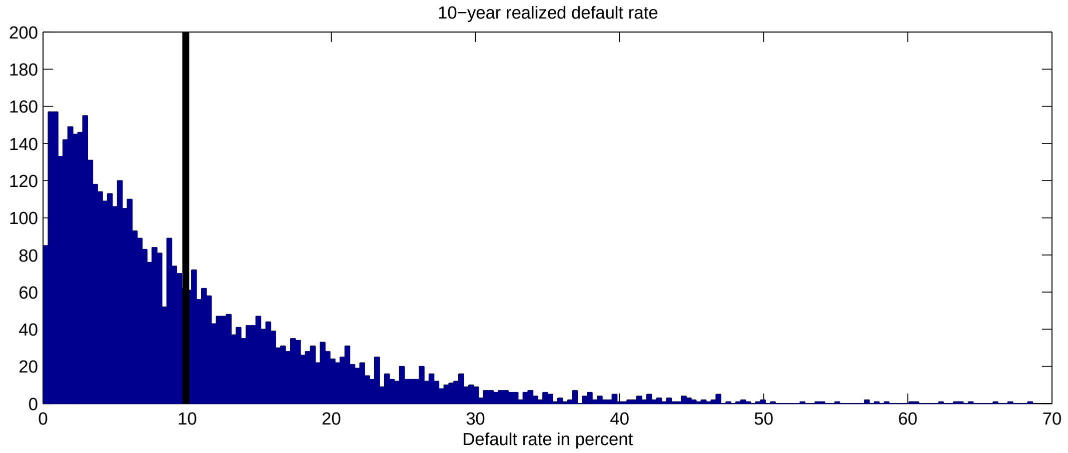



**Fig. 1** *Illustrating the convexity bias when calculating the spread in a structural model using average leverage and comparing it to the average spread.* It is a common approach to compare average actual spreads to model-implied spreads, where the model-implied spreads are calculated by using average firm variables. This introduces a bias because the spread in structural models is a non-linear function of firm variables. The figure illustrates the bias in case of leverage ratio for a typical BBB-rated firm. Asset volatility is 28%, dividend yield 5.2%, recovery rate 49.2%, and riskfree rate 5%



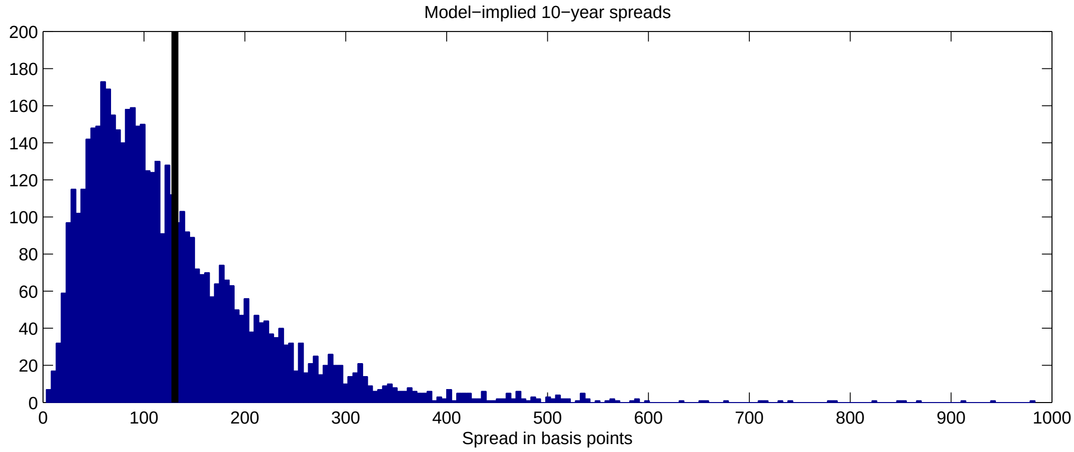




**Fig. 2** *Simulated default frequencies and model-implied spreads.* We assume that an index has 1,000 firms entering each month and the following month all firms exit and 1,000 new firms enter. This goes on for 20 years. All firms are identical when they enter the index. Each month we simulate the subsequent value of each firm and see if they are below the default threshold in 10 years. Half of the firm volatility is systematic and half is idiosyncratic. The idiosyncratic part is independent across firms in a given month and independent across firms in different months. Firm parameters are identical for all firms and chosen such that firms are typical BBB-rated firms. In each month, we calculate the realized default frequency on 10-year horizon and calculate the average across all months. The simulated realized average default probability is different from expected mainly because the economy has evolved differently than expected. We repeat this simulation 5,000 times and the top graph shows the distribution of realized 10-year cumulative default probabilities. The solid line is the expected cumulative default probability. For each of the 5,000 simulation we fit the Merton model to the realized default probability by finding the asset volatility that gives the realized default probability. With this asset volatility we calculate the model-implied 10-year spread. The bottom graph shows the distribution of model-implied spreads. The solid line is the actual spread.

