Public Information and Efficient Capital Investments: Implications for the Cost of Capital and Firm Values

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Abstract

In a standard financial economics model of asset pricing and value maximizing firms, we show better public information about firm-specific and economy-wide events facilitates a more efficient allocation of capital among firms as well as more efficient aggregate capital investments. Thus, investor welfare increases as the informativeness of public information increases. On the other hand, we also show that improvements in investor welfare due to better public information are often associated with a higher cost of capital and with lower firm values.

Keywords: Public Information; Efficient Resource Allocation; Investor Welfare; Cost of Capital; Firm Value.
1 Introduction

A well-functioning capital market should, through the equilibrium price system, (a) facilitate an efficient sharing of the risks in the economy among individually optimizing investors/consumers and (b) allocate the real resources in the economy to their most efficient use (over time, among firms, and within firms) when firms chose their actions to maximize firm value (cf. Debreu 1959). In single-person decision making settings, better pre-decision information is valuable if, and only if, this information leads the decision maker to change his actions (cf. Blackwell’s Theorem). In other words, there is no value of just knowing earlier (with time-additive preferences); but better information about the future consequences of current actions may facilitate more efficient action choices, i.e., the decision-facilitating role of information. Prior literature has established the same result holds in a capital market setting if beliefs are homogeneous, preferences are time-additive, and the capital market is (effectively dynamically) complete (see, e.g., Kunkel 1982, Hakansson, Kunkel, and Ohlson 1982, and Feltham and Christensen 1988). In this setting, better public information (such as accounting or macro economic information) is only valuable to investors if the change in posterior beliefs and the associated equilibrium price system lead value maximizing firms to change their capital investments.

Feltham and Christensen (1988) show firm-specific productivity information is valuable to investors trading in well-diversified portfolios because it facilitates a more efficient allocation of aggregate capital investments among firms. Firm-specific windfall information has no value to investors even though the market prices of individual firms depend on this information, because it does neither affect the marginal productivity of invested capital nor the value of well-diversified portfolios. Economy-wide information affecting the beliefs of future aggregate production/consumption is valuable to investors whether it pertains to the marginal productivity of invested capital or windfall states affecting only the level of future aggregate production/consumption. A change in expectations about the level of future production/consumption affects the expected marginal rates of substitution between future and current consumption and, thus, the state prices. This leads to changes in aggregate capital investments by value maximizing firms, i.e., a more efficient resource allocation over time.

We ask the question: Can we empirically evaluate public information system changes (such as changes in financial reporting regulation) in terms of the informativeness and the efficiency of the resource allocation by investigating the capital market reactions to such changes, for example, changes to firms’ cost of capital or changes to firm values at the announcement date? The answer is negative: the impact on shareholder wealth and on the required rate of return are both poor indicators of the impact on investor welfare.
The capital market reaction depends on the type of information, the investors’ preferences and on the production function. Even in the most basic consumption-based capital asset pricing model (CCAPM) with time-additive, constant relative risk aversion (CRRA) preferences and log-normally distributed aggregate consumption (conditional on the capital investment), the firms’ cost of capital and their value may increase or decrease following the announcement of a more informative public information system. For example, we show that more informative firm-specific productivity information generally increases the firms’ cost of capital, while the \textit{ex ante} firm values decrease (increase) for relative risk aversion parameters above (below) one. More informative economy-wide productivity information also increases the firms’ cost of capital unless the relative risk aversion parameter is slightly above one, while the \textit{ex ante} firm values decrease (increase) for relative risk aversion parameters above (below) one as for firm-specific productivity information.

The key to understanding these results is that, in a general equilibrium setting, changes in the capital market information structure affect not only value maximizing capital investments but also the equilibrium interest rates and the pricing kernel determining the equilibrium risk-adjustments to expected future payoffs. We apply the standard consumption-based capital asset pricing model (see, e.g., Rubinstein 1976, and Breeden 1986) in which firm value (and the price of any other asset) is determined by discounting the expected future payoffs minus a risk adjustment for the covariance between future payoffs and the pricing kernel with the zero-coupon interest rates from the term structure of interest rates. Hence, the impact of better public information on \textit{ex ante} firm values and, eventually, on the implied cost of capital given firm value and expected future payoffs, is determined by the impact on expected future payoffs, the risk adjustment to expected future payoffs, and on the term structure of interest rates. In the CCAPM with constant relative risk aversion, equilibrium interest rates are increasing in expected aggregate consumption growth and decreasing in the riskiness of aggregate consumption growth, and the pricing kernel is a measure of the scarcity of future aggregate consumption in different states.

Consider first an economy in which there are only diversifiable firm-specific risks such that the representative investor faces no risks. In this setting, there are no risk-adjustments to equilibrium interest rates and expected future payoffs. Hence, the \textit{ex ante} firm values are solely determined by the expected future payoffs discounted by equilibrium zero-coupon interest rates.

As noted above, better information about additive firm-specific windfall gains or losses has no consequence for efficient capital investments and, therefore, no impact on \textit{ex ante} firm values before such signals become publicly available. On the other hand, better information about firm-specific productivity events, which affects both the level of an individual
firm’s future payoff and its marginal productivity of invested capital, allows for a more efficient allocation of aggregate capital investments among firms. In other words, increasing the firm-specific productivity informativeness yields higher future aggregate payoffs for the same level of aggregate capital investments, i.e., the productivity of aggregate capital investments increases. For fixed aggregate capital investments, this implies that the marginal rate of substitution between future and current consumption decreases (which is the price of the zero-coupon bond). Consequently, the interest rate and, thus, the hurdle rate for an additional dollar invested to be valuable, increases.

A more efficient allocation of aggregate capital investments among firms not only increases future aggregate payoffs for fixed aggregate investments but also the marginal productivity of an additional aggregate dollar optimally invested in firms. Hence, there is a trade-off between smoothing the higher aggregate payoffs over time by reducing aggregate capital investments (i.e., the income effect) and taking advantage of the higher marginal productivity of invested capital by increasing aggregate capital investments (i.e., the substitution effect). The equilibrium outcome of this trade-off depends on the investors’ preferences and, in particular, on the investors’ elasticity of intertemporal substitution (EIS). With CRRA utility, the inverse EIS is equal to the constant relative risk aversion parameter ($\gamma$). In a somewhat similar setting, Merton (1969) shows consumption smoothing is the dominant concern when $\gamma$ is above one (EIS below one), while the marginal productivity gain is the dominant concern when $\gamma$ is below one (EIS above one). Consistent with this result, we show the equilibrium aggregate capital investments are decreasing (increasing) in the informativeness of firm-specific productivity information if $\gamma$ is above (below) one. This implies that the impact on equilibrium aggregate capital investments “dampens” (“amplifies”) the impact on the equilibrium interest rate if $\gamma > 1$ ($\gamma < 1$) compared to a setting in which aggregate capital investments are kept fixed as the informativeness of the firm-specific productivity information increases. In the dividing case with log-utility ($\gamma \rightarrow 1$), the concerns for consumption smoothing and the marginal productivity gain are perfectly offsetting such that the equilibrium aggregate capital investments are independent of the firm-specific productivity informativeness. In all cases with $\gamma > 0$, the equilibrium interest rate increases as the informativeness of firm-specific productivity information increases.

Stepping back one period before the firm-specific information becomes publicly available (i.e., to the ex ante date), aggregate and expected firm-specific capital investments decrease (increase) for $\gamma > 1$ ($\gamma < 1$) as the informativeness of the firm-specific productivity information system increases. Hence, the equilibrium spot interest rate for the first period increases for $\gamma > 1$ ($\gamma < 1$) due to higher (lower) aggregate consumption growth in the first period.

The higher equilibrium interest rates in both periods and the less efficient use of the
productivity gains (by reducing aggregate capital investments) for $\gamma > 1$ imply that the \textit{ex ante} firm values decrease, i.e., there is a negative stock market reaction to the announcement of a more informative firm-specific productivity information system. The \textit{ex ante} expected utility of the representative investor increases, since less is invested for higher future payoffs and since the consumption stream is more “smooth” over time. The firms’ \textit{ex ante} implied cost of capital is a (somewhat complicated) weighted average of the equilibrium spot interest rate and the equilibrium \textit{ex post} interest rate (from the capital investment date until the date of the future payoffs), where the weights are determined by the expected dividends at each date. Since both interest rates increase with firm-specific productivity informativeness, the \textit{ex ante} implied cost of capital also increases unless the weight on the lower of the two interest rates increases too much. In other words, the impact on the \textit{ex ante} cost of capital may, in general, depend on the shape on the equilibrium term structure of interest rates, which in turn may depend on the pattern of future consumption growth arising from other sources than capital investments.

The more efficient use of the productivity gains (by increasing aggregate capital investments) with $\gamma < 1$ more than offsets the increase in the second-period equilibrium interest rate such that the \textit{ex ante} firm values increase. In this case, there is a positive stock market reaction to the announcement of a more informative firm-specific productivity information system and, furthermore, the stock market reaction is positively related to the change in the representative investor’s \textit{ex ante} expected utility. Hence, the interpretation of a stock market reaction to an announcement of a change in information system in terms of welfare gains/losses is highly dependent on the size of the investors’ relative risk aversion (or, equivalently, the inverse EIS). The firms’ implied cost of capital generally increases with firm-specific productivity informativeness as when $\gamma > 1$. This makes sense, since an improved allocation of aggregate capital investments among firms and over time, in general, lowers the investors’ marginal utility of an additional dollar of future consumption and, thus, the hurdle rate for an additional dollar invested to be valuable must increase.

Secondly, consider an economy in which there are both diversifiable firm-specific risks and economy-wide productivity risks. The comparative statics with respect to increasing the informativeness of firm-specific productivity information are basically the same as discussed above for an economy without economy-wide risks. Contrary to firm-specific information, increasing the informativeness of economy-wide productivity information has an impact on both the risk premia in equilibrium interest rates and on the pricing kernel, which are both stochastic in the second period contingent on the specific economy-wide signal. The economy-wide productivity signal is modeled as the normally distributed posterior mean of the log of the productivity parameter, which is multiplied on a concave power function of
the capital investments, such that aggregate future payoffs are lognormally distributed given the productivity signal and the capital investment.

This implies that the impact of a higher economy-wide productivity signal is similar to the impact of increasing the informativeness of the firm-specific productivity information, i.e., a higher economy-wide productivity signal is associated with both a higher level of expected future aggregate payoffs as well as a higher expected productivity of invested capital. Hence, the equilibrium aggregate capital investments are decreasing (increasing) in the economy-wide productivity signal when $\gamma > 1$ ($\gamma < 1$) and more so, the more informative these signals are. In other words, more informative economy-wide productivity information reduces (increases) the risk in future payoffs and, thus, the risk in aggregate consumption when $\gamma > 1$ ($\gamma < 1$). Since the risk of future aggregate consumption is positively related to the representative investor’s incentive for precautionary savings in zero-coupon bonds, it follows that the equilibrium interest rates increase (decrease) when $\gamma > 1$ ($\gamma < 1$).

The risk-adjustment to expected future payoffs in the determination of \textit{ex ante} firm values is determined by the \textit{ex ante} covariance between the pricing kernel and the firms’ future payoffs. The countercyclicality (procyclicality) of equilibrium aggregate capital investments with the economy-wide productivity signal reduces (increases) the variation in the pricing kernel (as well as the variation in future aggregate payoffs) such that the risk-adjustment is reduced (increased) when $\gamma > 1$ ($\gamma < 1$) as the informativeness of the economy-wide productivity information increases. For both $\gamma > 1$ and $\gamma < 1$, the impact on the risk-adjustment dominates the impact on the expected payoffs such that the risk-adjusted expected future payoffs increase (decrease) when $\gamma > 1$ ($\gamma < 1$).

The impact of more informative economy-wide productivity information on equilibrium interest rates and risk-adjusted expected future payoffs thus go in opposite directions for the determination of the \textit{ex ante} firm values for both $\gamma > 1$ and $\gamma < 1$. We show that the impact on the equilibrium zero-coupon interest rates are the dominant force such that the equilibrium \textit{ex ante} firm values decrease (increase) when $\gamma > 1$ ($\gamma < 1$) as the informativeness of the economy-wide productivity information increases. Hence, as with firm-specific productivity information, the stock market reaction to the announcement of a more informative economy-wide productivity information system is negatively (positively) associated with the change in the investors’ equilibrium \textit{ex ante} expected utility (welfare) when $\gamma > 1$ ($\gamma < 1$).

We show the firms’ \textit{ex ante} implied cost of capital increases as the informativeness of the economy-wide productivity information increases unless the relative risk aversion is slightly above one. Hence, even if equilibrium interest rates decrease for $\gamma < 1$, the aggressive and procyclical use of the productivity signals in determining the equilibrium aggregate capital investments implies that the associated increase in the implied rate of return risk premium
more than offsets the reduction in interest rates. When $\gamma$ is somewhat larger than one, the impact of the less aggressive and countercyclical use of the productivity signals in determining the equilibrium aggregate capital investments on the implied rate of return risk premium is not large enough to offset the increase in interest rates. This changes when $\gamma$ is slightly above one such that the \textit{ex ante} implied cost of capital decreases as the informativeness of the economy-wide productivity information increases.

Our analysis has important implications for the usefulness of empirical studies of capital market reactions to changes in public information systems (such as changes in financial reporting regulation, e.g., IFRS adoption) to assess changes in the informativeness and the associated efficiency of the resource allocation in the economy among firms and over time. The firms’ \textit{ex ante} implied cost of capital generally increases as the efficiency of the resource allocation increases, but there are exceptions; for example, when the investors’ relative risk aversion is slightly above one and we are considering economy-wide productivity information. The stock market reaction in terms of changes in \textit{ex ante} firm values is negatively (positively) associated with the efficiency of the resource allocation when the investors’ relative risk aversion is above (below) one. Of course, the problem is that we do not know the investors’ relative risk aversion (or their elasticity of intertemporal substitution). At the very least, one should be very careful in how the results from such empirical studies are interpreted; for example, reductions in the firms’ cost of capital are generally a sign of less informative public information systems and less efficient resource allocation, while a positive stock market reaction must be interpreted similarly when the relative risk aversion parameter is above one (which seems to be the more relevant case empirically).

\textbf{Related literature}

In this section there will be some critical comments on the information and cost of capital literature focussing on the \textit{ex post} cost of capital in (a) pure exchange economies possibly with a constant riskfree interest rate, (b) production economies with essentially a single consumption date, (c) production economies with several substituting generations of investors forced to sell their assets at the end of each period and an exogenous riskfree interest rate. When investors are risk averse and alive for more than one period, and there are economy-wide risks, risk premia and equilibrium interest rates are intimately related through the investors’ first-order conditions for intertemporal portfolio choice...
2 The Economy

We consider a large economy with two periods in which investors consume at three dates $t = 0, 1, 2$, and trade long-lived assets at dates $t = 0$ and $t = 1$. There are three types of long-lived assets in the economy: $N \cdot J$ risky assets/firms in unit supply, and two zero-coupon bonds in zero net-supply maturing at $t = 1$ and $t = 2$, respectively. For simplicity, we assume that all $N$ firms of type $j = 1, ..., J$ share the same production technology such that within type $j$ firms variations in payoffs are solely due to variations in firm-specific investments and/or firm-specific risks. The firms’ dividends are denoted $d_{tjn}, t = 0, 1, 2, j = 1, ..., J, n = 1, ..., N$. The dividends at $t = 0$ are exogenously given, i.e., $d_{0jn} = \hat{d}_{0j}$, while the dividends at $t = 1$ are determined as the difference between exogenous payoffs from first-period production $\hat{d}_{1j}$ (which we, for simplicity, assume are riskless) and capital investments $q_{jn}$ for second-period production, i.e., $d_{1jn} = \hat{d}_{1j} - q_{jn}$. Each firm has an unmodelled manager (an automaton) who chooses capital investments at $t = 1$ to maximize the cum-dividend market value of the firm at that date. The firms’ dividends at $t = 2$ depend on the capital investments made at $t = 1$, an uncertain economy-wide state of nature $s_e \in S_e$ as well as on an uncertain firm-specific state of nature $s_{jn} \in S_{jn} = S_j$, i.e., $d_{2jn} = f_j(s_e, s_{jn}, q_{jn})$, where the production function $f_j(\cdot)$ is twice differentiable, strictly increasing and strictly concave function of $q_{jn}$. All uncertainty is depicted by the set of states $S = S_e \times \Pi_{j=1}^J \Pi_{n=1}^N S_{jn}$, over which investors have homogeneous prior beliefs given by the probability density function $\phi(s)$, $s \in S$. For simplicity, we assume that all elements of the states are independent, i.e., $\phi(s) = \phi_e(s_e) \cdot \Pi_{j=1}^J \Pi_{n=1}^N \phi_j(s_{jn}), s \in S$.\(^1\)

At $t = 0$, investors and managers hold the beliefs $\phi(s)$ over states $s \in S$ and at $t = 2$ the state is revealed. At $t = 1$, the investors’ and managers’ beliefs over states $s \in S$ depend on a common public signal $y \in Y$ and is represented by the conditional probability density function $\phi(s|y)$. The public signal consists of an economy-wide signal $y_e \in Y_e$ pertaining to the economy-wide state $s_e \in S_e$ and firm-specific signals $y_{jn} \in Y_{jn} = Y_j$ pertaining to the firm-specific state $s_{jn} \in S_{jn} = S_j$ for each firm such that $\phi(s|y) = \phi_e(s_e|y_e) \cdot \Pi_{j=1}^J \Pi_{n=1}^N \phi_j(s_{jn}|y_{jn}), s \in S, y \in Y$. We conjecture (and later verify) that in a large efficient economy, value-maximizing firm-specific capital investments at $t = 1$ depend only on the economy-wide signal $y_e \in Y_e$ and the firm-specific signal $y_{jn} \in Y_{jn}$, i.e., $q_{jn}(y) = q_j(y_e, y_{jn}), j = 1, ..., J, n = 1, ..., N, y \in Y$.

Since our focus is strictly on the role of public information for the efficient amount of aggregate capital investments and the allocation of this investment among firms, and not on

\(^1\)Feltham and Christensen (1988) consider the more general setting in which the firm-specific states (and signals) are only required to be independent conditional on the economy-wide state (and signal).
the role of public information for efficient risk sharing and implementation of the investors’

efficient intertemporal consumption plans, we assume, for simplicity, there are $N \cdot J$ investors,

who are identical in all respects such as endowments, beliefs, and preferences.\(^2\) Hence, we

may assume there is no net-trading in equilibrium and, thus, each investor receives at each
date the fraction $\frac{1}{NJ}$ of the dividends on the market market portfolio of firm shares in non-
zero net supply, i.e., the equilibrium consumption of a representative investor at each date
$t = 0, 1, 2$ is determined as $\frac{1}{NJ} \sum_{j=1}^{J} \sum_{n=1}^{N} d_{tjn}$. Our specification of the economy-wide and

firm-specific states and information, and the strong law of large numbers for independent
random variables, then implies that for a large economy (in which $N \rightarrow \infty$), the equilibrium

consumption for a representative investor can be expressed as\(^3\)

\[
c_0 = \frac{1}{J} \sum_{j=1}^{J} \tilde{d}_{0j} \equiv \tilde{d}_0, \tag{1a}
\]

\[
c_1(y_e) = \frac{1}{J} \sum_{j=1}^{J} \tilde{d}_{1j} - \frac{1}{J} \sum_{j=1}^{J} \mathbb{E}[q_j(y_e, y_{jn}) | y_e] \equiv \tilde{d}_1 - \frac{1}{J} \sum_{j=1}^{J} q_j^E(y_e) \equiv \tilde{d}_1 - \tilde{q}_0(y_e), \tag{1b}
\]

\[
c_2(s_e) = \frac{1}{J} \sum_{j=1}^{J} \mathbb{E}[f_j(s_e, s_{jn}, q_{jn}) | s_e] \equiv \frac{1}{J} \sum_{j=1}^{J} f_j^E(s_e) \equiv \tilde{f}_0(s_e). \tag{1c}
\]

Equations (1b) and (1c) reflect the fact that if investors hold well-diversified portfolios in a
large economy, then their consumption plans only depend on the economy-wide information
$y_e$ at $t = 1$, and the economy-wide state $s_e$ at $t = 2$, i.e., the impact of firm-specific information
and states is diversifiable. In order words, the representative investor’s equilibrium consumption at $t = 1$ and at $t = 2$ depend only on the average of the expected dividends conditional on the economy-wide signal and state, respectively, for the $J$ firm types.

