

# Collateral consistent derivatives pricing

FRIC Practitioner Seminar, CBS

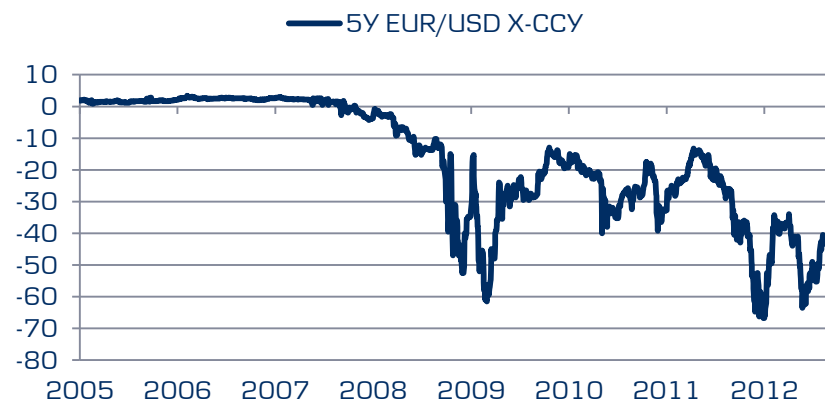
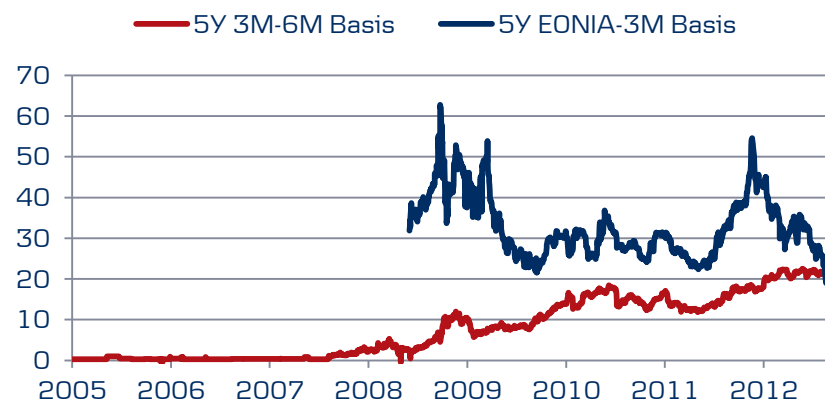
Martin D. Linderstrøm, [marlin@danskebank.dk](mailto:marlin@danskebank.dk)  
Counterparty Credit & Funding Risk

# Agenda

- The intuition behind collateral consistent pricing.
  - A benchmark case: A multi-currency calibration under EUR cash collateral.
- The complexities of a multi CSA book
  - Which collateral assumptions hold for calibration instruments?
  - The ISDA Standardized CSA approach
  - Market fragmentation between CCP cleared and bilateral trades?
- Case studies in curve calibration
  - What are reasonable bounds for forward curves?
  - Arbitrages in fragmented markets?
- Pricing and hedging discounting risks under different CSA regimes?
  - The collateral valuation adjustment.
  - The cheapest-to-deliver optionality in CSAs
  - Hedge ratios with and without optionality?

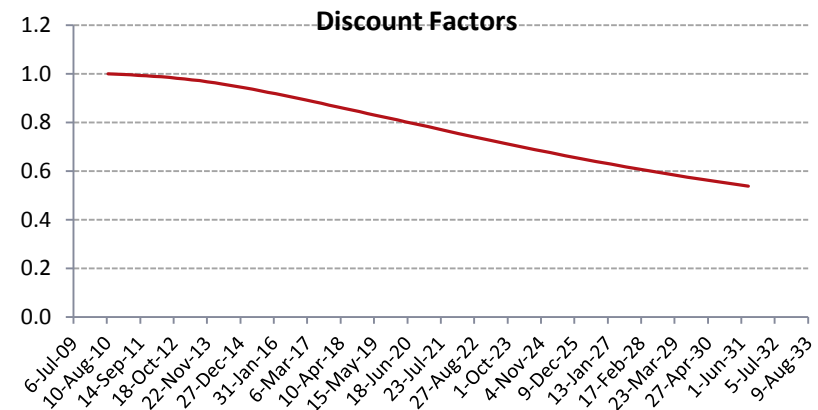
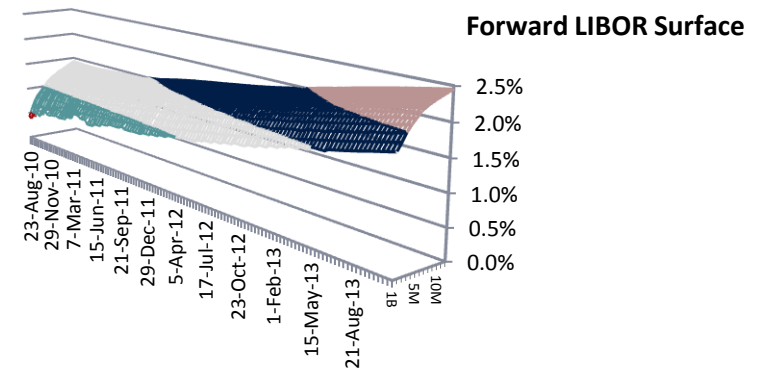
## Swaps in the old way

- In the “old” days (until Aug’07) many market participants had just one swap curve for each currency.
  - Forward rates - irrespective of tenor - were calculated on this.
  - Discount factors were also derived from this curve.
- This implicitly assumes:
  - No money market basis (e.g. 3s6s basis is zero).
  - No cross currency basis (e.g. EUR/USD basis is (close to) zero).
  - Traders can fund themselves at xIBOR.
  - Note that on a single curve, a Floating Rate Note trades at par at fixing time.
- These assumptions are no longer valid.



# Swaps in the new way

- Need for multiple projection curves for each currency.
  - We cannot compute 3M xIBOR and 6M xIBOR forwards on the same curve.
- Need for a single discounting curve for each currency.
  - This should reflect CCS spreads.
  - But what should be my anchor in terms of currency and credit premium?
  - If your trade is collateralised, you should discount with the collateral rate.
  - What is your collateral rate?