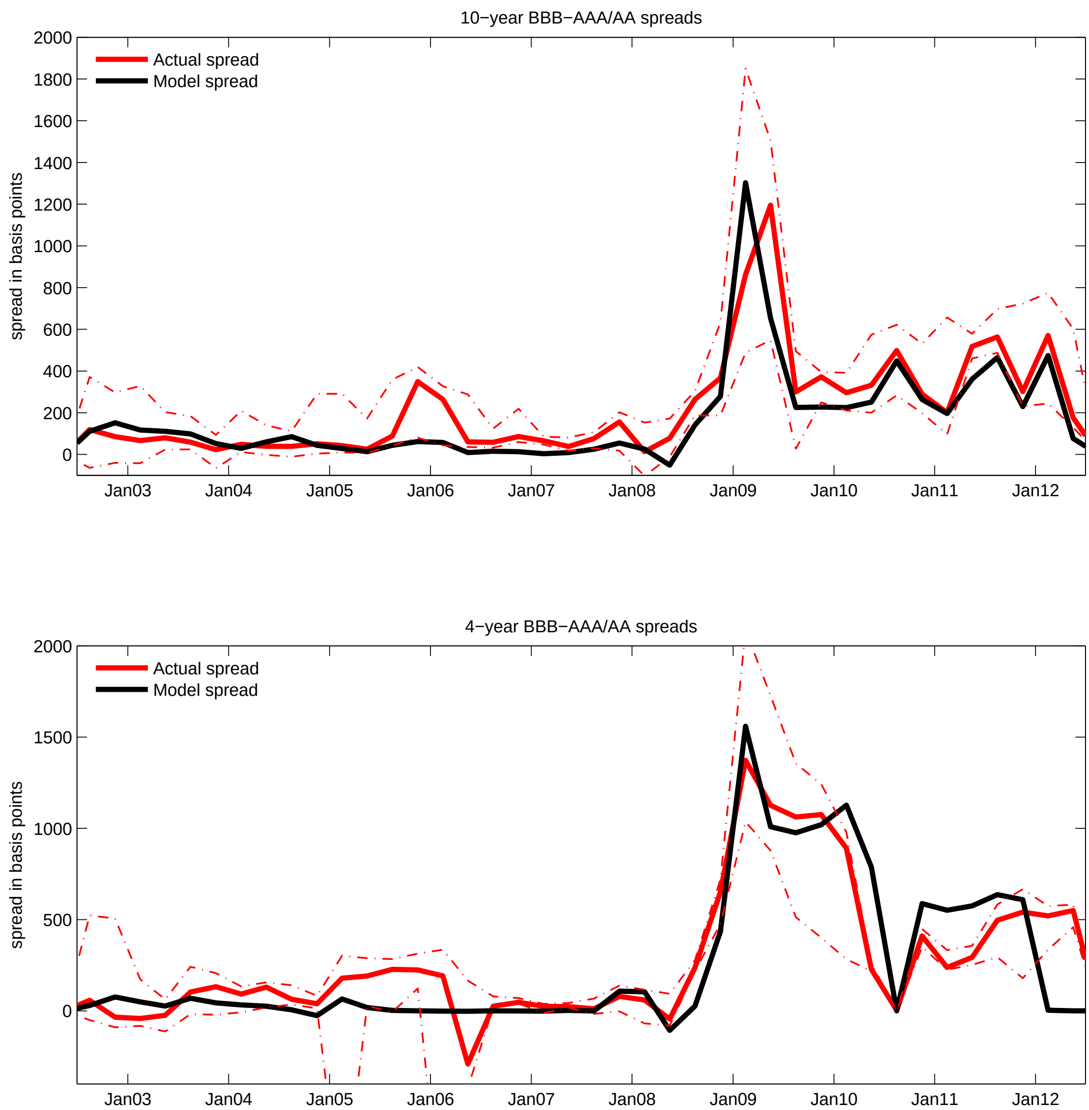



**Fig. 3** *BBB-AAA/AA corporate bond yield spreads.* This graph shows the time series variation of actual and model-implied AAA/AA-BBB spreads. On a quarterly basis all transactions in bonds rated AAA/AA and bonds rated BBB are collected, and the graph shows the volume-weighted median BBB spread minus the volume-weighted median AAA/AA spread. This is done for maturities 3-5y and 5-30y. Volume-weighted 10pct and 90pct quantiles are bootstrapped as explained in the text. The figure also shows the model-implied Merton spread, found by calculating the model-implied AA/AA-BBB spread computed in the same way as the actual spread. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