We assume the investors’ preferences are represented by identical time-additive utility functions on the form

\[
u(c_0, c_1, c_2) = \sum_{t=0}^{2} u_t(c_t),
\]

where $u_t(\cdot)$ is twice differentiable, strictly increasing, strictly concave, and has infinite marginal utility for consumption at the minimal level of consumption $c_0$, i.e., $\lim_{c_1 \downarrow c_2} u_t(c_t) = \infty$.

It then follows from the investors’ first-order conditions for optimal portfolio choice at dates
$t = 0$ and $t = 1$ that there exists a pricing kernel $\{m_{zt}\}$ such that the equilibrium ex-dividend

\(^2\)Feltham and Christensen (1988) consider the role of dynamic trading in a sufficiently varied set of
well-diversified portfolios as a mechanism to achieve efficient risk sharing, which in turn ensures Gorman
aggregation with CRRA preferences.

\(^3\)See Malinvaud (1972, 1972), Berninghaus (1977) and Feltham and Christensen (1988) for definitions of
equilibria and efficiency in large economies.
price of any long-lived asset with a sequence of dividends \( \{d_1, d_2\} \) is given by

\[
v_t = \sum_{\tau = t+1}^{2} B_{\tau t} E_t [m_{\tau t} d_\tau] = \sum_{\tau = t+1}^{2} B_{\tau t} \{E_t [d_\tau] + \text{Cov}_t [m_{\tau t}, d_\tau]\}, \quad t = 0, 1, \tag{2}
\]

where

\[
m_{\tau t} = \frac{u_\tau'(c_\tau)}{E_t [u_\tau'(c_\tau)]}, \quad E_t [m_{\tau t}] = 1,
B_{\tau t} = \exp \left[ -\iota_{\tau t} (\tau - t) \right] = \frac{E_t [u_\tau'(c_\tau)]}{u_\tau'(c_t)},
\]

and \( \iota_{\tau t} \) is the zero-coupon interest rate at date \( t \) for maturity \( \tau \), and \( E_t [\cdot] \) and \( \text{Cov}_t [\cdot] \) denote the conditional expectation and covariance, respectively, given information at date \( t \). Using that the investors’ equilibrium consumption plans are measurable with respect to the economy-wide signal/state, cf., equations (1b)–(1c), the pricing kernel is also measurable with respect to the economy-wide signal/state and, thus, the equilibrium prices can also be expressed as

\[
v_t = \sum_{\tau = t+1}^{2} B_{\tau t} E_t [m_{\tau t} E^e_\tau [d_\tau]] = \sum_{\tau = t+1}^{2} B_{\tau t} \{E_t [d_\tau] + \text{Cov}_t [m_{\tau t}, E^e_\tau [d_\tau]]\}, \quad t = 0, 1, \tag{3}
\]

where \( E^e_\tau [\cdot] \) and \( \text{Cov}^e_\tau [\cdot] \) denote the conditional expectation and covariance, respectively, given the economy-wide signal/state at date \( \tau > t \). In other words, the equilibrium pricing is such that only variations in future dividends due to economy-wide events command a risk premium—variations in future dividends due to firm-specific events are fully diversifiable.

The literature uses a varied set of cost of capital measures. One measure is the dollar risk premia, i.e.,

\[
\text{RP}_{\tau t} = -\text{Cov}_t [m_{\tau t}, E^e_\tau [d_\tau]], \quad t = 0, 1; \tau = 1, 2. \tag{4}
\]

This is the generally preferred concept for risk premia in valuation theory, since it is always well-defined (when there are no arbitrage opportunities in the capital market) and is consistent with contemporaneous multi-period asset pricing theory (see, e.g., Rubinstein 1976 and Christensen and Feltham 2009). Another concept, which is more directly aligned with required hurdle rates of returns for firms’ capital investments, is the expected rates of return on the firms’ stocks (whenever it is well-defined), i.e.,

\[
\mu_{t+1, t} \equiv \ln \left[ \frac{E_t [d_{t+1} + v_{t+1}]}{v_t} \right], \quad t = 0, 1. \tag{5}
\]
Christensen et al. (2010) denote $\mu_{21}$ the *ex-post* cost of capital and show that it depends on both the informativeness of the public information system at $t = 1$ and on the realized public signal itself and, thus, it is generally a stochastic variable as seen from $t = 0$. The expected rate of return at $t = 0$, i.e., $\mu_{10}$, is similarly denoted the *pre-posterior* cost of capital, and it generally depends on the informativeness of the public signals at $t = 1$. The *rate of return risk premia* are defined as the difference between the expected rates of returns and the riskless spot interest rate, i.e.,

$$\omega_{t+1,t} \equiv \mu_{t+1,t} - \iota_{t+1,t}, \quad t = 0, 1.$$  

(6)

Christensen et al. (2010) show that in a perfectly competitive exchange economy with homogeneous beliefs and time-additive preferences, the impacts of the informativeness of the public information systems on the *ex-post* cost of capital and the *pre-posterior* cost of capital are perfectly offsetting in the sense that the *ex ante* cost of capital at $t = 0$ is independent of the informativeness of the public information system at $t = 1$. The *ex ante* cost of capital is defined as the discount rate which makes the discounted expected future dividends equal to the equilibrium price of the risky asset at $t = 0$, i.e., $\rho$ is the discount rate solving the equation

$$v_0 = \sum_{\tau=1}^{2} \exp [-\rho\tau] E_0 [d_\tau].$$  

(7)

The *ex ante* cost of capital is also known as the implied cost of capital at $t = 0$. Since the stochastic dividends are fixed in an exchange economy, no impact of the public information system on the *ex ante* cost of capital is equivalent to no impact of the public information system on the *ex ante* asset prices.

In this article, we ask the question whether more informative public reporting at $t = 1$ yields more efficient capital investments and, thus, dividends streams (in terms of the *ex ante* expected utility of the representative investor) and if so, what will be the impact on the cost of capital and firm values? The measure of the efficiency of the public information system and the associated equilibrium capital investments is the certain annuity consumption stream $CE$ over the three consumption dates, which gives the representative investor the same utility as the equilibrium *ex ante* expected utility $EU$, i.e., $CE$ is a generalized *ex ante* certainty equivalent for our multi-period setting solving the equation

$$EU = \sum_{t=0}^{2} u_t(CE).$$  

(8)

Our primary cost of capital measure will be the *ex ante* cost of capital. It reflects the
constant required expected rate of return on capital raised from the capital market at $t = 0$ and used for capital investments or dividends at $t = 1$. It follows from Christensen et al. (2010) that any impact of the public information system on this measure is strictly due to the impact on the efficient allocation of capital investments among firms and over time in our production economy. We do not exclude the possibility that new capital is raised (as negative dividends) at $t = 1$, but that is a zero-NPV activity in a competitive capital market. Even though uncertainty about future consequences of current capital investments may have been reduced due to public information at $t = 1$, you get what you pay for. Similarly, the impact of the public information system on firm values will be measured by the impact on the \textit{ex ante} equilibrium firm values at $t = 0$ before any signals are released from the public information system.

3 Efficient Resource Allocation

In this section we study the impact of public information on the efficient allocation of capital among firms and on the optimal aggregate capital investments in the economy. We impose additional structure on the production functions and the set of states to clearly distinguish between states that affect the productivity of invested capital and states which merely affect the level of future payoffs independently of the amount of capital invested at $t = 1$. That is, we expand the description of the states to encompass both productivity and windfall states (see Feltham and Christensen 1988).

**Definition 1** $\theta_e \in \Theta_e$ and $\theta_{jn} \in \Theta_j$ constitute windfall states, and $\xi_e \in \Xi_e$ and $\xi_{jn} \in \Xi_j$ constitute productivity states if $s_e = (\theta_e, \xi_e)$ and $s_{jn} = (\theta_{jn}, \xi_{jn}), j = 1, \ldots, J; n = 1, \ldots, N$, and

(a) $\phi(s_e, s_{jn}) = \phi_e(\theta_e)\phi_e(\xi_e)\phi_j(\theta_{jn})\phi_j(\xi_{jn})$, \textit{i.e., independence},

(b) $f_j(s_e, s_{jn}, q_{jn}) = g_j(\theta_e, \theta_{jn}) + h_j(\xi_e, \xi_{jn}, q_{jn})$, \textit{i.e., separable production impact},

(c) $\xi_e$ and $\xi_{jn}$ influence both the level and slope of $h_j(\cdot)$ as a function of $q_{jn}$.

Feltham and Christensen (1988) show that public information at $t = 1$ about firm-specific productivity states $(\xi_{jn})$, economy-wide productivity states $(\xi_e)$, and economy-wide windfall states $(\theta_e)$ improves the efficiency of capital investments in the economy, while information about firm-specific windfall states $(\theta_{jn})$ has no impact on efficient capital investments. In this article, we impose a particular structure on the production functions to illustrate more
concretely how public information affects efficient capital investments, the cost of capital, and firm values. In particular, we make the assumption that

\[ g_j(\theta_e, \theta_{jn}) = a_j + b_j \theta_e + c_j \theta_{jn}, \quad b_j \geq 0, c_j \geq 0, \]

\[ h_j(\xi_e, \xi_{jn}; q_{jn}) = \exp\left[ \alpha_j + \beta_j \xi_e + \psi_j \xi_{jn} \right] q_{jn}^k, \quad k \in (0, 1), \beta_j \geq 0, \psi_j \geq 0, \]

where all states are standard normally distributed, i.e., \( \theta_e, \theta_{jn}, \xi_e, \xi_{jn} \sim N(0, 1) \). Given normally distributed states, we model (without loss of generality) the public signals at \( t = 1 \) as the posterior means of the associated states, i.e.,

\[ \theta_{e|y_{th}} \sim N(y_{th}, \sigma^2_{0\theta e}), \quad \theta_{jn|y_{th}} \sim N(y_{th}, \sigma^2_{0\theta j}), \quad \xi_{e|y_{\xi e}} \sim N(y_{\xi e}, \sigma^2_{0\xi e}), \quad \xi_{jn|y_{\xi jn}} \sim N(y_{\xi jn}, \sigma^2_{0\xi j}). \]

Hence, the public signals have pre-posterior distributions:

\[ y_{th} \sim N(0, \sigma^2_{0\theta e}), \quad y_{th} \sim N(0, \sigma^2_{0\theta j}), \quad y_{\xi e} \sim N(0, \sigma^2_{0\xi e}), \quad y_{\xi jn} \sim N(0, \sigma^2_{0\xi j}), \]

with the sum of the pre-posterior and the posterior variances being equal to the prior variances, i.e.,

\[ \sigma^2_{0\theta e} + \sigma^2_{0\theta j} = 1, \sigma^2_{0\theta j} + \sigma^2_{0\theta j} = 1, \quad \sigma^2_{0\xi e} + \sigma^2_{0\xi j} = 1, \sigma^2_{0\xi j} + \sigma^2_{0\xi j} = 1. \]

The informativeness of the public signals can thus be measured as the pre-posterior variances of the signals \( \sigma^2_{0(\cdot)} \) for a given prior variance (equal to one without loss of generality).\(^4\)

**Lemma 2** Consider two public information systems \( \eta' \) and \( \eta'' \) for which the pre-posterior variances of the signals \( \sigma^2_{0(\cdot)} \) are higher for \( \eta'' \) than for \( \eta' \). In this setting, \( \eta'' \) is more informative than \( \eta' \).

### 3.1 Efficient allocation within firm types

In this section we determine the efficient allocation of a fixed per firm aggregate capital investment \( q_j \) among firms of the same type for a given public signal \( y \in Y \). The key quantity for determining an efficient allocation of capital investments among firms is the marginal rate of transformation, i.e.,

\[ \text{MRT}_j(\xi_e, \xi_{jn}; q_{jn}) \equiv \frac{\partial h_j(\xi_e, \xi_{jn}; q_{jn})}{\partial q_{jn}} = k \exp\left[ \alpha_j + \beta_j \xi_e + \psi_j \xi_{jn} \right] q_{jn}^{k-1}. \]

\(^4\)All proofs are in Appendix C.
Note that the ratio of marginal rate of transformations for firms of the same type do not depend on the economy-wide state. Hence, we can determine the optimal allocation of $q_j$ independently of the economy-wide information and state. Since firm-specific risks do not command a risk premium (see equation (3)), the optimal allocation of $q_j$ is such that the conditional expected marginal rate of transformation is the same for all firms of type $j$ given the firms’ firm-specific productivity signals $y_{\xi_j}$. Solving for the optimal capital investment $q_j^*(q_j, y_{\xi_j})$ of firm $n$ of type $j$ in terms of the common conditional expected marginal rate of transformation for type $j$ firms, and using the fact that for a well-diversified portfolio of type $j$ firms, the average optimal capital investment must be equal to $q_j$, we get the following result.

**Lemma 3** Assume the per firm aggregate capital investment in type $j$ firms, $j = 1, \ldots, J$, is equal to $q_j$.

(a) The optimal investment in firm $n$ of type $j$ given the public firm-specific productivity signal $y_{\xi_j}$ is given by

$$q_j^*(q_j, y_{\xi_j}) = \exp \left[ \psi_j \left( \frac{y_{\xi_j}}{1 - k} \sigma_{0\xi_j}^2 \right) / (1 - k) \right] q_j,$$

and it is increasing in the firm-specific productivity signal $y_{\xi_j}$. It does not depend on economy-wide productivity information or on windfall information given $q_j$.

(b) The payoff from capital investments $h_j(\cdot)$ in a well-diversified portfolio of type $j$ firms is given by

$$h_j^E(\xi_e, q_j) = \exp \left[ \alpha_j + \beta_j \xi_e + \frac{1}{2} \psi_j^2 \left( 1 + \sigma_{0\xi_j}^2 \frac{k}{1 - k} \right) q_j \right] q_j^k,$$

and it is increasing in the informativeness of the firm-specific productivity signals, $\sigma_{0\xi_j}^2$.

Not surprisingly, firms receiving higher firm-specific productivity signals invest more than firms receiving lower firm-specific productivity signals—this is efficient allocation of capital investments among firms at work! The more efficient allocation of capital investments among firms of the same type increases the productivity of aggregate capital investments with a factor $\exp \left[ \frac{1}{2} \psi_j^2 \sigma_{0\xi_j}^2 \frac{k}{1 - k} \right]$ and, thus, more so when the firm-specific productivity signals are more informative, i.e., when the pre-posterior variance $\sigma_{0\xi_j}^2$ increases.
3.2 Efficient allocation among firm types

In this section we determine the efficient allocation of a fixed aggregate capital investment $\bar{q}_0$ per firm among the $J$ firm types, i.e., $\bar{q}_0 = \frac{1}{J} \sum_{j=1}^{J} q_j^1(\bar{q}_0)$. Having determined the efficient allocation of capital investments within firm types, the marginal rate of transformation for firms of type $j$ is

$$MRT_j(\xi_e; q_j) \equiv \frac{\partial h_j^E(\xi_e, q_j)}{\partial q_j} = k \exp \left[ \alpha_j + \beta_j \xi_e + \frac{1}{2} \psi_j^2 \left( 1 + \frac{\sigma_{0j}^2}{k(1 - k)} \right) \right] q_j^{k-1}. \tag{10}$$

To simplify the analysis, we consider throughout a setting in which the firm types have the same sensitivity with respect to the economy-wide productivity state $\xi_e$, i.e., $\beta_j = \beta, j = 1, \ldots, J$. This implies that the ratio of marginal rate of transformations for firms of different types do not depend on the economy-wide state and, thus, the efficient allocation of investments among firm types is independent of the economy-wide information and state. Hence, the marginal rate of transformation is common to all firm types. Solving for the optimal capital investment $q_j^1(\bar{q}_0)$ for firms of type $j$ in terms of the common marginal rate of transformation, we get the following result.

**Lemma 4** Suppose the aggregate capital investment per firm is equal to $\bar{q}_0$ and $\beta_j = \beta, j = 1, \ldots, J$.

(a) The optimal capital investment in firms of type $j$ is

$$q_j^1(\bar{q}_0) = \kappa_j \left( \sigma_{0j}^2 \right) \bar{q}_0, \tag{11}$$

where the expression for $\kappa_j \left( \sigma_{0j}^2 \right)$ is given in the proof. The fraction of aggregate capital investments invested in type $j$ firms $\kappa_j \left( \sigma_{0j}^2 \right)$ does not depend on the economy-wide and firm-specific signals, but it is an increasing function of the informativeness of the firm-specific productivity signals, $\sigma_{0j}^2$.

(b) The payoff from capital investments $h_j(\cdot)$ at $t = 2$ to the representative investor is

$$\bar{h}(\xi_e, \bar{q}_0) = \exp \left[ \pi + \beta \xi_e \right] \bar{q}_0^{\kappa}, \tag{12}$$

where the expression for $\pi$ is given in the proof. The payoff $\bar{h}(\xi_e, \bar{q}_0)$ is increasing in the economy-wide productivity signal $\xi_e$, and in the informativeness of the firm-specific productivity signals $\sigma_{0j}^2, j = 1, \ldots, J$.

Increasing the informativeness of firm-specific productivity information for a given firm type increases the productivity of investments within that firm type and, thus, this firm type
gets allocated a larger fraction of the aggregate investments in the economy. Increasing the informativeness of the firm-specific productivity information for all firm types increases the overall productivity of aggregate investments in the economy.