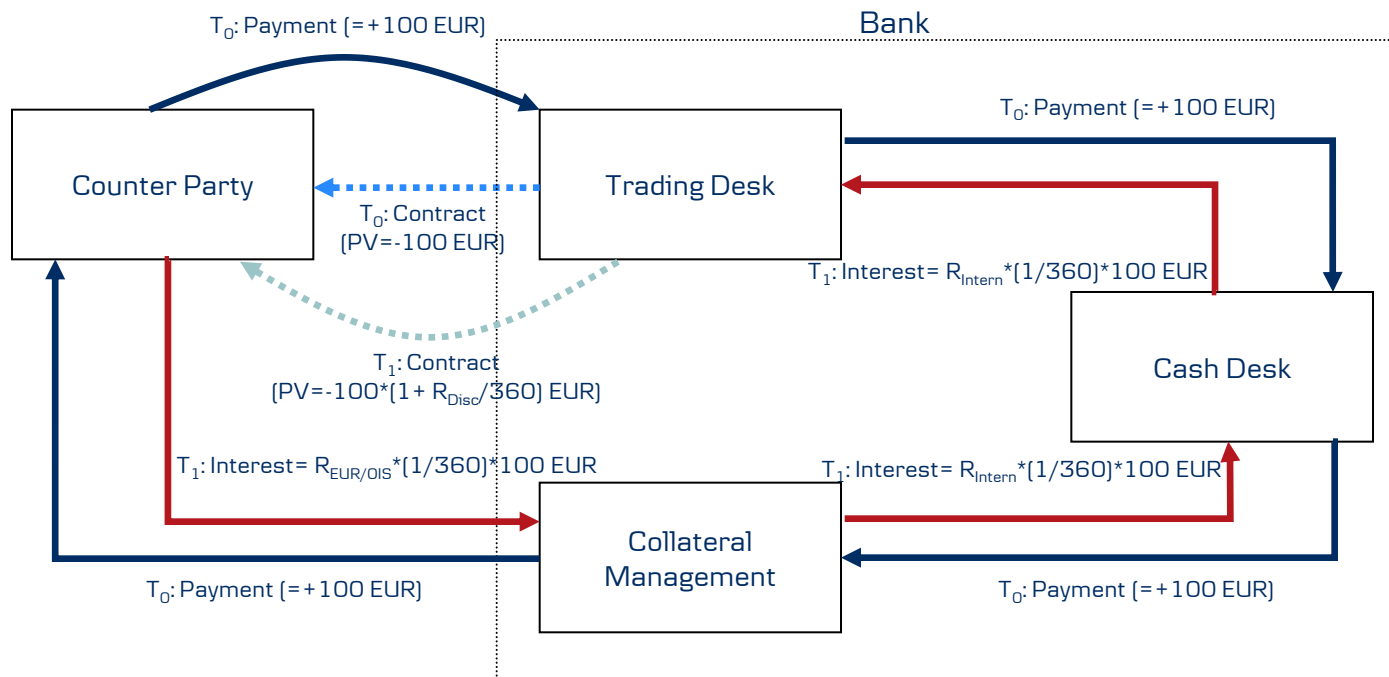


## The institutional setting

- The ISDA Master agreement
  - The legal umbrella underpinning netting.
  - Default and early termination provisions.
- ISDA Definitions
  - Sets standards for methodologies such as settlement of options, application of floating rates etc.
- ISDA Credit Support Annex (Credit Support Deed)
  - Defines the terms for collateralisation.
  - Sets Thresholds, Independent Amounts, Minimum Transfer Amounts and valuation frequency.
  - Eligible collateral and specifies interest earned.

# The intuition behind collateral consistent pricing

## Flow analysis: EUR Derivative - EUR Cash collateral

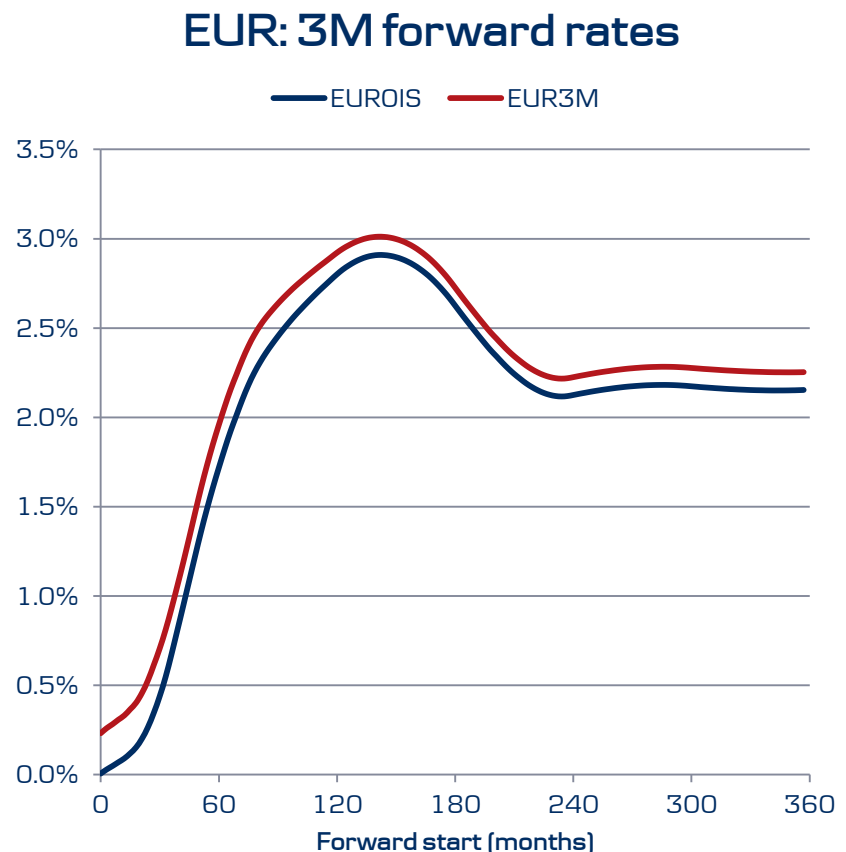


- Cash desk is passing through the liquidity - no haircuts or disagreement on valuation.
- Internal loop can be "closed" if  $r_{Intern} = r_{OIS}$
- See Piterbarg (2010).

- For the setup to be arbitrage free, the trader needs to be discounted at the rate his cash position earns, i.e.  $R_{Disc} = R_{OIS}$ .
- He could in principle hedge his cash exposure via an EONIA swap.

# A benchmark case: A multi-currency calibration under EUR cash collateral

- Stylised market:
  - Only IRSs against 3M xIBOR.
  - 3M xIBOR-OIS basis swaps.
  - X-CCY basis swaps against 3M XIBOR.
  - Only swap instruments 1-30Y.
- Setup
  - Separate forward and discounting curves.
  - Single collateral assumption – all products are EUR cash collateralised.
  - Want a CCS consistent valuation setup.
- Approach
  - Calibrate jointly EUR3M and EUROIS=EURDISC curves.

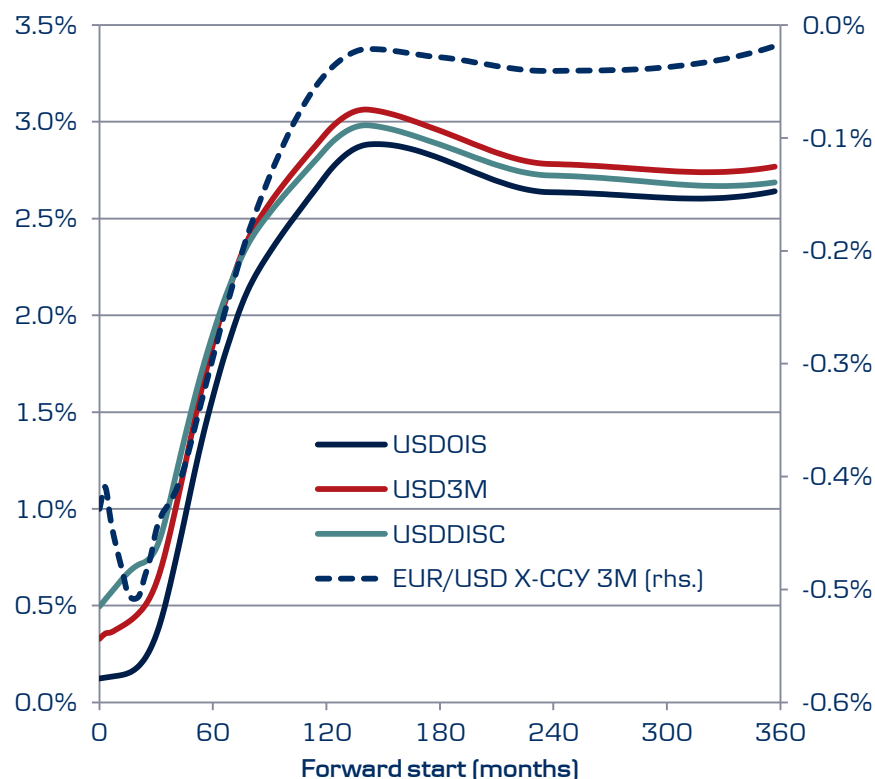




## Benchmark case – cont'd

- Approach cont'd:
  - Calibrate jointly USD3M, USDOIS and USDDISC curves...
  - ...requires EUR model as input since X-CCY legs have initial PV...
  - ...USDDISC curve is not dependent on USDOIS.
- Pricing implication
  - This creates a X-CCY dependence for the pricing of every USD cashflow.
  - Hedging tool for USD net liquidity is to trade USD fixed-EONIA float CCS...
  - ...this delivers the required EONIA floater to collateral mgmt.