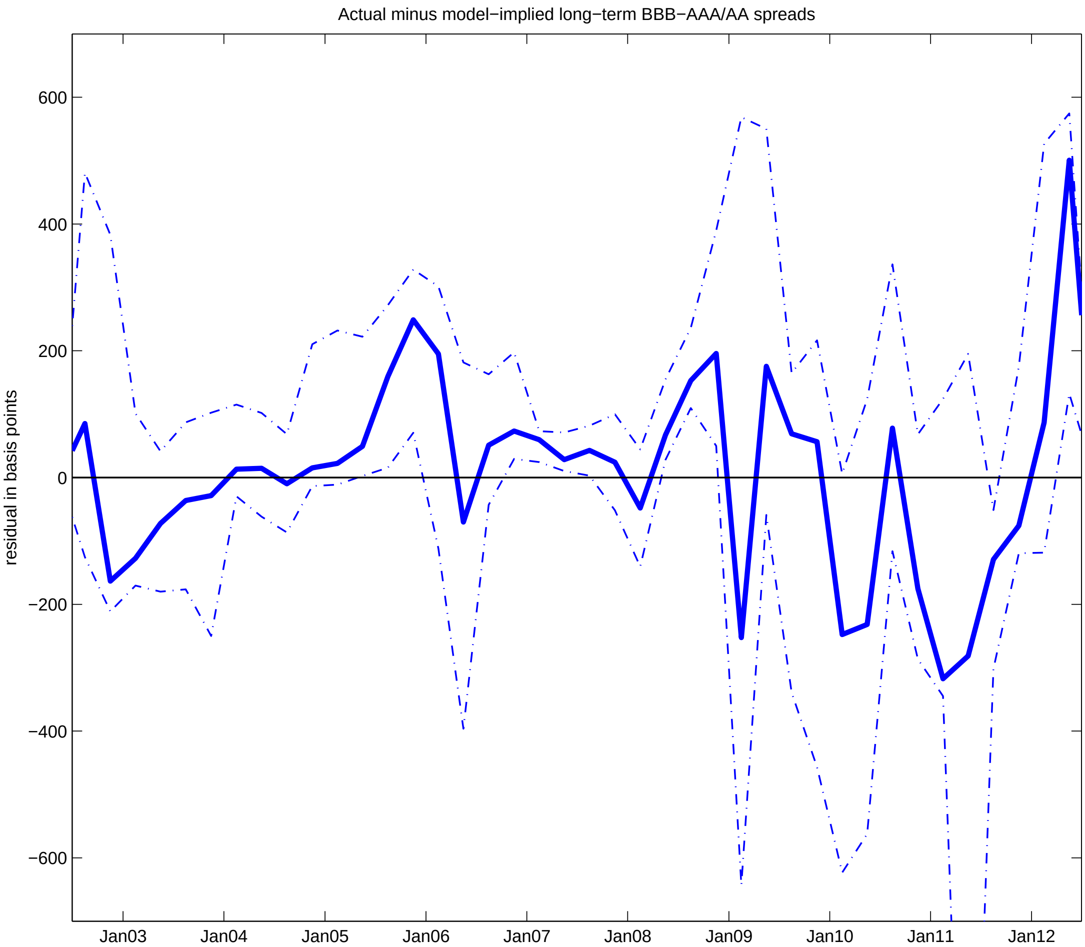



**Fig. 4** *Difference between actual and model-implied long-term BBB-AAA/AA spread.* On a quarterly basis all transactions in bonds rated AAA/AA and bonds rated BBB with a maturity of three years or more are collected. The graph shows the difference between the actual and model-implied BBB-AAA/AA spread. The BBB-AAA/AA spread is calculated as the volume-weighted median BBB spread minus volume-weighted median AAA/AA spread and volume-weighted 10pct and 90pct quantiles are bootstrapped as explained in the text.