### 3.3 Efficient allocation over time

The public productivity information at \( t = 1 \) affects the efficient allocation of aggregate capital investments among firms. When aggregate capital investments are allocated more efficiently, the future payoffs increase for a fixed aggregate capital investment, and the productivity of an additional aggregate dollar invested increase. In this section we determine how these facts affect the optimal aggregate investments and, thus, the optimal intertemporal allocation of aggregate consumption possibilities. An optimal aggregate investment maximizes the representative investor’s conditional expected utility given public information at \( t = 1 \), i.e.,

\[
q^*_o(y_e) = \arg \max_{q_o \in \mathbb{R}^+} U_1(y_e) = u_1(c_1^\dagger) + \mathbb{E} \left[ u_2(c_2^\dagger) | y_{\theta e}, y_{\xi e}, \bar{q}_o \right], 
\]

where \( c_t^\dagger \) denote consumption at date \( t = 1, 2 \) given an optimal allocation of the aggregate capital investment \( q_o \) among firms given in Lemma 3 and Lemma 4, i.e.,

\[
c_1^\dagger = \bar{d}_1 - \bar{q}_o, \quad c_2^\dagger = \bar{g}(\theta_e) + \bar{h}(\xi_e, \bar{q}_o),
\]

where \( \bar{g}(\theta_e) = \alpha + \beta \theta_e, \alpha = \frac{1}{J} \sum_{j=1}^J a_j \), and \( \bar{b} = \frac{1}{J} \sum_{j=1}^J b_j \). The first-order condition for an optimal aggregate investment \( q^*_o(y_e) \) given the economy-wide public signal \( y_e = (y_{\theta e}, y_{\xi e}) \) is

\[
\exp [-\iota^*_2(y_e)] \left[ \mathbb{E} \left[ \frac{\partial \bar{h}}{\partial \bar{q}_o} | y_e, \bar{q}_o(y_e) \right] + \mathbb{Cov} \left[ m^*_{21}, \frac{\partial \bar{h}}{\partial \bar{q}_o} | y_e, \bar{q}_o(y_e) \right] \right] = 1, 
\]

where

\[
m^*_{21}(y_e) = \frac{u_2'(c_2^*) | y_e, \bar{q}_o(y_e) |}{\mathbb{E} [u_2'(c_2^*) | y_e, \bar{q}_o(y_e)]}, \quad \exp [-\iota^*_2(y_e)] = \frac{\mathbb{E} [u_2'(c_2^*) | y_e, \bar{q}_o(y_e)]}{u_1'(c_1^*(y_e))}.
\]

Comparing to equation (3), we obtain the following result.

**Lemma 5** Suppose \( \beta_j = \beta, j = 1, ..., J \). The optimal aggregate capital investment \( q^*_o(y_e) \) maximizes the cum-dividend market value of the market portfolio taking the equilibrium riskless interest rate \( \iota^*_2(y_e) \) and pricing kernel \( m^*_{21}(y_e) \) as given. In turn, the equilibrium riskless interest rate \( \iota^*_2(y_e) \) and pricing kernel \( m^*_{21}(y_e) \) both depend on the optimal aggregate capital investment \( q^*_o(y_e) \).
(a) The optimal aggregate capital investment $\bar{q}_o(y_e)$ and its allocation among firms do not depend on the firm-specific windfall signals and their informativeness.

(b) The optimal aggregate capital investment $\bar{q}_o(y_e)$ does not depend on the firm-specific productivity signals, but may depend on the informativeness of these signals through the impact on the efficiency of its allocation among firms.

(c) The optimal aggregate capital investment $\bar{q}_o(y_e)$ may depend on both the economy-wide windfall and productivity signals and on their informativeness.

Firm-specific windfall information affects neither the allocation of aggregate capital investments among firms nor the equilibrium consumption of a representative investor holding a fraction of the market portfolio and, thus, has no impact on the optimal aggregate capital investments. Firm-specific productivity information facilitates a more efficient allocation of aggregate capital investments among firms and, thus, increases the productivity of aggregate capital investment as well as the level of aggregate future payoffs for a given aggregate capital investment. The former effect calls for higher aggregate capital investments, i.e., a substitution effect, while the latter effect calls for lower aggregate capital investments in order to smooth the productivity gains over the two periods, i.e., an income effect.

Economy-wide windfall information does not affect the allocation of aggregate capital investments among firms, but it affects the beliefs about future aggregate consumption possibilities. A favorable economy-wide windfall signal yields higher conditional expected future aggregate consumption possibilities, and more informative economy-wide windfall signals reduce the posterior variance of future economy-wide windfall gains. Both effects call for reduced aggregate capital investments. The former effect is due to smoothing of expected windfall gains over the two periods, i.e., an income effect, while the latter effect is due to a reduced demand for precautionary savings. Economy-wide productivity information have similar effects in addition to its impact on the productivity of aggregate capital investments, i.e., the substitution effect.

The net-impact on optimal aggregate capital investments of the substitution, the income, and the precautionary savings effects following from changes in the informativeness of firm-specific productivity information, the specific economy-wide signals, and their informativeness depends on the investors’ preferences. In Section 4, we consider standard time-additive CRRA preferences and study these trade-offs explicitly depending on the investors’ common relative risk aversion parameter.
3.4 Ex ante welfare

In the preceding subsections, we have characterized the efficient allocation of resources among firms and over time, and how that allocation may depend on specific signals and their informativeness. More informative firm-specific productivity signals increase payoffs on well-diversified portfolios for the same aggregate capital investment and are, thus, valuable to investors from an ex ante perspective if they prefer more to less. At the same time, the optimal aggregate capital investments may depend on both the informativeness of firm-specific productivity signals and the specific economy-wide signals and their informativeness. Since the optimal aggregate capital investment maximizes the conditional expected utility of the representative investor at $t = 1$ (see equation (13)), we can apply Lemma 2 and the Blackwell Theorem to shown that the investors’ ex ante expected utilities are weakly increasing in the pre-posterior variances of the public signals, i.e., public information is valuable to investors if, and only if, this information affects optimal capital investment choices.

Proposition 6 Consider two public information systems $\eta'$ and $\eta''$ for which the informativeness of the signals (i.e., pre-posterior variances $\sigma_{0j}^2$) are higher for $\eta''$ than for $\eta'$. In this setting, the investors’ ex ante expected utilities with $\eta''$ are at least as high as with $\eta'$. In particular, the investors’ ex ante expected utilities

(a) do not depend on the informativeness of the firm-specific windfall signals $\sigma_{0\theta j}^2, j = 1, \ldots, J$,
(b) increase in the informativeness of the firm-specific productivity signals $\sigma_{0\xi j}^2, j = 1, \ldots, J$,
(c) increase in the informativeness of the economy-wide windfall signals $\sigma_{0\theta e}^2$, and
(d) increase in the informativeness of the economy-wide productivity signals $\sigma_{0\xi e}^2$, unless the substitution, income, and precautionary savings effects balance such that the optimal aggregate capital investment is independent of the economy-wide productivity signals.

4 Implications for the Cost of Capital and Firm Values

In this section, we examine the implications for the cost of capital and ex ante firm values of the impact of the informativeness of public information on efficient capital investments. As shown in the preceding section, the efficient allocation of aggregate investments among firms do not depend on the investors’ preferences, but the optimal aggregate capital investment does depend on these preferences. Closed-form expressions for equilibrium interest rates and
risk premia can only be calculated for certain classes of utility functions and distributions of future payoffs. We use the standard model in consumption-based asset pricing with time-additive power utility, i.e.,

\[ u_t(c_t) = \exp \left[ -\delta t \right] \frac{1}{1-\gamma} c_t^{1-\gamma}, \quad c_t > 0, \gamma > 0, \gamma \neq 1. \]

Combined with lognormal distributions for consumption, this utility function yields closed-form solutions for asset prices. To preserve a lognormal distribution of consumption for the representative investor given public information at \( t = 1 \), we assume throughout the following analysis that the average fixed components of \( t = 2 \) dividends is equal to zero, i.e.,

\[ \bar{a} = \frac{1}{J} \sum_{j=1}^{J} a_j = 0, \]

and that there is no economy-wide windfall risk, i.e.,

\[ c_j = 0, \quad j = 1, ..., J. \]

As noted above, the impact of economy-wide windfall information is to facilitate improved consumption smoothing through an inverse relationship between the economy-wide windfall signal and the aggregate capital investments. That role of economy-wide windfall information is shared with economy-wide productivity information which, in addition, is informative about the productivity of aggregate investments. Hence, there is not much loss of generality in assuming that there is only economy-wide productivity information. In addition, the aggregate \( t = 2 \) dividends can only be lognormally distributed if the sensitivity of the economy-wide productivity state is the same for all firm types, i.e., we assume throughout the following analysis that

\[ \beta_j = \beta, \quad j = 1, ..., J. \]

Even though we make assumptions to ensure that the representative investor’s \( t = 2 \) consumption is lognormally distributed given information at \( t = 1 \), the prior distribution at \( t = 0 \) of the representative investor’s \( t = 2 \) consumption is not lognormal if the equilibrium aggregate investments at \( t = 1 \) varies with the economy-wide productivity signal at \( t = 1 \) (which is generally the case). Since there is no closed-form solution for equilibrium capital investments even with lognormally distributed aggregate \( t = 2 \) dividends given information at \( t = 1 \), these investments as well as the \textit{ex ante} asset prices and the \textit{ex ante} expected utility of the representative investor must be calculated numerically.

We illustrate our results using a setting with three firm types primarily distinguished
by the informativeness of their firm-specific productivity informativeness. The base case, in
which all firms have identical production functions and information systems, is given by the
parameters in Table 1.

<table>
<thead>
<tr>
<th>Exogenous payoffs</th>
<th>Production functions</th>
<th>Informativeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{d}_{0j}$</td>
<td>$d_{1j}$</td>
<td>$a_j$</td>
</tr>
<tr>
<td>29</td>
<td>50</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 1: Base case parameters for exogenous payoffs, production functions,
and informativeness of information systems.

We assume throughout that the rate of time preference is $\delta = 2\%$, but we vary the values
of the relative risk aversion parameter $\gamma$. Note that with time-additive power utility, the
relative risk aversion parameter $\gamma$ is also a measure of the inverse elasticity of intertemporal
substitution, i.e., a high relative risk aversion for stochastic variations in consumption at a
given date also reflects a strong desire for smoothing consumption across periods.

We start our analysis in Section 4.1 considering the impact of public information on
efficient capital investments and the *ex post* cost of capital, while Section 4.2 examines how
the informativeness of forthcoming public information affects the *ex ante* cost of capital and
firm values.

### 4.1 Efficient capital investments and *ex post* cost of capital

In this section, we examine the impact of increasing the informativeness of the public signal
$y = (y_{\xi\epsilon}, \{y_{\theta_{jn}}; y_{\xi_{jn}}\}_{j=1,\ldots,J; n=1,\ldots,N})$ at $t = 1$ on the equilibrium capital investments and the firms' *ex post* cost of capital. Consider first a given average aggregate capital investment $\bar{q}_o$ at $t = 1$. Given an optimal allocation of the aggregate capital investment $\bar{q}_o$ among firms, the representative investor’s consumption at $t = 1$ and $t = 2$ is given by (see Lemma 4)

$$c_1^1 = \bar{d}_1 - \bar{q}_o, \quad c_2^1 = \exp[\pi + \beta \xi\epsilon] \bar{q}_o^k.$$  

Conditional on the economy-wide productivity signal $y_{\xi\epsilon}$ and the average aggregate capital
investment $\bar{q}_o$, the representative investor’s $t = 2$ consumption is lognormally distributed,
i.e.,

$$\ln[c_2^1|y_{\xi\epsilon}; \bar{q}_o] \sim N(\pi + \beta y_{\xi\epsilon} + k \ln \bar{q}_o; \beta^2 \sigma^2_{\xi\epsilon}).$$

(15)
The second-period spot interest rate $i_{21}^{t}(y_{\xi}, \overline{q}_o)$ at $t = 1$ is determined as the marginal rate of substitution between $t = 1$ and $t = 2$ consumption (see equation (2)), i.e.,

$$i_{21}^{t}(y_{\xi}, \overline{q}_o) = -\ln \left[ \frac{\mathbb{E}[u_2'(c_2)|y_{\xi}, \overline{q}_o]}{u_1'(c_1)} \right].$$

Calculating the conditional expected marginal utility of $t = 2$ consumption and simplifying yields the following result.

**Lemma 7** Conditional on the economy-wide productivity signal $y_{\xi}$ and the average aggregate capital investment $\overline{q}_o$, the equilibrium second-period spot interest rate $i_{21}^{t}(y_{\xi}, \overline{q}_o)$ at $t = 1$ is determined as

$$i_{21}^{t}(y_{\xi}, \overline{q}_o) = \delta + \gamma \left( \mathbb{E} \left[ \ln[c_2]|y_{\xi}, \overline{q}_o \right] - \ln[c_1] \right) - \frac{1}{2} \gamma^2 \text{Var} \left[ \ln[c_2]|y_{\xi}, \overline{q}_o \right]$$

$$= \delta + \gamma \left( \pi + \beta y_{\xi} + k \ln[\overline{q}_o] - \ln[\overline{d}_1 - \overline{q}_o] \right) - \frac{1}{2} \gamma^2 \beta^2 \sigma^2_{1\xi}. \quad (16)$$

Hence, the second-period spot interest rate is determined as the rate of time preference $\delta$ plus the inverse elasticity of intertemporal substitution $\gamma$ times the (percentage) expected consumption growth minus a risk premium. The risk premium is determined as one-half times the relative risk aversion parameter $\gamma$ squared times the variance of (percentage) consumption growth. The intuition for the risk premium in the second-period spot interest rate is that more uncertainty about $t = 2$ consumption increases the precautionary demand for sure dollars at $t = 2$ (i.e., the riskless security) and, thus, in equilibrium, the spot interest rate must be lower.

Since an increase in the firm-specific productivity informativeness $\sigma^2_{0\xi j}$ increases the average productivity parameter of capital investments $\pi$ (see Lemma 4), the conditional expected consumption growth also increases, while the conditional variance of consumption growth is unaffected. Hence, the second-period spot interest rate $i_{21}^{t}(y_{\xi}, \overline{q}_o)$ increases as the informativeness of firm-specific productivity information $\sigma^2_{0\xi j}$ increases for any of the firm types $j = 1, ..., J$ (conditional on $y_{\xi}$ and $\overline{q}_o$). Similarly, the conditional expected consumption growth increases in both the economy-wide productivity signal $y_{\xi}$ and in the average aggregate capital investment $\overline{q}_o$, while the conditional variance of consumption growth is unaffected by these quantities. Hence, the second-period spot interest rate $i_{21}^{t}(y_{\xi}, \overline{q}_o)$ is an increasing function of both $y_{\xi}$ and $\overline{q}_o$. Furthermore, the second-period spot interest rate $i_{21}^{t}(y_{\xi}, \overline{q}_o)$ is an increasing function of the informativeness of the economy-wide productivity signal, i.e., $\sigma^2_{0\xi} = 1 - \sigma^2_{1\xi}$, due to a reduced uncertainty about $t = 2$ consumption and, thus, a lower demand for precautionary savings and, hence, the risk premium in the second-period
spot interest rate is reduced.

The second-period pricing kernel \( m^{1\downarrow}_{21}(\xi, y_{\xi}, \overline{q}_o) \) determining the \textit{ex post} risk premia in equilibrium security prices at \( t = 1 \) is determined as the ratio between the marginal utility of \( t = 2 \) consumption and the expected marginal utility of \( t = 2 \) consumption conditional on the economy-wide productivity signal \( y_{\xi} \) and the average aggregate capital investment \( \overline{q}_o \) (see equation (2)), i.e.,

\[
m^{1\downarrow}_{21}(\xi, y_{\xi}, \overline{q}_o) = \frac{u'_{2}(c^{1\downarrow}_{2}(\xi, \overline{q}_o))}{E[u'_{2}(c^{1\downarrow}_{2})|y_{\xi}, \overline{q}_o]},
\]

Using the lognormal distribution property of \( t = 2 \) consumption and completing the calculations yields the following result.

\textbf{Lemma 8} Conditional on the economy-wide productivity signal \( y_{\xi} \) and the average aggregate capital investment \( \overline{q}_o \), the equilibrium pricing kernel \( m^{1\downarrow}_{21}(y_{\xi}, \overline{q}_o) \) at \( t = 1 \) is determined as

\[
m^{1\downarrow}_{21}(\xi, y_{\xi}, \overline{q}_o) = \exp \left[ \gamma \left( E \left[ \ln[c^{1\downarrow}_{2}] | y_{\xi}, \overline{q}_o \right] - \ln[c^{1\downarrow}_{2}(\xi, \overline{q}_o)] \right) - \frac{1}{2} \gamma^2 \sigma^2 \right].
\]

Since the investors’ \( t = 2 \) consumption is increasing in the economy-wide productivity state \( \xi \), the pricing kernel \( m^{1\downarrow}_{21}(\xi, y_{\xi}, \overline{q}_o) \) is decreasing in \( \xi \) and, thus, the pricing kernel is a measure of the “scarcity” of consumption at \( t = 2 \) with an expected value equal to one conditional on the the economy-wide productivity signal \( y_{\xi} \). Note that the pricing kernel is independent of the average aggregate investment \( \overline{q}_o \) in our “lognormal setting” and, thus, the equilibrium pricing kernel with an optimal average aggregate capital investment is

\[
m^{*\downarrow}_{21}(\xi, y_{\xi}) = m^{1\downarrow}_{21}(\xi, y_{\xi}, \overline{q}_o).
\]

The equilibrium ex-dividend price \( v^{1\downarrow}_{1jn} \) of firm \( n \) of type \( j \) at \( t = 1 \) given the firm-specific signals \( (y_{\theta_{jn}}, y_{\xi_{jn}}) \), the economy-wide productivity signal \( y_{\xi} \), and the average aggregate capital investment \( \overline{q}_o \) is given by equation (3) as

\[
v^{1\downarrow}_{1jn} = v^{1\downarrow}_{1j}(y_{\theta_{jn}}, y_{\xi_{jn}}, y_{\xi}, \overline{q}^{1\downarrow}_j(\cdot))
\]

\[
= \exp[-i^{\downarrow}_{1j}] \left[ E[d^{1\downarrow}_{2j}(y_{\theta_{jn}}, y_{\xi_{jn}}, y_{\xi}, \overline{q}^{1\downarrow}_j(\cdot)) - RP^{\downarrow}_{21jn}\right], \quad (17)
\]
where the conditional expected $t = 2$ dividend is

$$E[d_{2j}^t | y_{\theta jn}, y_{\xi jn}, y_{\xi e}, q_j^t(\cdot)] = a_j + c_j y_{\theta jn} + E[h_j^t | y_{\xi jn}, y_{\xi e}, q_j^t(\cdot)]$$

$$= a_j + c_j y_{\theta jn} + \exp \left[ \alpha_j + \psi_j \left( y_{\xi jn} + \frac{1}{2} \psi_j \sigma^2_{1\xi j} \right) + \beta \left( y_{\xi e} + \frac{1}{2} \beta \sigma^2_{1\xi e} \right) \right] \left( q_j^t(\cdot) \right)^k, \quad (18)$$

and where the equilibrium capital investment function $q_j^t(\cdot) \text{ (for fixed } \overline{q}_o) \text{ is given by (9) and (11) in Lemma 3 and Lemma 4 as}$

$$q_j^t(y_{\xi jn}, y_{\xi e}, \overline{q}_o) = \exp \left[ \psi_j \left( y_{\xi jn} - \frac{1}{2} \frac{\psi_j}{1-k} \sigma^2_{1\xi j} \right) / (1-k) \right] \kappa_j \left( \sigma^2_{0\xi j} \right) \overline{q}_o.$$