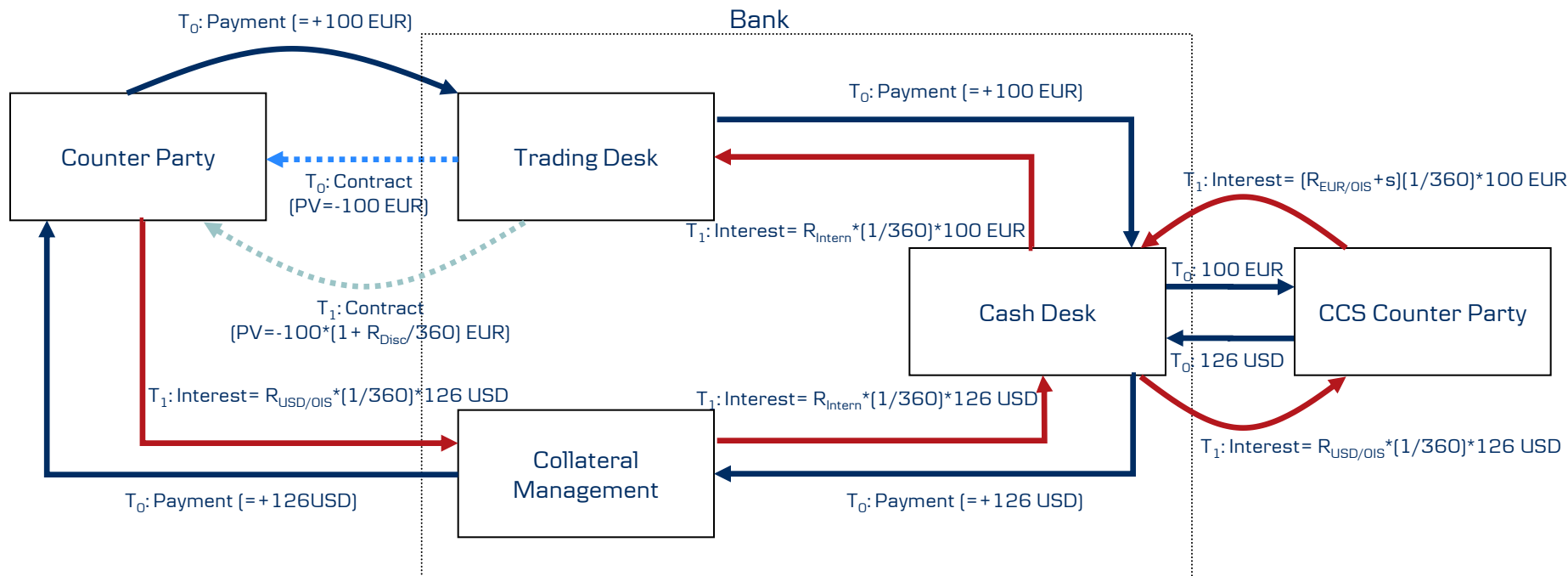
USD: 3M forward rates





# The intuition behind collateral consistent pricing (cont.)

## Flow analysis: EUR Derivative - USD Cash collateral



- To produce the collateral posting in USD an Eonia/Fed-Funds CCS is entered.
- Notice that there is a spread  $s$  on the EUR leg!
- See Piterbarg (2012).

- The discount rate needs to reflect the spread in the CCS.
- In reality there may be multiple currencies, and hence a cheapest-to-deliver option for the collateral poster!

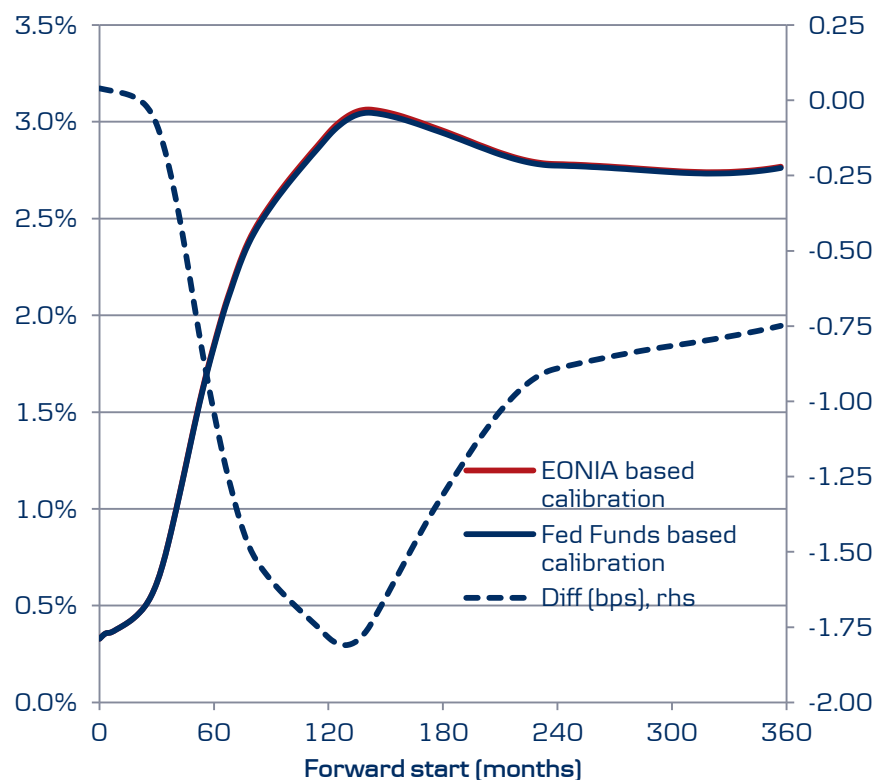
# Calibration instrument assumptions

- Fundamentals
  - What do we mean by calibration instruments?
  - Our model tells where to price one product relative to others...
  - ...so we should calibrate it market prices at which we can execute hedges.
- "The Market"
  - How is "The Market" collateralised?
  - No single answer...
  - ...CSAs are bilateral agreements - and they vary substantially.
  - CCP collateralisation rules are however very clear.
- My calibration should depend carefully on the collateral assumptions that I will face once I start using the calibration instruments for hedging.
  - Each market segment offers one source of risk - but can be collateralised differently:
  - On several CCPs EUR trades are EONIA collateralised, USD trades are FF collateralised...
  - ...the same goes for the ISDA Standardised CSA.
  - But what holds true for FX products?