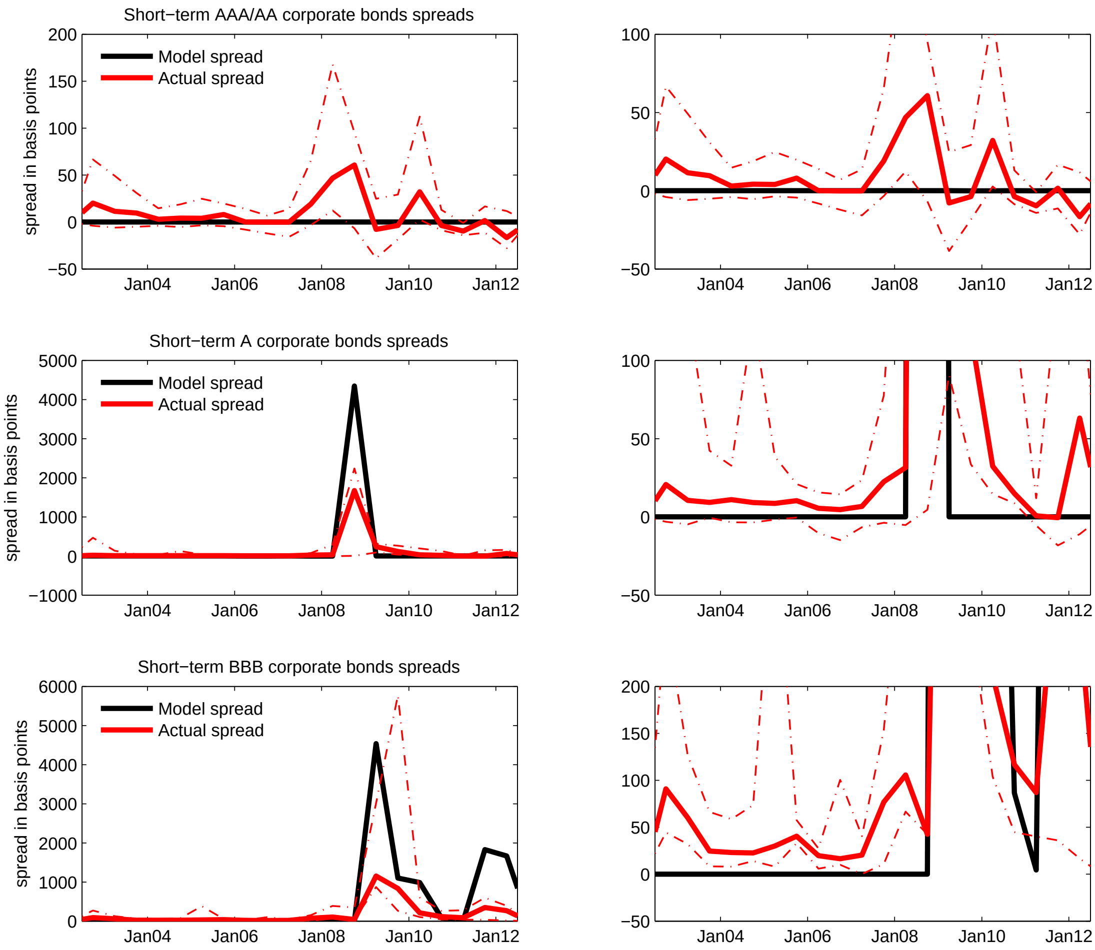



**Fig. 5** *Short-term corporate bond yield spreads.* This graph shows the time series variation of actual and model-implied short-term industrial corporate bond spreads. On a semi-annual basis all transactions in bonds maturing within one year on transaction day are collected. The Figure shows - for ratings AAA/AA, A, and BBB - the volume-weighted median actual spread along with volume-weighted 10pct and 90pct quantile. The figure also shows the model-implied Merton spread, found by calculating the model-implied spread for each transaction and computing the volume-weighted median. The figures on the right-hand side are the identical to the figures on the left-hand side except that the scale on the y-axis is different. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