The conditional dollar risk premium $RP_{21jn}^t$ is given by equation (4) as

$$RP_{21jn}^t = RP_{21j}^t(y_{\theta jn}, y_{\xi jn}, y_{\xi e}, q_j^t(\cdot))$$

$$= -\text{Cov} \left[ m_{21}, E \left[ d_{2j}^t | y_{\theta jn}, y_{\xi jn}, \xi_e, q_j^t(\cdot) \right] | y_{\theta jn}, y_{\xi jn}, y_{\xi e}, q_j^t(\cdot) \right], \quad (19)$$

and the rate of return risk premium $\omega_{21jn}^t$ is given by equations (5) and (6) as

$$\omega_{21jn}^t = \omega_{21j}(y_{\theta jn}, y_{\xi jn}, \xi_e, q_j^t(\cdot)) = \mu_{21j}(y_{\theta jn}, y_{\xi jn}, \xi_e, q_j^t(\cdot)) - \iota_{21}(y_{\xi e}, \overline{q}_o)$$

$$= \ln \left[ \frac{E \left[ d_{2j}^t | y_{\theta jn}, y_{\xi jn}, \xi_e, q_j^t(\cdot) \right]}{\iota_{1j}^t(y_{\theta jn}, y_{\xi jn}, y_{\xi e}, q_j^t(\cdot))} \right] - \iota_{21}(y_{\xi e}, \overline{q}_o). \quad (20)$$

**Lemma 9** Conditional on public information at date $t = 1$, it holds that:

- **(a)** The dollar risk premium $RP_{21j}^t$ for firm $n$ of type $j$ at $t = 1$ is given as

$$RP_{21jn}^t = \left\{ 1 - \exp \left[ -\gamma \beta^2 \sigma^2_{1\xi e} \right] \right\} E[h_j^t | y_{\xi jn}, y_{\xi e}, q_j^t(\cdot)]. \quad (21)$$

- **(b)** The one-period ahead expected rate of return $\mu_{21jn}^t$ of firm $n$ of type $j$ at $t = 1$ is given as

$$\mu_{21jn}^t = \iota_{21}(y_{\xi e}, \overline{q}_o) + \omega_{21jn}^t,$$

where the rate of return risk premium $\omega_{21jn}^t$ is given as

$$\omega_{21jn}^t = \ln \left[ \frac{a_j + c_j y_{\theta jn} + E[h_j^t | y_{\xi jn}, y_{\xi e}, q_j^t(\cdot)]}{a_j + c_j y_{\theta jn} + \exp \left[ -\gamma \beta^2 \sigma^2_{1\xi e} \right] E[h_j^t | y_{\xi jn}, y_{\xi e}, q_j^t(\cdot)]} \right]. \quad (22)$$

- **(c)** The one-period ahead expected rate of return $\overline{\mu}_{21}^t$ of the market portfolio of firms at
The firms’ \textit{ex post} dollar risk premia depend on both the average aggregate capital investment, $q_o$, and on the firm-specific and economy-wide productivity signals, $y_{Ejn}$ and $y_{Ee}$, and is, thus, non-trivial random variables as seen from an \textit{ex ante} perspective at $t = 0$. This is because the level of dividends covarying with the economy-wide productivity state at $t = 2$ depend on and are increasing in these signals. However, the fixed component of $t = 2$ dividends $a_j$ and the firm-specific windfall signal $y_{ojn}$ do not affect the dollar risk premium, since the latter is independent of the economy-wide productivity state at $t = 2$ and is, thus, fully diversifiable for investors.

On the other hand, the firms’ \textit{ex post} rate of return risk premia depend on both the fixed component of $t = 2$ dividends $a_j$ and the firm-specific windfall signal $y_{ojn}$. Since $\beta \geq 0$ and $c_j \geq 0$, the \textit{ex post} rate of return risk premium of firm $n$ of type $j$ decreases as $a_j$ increases and as $y_{ojn}$ increases. This reflects the fact that as the fraction of expected $t = 2$ dividends which does not add to the dollar risk premium increases, the lower is the required expected rate of return. Hence, this is a similar impact on the required expected rate of return as the impact on the required rate return on a stock when leverage is reduced. The firm-specific productivity signal $y_{Ejn}$, on the other hand, increases the required expected rate of return, since it increases the dollar risk premium. Appendix A develops this result further expressing a firm’s required expected rate of return as a weighted average of required expected rates of returns on firm-specific “windfall claims” and “productivity claims.”

Since we assume that the average fixed component of $t = 2$ dividends is equal to zero, i.e., $\bar{a} = 0$, and firm-specific windfall risks are fully diversifiable, this “leverage effect” on the rate of return risk premia for individual firms has no impact on the $t = 2$ dividends of the market portfolio (nor on a well-diversified portfolio of type $j$ firms) and, therefore, no impact on the rate of return risk premium of the market portfolio (or a well-diversified portfolio of type $j$ firms). In particular, the rate of return risk premium of the market portfolio does neither depend on firm-specific signals nor on economy-wide signals or on the average aggregate capital investment. This implies that the equilibrium rate of return risk premium on the market portfolio for an equilibrium average aggregate capital investment is $\bar{\omega}_{21} = \bar{\omega}_{21}^\dagger$. 

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Combined with Lemma 7, the one-period ahead expected rate of return $\bar{\mu}_{21}^{\dagger}$ of the market portfolio of firms at $t = 1$ is given as

$$\bar{\mu}_{21}^{\dagger} = \delta + \gamma (\bar{\pi} + \beta y_{\xi e} + k \ln[\bar{q}_o] - \ln[\bar{d}_1 - \bar{q}_o]) + \gamma \beta^2 \sigma_{1\xi e}^2 \left( 1 - \frac{1}{2} \gamma \right).$$

(22)

Hence, the one-period ahead expected rate of return $\bar{\mu}_{21}^{\dagger}$ of the market portfolio of firms at $t = 1$ is increasing in the informativeness of the economy-wide productivity signal $\sigma_{0\xi e}^2 = 1 - \sigma_{1\xi e}^2$ if $\gamma > 2$, ceteris paribus.

However, the equilibrium average aggregate capital investment $\bar{q}_o$ also depends on the informativeness of the economy-wide productivity signal $\sigma_{0\xi e}^2$ and on the economy-wide productivity signal $y_{\xi e}$ itself. The equilibrium average aggregate capital investment $\bar{q}_o$ is by Lemma 5 determined as the value maximizing investment for the market portfolio taking the second-period spot interest rate and the pricing kernel as given, i.e., $\bar{q}_o$ is determined implicitly by equation (14) as

$$\exp [\bar{\pi}^{\dagger}_2 (y_{\xi e})] = \exp [\bar{\pi}^{\dagger}_2 (y_{\xi e}, \bar{q}_o^{\dagger} (y_{\xi e}))]$$

$$= k (\bar{q}_o^{\dagger} (y_{\xi e}))^{k-1} \exp [\bar{\pi}] \left[ \text{E} [\exp [\beta \xi_e] | y_{\xi e}] + \text{Cov} [m_{21}^{\dagger}, \exp [\beta \xi_e] | y_{\xi e}] \right].$$

(23)

Using the expression for the interest rate in Lemma 7, and using the expression for the pricing kernel in Lemma 8 to compute the last term on the right-hand side of (23) yield the following result.

**Lemma 10** Conditional on public information at date $t = 1$, the equilibrium average aggregate capital investment $\bar{q}_o^{\dagger} (y_{\xi e})$ is determined implicitly by the equation

$$\bar{\pi}^{\dagger}_2 (y_{\xi e}) = \ln [k] - (1 - k) \ln [\bar{q}_o^{\dagger} (y_{\xi e})] + \bar{\pi} + \beta (y_{\xi e} + \frac{1}{2} \beta (1 - 2 \gamma) \sigma_{1\xi e}^2),$$

where

$$\bar{\pi}^{\dagger}_2 (y_{\xi e}) = \delta + \gamma (\bar{\pi} + \beta y_{\xi e} + k \ln[\bar{q}_o^{\dagger} (y_{\xi e})] - \ln[\bar{d}_1 - \bar{q}_o^{\dagger} (y_{\xi e})]) - \frac{1}{2} \gamma^2 \beta^2 \sigma_{1\xi e}^2.$$

(24)

(25)

Having determined the equilibrium spot interest rate, $\bar{\pi}^{\dagger}_2 (y_{\xi e})$, the equilibrium rate of return risk premia, $\omega_{21jn}$, the equilibrium average aggregate capital investment, $\bar{q}_o^{\dagger} (y_{\xi e})$, and its allocation among firms, the following two subsections examine the impact on these quantities of varying the informativeness of firm-specific and economy-wide productivity information, respectively.
4.1.1 Firm-specific information

Note from Lemma 10 that the informativeness of firm-specific productivity information neither affects the risk premium in the equilibrium interest rate \((\frac{1}{2}\gamma^2 \beta^2 \sigma^2_{1\xi e})\) nor the risk-adjustment to the expected marginal productivity of equilibrium average aggregate investments \((\frac{1}{2}\beta^2 (1 - 2\gamma) \sigma^2_{1\xi e})\). Hence, the impact of firm-specific productivity informativeness on the equilibrium interest rate and the risk-adjusted expected marginal productivity of average aggregate investments is exclusively through the impact on the average productivity parameter \(\pi\) of aggregate capital investments.

As noted above, increasing the informativeness of firm-specific productivity information \(\sigma^2_{0\xi j}\) for any of the firm types \(j = 1, ..., J\) improves the allocation of aggregate capital investments among firms as reflected by an increase in the average productivity parameter of capital investments \(\overline{\pi}\) (see Lemma 4). For fixed aggregate capital investments, an increase in the average productivity parameter of capital investments increases the expected consumption growth and, thus, increases the interest rate. Improving the allocation of given aggregate capital investments among firms increases the level of future consumption but leaves current consumption unchanged. This suggests that aggregate investments should be reduced in order to smooth the productivity gains in consumption over the two periods (as reflected by the inverse elasticity of intertemporal substitution \(\gamma\)). However, improving the allocation of aggregate capital investments also increases the marginal return of an additional aggregate dollar invested, which suggests that aggregate investment should be increased. The latter (substitution effect) suggests that the equilibrium interest rate increases further compared to a setting with fixed aggregate investments, while the former (income effect) suggests that the impact on the equilibrium interest rate is reduced compared to a setting with fixed aggregate capital investments.

The following result shows that the equilibrium ex post cost of capital on the market portfolio increases as the firm-specific productivity information becomes more informative in any of the \(J\) firm types, while the impact on the equilibrium average aggregate capital investment depends on the investors’ inverse elasticity of intertemporal substitution \(\gamma\).

**Proposition 11** Conditional on public information at date \(t = 1\), it holds that:

(a) The equilibrium interest rate \(\bar{r}^*_{121}\) at \(t = 1\) is increasing in the informativeness of firm-specific productivity information \(\sigma^2_{0\xi j}\) for firm type \(j = 1, ..., J\).

(b) The equilibrium rate of return risk premium on the market portfolio \(\bar{\pi}^*_{121}\) is independent of the informativeness of firm-specific productivity information \(\sigma^2_{0\xi j}\) for firm type \(j = 1, ..., J\).
(c) The equilibrium one-period ahead expected rate of return \( \pi_{21} \) of the market portfolio of firms is increasing in the informativeness of firm-specific productivity information \( \sigma^2_{\delta j} \) for firm type \( j = 1, \ldots, J \).

(d) The equilibrium average aggregate capital investment \( q^*_o \) is strictly decreasing (increasing) in the informativeness of firm-specific productivity information \( \sigma^2_{\delta j} \), \( j = 1, \ldots, J \), if, and only if, the inverse elasticity of intertemporal substitution \( \gamma \) is larger (smaller) than one.

The rate of return risk premium on the market portfolio \( \omega^*_{21} = \gamma \beta^2 \sigma^2_{\xi e} \) is independent of firm-specific productivity informativeness and, thus, the impact of firm-specific productivity informativeness on the one-period ahead expected rate of return on the market portfolio \( \pi^*_{21}(y_{\xi e}) = \pi^*_{21}(y_{\xi e}) + \omega^*_{21} \) (i.e., the equilibrium ex post cost of capital on the market portfolio) is exclusively through the equilibrium interest rate \( \pi^*_{21} \).

The optimal trade-off in determining the optimal aggregate capital investments between the impact of more informative firm-specific productivity information on the level and on the marginal productivity of capital investments, i.e., between consumption smoothing and increased returns on an additional aggregate dollar invested, depends on the investors’ inverse elasticity of intertemporal substitution. The optimal trade-off is such that optimal capital investments increase in the informativeness of firm-specific productivity information if the incentives for intertemporal consumption smoothing is relatively minor (i.e., \( \gamma < 1 \)), while the opposite result obtains if the incentive for consumption smoothing is a main concern (i.e., \( \gamma > 1 \)). The dividing case is log-utility (i.e., \( \lim_{\gamma \to 1} \frac{1}{1-%e\gamma} c^{1-%e\gamma}_t = \ln(c_t) \)) in which case there is no impact of the informativeness of firm-specific productivity information on equilibrium aggregate capital investments.\(^5\)

Even though the equilibrium aggregate investments may decrease as the firm-specific productivity information becomes more informative (due to the investors’ consumption smoothing incentives), the impact on the equilibrium expected consumption growth and, thus, on the equilibrium interest rate is still positive. That is, the productivity gains from more efficient allocations of aggregate capital investments among firms more than offset the negative impact on future payoffs of the reduction in equilibrium aggregate capital investments. This is illustrated in Figure 1. While the equilibrium aggregate capital investments may decrease or increase (depending on whether \( \gamma > 1 \) or \( \gamma < 1 \)), the equilibrium one-period ahead expected rate of return on the market portfolio increases as the firm-specific productivity informativeness increases due to a more efficient allocation of aggregate capital investments.

\(^5\)These results are closely related to the income and substitution effects of an exogenous increase in the expected return on a risky asset for the optimal portfolio choice with a riskless and a risky asset first analyzed by Merton (1969) for power utility and lognormally distributed payoff on the risky asset.
Figure 1: Second-period interest rate and \textit{ex post} one-period ahead expected rate of return on the market portfolio (ERR MP) (left panel) and equilibrium aggregate capital investments (right panel) as functions of firm-specific productivity informativeness ($\sigma_{0\xi}^2$) for identical firm types given by the parameters in Table 1. Both panels show the results for $\gamma = 0.5$ and for $\gamma = 2$. In all cases, the economy-wide productivity signal is equal to its prior mean ($y_{\xi e} = 0$).

among firms and, thus, a higher equilibrium second-period interest rate. Of course, the rate of return risk premium on the market portfolio $\pi_{21}^e$ increases as the relative risk aversion $\gamma$ increases.

4.1.2 Economy-wide information

It follows from Lemma 10 that the equilibrium average aggregate capital investment $\bar{q}_0^*$ depends on the informativeness of the economy-wide productivity signal $\sigma_{0\xi}^2$ and on the economy-wide productivity signal $y_{\xi e}$ itself. The following proposition establishes this dependence and the consequences for the equilibrium interest rate and the \textit{ex post} cost of capital on the market portfolio.

Proposition 12 \textit{Conditional on public information at date $t = 1$, it holds that:}

(a) The equilibrium interest rate $i_{21}^*$ at $t = 1$ is increasing in the economy-wide productivity signal $y_{\xi e}$, and if the inverse elasticity of intertemporal substitution $\gamma$ is larger than one-half also in the informativeness of economy-wide productivity information $\sigma_{0\xi}^2$.

(b) The equilibrium rate of return risk premium on the market portfolio $\pi_{21}^e$ is independent of the economy-wide productivity signal $y_{\xi e}$ and the aggregate capital investment $\bar{q}_0^*$, but it is decreasing in the informativeness of economy-wide productivity information $\sigma_{0\xi}^2$.

(c) The equilibrium one-period ahead expected rate of return $\pi_{21}^e$ of the market portfolio of firms is increasing in the economy-wide productivity signal $y_{\xi e}$, but it may be decreasing or increasing in the informativeness of economy-wide productivity information $\sigma_{0\xi}^2$. 

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Figure 2: Second-period interest rate and \textit{ex post} one-period ahead expected rate of return on the market portfolio (ERR MP) (left panel) and equilibrium aggregate investments (right panel) as functions of the economy-wide productivity signal for identical firm types given by the parameters in Table 1. Both panels show the results for $\gamma = 0.5$ and for $\gamma = 2$.

(d) The equilibrium average aggregate capital investment $\overline{q}_0^*$ is decreasing (increasing) in the economy-wide productivity signal $y_{\xi e}$ if, and only if, the inverse elasticity of intertemporal substitution $\gamma$ is larger (smaller) than one. The equilibrium average aggregate capital investment $\overline{q}_0^*$ is decreasing in the informativeness of economy-wide productivity information $\sigma_{0\xi e}^2$.

It follows from Lemma 10 that an increase in the economy-wide productivity signal $y_{\xi e}$ has the same impact on interest rates and equilibrium aggregate capital investments as an increase in the average productivity parameter $\overline{\pi}$ of aggregate capital investments. Figure 2 shows how the equilibrium second-period interest rate, the one-period ahead expected rate of return on the market portfolio (left panel), and the equilibrium aggregate investments (right panel) depend on the economy-wide productivity signal for $\gamma > 1$ and for $\gamma < 1$. The expected marginal productivity of investments and the expected level of $t = 2$ payoffs of all firm types increases with the economy-wide productivity signal. Hence, an increase in the economy-wide productivity signal $y_{\xi e}$ has the same impact on interest rates and equilibrium aggregate investments as an increase in the informativeness of the firm-specific productivity signals (compare to Figure 1). While the dollar risk premium on the market portfolio increases with the economy-wide productivity signal, the rate of return risk premium is independent of this signal (see Lemma 9(a) and (c)).

Figure 3 shows how the equilibrium second-period interest rate, the one-period ahead expected rate of return on the market portfolio (left panel) depends on the economy-wide productivity informativeness for an exogenous aggregate capital investment equal to the equilibrium aggregate investment for $\sigma_{0\xi e}^2 = 0$ (shown in the right panel). The economy-wide productivity signal is equal to its prior mean ($y_{\xi e} = 0$) in both panels.
Figure 3: Second-period interest rate and ex post one-period ahead expected rate of return on the market portfolio (ERR MP) (in the left panel) for an exogenous aggregate investment ($\bar{q}_0 = \bar{q}_e(\sigma^2 e = 0)$) (shown in the right panel) as functions of the economy-wide productivity informativeness for identical firm types given by the parameters in Table 1. Both panels show the results for $\gamma = 0.5$, $\gamma = 2$, and for $\gamma = 4$. The economy-wide productivity signal is equal to its prior mean ($y_{t\xi} = 0$) in both panels.

The equilibrium second-period interest rate is increasing in the economy-wide productivity informativeness for all values of the relative risk aversion $\gamma$ due to a lower demand for precautionary savings as the second-period uncertainty decreases (see Lemma 7). On the other hand, the rate of return risk premium $\omega_{21}^\dagger$ on the market portfolio decreases in the economy-wide productivity informativeness (independently of the economy-wide productivity signal $y_{t\xi}$) such that the one-period ahead expected rate of return on the market portfolio may decrease (increase) for low (high) values of the relative risk aversion $\gamma < 2$ ($\gamma > 2$), while the two effects are exactly offsetting for $\gamma = 2$ (see equation (22)).