# Changing the assumptions

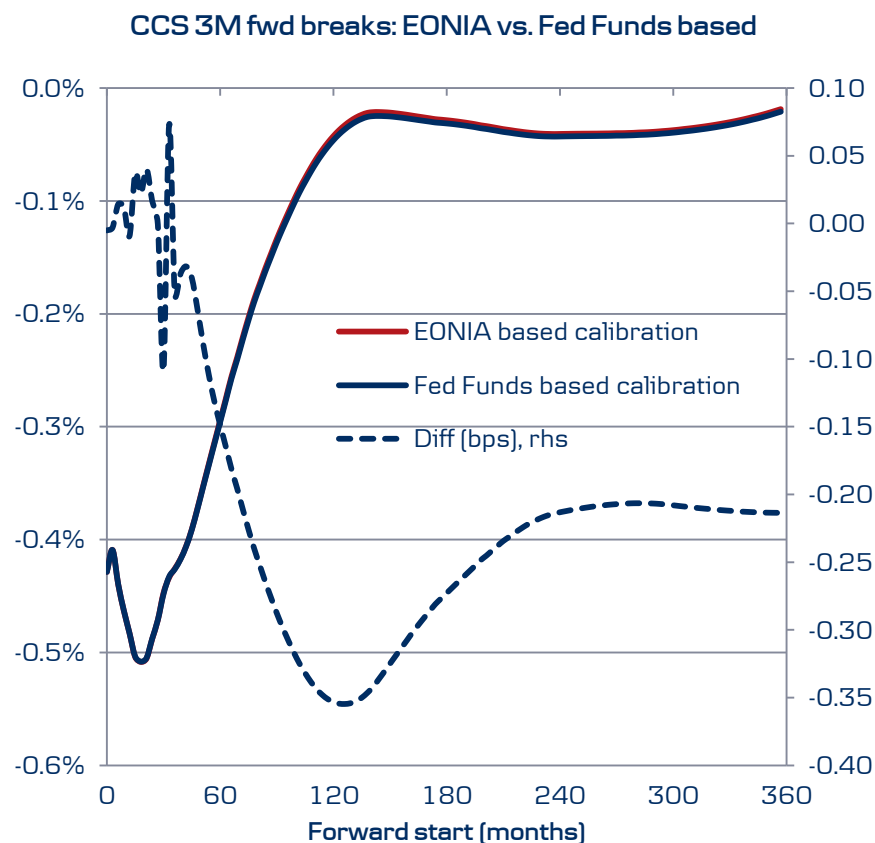
- Back to USD:
  - Let us instead calibrate by using Fed Funds discounting...
  - ...most of "The Market" for USD swaps clears via LCH...
  - We are using the same market quotes for spot instruments...
  - ...but see slight changes in the 3M Fwd curve for 3M USD LIBOR.
- Intuition:
  - A par-swap rate is a weighted average of xIBOR forward rates.
  - Changing the discounting assumption alters the weighting of the individual fwd xIBOR rates.
  - A typical swap market calibration has many degrees of freedom.
- Conclusion:
  - Depending on your assumptions, you can easily misprice forward starting swaps with 1.0-1.5 bps.
  - This is huge in a market that trades with bid-offer spreads in the 0.25-1 bps range.

USD3M Fwd: EONIA vs. Fed Funds based



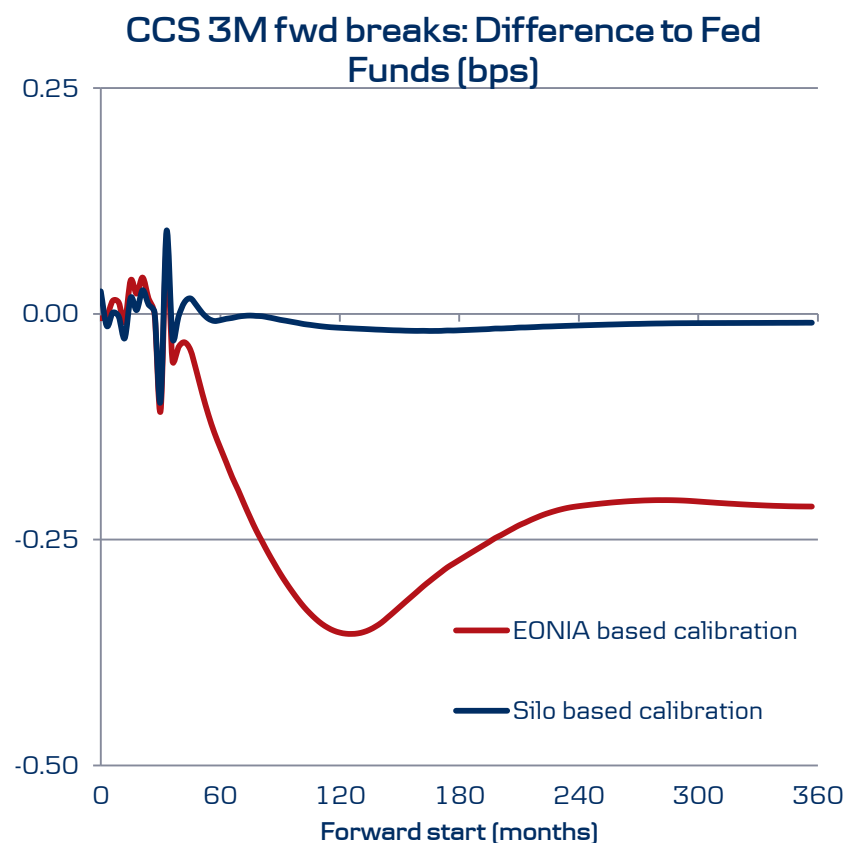
## Changing the assumptions – cont'd

- Cross currency swaps:
  - The same effect holds true for CCSs.
  - In most markets, the fwd curves for the CCS breaks are less steep than xIBOR fwd curves...
  - ...this means that the discounting effect is smaller.
- ISDA Standardised CSA:
  - Is promoting USD cash collateral for FX products incl. CCS...
  - ...so Fed Funds discounting must be right...
  - ...but what about the fwd curves needed to price up this product?
  - This introduces a multi-step calibration requirement...
  - ...need to calibrate "silo" models first and subsequently introduce a new discounting curve.