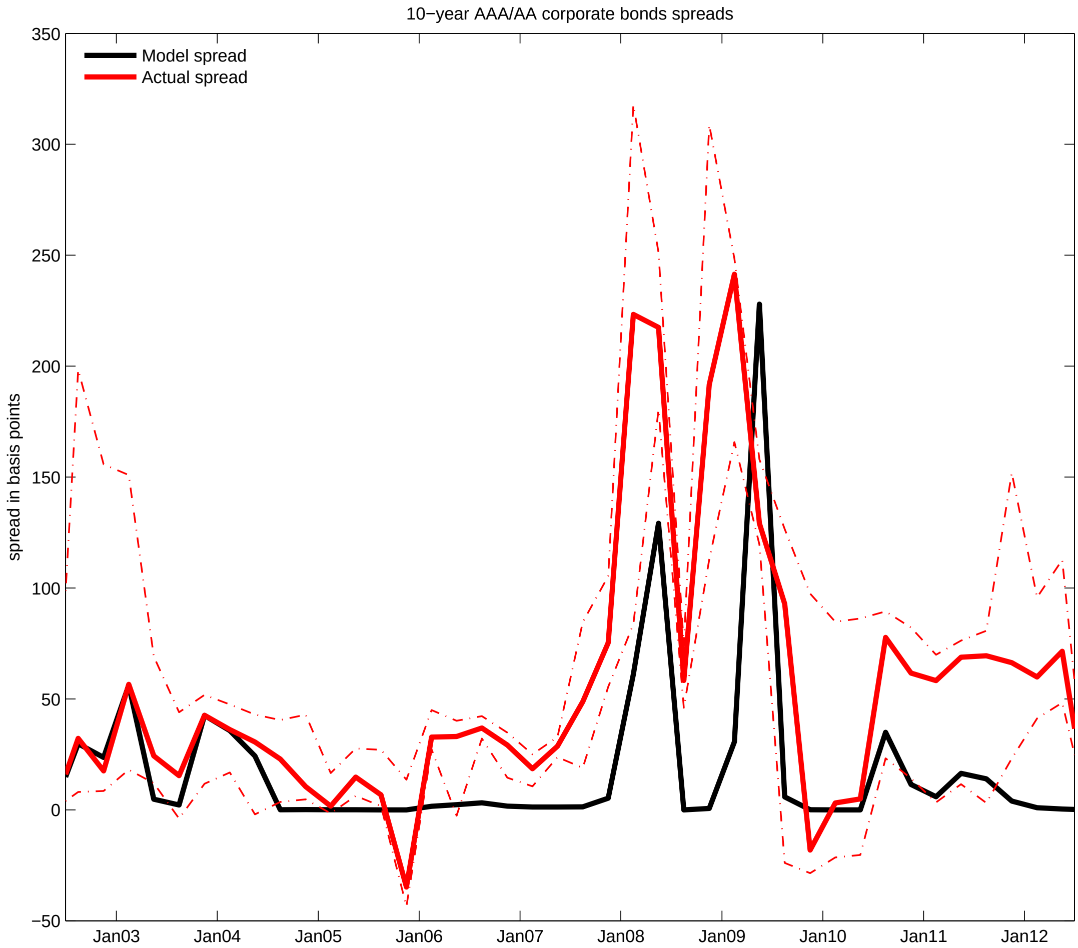



**Fig. 6** *10-year AAA/AA corporate bond yield spreads.* This graph shows the time series variation of actual and model-implied short-term industrial AAA/AA corporate bond spreads. On a quarterly basis all transactions in bonds with maturity between 5 and 30 years on transaction day are collected. The figure shows the volume-weighted median actual spread along with volume-weighted 10pct and 90pct quantiles. The figure also shows the model-implied Merton spread, found by calculating the model-implied spread for each transaction and computing the volume-weighted median. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