Figure 4 shows the equilibrium second-period interest rate, the one-period ahead expected rate of return on the market portfolio (left panel), and the equilibrium aggregate capital investment (right panel) as functions of the economy-wide productivity informativeness (for an economy-wide productivity signal equal to its prior mean ($y_{t\xi} = 0$)). The equilibrium aggregate investment decreases as the economy-wide productivity informativeness increases for all values of the the relative risk aversion (Proposition 12(d)). This is due to a reduction in the uncertainty about $t = 2$ consumption and, thus, a lower expected marginal utility of $t = 2$ consumption ceteris paribus (which is reflected in a higher second-period interest due to a lower demand for precautionary savings, see Figure 3, left panel). The lower expected marginal utility of $t = 2$ consumption shifts the investment incentives in order to increase consumption at $t = 1$ and decrease consumption at $t = 2$, i.e., decrease aggregate investments at $t = 1$. 
Figure 4: Second-period interest rate and *ex post* one-period ahead expected rate of return on the market portfolio (ERR MP) (in the left panel) for equilibrium aggregate investments ($\bar{\sigma}_{0}^{2}(\sigma_{0\ell_{c}}^{2})$) (shown in the right panel) as functions of the economy-wide productivity informativeness for identical firm types given by the parameters in Table 1. Both panels show the results for $\gamma = 0.5$, $\gamma = 2$, and for $\gamma = 4$. The economy-wide productivity signal is equal to its prior mean ($y_{t\ell_{c}} = 0$) in both panels.

The reduction in equilibrium aggregate investments “dampens” the impact of an increased economy-wide productivity informativeness on the equilibrium second-period interest rate compared to a setting with an exogenous aggregate investment. For $\gamma = \frac{1}{2}$, the impact is completely neutralized, while the equilibrium second-period interest rate still increases in the economy-wide productivity informativeness for $\gamma > \frac{1}{2}$ (Proposition 12(a)). The equilibrium rate of return risk premium $\bar{\pi}_{21}$ on the market portfolio is unaffected by changes in equilibrium aggregate investments (Proposition 12(b)). The “dampening” effect of reduced equilibrium investments on the equilibrium *ex post* interest rate is such that the equilibrium one-period ahead expected rate of return on the market portfolio, i.e., the equilibrium *ex post* cost of capital for aggregate investments $\bar{\omega}_{21}$, is decreasing in the informativeness of the economy-wide productivity information $\sigma_{0\ell_{c}}^{2}$ for all values of the relative risk aversion $\gamma$ shown in Figure 4 contrary to the setting with an exogenous aggregate investment shown in Figure 3.

It may seem puzzling that the equilibrium aggregate investment decreases at the same time as the *ex post* cost of capital for aggregate investments $\bar{\omega}_{21}$ decreases when the economy-wide productivity information becomes more informative. The reason is that increased economy-wide productivity informativeness decreases the remaining uncertainty about the economy-wide productivity state revealed at $t = 2$. Since the payoffs at $t = 2$ is a convex function of the economy-wide productivity state, the *expected payoffs* at $t = 2$ on the market portfolio decreases as the remaining uncertainty about the economy-wide productivity state is reduced. Hence, there is both a denominator effect (on the *ex post* cost of capital) and
a numerator effect (on the expected payoffs) of changes in the economy-wide productivity informativeness.

For a fixed aggregate investment, the \textit{ex ante} distribution of \( t = 1 \) and \( t = 2 \) payoffs are unaffected by the informativeness of the economy-wide productivity signals. From the above comparative statics, it follows that the equilibrium aggregate investments depend not only on the informativeness of the economy-wide productivity signals but also on the signal itself. For a given economy-wide productivity signal, the aggregate investment decreases as the informativeness of the signal increases, however, the distribution of signals also changes, which makes the impact on \textit{ex ante} expected \( t = 1 \) and \( t = 2 \) payoffs unclear. That is, the impact of altering the informativeness of the economy-wide productivity information system on, for example, equilibrium capital investments is best assessed from an \textit{ex ante} perspective at \( t = 0 \).

4.2 \textbf{Ex ante cost of capital and firm values}

Most of the literature on information and the cost of capital has taken a pure exchange economy as a starting point, i.e., an economy in which capital investments (and other production choices) do not change as the informativeness of the information system changes. In Appendix B, we reconcile our analysis with the key result of this literature: any reduction in the \textit{ex post} cost of capital due to more economy-wide information is perfectly offset by an equal increase in the pre-posterior cost of capital leaving the \textit{ex ante} cost of capital and firm values unchanged (see Christensen et al. 2010). Hence, any impact on the \textit{ex ante} cost of capital and firm values must be through changes in the equilibrium capital investment policies as the informativeness of the information system changes. In this section, we first analyze the effects of increased firm-specific productivity informativeness on the \textit{ex ante} cost of capital and firm values and, subsequently, we analyze the effects of increased informativeness of the economy-wide productivity information on the \textit{ex ante} cost of capital and firm values.

4.2.1 \textbf{Firm-specific information}

The equilibrium spot and zero-coupon interest rates at date \( t \) is determined by the expected marginal rate of substitution between future and current equilibrium consumption, i.e.,

\[
\tau^*_t = -\ln \frac{\text{E}_t [u'_t (c^*_t)]}{u'_t (c^*_t)} / (\tau - t).
\]

Since equilibrium consumption at \( t = 1 \) and at \( t = 2 \) are not lognormally distributed given the prior beliefs when capital investments are based on economy-wide information, we cannot
explicitly calculate the expected marginal utility of $t = 1$ and $t = 2$ consumption. However, Proposition 12 contains information sufficient for determining the impact of altered informativeness of firm-specific productivity information on spot and zero-coupon interest rates at $t = 0$.

From Proposition 12(d) we know that equilibrium investments $\bar{q}_j^*(y_{\xi e})$ are decreasing (increasing) in the informativeness of firm-specific productivity information $\sigma^2_{0\xi j}$, $j = 1, \ldots, J$ if, and only if, the inverse elasticity of intertemporal substitution $\gamma$ is larger (smaller) than one. As the distribution over the economy-wide signals, $y_{\xi e}$, is unaffected by the precision of firm-specific information, the changes to equilibrium investments implies that the equilibrium spot interest rate $\nu^*_1$ is increasing (decreasing) in the informativeness of firm-specific productivity information if $\gamma > 1$ ($\gamma < 1$).

The fact that more is invested for any economy-wide signal when $\gamma < 1$ implies more is available for consumption in any state at $t = 2$. This in turn implies that the zero-coupon rate $\nu^*_2$ is increasing in the informativeness of firm-specific productivity information when $\gamma < 1$. From Proposition 12(a) we know that $\nu^*_2(y_{\xi e}) = \ln(u'_1(d_1 - \bar{q}_o^*(y_{\xi e}))) - \ln(E[u'_2(c^*_2)|y_{\xi e}])$ is increasing in the informativeness of firm-specific productivity information regardless of the size of $\gamma$. When $\gamma > 1$, less is invested at $t = 1$ as the informativeness of firm-specific productivity information increases for any economy-wide signal $y_{\xi e}$. This implies that $\ln(u'_1(d_1 - \bar{q}_o^*(y_{\xi e})))$ is decreasing, but as $\nu^*_2(y_{\xi e})$ is increasing, $\ln(E[u'_2(c^*_2)|y_{\xi e}])$ must be decreasing even more. In sum, this implies that $E[u'_2(c^*_2)|y_{\xi e}]$ is decreasing in the informativeness of firm-specific productivity information for any $y_{\xi e}$ and, thus, so is $E[u'_2(c^*_2)] = E[E[u'_2(c^*_2)|y_{\xi e}]]$, since the prior distribution of the economy-wide signals $y_{\xi e}$ is unaffected by the precision of firm-specific information. This implies that the zero-coupon rate $\nu^*_2$ is increasing in the informativeness of firm-specific productivity information when $\gamma > 1$.

**Proposition 13** At the ex ante date $t = 0$, the comparative statics for the equilibrium interest rates with respect to the informativeness of firm-specific information are as follows:

(a) The equilibrium spot interest rate $\nu^*_1$ is increasing (decreasing) in the informativeness of firm-specific productivity information $\sigma^2_{0\xi j}$, $j = 1, \ldots, J$, if $\gamma > 1$ ($\gamma < 1$).

(b) The equilibrium zero-coupon interest rate $\nu^*_2$ is increasing in the informativeness of firm-specific productivity information $\sigma^2_{0\xi j}$, $j = 1, \ldots, J$.

Given the equilibrium ex-dividend price of a type $j$ firm at $t = 0$, the equilibrium implied cost of capital at $t = 0$ for type $j$ firms $\rho^*_j$ is determined by equation (7) as the discount rate solving the equation

$$v^*_0 = E[d^*_1|q^*_j(\cdot)] \exp[-\rho^*_j] + E[d^*_2|q^*_j(\cdot)] \exp[-2\rho^*_j]$$
and, in particular, the implied cost of capital on the market portfolio $\overline{p}^*$ is determined as the discount rate solving the equation

$$\overline{v}_0^* = E[\overline{d}_1 - \overline{q}_0^*(y_{\xi e})] \exp [-\overline{p}^*] + E[\overline{h}(\xi_e, \overline{q}_0^*)] \exp [-2\overline{p}^*].$$

(26)

Having determined the equilibrium capital investments at $t = 1$, we must first determine the equilibrium ex ante firm values, $v_{0j}^*$ and $\overline{v}_0^*$, in order to determine the implied cost of capital measures. For brevity, we only consider the implied cost of capital on the market portfolio. The ex ante price of the market portfolio is the risk-adjusted expected net-dividends discounted with the risk-free interest rates:

$$\overline{v}_0^* = \exp [-\iota_{10}^*] \left[ E \left[ \overline{d}_1 - \overline{q}_0^*(y_{\xi e}) \right] + \text{Cov}_0 \left[ m_{10}^*, \overline{d}_1 - \overline{q}_0^*(y_{\xi e}) \right] \right] + \exp [-2\iota_{20}^*] \left[ E \left[ \overline{h}(\xi_e, \overline{q}_0^*) \right] + \text{Cov}_0 \left[ m_{20}^*, \overline{h}(\xi_e, \overline{q}_0^*) \right] \right].$$

(27)

Studying equation (27), we find the following result.

**Proposition 14** The ex ante value of the market portfolio is decreasing in firm-specific productivity informativeness when $\gamma > 1$, and increasing in firm-specific productivity informativeness when $\gamma < 1$.

Changes in firm-specific productivity informativeness lead to changes in capital investments, to changes in interest rates, to changes in risk premia, to changes in firm values, and eventually to changes in the implied cost of capital. There is no one-to-one relationship between the ex ante firm values and the implied cost of capital, but when the inverse elasticity of intertemporal substitution $\gamma$ is larger than one, the implied cost of capital on the market portfolio is indeed increasing in firm-specific productivity informativeness.

**Proposition 15** When $\gamma > 1$, the implied cost of capital on the market portfolio $\overline{p}^*$ is increasing in firm-specific productivity informativeness, $\sigma_{0\xi j}^2$.

The reason is that a more efficient allocation of capital across firms allows for less to be invested for a larger effect. Thus, when $\gamma > 1$, consumption is increasing at both $t = 1$ and at $t = 2$, which at $t = 0$ leads to a lower willingness-to-pay for future dollars/consumption. As demonstrated in Proposition 14, the reduced willingness-to-pay for future consumption dominates the effect of increased future risk-adjusted expected dividends and, as a result, the ex ante value of the market portfolio is decreasing in firm-specific productivity informativeness. Simply put: as the implied cost of capital is the discount rate which when used for discounting future expected dividends returns the equilibrium market price, that rate
must be increasing—price is falling while expected dividends at both \( t = 1 \) and \( t = 2 \) are increasing.

When \( \gamma < 1 \), the \textit{ex ante} value of the market portfolio, \( \overline{v}_0^* \), is increasing in firm-specific productivity informativeness as is the expected aggregate \( t = 2 \) dividend, \( E \left[ \overline{h}(\zeta_\iota, \overline{q}_o(y_{\iota\xi})) \right] = E \left[ \pi(\overline{q}_o(y_{\iota\xi}))^k \exp \left[ \beta(y_{\iota\xi} + \frac{1}{2}\sigma_{\iota\xi}^2) \right] \right] \). Contrary to this, the expected aggregate \( t = 1 \) dividend, \( E [d_1 - \overline{q}_o(y_{\iota\xi})] \), decreases, since \( \overline{q}_o(y_{\iota\xi}) \) is increasing in \( \sigma_{0\iota\xi}^2 \) for any \( y_{\iota\xi} \) when \( \gamma < 1 \). Thus, we have a situation in which the \textit{ex ante} price increases, while dividends at \( t = 1 \) and \( t = 2 \) move in opposite directions making the impact on the implied cost of capital unclear. Now, since consumption at \( t = 1 \), \( d_1 - \overline{q}_o(y_{\iota\xi}) \), is decreasing, while second-period consumption, \( E [\overline{h}(\cdot)] \), is increasing, intuition might indicate that the implied cost of capital could decrease if the spot interest rate is higher than the zero-coupon rate—more weight is placed on the low interest rate. However, we have \textit{not} been able to generate examples in which the implied cost of capital on the market portfolio is \textit{decreasing} in firm-specific productivity informativeness.

Figure 5 illustrates that the implied cost of capital for the market portfolio is increasing in firm-specific productivity informativeness given the parameters in Table 1, both when \( \gamma > 1 \) and when \( \gamma < 1 \). The same type of comparative statics hold for firm \( j \) (and for firms \( j' \neq j \)) when we only vary the firm-specific productivity informativeness of firm \( j \), \( \sigma_{0j}^2 \).

In a setting in which there is no economy-wide information revealed at \( t = 1 \), the one-period ahead expected rate of returns at \( t = 0 \) (see equation (5)) on individual firms and on the market portfolio are all equal to the spot interest rate and, thus, \( \mu_{10j}^* = \overline{p}_{10}^* = \iota_{10}^* \) is increasing in firm-specific productivity informativeness \( \sigma_{0j}^2 \) when \( \gamma > 1 \) and decreasing in \( \sigma_{0j}^2 \) when \( \gamma < 1 \) (see Proposition 13(a)). However, when economy-wide information is
lognormally distributed when there is economy-wide information released at $t = 1$ and aggregate capital investments vary with this information, the one-period ahead expected rate of return differs from the spot interest rate. This makes it difficult to determine analytically the impact on the one-period ahead expected rate of return on an individual firm or on the market portfolio of an increase in firm-specific productivity informativeness. The problem is that neither cum-dividend prices nor consumption are difficult to determine analytically the impact on the one-period ahead expected rate of return on an individual firm or on the market portfolio of an increase in firm-specific productivity informativeness. The problem is that neither cum-dividend prices nor consumption are 

released at $t = 1$ and aggregate capital investments vary with this information, the one-period ahead expected rate of return differs from the spot interest rate. This makes it difficult to determine analytically the impact on the one-period ahead expected rate of return on an individual firm or on the market portfolio of an increase in firm-specific productivity informativeness. The problem is that neither cum-dividend prices nor consumption are lognormally distributed when there is economy-wide information released at $t = 1$ and, thus, calculating the expected cum-dividend price or the dollar risk premium analytically is impossible. Nevertheless, Figure 5 shows that the comparative statics for the setting with no economy-wide information carry over to our numerical examples with economy-wide risks at $t = 1$: the one-period ahead expected rate of return on the market portfolio and on individual firms tend to move in sync with the spot interest rate when firm-specific productivity informativeness changes. The reason is that the first-period dollar risk premium is small relative to the expected cum-dividend firm value and, thus, the first-period rate of return risk premium is small as well.

We now return to the $t = 0$ value of the market portfolio, and compare it to the investors’ ex ante expected utility as measured by their ex ante certainty equivalent defined in equation (8). Figure 6 illustrates the impact of firm-specific productivity informativeness in a setting with identical firm types characterized by the parameters in Table 1. The figure depicts the equilibrium ex ante value of the market portfolio and the equilibrium ex ante certainty equivalent as a function of the informativeness of the firm-specific productivity information $\sigma_{0GJ}^2$ (in the left panel for $\gamma = 2$ and in the right panel for $\gamma = 0.5$). As noted in Proposition 14, the ex ante value of the market portfolio is decreasing in the informativeness of the firm-specific productivity information when $\gamma > 1$, while the ex ante value of the market portfolio is increasing in the informativeness of the firm-specific productivity information
when $\gamma < 1$. The intuition is that even though a more efficient allocation of capital leads to increasing future payoffs, it also reduces the willingness-to-pay for an additional future dollar due to the investors’ decreasing marginal utility of consumption. When $\gamma > 1$, this reduced willingness-to-pay for future consumption dominates the effect of increased future dividends and, consequently, \textit{ex ante} prices fall. The opposite obtains when $\gamma < 1$.

As a more efficient allocation of capital among firms and across time can be achieved when firm-specific productivity information is improved, it is clear that the \textit{ex ante} expected utility is increasing in the informativeness of firm-specific productivity information regardless of the specifics of the investors’ preferences (see Proposition 6). Hence, the \textit{ex ante} change in market prices is a poor indicator of welfare implications of information system changes, and especially so when $\gamma > 1$. In other words, “shareholder wealth” and “shareholder welfare” are two fundamentally different concepts. In a general equilibrium analysis of information system choice in a production economy, the distributions of future payoffs change but the basic state prices used to value these payoffs also change. Investors always benefit from a more efficient use of scarce resources in the economy, but the resulting drop in the willingness-to-pay for an additional future dollar imply that current wealth decreases when $\gamma > 1$. The reduced wealth is inconsequential to the representative investor holding a constant fraction of the market portfolio (or, more generally, to individual investors holding equilibrium endowments, see, e.g., Feltham and Christensen 1988).

4.2.2 Economy-wide information

When firm-specific productivity informativeness changes capital investments, expected dividends at $t = 1$ and $t = 2$ also change. One thing that does not change though is the uncertainty pertaining to the economy-wide signals and states. That is, the distribution over economy-wide posteriors and the conditional distribution of economy-wide states are unaffected by firm-specific productivity information, and so is the \textit{ex post} rate of return risk premium on the market portfolio. This obviously ceases to hold when we consider changes in the informativeness of economy-wide productivity information. Nevertheless, the comparative statics for \textit{ex ante} firm values with respect to firm-specific productivity informativeness carry over unchanged to the setting with varying economy-wide productivity informativeness.

**Proposition 16** \textit{The \textit{ex ante} value of the market portfolio is decreasing in economy-wide productivity informativeness when $\gamma > 1$, and increasing in economy-wide productivity informativeness when $\gamma < 1$.}

Since analytical results are difficult to obtain when aggregate consumption is not log-normally distributed with prior beliefs, the following discussion of the economics behind Proposition 16
is based on numerical investigations (for a large set of varying parameters). The discussion is structured around the fundamental asset pricing relation (3): the *ex ante* value of the market portfolio is equal to the sum of future risk-adjusted expected aggregate dividends discounted with zero-coupon interest rates.