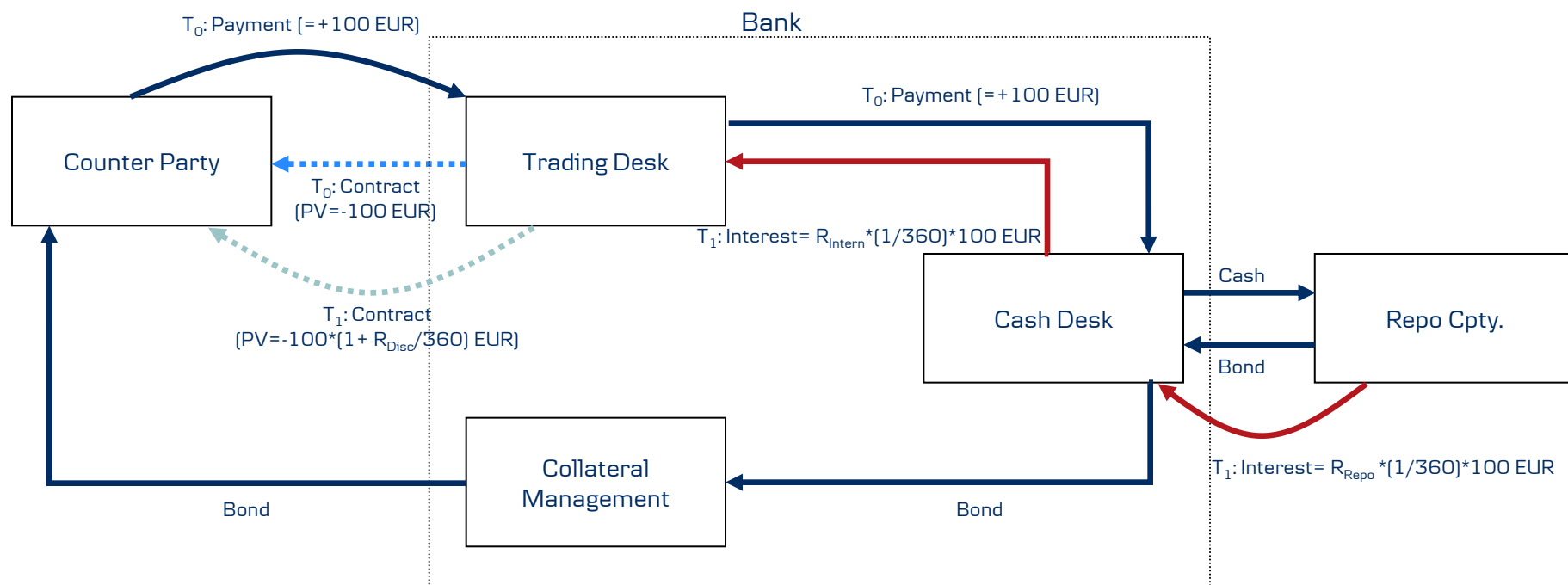
## Changing the assumptions – cont'd

- An aside on CCSs:
  - The basic building block for CCSs is in itself tricky...
  - ...MtM FX resets or constant notionals?
  - Should FX-Basis correlation be included?
  - Does the market standard CCS product rather warrant a full hybrid model?
- Conclusion:
  - The full sequential calibration of the silo-based model matters in certain curve segments.
  - Is obviously dependent on interpolation settings...
  - ...but for plausible choices, the difference in a 5Y5Y EUR/USD CCS can be up 0.25 bps.



# The intuition behind collateral consistent pricing

## Flow analysis: EUR Derivative - EUR security collateral

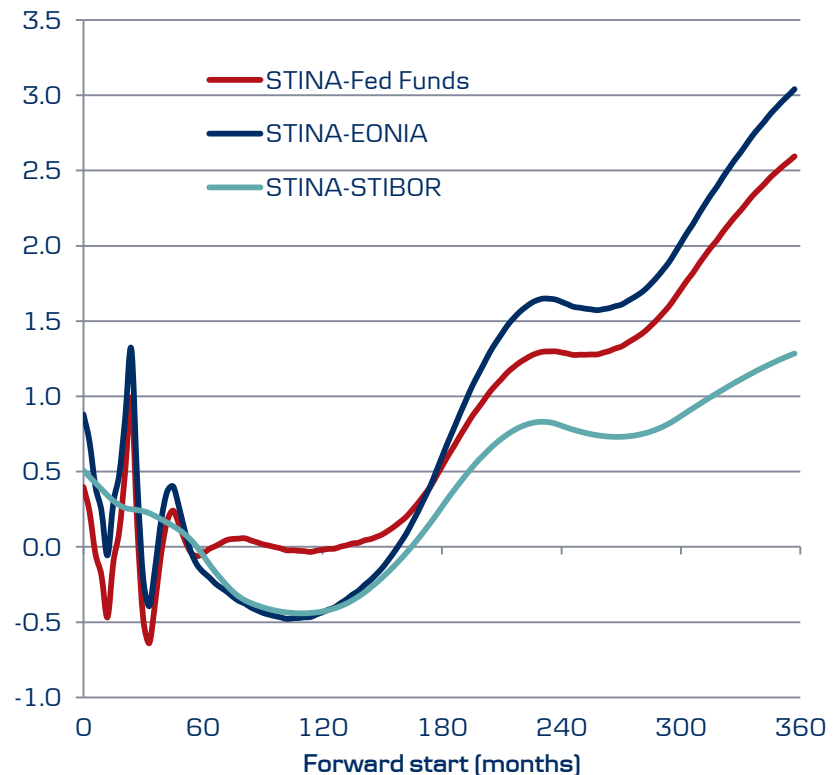


- Security collateral can be financed at their respective repo rate.
- Note the role of haircuts: Cash desk potentially receives one, but collateral management will have to provide one in the CSA. Only differences in haircuts matter - and then becomes a question of unsecured funding rates.

# Case study: Potential for market fragmentation in SEK

- CCP vs. Bilateral:
  - Clearing is not standard in all markets - yet.
  - In SEK, a large share of the IRS market is cleared but much is still bilateral.
  - Among the market makers security collateral is allegedly common place...
  - ...and some of this is closer to STIBOR funded.
- CCP valuation vs. cash accrual
  - LCH.SwapClear uses STIBOR discounting for VM calculation...
  - ...but still pays T/N rate on SEK cash.
  - First order (accrual rate) vs. second order (accrual balance) effect.
- Conclusion:
  - If there is still only one broker price, there should be fragmentation in the forward swap market.
  - Screen prices should be different.

SEK3M: Fwd curve diffs (bps)

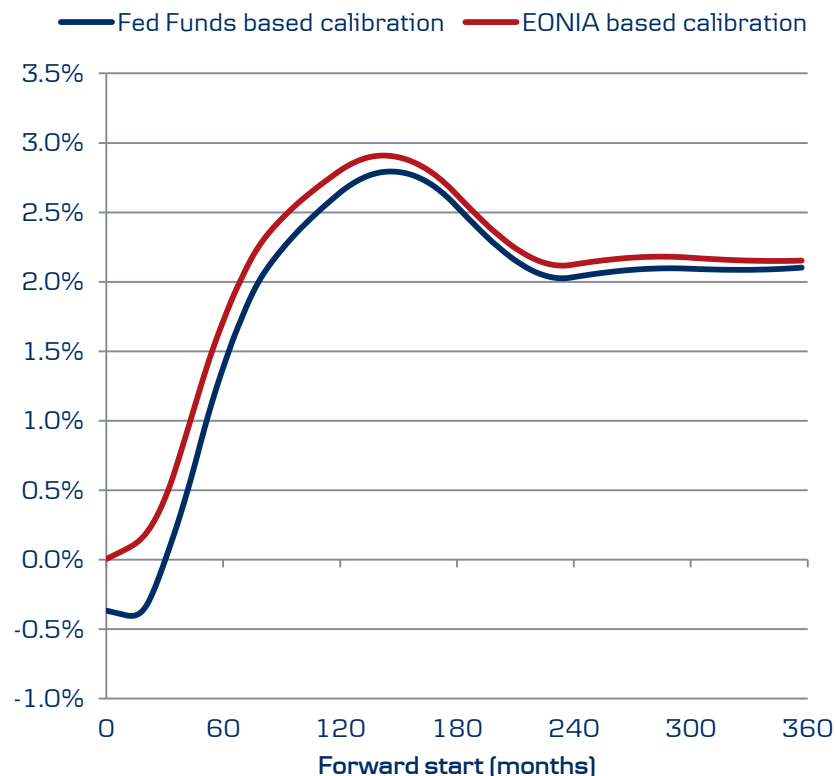




# Collateral valuation adjustments

- CSA optionality:
  - Many (older) CSAs contain long lists of eligible collateral.
  - If collateral can be freely substituted, this creates a cheapest-to-deliver option for the posting party.
  - This creates a need for an effective discount curve - created from more than one curve.
- Example:
  - Can choose between placing EUR cash earning EONIA and USD cash earning Fed Funds.
  - This is effectively a series of call options on the EONIA-FF CCS spread.
- Intrinsic value of CSA option:
  - Find the upper convolution of the EONIA disc curve and the Fed Funds adjusted curve (in fwd terms).
  - Use these forward rates to generate effective discount curve.
  - In the specific example, it is expected to be cheapest to deliver EUR for all 30Y years...
  - ...but there is a risk that USD will be cheaper.