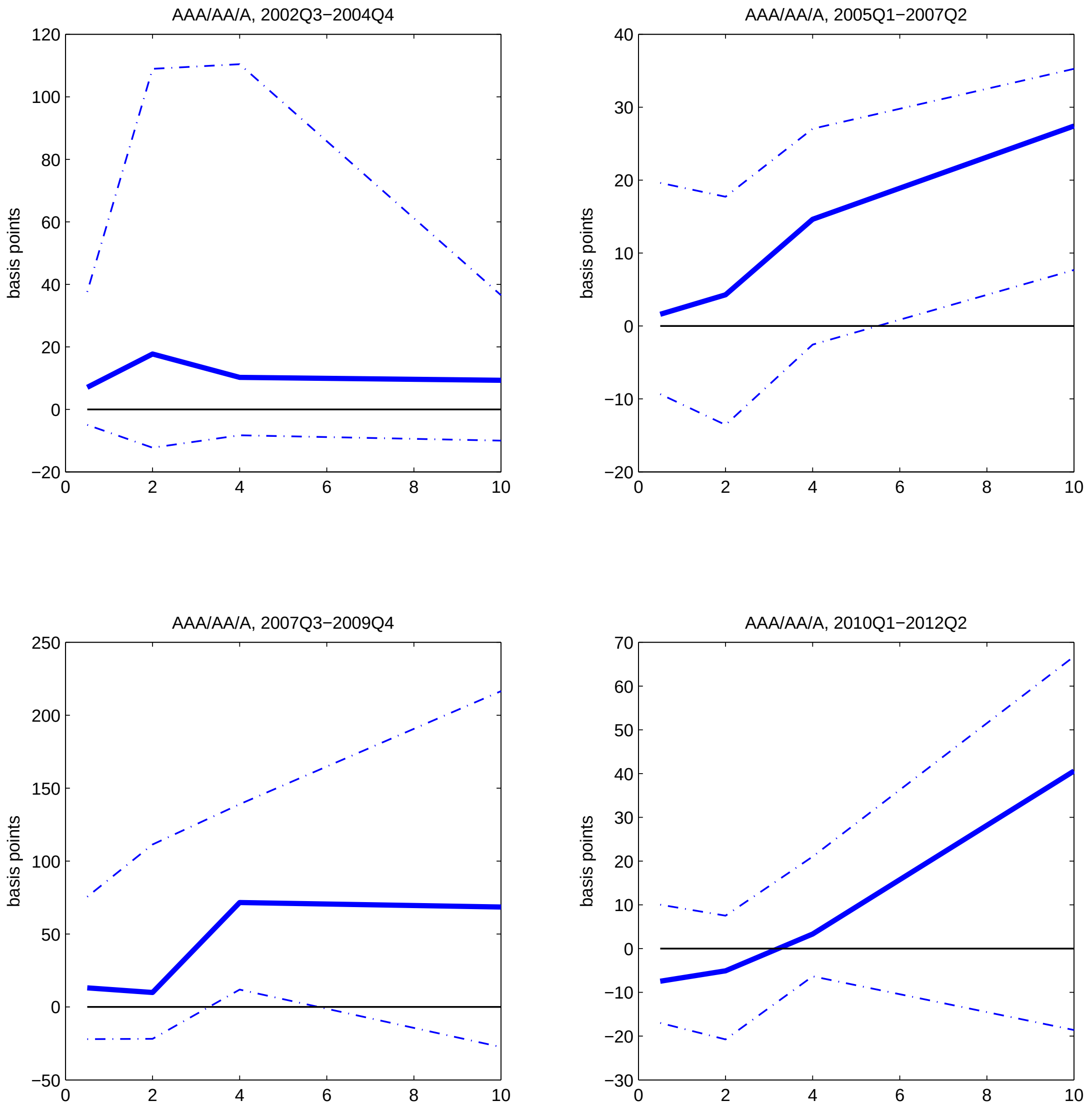



**Fig. 7** *Part of the corporate bond spread not explained by the Merton model, AAA/AA/A-rated bonds* For each transaction the pricing residual is defined as the actual spread minus the spread implied by the Merton model. The sample period is divided into four subperiods and for maturities 0-1y, 1-3y, 3-5y, and 5-30y the volume-weighted median residual along with 10pct and 90pct quantiles are calculated. The graph shows the result for all bonds with rating AAA,AA, or A. Bond transactions cover the period 2002Q3-2012Q2 and are from TRACE.