As demonstrated in Proposition 12(d), equilibrium capital investments given the economy-wide signal, $q_o^\ast(y_{\xi e})$, decrease as the economy-wide productivity informativeness increases (when $\gamma \neq 1$). In our numerical examples, this implies that the expected capital investment, $E[q_o^\ast(y_{\xi e})]$, is decreasing and, thus, expected first-period consumption, $E[\overline{d}_1 - q_o^\ast(y_{\xi e})]$, is increasing in economy-wide productivity informativeness—this is not a foregone conclusion as the prior distribution of the economy-wide signals $y_{\xi e}$ is changing when we change the economy-wide productivity informativeness. Though the expected capital investment is reduced, this does not imply that the expected second-period aggregate dividend, $E[\overline{h}(\xi_e, q_o^\ast)]$, is decreasing as well. In our numerical examples, it holds that $E[\overline{h}(\xi_e, q_o^\ast)]$ is increasing when $\gamma < 1$ and decreasing when $\gamma > 1$.

When $\gamma$ is large ($\gamma > 1$), the investors are relatively risk averse and, thus, with time-additive preferences they also have a strong desire for consumption smoothing. The investors’ desire for consumption smoothing dominates the desire to exploit the production opportunities efficiently such that the aggregate capital investment is decreasing in productivity, i.e., $d\overline{q}_o^\ast(y_{\xi e})/dy_{\xi e} < 0$ (see Proposition 12(d)). In other words, a high (low) productivity signal, which means that the level of expected future consumption and the productivity of capital investments are both high (low), leads to lower (higher) capital investments and, thus, a reduction (an increase) in $t = 2$ dividends. From an *ex ante* perspective, this optimal capital investment policy implies that the uncertainty in second-period consumption is reduced (compared to a setting with fixed capital investment) and more so, the more informative is the economy-wide productivity signals about the economy-wide productivity states at $t = 2$.

As a consequence, the dollar risk premium for $t = 2$ dividends decreases, and it decreases more than the expected dividend such that the risk-adjusted expected dividend increases as the economy-wide productivity informativeness increases. The variation in first-period dividends also increases, but the impact on the first-period dollar risk premium is not sufficient to dominate the increase in expected first-period dividends due to lower expected capital investments. Hence, the risk-adjusted expected dividends increase in both periods as the economy-wide productivity informativeness increases.

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6 As the capital investment contains a (large) precautionary savings motive (when $\gamma$ is large), capital investments are more sensitive to a reduction in *ex post* uncertainty the larger is $\gamma$.

7 As argued below, the reason for this is that $d\overline{q}_o^\ast(y_{\xi e})/dy_{\xi e} < 0$ when $\gamma > 1$, and $d\overline{q}_o^\ast(y_{\xi e})/dy_{\xi e} > 0$ when $\gamma < 1$ (see Proposition 12(d)). That is, lower expected investments are invested “less efficiently” when $\gamma > 1$—this naturally leads to lower expected $t = 2$ payoffs—and “more efficiently” when $\gamma < 1$.  

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Since risk-adjusted expected dividends on the market portfolio increase in both periods, when the informativeness of the economy-wide productivity signals increases, the \textit{ex ante} value of the market portfolio would also be increasing if there were no impact on interest rates. However, a high risk aversion ($\gamma > 1$) also implies that the desire to save for precautionary reasons is strong such that there is a large positive impact on interest rates of reduced uncertainty in future consumption. The impact of the reduction in the uncertainty of $t = 2$ consumption, following from the optimal capital investment policy, on the equilibrium $t = 2$ zero-coupon interest rate, and the impact of increased expected consumption growth in the first period, imply that the increase in both the spot and the $t = 2$ zero-coupon equilibrium interest rates are strong enough to swamp the increase in risk-adjusted expected dividends at $t = 1$ and at $t = 2$, i.e., the \textit{ex ante} value of the market portfolio is decreasing in economy-wide productivity informativeness when $\gamma > 1$.\textsuperscript{8} These effects are larger, the larger is the inverse elasticity of intertemporal substitution, $\gamma$.

When $\gamma$ is large, the drop in the \textit{ex ante} value of the market portfolio $\pi_0^*$ due to rapidly increasing interest rates, as informativeness increases, is large relative to i) the increase in first-period expected dividends and ii) the drop in second-period expected dividends.\textsuperscript{9} Studying equation (26), it is easy to see this requires the implied cost of capital to be increasing. On the other hand, when $\gamma$ is larger than one but not too large, interest rates are increasing more modestly when informativeness is increased and, thus, the drop in $\pi_0^*$ and the increase in expected first-period dividends are modest relative to the drop in expected second-period dividends. Our numerical investigations suggest that there is a cutoff for the relative risk aversion $\gamma^* > 1$ for which the implied cost of capital is increasing as the informativeness of the economy-wide productivity signals increase when $\gamma > \gamma^*$, and decreasing when $\gamma \in (1, \gamma^*)$.

We illustrate these effects in Figure 7 for a setting with identical firm types characterized by the parameters in Table 1. The figure depicts the equilibrium spot and zero-coupon interest rates as well as the implied cost of capital on the market portfolio as functions of the informativeness of the economy-wide productivity information $\sigma_{06e}^2$—in the left panel for $\gamma = 2$ and in the right panel for $\gamma = 4$. When $\gamma = 2$, the implied cost of capital on the market portfolio is decreasing in the informativeness of the economy-wide productivity information and when $\gamma = 4$, the implied cost of capital on the market portfolio is increasing.

\textsuperscript{8}For some parameters, the risk-adjusted expected first-period dividend is decreasing—this does of course not lead to an increasing $t = 0$ value.

\textsuperscript{9}From the proof of Proposition 17 in Appendix B, it can be seen that if $q_0$ is fixed, then $d(E[d_2] - RP)/d\sigma_{06e}^2 = -(E[d_2] - RP)(1/2 - \gamma)\beta^2$ while $d\pi_1/d\sigma_{06e}^2 = -\pi_1 1/2 \beta^2 (1 - \gamma)^2 < 0$. That is, as uncertainty is reduced, the risk-adjusted expected dividend increases for $\gamma > 1/2$. However, price still drops and the drop in price is larger, the larger is $\gamma$. This carries over to $\pi_0^*$, but only when $\gamma > 1$.  

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Figure 7: *Ex ante* interest rates, implied cost of capital, and one-period ahead expected returns as functions of economy-wide productivity informativeness ($\sigma_{\xi}^2$) for identical firm types given by the parameters in Table 1. The left panel shows the results for $\gamma = 2$, while the right panel shows the results for $\gamma = 4$.

in the informativeness of the economy-wide productivity information. From the two panels, it is also evident interest rates are increasing much more rapidly in the informativeness of economy-wide productivity information when $\gamma = 4$ than when $\gamma = 2$. Note also that the one-period ahead expected rate of return on the market portfolio follows the spot interest rate closely due to a low first-period dollar risk premium as in the setting with varying firm-specific productivity informativeness.$^{10}$

When $\gamma < 1$, the investors are not very risk averse and, importantly, they have a weak demand for consumption smoothing. In this setting, the investors’ desire for consumption smoothing is dominated by the desire to exploit the production opportunities efficiently such that capital investments are increasing in productivity, i.e., $d\pi\rho(y_{\xi e})/dy_{\xi e} < 0$ (see Proposition 12(d)). This has two consequences. First, lower expected capital investments are invested “more efficiently” leading to higher expected $t = 2$ dividends. Second, from an *ex ante* perspective, the optimal capital investment policy amplifies the the uncertainty in second-period consumption (compared to a setting with fixed capital investment) and more

$^{10}$If we vary economy-wide productivity informativeness for the parameters in Table 1, then the implied cost of capital is decreasing when $\gamma$ is between one and roughly 2.75 and increasing for higher levels of risk aversion. Our examples suggest that the distribution of marginal utilities determine whether the implied cost of capital is increasing or decreasing in economy-wide productivity informativeness. One way of adjusting marginal utilities is through $\gamma$ but another is through the marginal productivity of capital parameter, $k$. If $k$ is reduced from 0.75 to 0.40, interest rates are low—partly due to low expected consumption growth but also due to a “high” probability of low $t = 2$ dividends. In this setting, if economy-wide productivity informativeness is increased, $\pi\rho$ drops rapidly relative to the increase in $E[\delta_t - g_{t}]$ and the drop in $E[h(\xi_e, \xi_e^\infty)]$. In that case, the implied cost of capital is increasing in economy-wide productivity informativeness also when $\gamma = 2$. Hence, it would seem that the choice of parameters determine whether the implied cost of capital is increasing or decreasing in economy-wide productivity informativeness when $\gamma > 1$. This may be so, but it should be added that generating examples of increasing implied cost of capital becomes increasingly arduous as $\gamma$ falls below two.
so, the more informative is the economy-wide productivity signals about the economy-wide productivity states at $t = 2$. As a result, the dollar risk premium for $t = 2$ dividends increases, and it increases more than the expected dividend such that the risk-adjusted expected dividend decreases as the economy-wide productivity informativeness increases.

The variation in first-period dividends also increases, and the impact of the increased demand for precautionary savings on equilibrium interest rates is again the dominating force (as for $\gamma = 0.5$) such that the expected dividend decreases as the economy-wide productivity informativeness increases. Hence, risk-adjusted expected dividends decrease in both periods as the economy-wide productivity informativeness increases.

The increased variation in both $t = 1$ and $t = 2$ aggregate dividends induced by the optimal capital investment policy, also increase the demand for precautionary savings. The impact of the increased demand for precautionary savings on equilibrium interest rates is larger than the impact of increased consumption growth for both periods such that the equilibrium spot and $t = 2$ zero-coupon interest rates both decrease. The drop in equilibrium interest rates is again the dominating force (as for $\gamma > 1$) such that the ex ante value of the market portfolio is increasing in economy-wide productivity informativeness when $\gamma < 1$ even though risk-adjusted expected dividends decrease in both periods. The increase in the ex ante value of the market portfolio $\pi_0^*$ is modest compared to the increase in expected $t = 1$ and $t = 2$ aggregate dividends, implying the implied cost of capital is increasing as the informativeness of the economy-wide productivity signals increase. This effect is stronger the more risk tolerant is the representative investor (or the lower is $\gamma$)

We illustrate these effects in Figure 8 for a setting with identical firm types characterized by the parameters in Table 1. The left panel of the figure depicts the equilibrium spot and zero-coupon interest rates as well as the implied cost of capital on the market portfolio as...
functions of the informativeness of the economy-wide productivity information $\sigma_{\xi e}^2$ when $\gamma = 0.5$. When $\gamma = 0.5$, the implied cost of capital on the market portfolio is increasing in the informativeness of the economy-wide productivity information, while interest rates are decreasing. For comparison, the right panel depicts the associated results for the limiting case for $\gamma = 0$ (i.e., for log utility) in which interest rates and the implied cost of capital are all independent of the informativeness of the economy-wide productivity information. Of course, the key for these results is that the equilibrium capital investments are independent of the economy-wide productivity signals in this setting (see Proposition 12(d)) and, thus, also independent of the informativeness of these signals. In other words, we are effectively back to an exchange economy in which there is no impact of public information on ex ante firm values and the cost of capital.

Until now we have only considered the impact on the market portfolio of altering the informativeness of economy-wide productivity information. If the structure of individual firms are similar to the market portfolio, the impacts on individual firms are similar to the impact on the market portfolio. However, a closer look at individual firms reveals that the value and the implied cost of capital on a firm is influenced significantly by the fixed component of dividends associated with that firm. If the fixed components are large compared to the stochastic components of the firm’s payoffs at $t = 1$ or $t = 2$, then the firm’s implied cost of capital will tend to move in the same direction as interest rates, i.e., decreasing for $\gamma < 1$ and increasing for $\gamma > 1$. This has the surprising consequence that the implied cost of capital may be increasing (decreasing) for all firm types (with $\bar{a} = 0$) but decreasing (increasing) for the market portfolio!

Similarly, even though the one-period ahead expected rate of return on the market portfolio at $t = 0$ follows the equilibrium spot rate closely (for both $\gamma < 1$ and $\gamma > 1$), this does not necessarily hold for the individual firm types. Take for example the zero-coupon bond maturing at $t = 2$; at $t = 1$ the value of this bond, $v^*_{t,1+2}(y_{\xi e})$, is decreasing in the economy-wide signal $y_{\xi e}$ (see Proposition 12(a)). As consumption at $t = 1$ is increasing (decreasing) in the economy-wide signal $y_{\xi e}$ when $\gamma > 1$ ($\gamma < 1$), the covariance between the $t = 1$ value of the zero-coupon bond and the pricing kernel, $m^*_{t,0} = u'_1(c^*_1)/E_0[u'_1(c^*_1)]$, is positive (negative) when $\gamma > 1$ ($\gamma < 1$). This implies the one-period ahead expected rate of return at $t = 0$, $\mu^*_{t,1+2} = \mu^*_{t,0} + \ln \left( \frac{E[v^*_{t,1+2}]}{E[v^*_{t,1+2}] + Cov \left[ u'_1(d_1 - \bar{v}_0)/E \left[ u'_1(d_1 - \bar{v}_0) \right] , v^*_{t,1+2} \right]} \right)$, contains a negative (positive) rate of return risk premium when $\gamma > 1$ ($\gamma < 1$). Furthermore, the absolute value of the rate of return risk premium is increasing in the informativeness of the economy-wide productivity information. This implies the first-period rate of return risk
Figure 9: *Ex ante* firm values and certainty equivalents as functions of economy-wide productivity informativeness ($\sigma_{0\xi_e}^2$) for identical firm types given by the parameters in Table 1. The left panel shows the results for $\gamma = 2$, while the right panel shows the results for $\gamma = 0.5$.

premium on the zero-coupon bond maturing at $t = 2$ is moving in the opposite direction of the spot interest rate. In our numerical examples, the impact of increased informativeness on the rate of return risk premium dominates the impact on the spot interest rate such that the one-period ahead expected rate of return on a zero-coupon bond is increasing (decreasing) in the informativeness of economy-wide productivity information when $\gamma < 1$ ($\gamma > 1$). This result carries over to risky assets provided the fixed component of the firm's second-period payoffs, $a_j$, is sufficiently large.

Returning to the *ex ante* value of the market portfolio, it is impacted by changes in economy-wide productivity informativeness in a manner similar to how the *ex ante* value is impacted by changes in firm-specific productivity informativeness. This is illustrated in Figure 9 in a setting with identical firm types characterized by the parameters in Table 1. The figure depicts the equilibrium *ex ante* value of the market portfolio and the equilibrium *ex ante* certainty equivalent as functions of the informativeness of the economy-wide productivity signal $\sigma_{0\xi_e}^2$; in the left panel for $\gamma = 2$ and in the right panel for $\gamma = 0.5$. When $\gamma > 1$, the *ex ante* value of the market portfolio is decreasing in the informativeness of the economy-wide productivity signal and when $\gamma < 1$, the *ex ante* value of the market portfolio is increasing in the informativeness of the economy-wide productivity signal. Of course, the equilibrium *ex ante* certainty equivalent is increasing in the informativeness of the economy-wide productivity signal for all values of $\gamma$. Again, *ex ante* market values of firms are poor indicators of welfare implications of information system changes, and especially so when $\gamma > 1$. 
5 Conclusion

We have shown this and that, but all our results will probably change dramatically if we allow for...

Appendices

A Expected Rates of Returns and Firm-specific Risks

We can express the equilibrium ex-dividend price $v_{1jn}^\dagger$ of firm $n$ of type $j$ at $t = 1$ in equation (17) as the sum of the equilibrium values of two claims, a “windfall claim” and a “productivity claim” with only the latter commanding a dollar risk premium, i.e.,

$$v_{1jn}^\dagger = \exp[-\xi_{21}]E[g_j|y_{\theta_{jn}}] + \exp[-\xi_{21}] \left[ E[h_j^\dagger|y_{\xi_{jn}}, y_{\xi}, q_j^\dagger(\cdot)] - RP_{21jn}^\dagger \right] \equiv v_{1jn}^{g\dagger} + v_{1jn}^{h\dagger}.$$  

The “windfall claim” has a one-period ahead expected rate of return $\mu_{21jn}^{g\dagger}$ equal to the spot interest rate, i.e.,

$$\mu_{21jn}^{g\dagger} = \ln \left( E[g_j|y_{\theta_{jn}}]/v_{1jn}^{g\dagger} \right) = \nu_{21},$$  

whereas the “productivity claim” has a one-period ahead expected rate of return $\mu_{21jn}^{h\dagger}$ equal to the spot interest rate plus the rate of return risk premium $\omega_{21}$ on the market portfolio, i.e.,

$$\mu_{21jn}^{h\dagger} = \ln \left( E[h_j^\dagger|y_{\xi_{jn}}, y_{\xi}, q_j^\dagger(\cdot)]/v_{1jn}^{h\dagger} \right) = \nu_{21} + \omega_{21}.$$  

Defining one-period ahead gross rates of returns as $R_{21} \equiv d_2/v_1$, we may therefore write the one-period ahead expected gross rate of return of firm $n$ of type $j$ at $t = 1$ as

$$E_1[R_{21jn}^\dagger] = \frac{v_{1jn}^{g\dagger}}{v_{1jn}^\dagger} E[g_j|y_{\theta_{jn}}] + \frac{v_{1jn}^{h\dagger}}{v_{1jn}^\dagger} E[h_j^\dagger|y_{\xi_{jn}}, y_{\xi}, q_j^\dagger(\cdot)] = \frac{v_{1jn}^{g\dagger}}{v_{1jn}^\dagger} R_{21} + \frac{v_{1jn}^{h\dagger}}{v_{1jn}^\dagger} E_1[R_{21}^m],$$  

where $R_{21}^\dagger = \exp[\nu_{21}]$ is the gross spot riskless rate of return and $E_1[R_{21}^m] = \exp[\nu_{21} + \omega_{21}]$ is the one-period ahead expected gross rate of return on the market portfolio. Defining asset betas as $\beta_{21} \equiv \text{Cov}_1[R_{21}, R_{21}^m]/\text{Var}_1[R_{21}^m]$, we can therefore also write the one-period ahead expected gross rate of return of firm $n$ of type $j$ at $t = 1$ as

$$E_1[R_{21jn}^\dagger] = R_{21}^\dagger + \beta_{21jn} \left( E_1[R_{21}^m] - R_{21}^\dagger \right), \quad \beta_{21jn} = \frac{v_{1jn}^{g\dagger}}{v_{1jn}^\dagger} \cdot 0 + \frac{v_{1jn}^{h\dagger}}{v_{1jn}^\dagger} \cdot 1.$$  

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Figure 10: Ex post one-period ahead expected rate of return on an individual firm as functions of the firm-specific windfall signal $y_{\theta jn}$ (left panel) and the firm-specific productivity signal $y_{kjn}$ (right panel) for identical firm types given by the parameters in Table 1. Both panels show the results for $\gamma = 2$ and an economy-wide productivity signal equal to its prior mean ($y_{\xi e} = 0$).

In other words, the beta on firm $n$ of type $j$ at $t = 1$ is a weighted average of the beta on the riskless asset ($\beta_{21}^{ft} = 0$), and the beta on the market portfolio ($\beta_{21}^{m1} = 1$). The weights on the two betas depend on the fraction the “windfall claim” comprises of the total firm value. That fraction is increasing in the firm-specific windfall signal $y_{\theta jn}$ (and in the fixed component $a_j$ of $t = 2$ dividends). Hence, although firm-specific windfall risk does not command a dollar risk premium, firm-specific windfall information does affect equilibrium one-period ahead expected gross rates of returns for individual firms and, thus, these expected rates of returns are stochastic as seen from a $t = 0$ perspective (even if the second-period spot interest rate $i_{21}^{\dagger}$ were to be deterministic).