## 3M forward EURDISC rates



# The intuition behind collateral consistent pricing (cont.)

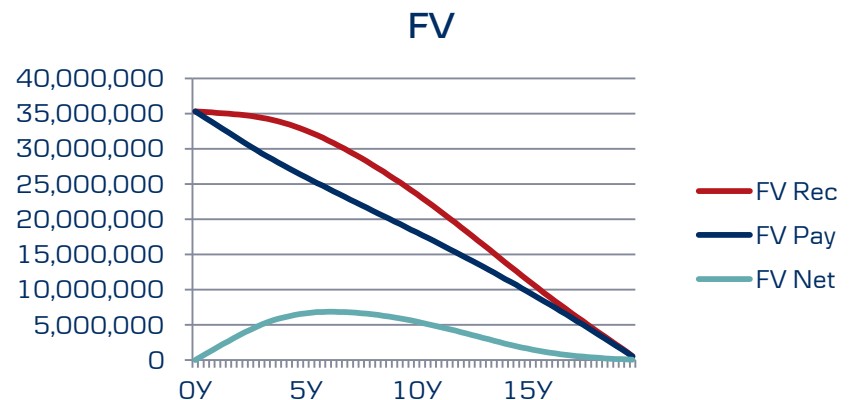
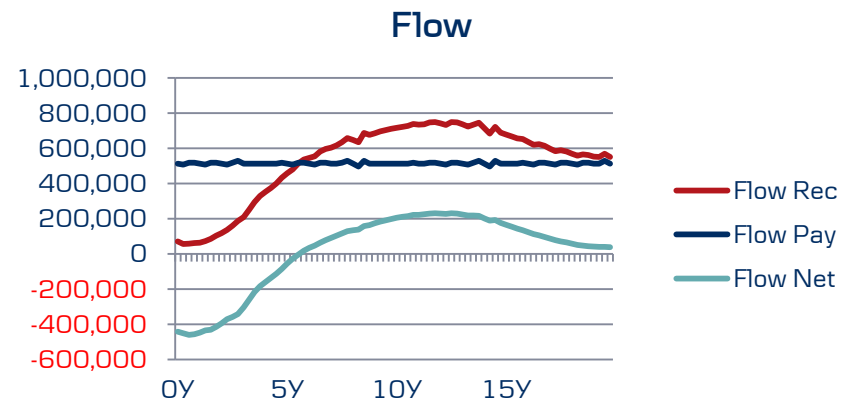
## Expected collateral flow – 100M EUR 20Y IRS Payer

- Net Flow

- Take forward Euribor rates and par fixed rate as given, assume EUR OIS discounting.
- Forward curve is upward sloping
- We pay out net the first 5 years, and receive net the last 15 years.

- Future Value as expected collateral balance.

- Starts and ends at zero for the ATM trade.
- Increases since we are owed more and more.
- Decreases when we start to receive.



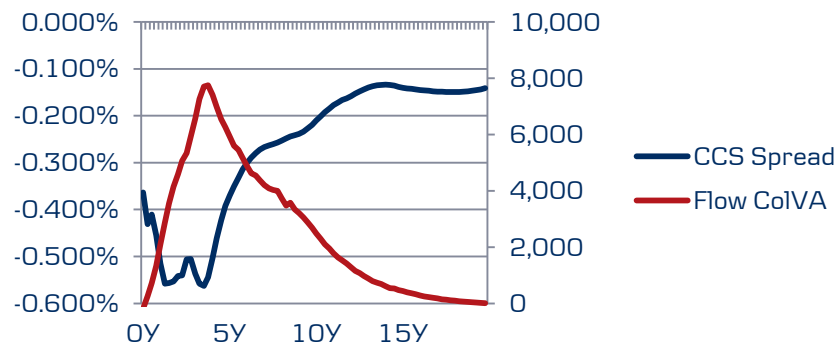
# The intuition behind collateral consistent pricing (cont.)

## Forward Cross Currency Basis Spreads – 1 Y Forward CCS next 20Y

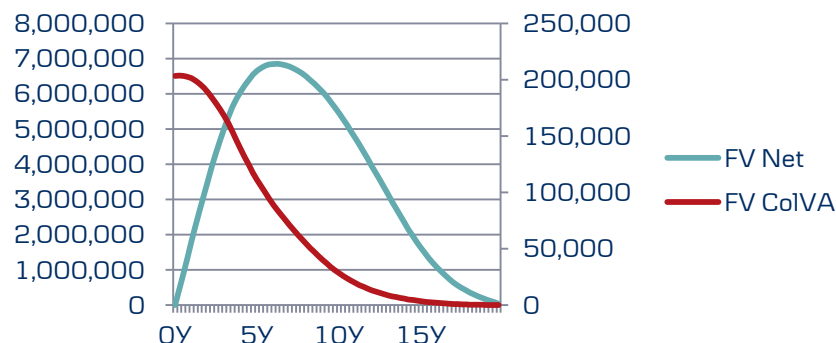
- CoIVA

- Consider the Collateral Valuation Adjustment if collateral should be posted in USD Cash rather than EUR Cash.
- User the FV Net as the CCS notional profile, compute the value of paying the spread.
- The spread is determined through the CCS with the Fed Funds rate flat on the one leg and Eonia plus a spread on the other.

Flow CoIVA



FV CoIVA

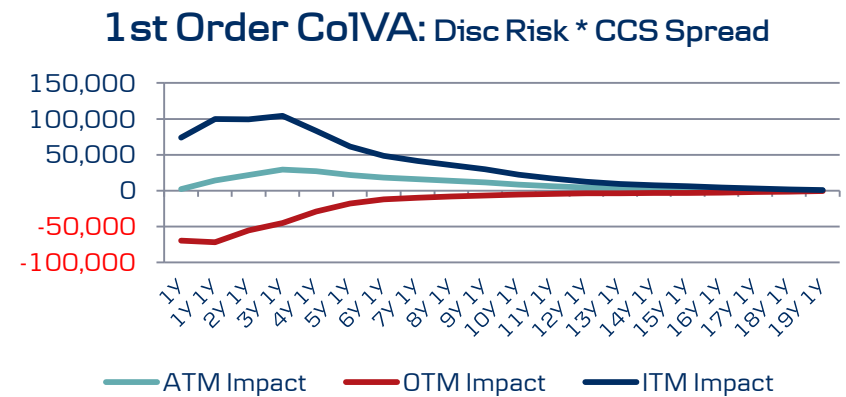
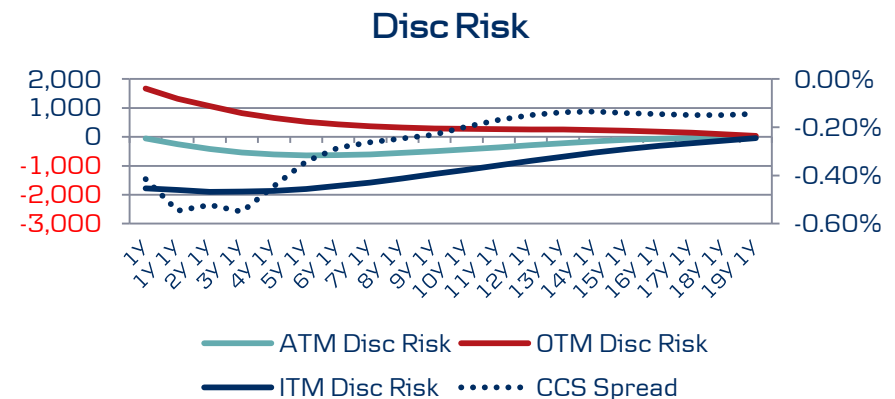