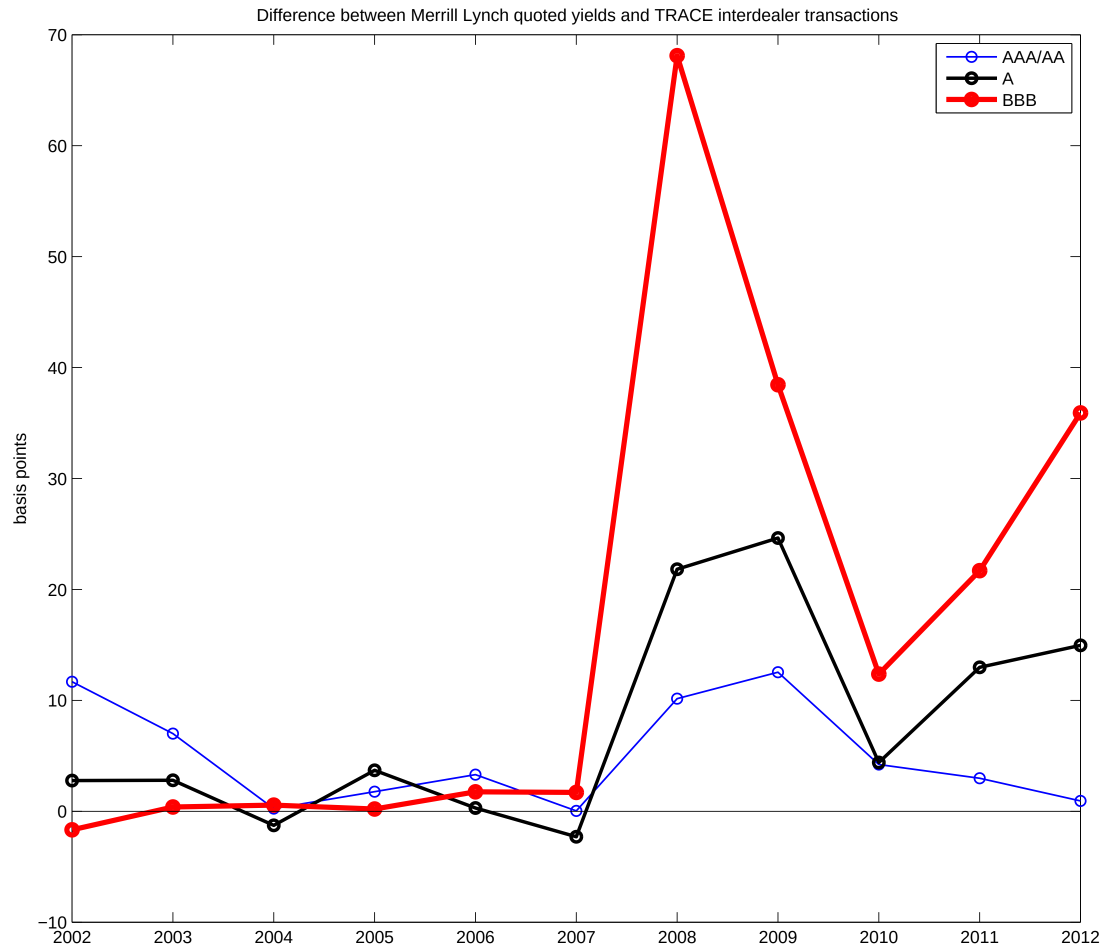
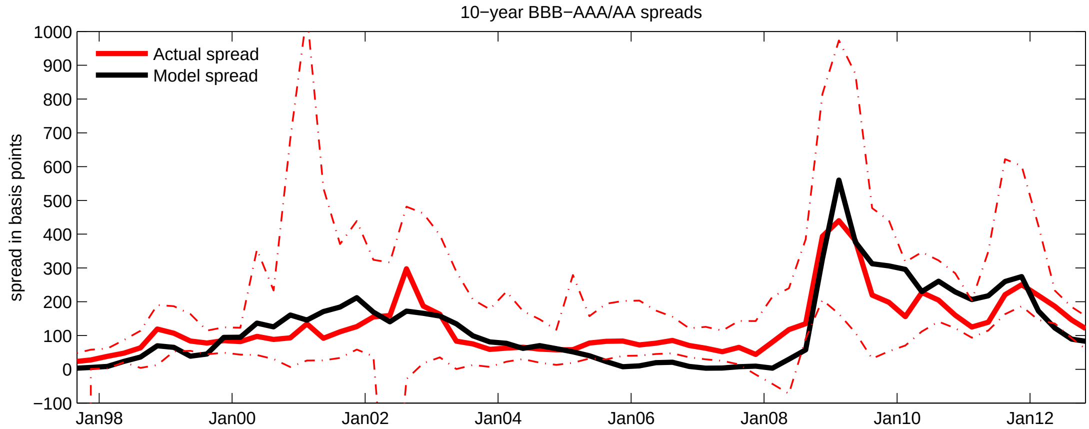



**Fig. 8** *Difference between Merril Lynch quoted yields and yields in TRACE interdealer transactions.* On a semi annual basis all interdealer transactions in TRACE are matched with a Merrill Lynch quote on the same day, if a quote exists. The difference between the Merrill Lynch quoted yield and the TRACE yield is calculated and the graph shows the semi-annual volume-weighted mean. The yield difference is winsorized at ±500 basis points. This is done separately for AAA/AA-, A-, and BBB-rated bonds.