We illustrate these results in Figure 10. The left panel illustrates that a firm’s ex post cost of capital is decreasing in its firm-specific windfall signal for all values of the firm-specific productivity signal. This follows from the fact that the fraction of the firm’s total payoffs which does not command a risk premium increases with the firm-specific windfall signal.

The right panel of Figure 10 illustrates that a firm’s ex post cost of capital is decreasing (increasing) in its firm-specific productivity signal for negative (positive) firm-specific windfall signals, but it does approach the one-period ahead expected rate of return of the market portfolio as the firm-specific productivity signal becomes sufficiently large. On the other hand, if the firm-specific windfall signal is equal to its prior mean of zero, the firm’s ex post cost of capital is independent of its firm-specific productivity signal and equal to the equilibrium one-period ahead expected rate of return of the market portfolio. Considering the payoffs of the firm as a sum of a “windfall claim” (which does not command a risk premium) and a “productivity claim” (which has a one-period ahead expected rate of return equal to
that of the market portfolio independently of the firm-specific productivity signal), these comparative statics follows from the sign and the size of the fraction the “windfall claim” comprises of total firm value. For example, if the windfall signal is negative, the expected windfall payoffs at $t = 2$ are negative and, thus, the value of the “windfall claim” at $t = 1$ is negative and, hence, the firm can be considered as a levered position in the “productivity claim” implying that the firm’s ex post cost of capital is larger than the one-period ahead expected rate of return of the “productivity claim.” If the firm-specific productivity signal is sufficiently low, the value of the firm at $t = 1$ becomes negative for sufficiently negative values of the firm-specific windfall signal in which case, the one-period ahead expected rate of return is not well-defined.

B  Exchange Economy

To reconcile our analysis with that of a pure exchange economy, we assume in this appendix that consumption, trade and investment take place as described but that an additional round of trading is interjected after $t = 1$ but before $t = 2$. Let $t = \tilde{t}$ denote the date at which this extra round of trading takes place, and assume additional information $\gamma$ is released after trading at $t = 1$ but prior to trading at $t = \tilde{t}$. The additional information feeds the posterior beliefs such that

$$\xi_e | \gamma_e \sim N(\gamma_e, \sigma^2_{1\xi_e}), \xi_j | \gamma_j \sim N(\gamma_j, \sigma^2_{1\xi_j}), \theta_j | \gamma_{\theta j} \sim N(\gamma_{\theta j}, \sigma^2_{1\theta j})$$

with pre-posterior distributions characterized by

$$\gamma_e | \gamma_e \sim N(y_e, \sigma^2_{1\xi_e} - \sigma^2_{1\xi_e}), \gamma_j | \gamma_j \sim N(y_j, \sigma^2_{1\xi_j} - \sigma^2_{1\xi_j}), \gamma_{\theta j} | \gamma_{\theta j} \sim N(y_{\theta j}, \sigma^2_{1\theta j} - \sigma^2_{1\theta j}).$$

This specification allows us to examine the impact of additional public information, $\sigma^2_{1(\cdot)} - \sigma^2_{1(\cdot)}$, keeping the equilibrium capital investments $q^*(y)$ at $t = 1$ fixed and, thus, given these capital investments, our economy from $t = 1$ to $t = 2$ is an exchange economy.

Investors trade at $t = \tilde{t}$ in the marketable assets and as no consumption occurs at $t = \tilde{t}$, only the relative prices of assets are meaningfully defined at this date. We choose without loss of generality the zero-coupon bond maturing at $t = 2$ as the numeraire such that this asset has a unit price at $t = \tilde{t}$. Hence, the $t = \tilde{t}$ price of asset $n$ of type $j$ is (see (17) and
the \( t = \hat{1} \) price of a well-diversified portfolio of type \( j \) firms is (by applying the law of large numbers)

\[
v_{\hat{1}jn}^* = E \left[ d_{jn} \mid \hat{y}_{tjn}, \hat{\xi}_{jn}, \hat{\xi}_e, q_j^*(y) \right] - RP_{2\hat{1}jn} = a_j + c_j \hat{y}_{tjn} + E[h_j \mid \hat{y}_{tjn}, \hat{\xi}_e, q_j^*(y)] \exp \left[ -\gamma \beta^2 \sigma^2_{1\xi_e} \right],
\]

and the \( t = \hat{1} \) price of the market portfolio is

\[
v_{\hat{1}}^* = \frac{1}{N} \sum_{n=1}^{N} v_{\hat{1}jn}^* = E_{\hat{y}_{t\xi}, \hat{\xi}_e} \left[ a_j + c_j \hat{y}_{tjn} + E[h_j \mid \hat{y}_{tjn}, \hat{\xi}_e, q_j^*(y)] \exp \left[ -\gamma \beta^2 \sigma^2_{1\xi_e} \right] \right] \\
= a_j + \exp \left[ -\gamma \beta^2 \sigma^2_{1\xi_e} \right] \exp \left[ \alpha_j + \beta(\hat{\xi}_e + \frac{1}{2} \beta \sigma^2_{1\xi_e}) + \frac{1}{2} \theta_j^2 \left( 1 + \sigma^2_{0\xi_j} \frac{k}{1-k} \right) \right] \\
\times (\kappa_j (\sigma^2_{0\xi_j}) \bar{q}_e(\xi_e))^k,
\]

where \( \bar{p} \) depends only on the firm-specific productivity informativeness at \( t = 1 \). That is, firm-specific information arriving after \( t = 1 \) is fully diversifiable such that the \( t = \hat{1} \) price of a well-diversified portfolio of type \( j \) firms is unaffected by the (additional) firm-specific information even though the \( t = \hat{1} \) price on any individual firm will depend on this information. Equivalently, the dollar risk premium \( RP_{2\hat{1}jn} \) and the one-period ahead expected rate of return \( \mu_{2\hat{1}jn} \) on any individual firm depends on the firm-specific signals and their precision. But the dollar risk premium \( RP_{2\hat{1}j} \) and the one-period ahead expected rate of return \( \mu_{2\hat{1}j} \) on a well-diversified portfolio of type \( j \) firms or the market portfolio are left unaffected by additional firm-specific information. Contrary to firm-specific information, economy-wide information does not diversify, and the additional economy-wide information reduces the posterior uncertainty which serves to reduce the dollar risk premium and, thus, also the \emph{ex post} rate of return risk premium \( \bar{\sigma}_{2\hat{1}} \) on the market portfolio as illustrated in Figure 3.

At any point in time, equilibrium prices are determined by the representative investor’s first-order conditions as in (3) and, hence, prices at \( t = 0 \) and at \( t = 1 \) are unaffected by the additional information and trading at \( t = \hat{1} \). In other words, the \emph{ex ante} equilibrium prices (and, thus, the \emph{ex ante} cost of capital) of individual securities are independent of the informativeness of the information system at \( t = \hat{1} \) (see also Christensen et al. 2010). Since the capital investments are not affected by the additional information at \( t = \hat{1} \), the
equilibrium consumption plan and *ex ante* expected utility of the representative investor are unaffected as well.

In equilibrium, there can be no arbitrage in the periods between any two subsequent trading dates (or any other two trading dates). At \( t = 1 \), the possible events at \( t = \hat{1} \) are the posteriors \( \hat{y} \), and the payoffs of the securities are the asset prices at \( t = \hat{1} \). It follows there is no arbitrage in the period from \( t = 1 \) to \( t = \hat{1} \) if, and only if, a pricing kernel, \( m_{11} \), exists such that

\[
v_{1jn} = v_{1jn}(y_{\theta jn}, y_{\xi jn}, y_{\kappa e}, q_j^*(y))
\]

\[
= \exp \left[ -\gamma \frac{\hat{1}}{1} \right] \left[ E \left[ v_{1jn} | y_{\theta jn}, y_{\xi jn}, y_{\kappa e} \right] + \text{Cov} \left[ v_{1jn}, m_{11} | y_{\theta jn}, y_{\xi jn}, y_{\kappa e} \right] \right].
\]

Pricing the \( t = 2 \) zero-coupon bond using this relation implies \( \gamma \frac{\hat{1}}{1} = \gamma \frac{2}{1} y_{\kappa e} / (\hat{1} - 1) \), and pricing asset \( n \) of type \( j \) implies

\[
v_{1jn} = \exp \left[ -\gamma \frac{\hat{1}}{1} \right] \left[ a_j + c_j y_{\theta jn} + E[h_j | y_{\xi jn}, y_{\kappa e}, q_j^*(y)] \exp \left[ -\gamma \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \right] - \text{RP}_{11jn} \right]
\]

and, thus, the dollar risk premium between \( t = 1 \) and \( t = \hat{1} \) equals

\[
\text{RP}_{11jn} = -\text{Cov} \left[ v_{1jn}, m_{11} | y_{\theta jn}, y_{\xi jn}, y_{\kappa e} \right]
\]

\[
= E[h_j | y_{\xi jn}, y_{\kappa e}, q_j^*(y)] \exp \left[ -\gamma \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \right] \left[ \exp \left[ \gamma \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \right] - 1 \right]
\]

for an individual asset, and

\[
\overline{\text{RP}}_{11} = \exp \left[ -\gamma \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \right] \exp \left[ \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \right] \left[ \exp \left[ \gamma \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \right] - 1 \right]
\]

for the market portfolio. This implies the one-period ahead expected rate of return \( \mu_{11jn} \) of firm \( n \) of type \( j \) at \( t = 1 \) is

\[
\mu_{11jn} = \gamma \frac{\hat{1}}{1} y_{\kappa e} + \omega_{11jn} = \gamma \frac{2}{1} y_{\kappa e} / (\hat{1} - 1) + \gamma \frac{\hat{1}}{1} y_{\kappa e},
\]

where the rate of return risk premium \( \omega_{11jn} \) is given as

\[
\omega_{11jn} = \left\{ \ln \left[ \frac{E \left[ v_{1jn} | y_{\theta jn}, y_{\xi jn}, y_{\kappa e}, q_j^*(y) \right]}{v_{1jn}(y_{\theta jn}, y_{\xi jn}, y_{\kappa e}, q_j^*(y))} \right] - \gamma \frac{\hat{1}}{1} y_{\kappa e} \right\} / (\hat{1} - 1)
\]

\[
= \ln \left[ \frac{a_j + c_j y_{\theta jn} + E[h_j | y_{\xi jn}, y_{\kappa e}, q_j^*(y)] \exp \left[ -\gamma \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \right]}{a_j + c_j y_{\theta jn} + E[h_j | y_{\xi jn}, y_{\kappa e}, q_j^*(y)] \exp \left[ -\gamma \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \frac{\hat{1}}{1} \right]} \right] / (\hat{1} - 1).
\]

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This yields the following result.

**Proposition 17** Conditional on public information at date \( t = 1 \), \( y = \left( y_{\xi e}, \{y_{jn}, y_{jn} \}_{j=1, \ldots, J; n=1, \ldots, N} \right) \), it holds for the market portfolio that\(^{11}\)

(a) The one-period ahead expected rate of return risk premium between \( t = 1 \) and \( t = 2 \) is

\[
\bar{\omega}_{21} = \frac{\gamma \beta^2 \sigma^2_{1\xi e}}{(2 - \hat{1})}.
\]

(b) The one-period ahead expected rate of return risk premium between \( t = 1 \) and \( t = \hat{1} \) is

\[
\bar{\omega}_{11} = \frac{\gamma \beta^2 (\sigma^2_{1\xi e} - \sigma^2_{1\xi e})}{(1 - 1)}.
\]

(c) The sum of the rate of return risk premia between \( t = 1 \) and \( t = \hat{1} \) and \( t = 2 \) equals the rate of return risk premium between \( t = 1 \) and \( t = 2 \):

\[
\bar{\omega}_{21} = \bar{\omega}_{11}(1 - 1) + \bar{\omega}_{21}(2 - \hat{1}) = \gamma \beta^2 \sigma^2_{1\xi e}.
\]

(d) Conditional on \( y_{\xi e} \), the sum of the dollar risk-premium between \( t = 1 \) and \( t = \hat{1} \) and the expected dollar risk premium between \( t = \hat{1} \) and \( t = 2 \) equals the dollar risk-premium between \( t = 1 \) and \( t = 2 \):

\[
RP_{11} + E \left[ RP_{21} | y_{\xi e}, \hat{q}_o(y_{\xi e}) \right] = RP_{21}.
\]

Thus, while increasing the informativeness of the additional economy-wide productivity information at \( t = \hat{1} \), i.e., decreasing the posterior variance \( \hat{\sigma}^2_{1\xi e} \), reduces both the rate of return risk premium and the expected dollar risk premium between \( t = \hat{1} \) and \( t = 2 \), it also increases the rate of return risk premium and the expected dollar risk premium between \( t = 1 \) and \( t = \hat{1} \) such that the informativeness of the additional information neither impacts the rate of return risk premium nor the dollar risk premium between \( t = 1 \) and \( t = 2 \). That is, the informativeness of the public information at \( t = \hat{1} \) only serves to allocate the total risk in dividends between the returns in the period preceding the signal (pre-posterior risk) and the period succeeding the signal (posterior risk).

\(^{11}\)While the additivity of the dollar risk premia in (d) also holds for individual firms, the additivity of rate of return risk premia in (c) only holds for individual firms if the fixed component of \( t = 2 \) dividends \( a_j \) is equal to zero and there is no firm-specific windfall risk, i.e., \( c_j = 0 \), in which case (a) and (b) also give the rate of return risk premia for individual firms. Nevertheless, the *ex ante* cost of capital (however measured) at \( t = 1 \) or \( t = 0 \) is always unaffected by the additional information at \( t = \hat{1} \), since the equilibrium prices at these dates and the dividends are unaffected by this information.
C Proofs

Proof of Lemma 2: Given the independence of the different types of states and signals, consider without loss of generality a setting with only economy-wide productivity states and information and two information systems $\eta'$ with signals $y'_{\xi e} \sim N(0, (\sigma'_{0\xi e})^2)$ and $\eta''$ with signals $y''_{\xi e} \sim N(0, (\sigma''_{0\xi e})^2)$ such that $(\sigma''_{0\xi e})^2 > (\sigma'_{0\xi e})^2$. Since $\xi_e \sim N(0, 1)$, we can make an orthogonal decomposition of $\xi_e$ into three zero-mean independent normally distributed variables such that

$$\xi_e = \varepsilon_1 + \varepsilon_2 + \varepsilon, \quad y'_{\xi e} = \varepsilon_1, \quad y''_{\xi e} = \varepsilon_1 + \varepsilon_2,$$

where $\varepsilon_2 \sim N \left(0, (\sigma''_{0\xi e})^2 - (\sigma'_{0\xi e})^2\right)$. Since

$$\phi (\xi_e | y'_{\xi e}, y''_{\xi e}) = \phi (\xi_e | \varepsilon_1, \varepsilon_1 + \varepsilon_2) = \phi (\xi_e | \varepsilon_1 + \varepsilon_2) = \phi (\xi_e | y''_{\xi e}),$$

it follows that $\eta'$ is a garbling of $\eta''$, while the converse does not hold when $(\sigma''_{0\xi e})^2 > (\sigma'_{0\xi e})^2$. It then follows from Marschak and Miyasawa (1968, Theorem 6.2) that $\eta''$ is more informative than $\eta'$ in the sense of Blackwell (1953).

Proof of Lemma 3: (a): Given normally distributed states and signals, the conditional expected marginal rate of transformation for firm $n$ of type $j$ given the firm-specific productivity signal $y_{\xi j n}$ and the economy-wide productivity state is

$$E[MRT_j | y_{\xi j n}, \xi_e; q_j n] = k \exp \left[\alpha_j + \beta_j \xi_e + \psi_j (y_{\xi j n} + \frac{1}{2} \psi_j \sigma^2_{j \xi j})\right] q_j^{k-1}.$$

As discussed above the lemma, an optimal allocation of $q_j$, $\left\{ q_j^f (q_j, y_{\xi j n}) \right\}_{n=1,\ldots,N}$, among firms of type $j$ is such that there exist a common conditional expected marginal rate of transformation for firms of type $j$, $Q_j (\xi_e, q_j)$, such that

$$E \left[ MRT_j | y_{\xi j n}, \xi_e; q_j^f (q_j, y_{\xi j n}) \right] = Q_j (\xi_e, q_j), \quad n = 1, \ldots, N,$$

implying that

$$q_j^f (q_j, y_{\xi j n}) = \left( \frac{Q_j (\xi_e, q_j)}{k} \right)^{1/(k-1)} \exp \left[ (\alpha_j + \beta_j \xi_e + \psi_j (y_{\xi j n} + \frac{1}{2} \psi_j \sigma^2_{j \xi j}) \right] / (1 - k) \right].$$

Since the per firm aggregate capital investment $q_j$ is fixed, the law of large numbers implies that

$$q_j^E (q_j) = \lim_{N \to \infty} \frac{1}{N} \sum q_j^f (q_j, y_{\xi j n}) = E \left[ q_j^f | q_j \right] = q_j.$$

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Inserting the expression for $q_j^\dagger(q_j, y_{\xi jn})$ and calculating the expectation with respect to the firm-specific signal yield

$$E\left[q_j^\dagger q_j\right] = \left(\frac{Q_j(\xi_e, q_j)}{k}\right)^{1/(k-1)} \exp \left[\left(\alpha_j + \beta_j \xi_e + \psi_j \left(\frac{\psi_j^2}{2(1-k)\sigma_{\xi j}^2} + \frac{1}{2} \psi_j \sigma_{1\xi j}^2\right)\right) / (1-k)\right]$$

$$= q_j,$$

and, thus,

$$\left(\frac{Q_j(\xi_e, q_j)}{k}\right)^{1/(k-1)} \exp \left[\left(\alpha_j + \beta_j \xi_e\right) / (1-k)\right]$$

$$= \exp \left[-\psi_j \left(\frac{\psi_j^2}{2(1-k)\sigma_{\xi j}^2} + \frac{1}{2} \psi_j \sigma_{1\xi j}^2\right) / (1-k)\right] q_j.$$

Inserting this expression back into the expression for $q_j^\dagger(q_j, y_{\xi jn})$ yields

$$q_j^\dagger(q_j, y_{\xi jn}) = \exp \left[-\psi_j \left(\frac{\psi_j^2}{2(1-k)\sigma_{\xi j}^2} + \frac{1}{2} \psi_j \sigma_{1\xi j}^2\right) / (1-k) + \psi_j (y_{\xi jn} + \frac{1}{2} \psi_j \sigma_{1\xi j}^2) / (1-k)\right] q_j$$

$$= \exp \left[\psi_j (y_{\xi jn} - \frac{1}{2} \psi_j \sigma_{\xi j}^2) / (1-k)\right] q_j,$$

which is equation (9). The comparative statics follow directly from this expression.