# The intuition behind collateral consistent pricing (cont.)

Discount Curve Risk wrt 1Y Forward CCS Spreads - 100M EUR 20Y IRS Payer

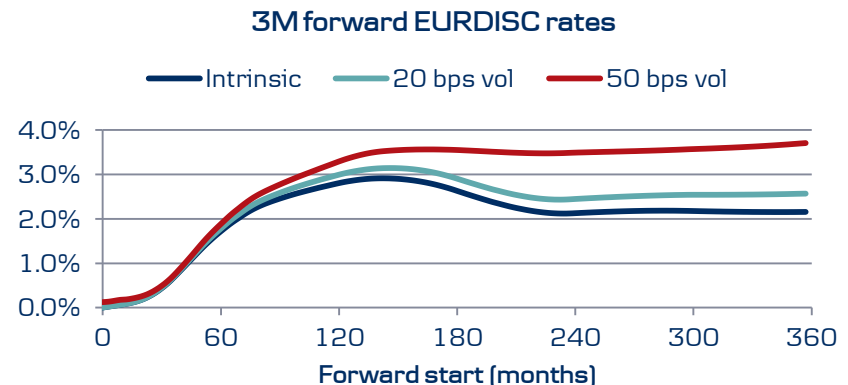
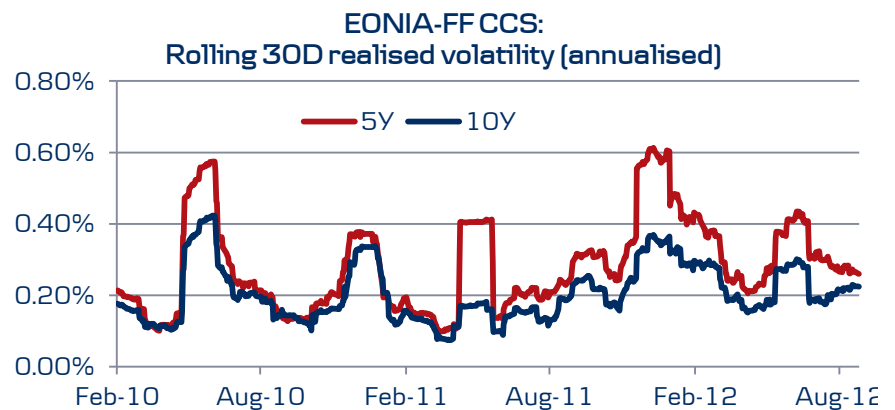
- Compute Discount Curve Risk wrt. 1Y Fwd swaps to derive 1st order CoIVA impact estimate from shifting collateral type.
- Example continued:
  - ATM, ITM (ATM-100bp), OTM(ATM+ 100bp)
  - Positive FV implies negative Fwd Disk Risk.
  - ITM/OTM have the extra disk risk from an annuity.
  - Result:

ATM	OTM	ITM
Impact	Impact	Impact
203k EUR	-356k EUR	761k EUR



# Option adjusted collateral consistent pricing

- Realised volatility on CCS spreads:
  - Spot (normal) volatility is in the 20-50 bps range on an annualised basis.
  - Forward spreads are however less volatile.
- How to include volatility?
  - Simple model, can only EUR or USD cash.
  - Assume Gaussian model.
  - Collateral poster is long a series of caplets on CCS breaks, struck at 0 bps.
- Conclusion
  - Given the shape of the CCS fwd break curve, the short expiries are deep OTM...
  - ...little effect on effective discounting curve.
  - But significant increases for long dated expiries (closer to ATM and higher vega).



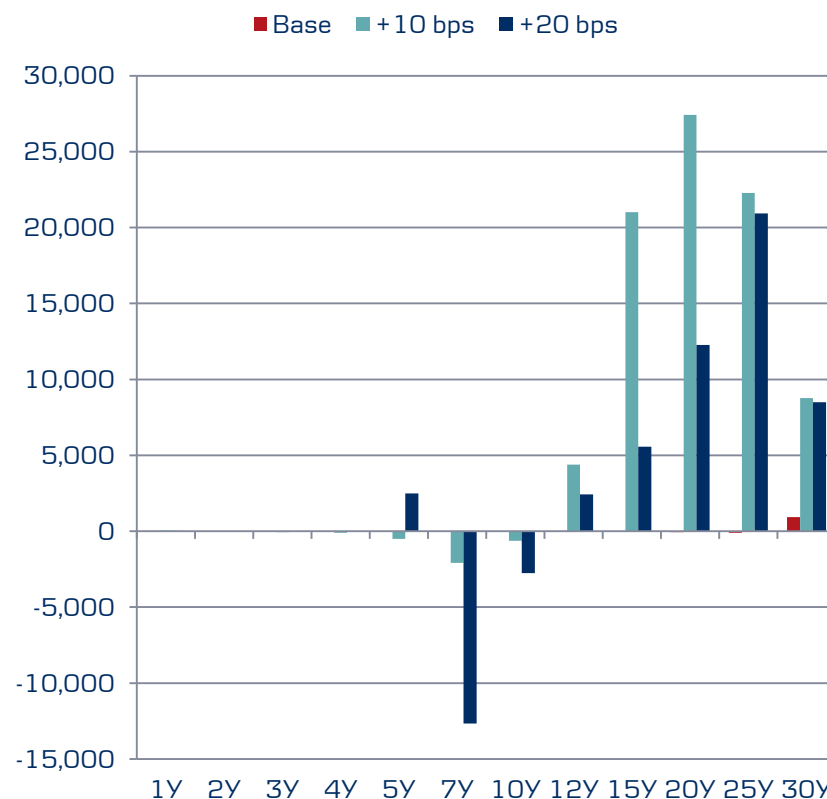
# Option adjusted collateral consistent pricing – cont’d

- Theory:
  - Fujii & Takahashi (2011) and Piterberg (2012)
- Example:
  - 30Y EUR Payer, 100m 250 bps OTM.
- Risk:
  - Using the intrinsic approach, not CCS hedge is required (EUR trade, EUR cash is CTD with certainty).
  - But this will change as basis spreads increase → Risk will "jump".
  - Stability in hedges is an important argument for developing CTD models...
  - ...especially in "naive" bump-and-re-run" mode.

Model	PV Initial	Difference
Intrinsic CTD	-46.67m	-
Option adj. CTD, 20 bps	-45,80m	878k
Option adj. CTD, 50 bps	-43.92m	2.756k

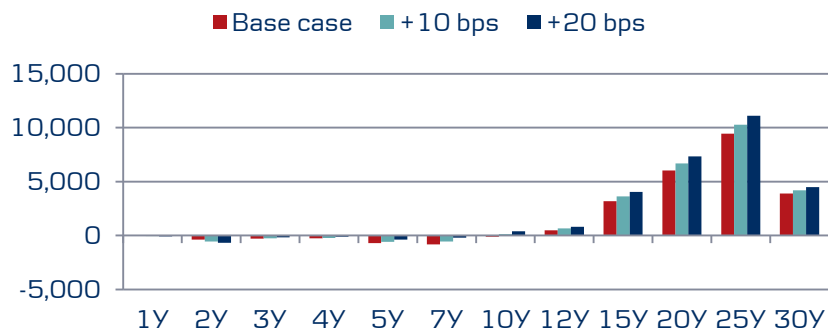
Note, this is a typical pension fund trade - a difference of 6% of the PV of derivatives can mean insolvency.