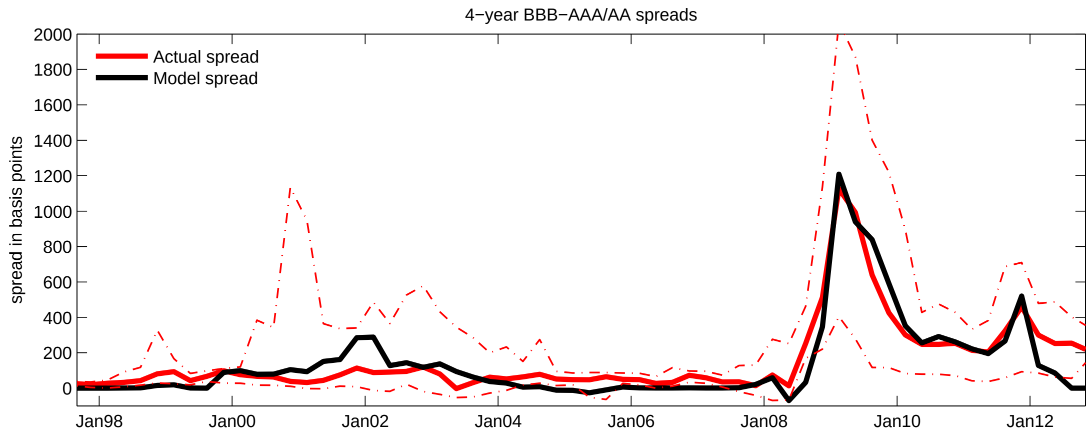




**Fig. 9** *BBB-AAA/AA corporate bond yield spreads using Merrill Lynch quotes.* This graph shows the time series variation of actual and model-implied AAA/AA-BBB spreads. On a quarterly basis all daily quotes in bonds rated AAA/AA and bonds rated BBB are collected, and the graph shows the median BBB spread minus the median AAA/AA spread. This is done for maturities 2.5-5.5y and 5-30y. 10pct and 90pct quantiles are bootstrapped as in Figure 3. The figure also shows the model-implied Merton spread, found by calculating the model-implied AA/AA-BBB spread computed in the same way as the actual spread. Daily bond quotes are from Merrill Lynch and cover the period 1997Q1-2012Q2.

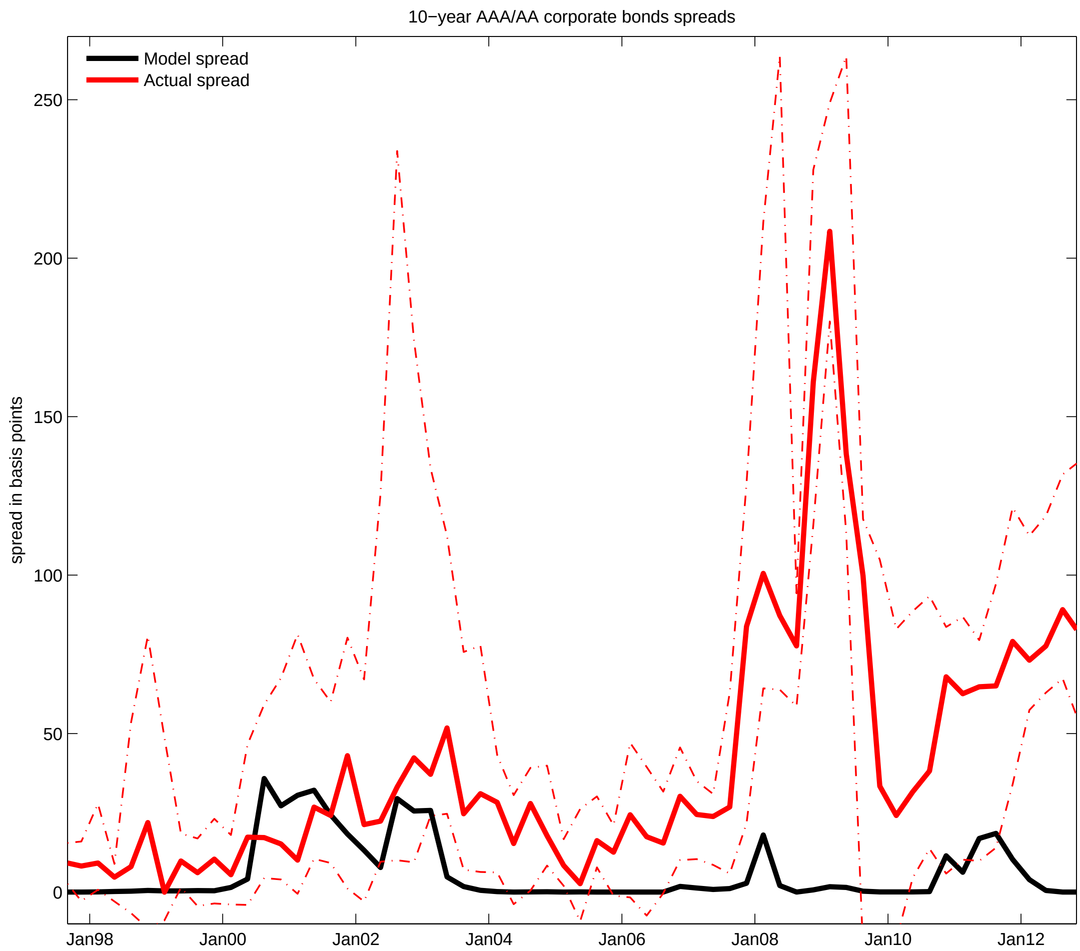



**Fig. 10** *10-year AAA/AA corporate bond yield spreads using Merrill Lynch quotes..* This graph shows the time series variation of actual and model-implied short-term industrial AAA/AA corporate bond spreads. On a quarterly basis all daily Merrill Lynch quotes in bonds with maturity between 5 and 30 years on transaction day are collected. The figure shows the median actual spread along with 10pct and 90pct quantiles. The figure also shows the model-implied Merton spread, found by calculating the model-implied spread for each quote and computing the median. Daily bond quotes are from Merrill Lynch and cover the period 1997Q1-2012Q2.