Intrinsic (0 bps): CCS Dv01 (EUR)

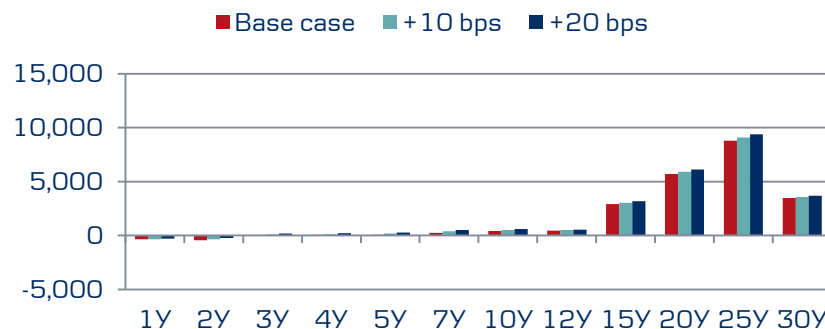


# Option adjusted collateral consistent pricing – cont'd

Option adj (20 bps): CCS risk (EUR)



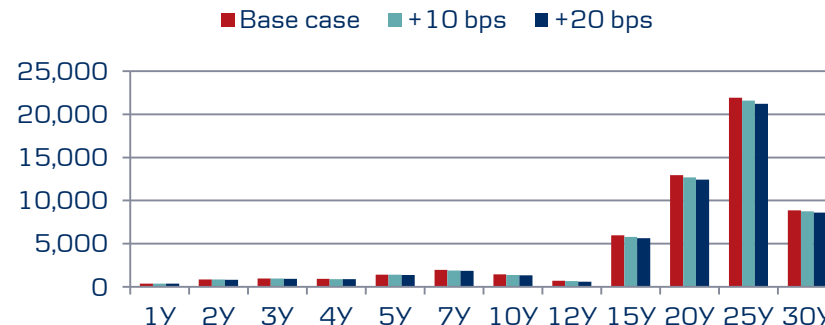
Option adj (50bps): CCS risk (EUR)



- Option adjusted discount deltas:

- Results in stable hedges.
- Intuition fits well against USD cash-only benchmark case.

USD cash only: CCS risk (EUR)





## Conclusion

- There is a direct link between collateral terms and discount factors.
- This is important – it is not just for market makers in derivatives.
- It is not trivial to construct collateral consistent swap curves – and arbitrages are sometimes not far away.
- The “poor man’s” collateral consistent approach can bring most market participants far.
- While the value of CTD options embedded in CSAs is debatable – the risk implications are clear.

## References

- Piterbarg, V. (2010), "Funding beyond discounting: Collateral agreements and derivatives pricing", Risk Magazine February, pp.97-102
- Fujii, M. & Takahashi, A. (2011), "Choice of collateral currency", Risk Magazine January, pp. 120-125
- Piterbarg, V. (2012), "Cooking with collateral", Risk Magazine, pp. 58-63

# Disclosure

This presentation has been prepared by Danske Research, a division of Danske Bank A/S ("Danske Bank").

## Analyst certification

Each research analyst responsible for the content of this research report certifies that the views expressed in the research report accurately reflect the research analyst's personal view about the financial instruments and issuers covered by the research report. Each responsible research analyst further certifies that no part of the compensation of the research analyst was, is or will be, directly or indirectly, related to the specific recommendations expressed in the research report.

## Regulation

Danske Bank is authorized and subject to regulation by the Danish Financial Supervisory Authority and is subject to the rules and regulation of the relevant regulators in all other jurisdictions where it conducts business. Danske Bank is subject to limited regulation by the Financial Services Authority (UK). Details on the extent of the regulation by the Financial Services Authority are available from Danske Bank upon request.

The research reports of Danske Bank are prepared in accordance with the Danish Society of Financial Analysts' rules of ethics and the recommendations of the Danish Securities Dealers Association.

## Conflicts of interest

Danske Bank has established procedures to prevent conflicts of interest and to ensure the provision of high quality research based on research objectivity and independence. These procedures are documented in the research policies of Danske Bank. Employees within the Danske Bank Research Departments have been instructed that any request that might impair the objectivity and independence of research shall be referred to the Research Management and the Compliance Department. Danske Bank Research Departments are organised independently from and do not report to other business areas within Danske Bank.

Research analysts are remunerated in part based on the over-all profitability of Danske Bank, which includes investment banking revenues, but do not receive bonuses or other remuneration linked to specific corporate finance or debt capital transactions.

## Financial models and/or methodology used in this research report

Calculations and presentations in this research report are based on standard econometric tools and methodology as well as publicly available statistics for each individual security, issuer and/or country. Documentation can be obtained from the authors upon request.

## Risk warning

Major risks connected with recommendations or opinions in this research report, including as sensitivity analysis of relevant assumptions, are stated throughout the text.

## First date of publication

Please see the front page of this research report for the first date of publication. Price-related data is calculated using the closing price from the day before publication.

## General disclaimer

This presentation has been prepared by Danske Markets (a division of Danske Bank A/S). It is provided for informational purposes only and should be viewed solely in conjunction with the oral presentation provided by Danske Markets and/or Danske Markets Inc. It does not constitute or form part of, and shall under no circumstances be considered as, an offer to sell or a solicitation of an offer to purchase or sell any relevant financial instruments (i.e. financial instruments mentioned herein or other financial instruments of any issuer mentioned herein and/or options, warrants, rights or other interests with respect to any such financial instruments) (“Relevant Financial Instruments”).

The presentation has been prepared independently and solely on the basis of publicly available information which Danske Bank considers to be reliable. Whilst reasonable care has been taken to ensure that its contents are not untrue or misleading, no representation is made as to its accuracy or completeness, and Danske Bank, its affiliates and subsidiaries accept no liability whatsoever for any direct or consequential loss, including without limitation any loss of profits, arising from reliance on this presentation.

The opinions expressed herein are the opinions of the research analysts responsible for the presentation and reflect their judgment as of the date hereof. These opinions are subject to change, and Danske Bank does not undertake to notify any recipient of this presentation of any such change nor of any other changes related to the information provided in the presentation.

Danske Bank, its affiliates, subsidiaries and staff may perform services for or solicit business from any issuer mentioned herein and may hold long or short positions in, or otherwise be interested in, the financial instruments mentioned herein. The Equity and Corporate Bonds analysts of Danske Bank and undertakings with which the Equity and Corporate Bonds analysts have close links are, however, not permitted to invest in financial instruments which are covered by the relevant Equity or Corporate Bonds analyst or the research sector to which the analyst is linked.

Danske Bank is authorized and subject to regulation by the Danish Financial Supervisory Authority and is subject to the rules and regulation of the relevant regulators in all other jurisdictions where it conducts business. Danske Bank is subject to limited regulation by the Financial Services Authority (UK). Details on the extent of the regulation by the Financial Services Authority are available from Danske Bank upon request.

This presentation is not intended for retail customers in the United Kingdom or the United States.

This presentation is protected by copyright and is intended solely for the designated addressee. It may not be reproduced or distributed, in whole or in part, by any recipient for any purpose without Danske Bank's prior written consent.

## Disclaimer related to presentations to U.S. customers

In the United States this presentation is presented by Danske Bank and/or Danske Markets Inc., a U.S. registered broker-dealer and subsidiary of Danske Bank. In the United States the presentation is intended solely to "U.S. institutional investors" as defined in SEC Rule 15a-6.

Danske Bank is not subject to U.S. rules with regard to the preparation of research reports and the independence of research analysts. In addition, the research analysts of Danske Bank who have prepared this presentation are not registered or qualified as research analysts with the NYSE or FINRA, but satisfy the applicable requirements of a non-U.S. jurisdiction.

Any U.S. investor recipient of this presentation who wishes to purchase or sell any Relevant Financial Instrument may do so only by contacting Danske Markets Inc. directly and should be aware that investing in non-U.S. financial instruments may entail certain risks. Financial instruments of non-U.S. issuers may not be registered with the U.S. Securities and Exchange Commission and may not be subject to the reporting and auditing standards of the U.S. Securities and Exchange Commission.