(b): The payoff from capital investments $h_j(\cdot)$ at $t = 2$ on a well-diversified portfolio of type $j$ firms, in which the per firm aggregate capital investment $q_j$ at $t = 1$ is efficiently allocated, is given by

$$h^E_j(\xi_e, q_j) = \lim_{N \to \infty} \frac{1}{N} \sum h_j(\xi_e, \xi_{jn}, q_j^\dagger(q_j, y_{\xi jn})) = E\left[h_j^\dagger q_j\right]$$

$$= E\left[\left[\alpha_j + \beta_j \xi_e \psi_j \xi_{jn}\right] \exp \left[\left(k \psi_j \left(y_{\xi jn} - \frac{1}{2} \frac{\psi_j^2}{(1-k)\sigma_{\xi j}^2}\right) / (1-k)\right) q_j^k\right]\right]$$

$$= E\left[\left[\psi_j \xi_{jn} + k \psi_j \left(y_{\xi jn} - \frac{1}{2} \frac{\psi_j^2}{(1-k)\sigma_{\xi j}^2}\right) / (1-k)\right] \exp \left[\left[\alpha_j + \beta_j \xi_e\right] q_j^k\right]\right].$$
The law of iterated expectations and \( \sigma^2_{ij} = 1 - \sigma^2_{0ij} \) yield

\[
E \left[ \exp \left( \psi_j \xi_{jn} + k \psi_j \left( y_{ij} - \frac{1}{2} \frac{\psi_j}{1-k} \sigma^2_{0ij} \right) / (1-k) \right) \right] \\
= E \left[ E \left[ \exp \left( \psi_j \xi_{jn} + k \psi_j \left( y_{ij} - \frac{1}{2} \frac{\psi_j}{1-k} \sigma^2_{0ij} \right) / (1-k) \right) \mid y_{ij} \right] \right] \\
= E \left[ \exp \left( \psi_j \left( y_{ij} + \frac{1}{2} \frac{\psi_j}{1-k} \sigma^2_{1ij} \right) + k \psi_j \left( y_{ij} - \frac{1}{2} \frac{\psi_j}{1-k} \sigma^2_{0ij} \right) / (1-k) \right) \right] \\
= E \left[ \exp \left( \frac{\psi_j}{1-k} y_{ij} + \frac{1}{2} \frac{\psi_j}{1-k} \sigma^2_{1ij} - \frac{1}{2} \frac{k \psi_j^2}{(1-k)^2} \sigma^2_{0ij} \right) \right] \\
= \exp \left[ \frac{1}{2} \frac{\psi_j^2}{1-k} \sigma^2_{0ij} + \frac{1}{2} \frac{\psi_j}{1-k} \left( 1 - \sigma^2_{0ij} \right) - \frac{1}{2} \frac{k \psi_j^2}{(1-k)^2} \sigma^2_{0ij} \right] \\
= \exp \left[ \frac{1}{2} \psi_j^2 \left( 1 + \sigma^2_{0ij} \frac{k}{1-k} \right) \right].
\]

Collecting terms yields (10), and the comparative static follows directly from this expression.

**Proof of Lemma 4:** (a): As discussed above the lemma, an optimal allocation of \( q_o \), \( \{ q_j^1(\bar{q}_o) \}_{j=1,...,J} \), among firm types is such that there exist a common marginal rate of transformation, \( Q(\xi_e; \bar{q}_o) \), such that

\[
MRT_j(\xi_e; q_j^1(\bar{q}_o)) = Q(\xi_e; \bar{q}_o), \quad j = 1, ..., J,
\]

implying that

\[
q_j^1(\bar{q}_o) = \left( \frac{Q(\xi_e, \bar{q}_o)}{k} \right)^{1/(k-1)} \exp \left[ \left( \alpha_j + \beta \xi_e + \frac{1}{2} \psi_j^2 \left( 1 + \sigma^2_{0ij} \frac{k}{1-k} \right) \right) / (1-k) \right].
\]

Since \( \bar{q}_o = \frac{1}{J} \sum_{j=1}^J q_j^1(\bar{q}_o) \), we get

\[
\left( \frac{Q(\xi_e, \bar{q}_o)}{k} \right)^{1/(k-1)} \exp [\beta \xi_e] = \frac{1}{J} \sum_{j=1}^J \exp \left[ \left( \alpha_j + \frac{1}{2} \psi_j^2 \left( 1 + \sigma^2_{0ij} \frac{k}{1-k} \right) \right) / (1-k) \right].
\]

Substituting this expression back into the expression for \( q_j^1(\bar{q}_o) \), we get (11), where

\[
\kappa_j(\sigma^2_{0ij}) = \frac{\exp \left[ \left( \alpha_j + \frac{1}{2} \psi_j^2 \left( 1 + \sigma^2_{0ij} \frac{k}{1-k} \right) \right) / (1-k) \right]}{\frac{1}{J} \sum_{j=1}^J \exp \left[ \left( \alpha_j + \frac{1}{2} \psi_j^2 \left( 1 + \sigma^2_{0ij} \frac{k}{1-k} \right) \right) / (1-k) \right]}.
\]

The comparative statics follow directly from this expression.
(b): The payoff from capital investments \( h_j(\cdot) \) at \( t = 2 \) to the representative investor is given by

\[
\overline{h}(\xi_e, \overline{q}_o) = \frac{1}{J} \sum_{j=1}^{J} h_j^E(\xi_e, q_j^o(\overline{q}_o))
\]

\[
= \frac{1}{J} \sum_{j=1}^{J} \exp \left[ \alpha_j + \beta \xi_e + \frac{1}{2} \psi_j^2 \left( 1 + \sigma_{0j}^2 \frac{k}{1-\kappa} \right) \right] (\kappa_j \left( \sigma_{0j}^2 \overline{q}_o \right)^k
\]

\[
= \frac{1}{J} \sum_{j=1}^{J} \frac{\exp \left[ \left( \alpha_j + \frac{1}{2} \psi_j^2 \left( 1 + \sigma_{0j}^2 \frac{k}{1-\kappa} \right) \right) / (1 - \kappa) \right]}{\left( \frac{1}{J} \sum_{j=1}^{J} \exp \left[ \left( \alpha_j + \frac{1}{2} \psi_j^2 \left( 1 + \sigma_{0j}^2 \frac{k}{1-\kappa} \right) \right) / (1 - \kappa) \right] \right)^k} \exp \left[ \beta \xi_e \right] \overline{q}_o^k
\]

which is (12), with

\[
\exp \left[ \overline{\pi} \right] = \frac{1}{J} \sum_{j=1}^{J} \frac{\exp \left[ \left( \alpha_j + \frac{1}{2} \psi_j^2 \left( 1 + \sigma_{0j}^2 \frac{k}{1-\kappa} \right) \right) / (1 - \kappa) \right]}{\left( \frac{1}{J} \sum_{j=1}^{J} \exp \left[ \left( \alpha_j + \frac{1}{2} \psi_j^2 \left( 1 + \sigma_{0j}^2 \frac{k}{1-\kappa} \right) \right) / (1 - \kappa) \right] \right)^k}.
\]

Clearly, \( \overline{h}(\xi_e, \overline{q}_o) \) is increasing in the economy-wide productivity signal \( \xi_e \). Differentiating \( \exp \left[ \overline{\pi} \right] \) with respect to \( z_j \equiv \exp \left[ \left( \alpha_j + \frac{1}{2} \psi_j^2 \left( 1 + \sigma_{0j}^2 \frac{k}{1-\kappa} \right) \right) / (1 - \kappa) \right] \) yields

\[
\frac{d \exp \left[ \overline{\pi} \right]}{dz_j} = \frac{d}{dz_j} \frac{1}{J} \sum_{j=1}^{J} \frac{z_j}{\left( \frac{1}{J} \sum_{j=1}^{J} z_j \right)^k} = \frac{1}{J} \left( \frac{1}{J} \sum_{j=1}^{J} z_j \right)^{k-1} \frac{1}{J} \frac{\frac{1}{J} \sum_{j=1}^{J} z_j}{\left( \frac{1}{J} \sum_{j=1}^{J} z_j \right)^{2k}}
\]

\[
= \frac{1}{J} \left( \frac{1}{J} \sum_{j=1}^{J} z_j \right)^{k-1} \frac{1}{J} \frac{z_j}{\left( \frac{1}{J} \sum_{j=1}^{J} z_j \right)^k} > 0,
\]

where the last inequality follows from \( z_j > 0 \) for all \( j \) and, thus, \( z_j / \sum_{j=1}^{J} z_j \leq 1 \), and \( \kappa < 1 \). Since \( z_j \) is increasing in \( \sigma_{0j}^2 \), \( \overline{h}(\xi_e, \overline{q}_o) \) is increasing in \( \sigma_{0j}^2 \) for all \( j = 1, \ldots, J \). ■

**Proof of Lemma 5:** It follows from equation (3) that the cum-dividend market value of the market portfolio (per investor) at \( t = 1 \) given the public economy-wide signal \( y_e \) and aggregate capital investment \( \overline{q}_o \) is

\[
V_{1}^m(\overline{q}_o)
\]

\[
= \overline{d}_1 - \overline{q}_o + \exp \left[ -\nu_{21}^*(y_e) \right] \left[ E_t \left[ \overline{g}(\theta_e) + \overline{H}(\xi_e, \overline{q}_o)|y_e \right] + \text{Cov} \left[ m_{21}^*(y_e), \overline{g}(\theta_e) + \overline{H}(\xi_e, \overline{q}_o)|y_e \right] \right],
\]

where the equilibrium interest rate \( \nu_{21}^*(y_e) \) and pricing kernel \( m_{21}^*(y_e) \) are taken as given. From this expression, it follows directly that the first order-condition for a value maximizing
aggregate capital investment is identical to the first-order condition for an optimal aggregate capital investment for the representative investor, i.e., (14). On the other hand, both the equilibrium interest rate and the equilibrium pricing kernel depend on the optimal aggregate capital investment, since the representative investor’s marginal utilities depend on this investment.

(a): Neither the firm-specific windfall signals nor their informativeness affect the conditional expected marginal rate of transformation of firms and, therefore, this type of information does not affect the efficient allocation of aggregate capital investments among firms (see Lemmas 3 and 4). Furthermore, this type of information does not affect beliefs about equilibrium consumption per capita. Hence, it has no impact on the equilibrium interest rates and the pricing kernel and, therefore, no impact on optimal aggregate capital investments.

(b): Equilibrium consumption per capital does not depend on the firm-specific productivity signals per se and, therefore, these signals have no impact on optimal aggregate investments. On the other hand, the informativeness of the firm-specific productivity signals affects the productivity parameter \( \bar{\pi} \) of aggregate capital investments (see Lemma 4) and, thus, may affect optimal aggregate capital investments.

(c): Clearly, economy-wide productive information affects the expected marginal rate of transformation of aggregate capital investments and, thus, it enters directly in the first-order condition (14) for optimal aggregate capital investments. Economy-wide windfall information does not affect the marginal rate of transformation of aggregate capital investments, but it affects beliefs about future equilibrium consumption per capital and, thus, it may affect both the equilibrium interest rate and the pricing kernel and thereby the optimal aggregate capital investment.

Proof of Proposition 2?: (a): The optimal aggregate capital investment is given implicitly by (27). Differentiating both sides of this equation with respect to the informativeness of the firm-specific productivity information for type \( j \) firms \( \sigma_{0j}^2 \) yields that

\[
\frac{\partial \nu_{21}^*}{\partial \sigma_{0j}^2} = \gamma \left\{ \frac{1}{\pi (\bar{\pi})^k} \left( \frac{\partial \bar{\pi}}{\partial \sigma_{0j}^2} (\bar{\pi})^k + k \bar{\pi} (\bar{\pi})^k - 1 \frac{\partial \bar{\pi}}{\partial \sigma_{0j}^2} \right) + \frac{1}{d_1 - \bar{\sigma}_o^* \frac{\partial \bar{\pi}}{\partial \sigma_{0j}^2}} \right\} = \frac{1}{\pi} \frac{\partial \bar{\pi}}{\partial \sigma_{0j}^2} - (1 - k) \frac{1}{\bar{\sigma}_o^*} \frac{\partial \bar{\pi}}{\partial \sigma_{0j}^2} \cdot (28)
\]

Assume \( \frac{\partial \nu_{21}^*}{\sigma_{0j}^2} \leq 0 \). Then the right-hand side is also negative and, thus,

\[
\frac{\partial \bar{\pi}}{\partial \sigma_{0j}^2} \geq \frac{\bar{\pi}}{(1 - k) \bar{\pi} \sigma_{0j}^2}.
\]
Substituting this into the left-hand side yields

$$
\frac{\partial \tau_{21}^*}{\partial \sigma_{0\xi_j}^2} \geq \frac{\gamma}{\pi} \frac{1}{(\bar{q}_o^*)^k} \left( \frac{\partial \bar{q}_o^*}{\partial \sigma_{0\xi_j}^2} (\bar{q}_o^*)^k + k \pi (\bar{q}_o^*)^{k-1} \bar{q}_o^* \frac{\partial \bar{q}_o^*}{(1-k) \pi \partial \sigma_{0\xi_j}^2} \right) + \frac{\gamma}{d_1 - \bar{q}_o^*} \frac{1}{(1-k) \pi \partial \sigma_{0\xi_j}^2} \bar{q}_o^*
$$

$$
= \gamma \left\{ \frac{1}{\pi} \frac{1}{(\bar{q}_o^*)^k} \left( (\bar{q}_o^*)^k + k \pi (\bar{q}_o^*)^{k-1} \bar{q}_o^* \right) \frac{\partial \bar{q}_o^*}{(1-k) \pi \partial \sigma_{0\xi_j}^2} + \frac{1}{d_1 - \bar{q}_o^*} \frac{\partial \bar{q}_o^*}{(1-k) \pi \partial \sigma_{0\xi_j}^2} \right\} \frac{\partial \bar{q}_o^*}{\partial \sigma_{0\xi_j}^2} > 0,
$$

where the last inequality follows from the fact that all terms in the brackets are strictly positive and the fact that $\partial \bar{q}_o^*/\partial \sigma_{0\xi_j}^2 > 0$. Hence, a contradiction is obtained to the assumption that $\partial \tau_{21}^*/\partial \sigma_{0\xi_j}^2 \leq 0$ and, thus, $\partial \tau_{21}^*/\partial \sigma_{0\xi_j}^2 > 0$, and this proves (a).

(b): Collecting terms including $\partial \bar{q}_o^*/\partial \sigma_{0\xi_j}^2$ in (28) yields

$$
\left\{ \gamma \left( k \pi (\bar{q}_o^*)^{k-1} + \frac{1}{d_1 - \bar{q}_o^*} \right) + (1-k) \frac{1}{\bar{q}_o^*} \right\} \frac{\partial \bar{q}_o^*}{\partial \sigma_{0\xi_j}^2} = (1-\gamma) \frac{\partial \bar{q}_o^*}{\partial \sigma_{0\xi_j}^2}.
$$

Since all terms in the brackets on the left-hand side are strictly positive and the fact that $\frac{1}{\pi \partial \sigma_{0\xi_j}^2} > 0$, it follows that $\frac{\partial \bar{q}_o^*}{\partial \sigma_{0\xi_j}^2} > (\theta)0$ if, and only if $\gamma < (\theta)1$, and this proves (b).

Proof of Proposition ??: (a): Since there are no risk premia in the equilibrium prices at $t = 1$ (see (?)) and at $t = 0$ (see (?)), it follows directly from (5) and (6) that $\mu_{t_{10j}} = c_{10i}^*$. It follows from Proposition ??(b) that equilibrium consumption at $t = 1$, i.e., $c_{1j}^* = d_1 - \bar{q}_o^*$, is increasing (decreasing) in $\sigma_{0\xi_j}^2$ if $\gamma > 1$ ($\gamma < 1$) and, thus, it follows from (??) that $c_{1j}^*$ is increasing (decreasing) in $\sigma_{0\xi_j}^2$ if $\gamma > 1$ ($\gamma < 1$).

(b): Since $c_{20j}^* = (c_{10j}^* + \tau_{21j}^*)/2$, if follows from (a) and Proposition ??(a) that $c_{20j}^*$ is increasing in $\sigma_{0\xi_j}^2$ if $\gamma > 1$. If $\gamma < 1$, $\bar{q}_o^*$ and the average $t = 2$ dividend $c_{2j}^*(\bar{q}_o^*)$ for any given $\bar{q}_o^*$ are both increasing in $\sigma_{0\xi_j}^2$ and, thus, equilibrium consumption $c_{2j}^*$ at $t = 2$ is increasing in $\sigma_{0\xi_j}^2$. It then follows from (??) that $c_{20j}^* = \delta + \gamma (\ln [c_{2j}^*] - \ln [c_{0j}])$ is increasing in $\sigma_{0\xi_j}^2$.

(c): Since the dividend ratio is assumed to be positive, the implied cost of capital $\rho_j^*$ is by Descartes’ rule of signs given by (??). Let the quantity $Q_j$ be defined as

$$
Q_j \equiv -dr_j^* + \sqrt{(dr_j^*)^2 + 4 [\exp [-\tau_{10j}^* dr_j^*] + \exp [-2t_{20j}^*]]} > 0.
$$

If follows from (??) that $\rho_j^*$ is increasing in the firm-specific productivity informativeness $\sigma_{0\xi_j}^2$ if $\partial Q_j/\partial \sigma_{0\xi_j}^2 < 0$. Given $\gamma > 1$, it follows from (a) and (b) that $\partial \tau_{21j}^*/\partial \sigma_{0\xi_j}^2 > 0$ and
$\partial t_{20}/\partial \sigma_{0\xi_j}^2 > 0$ and, thus,

\[
\frac{\partial Q_j}{\partial \sigma_{0\xi_j}^2} = -\frac{\partial dr_j^*}{\partial \sigma_{0\xi_j}^2} + \frac{1}{2 (Q_j + dr_j^*)} \left\{ (2dr_j^* + 4 \exp \left[ -t_{10}^* \right] ) \frac{\partial dr_j^*}{\partial \sigma_{0\xi_j}^2} \right\} \]

\[
- \frac{1}{2 (Q_j + dr_j^*)} \left\{ 4 \exp \left[ -t_{10}^* \right] \frac{\partial t_{10}^*}{\partial \sigma_{0\xi_j}^2} dr_j^* + 8 \exp \left[ -2t_{20}^* \right] \frac{\partial t_{20}^*}{\partial \sigma_{0\xi_j}^2} \right\} \]

\[
< -\frac{\partial dr_j^*}{\partial \sigma_{0\xi_j}^2} + \frac{1}{(Q_j + dr_j^*)} \left( dr_j^* + 2 \exp \left[ -t_{10}^* \right] \right) \frac{\partial dr_j^*}{\partial \sigma_{0\xi_j}^2}.
\]

Since $\partial dr_j^*/\partial \sigma_{0\xi_j}^2 < 0$, it then follows that $\partial Q_j/\partial \sigma_{0\xi_j}^2 < 0$ if

\[
(dr_j^*)^2 + 4 \exp \left[ -2t_{10}^* \right] + 4 \exp \left[ -t_{10}^* \right] dr_j^* > (dr_j^*)^2 + 4 \exp \left[ -t_{10}^* \right] dr_j^* + \exp \left[ -2t_{20}^* \right].
\]

or equivalently (since all terms are positive),

\[
(dr_j^*)^2 + 4 \exp \left[ -2t_{10}^* \right] + 4 \exp \left[ -t_{10}^* \right] dr_j^* > (dr_j^*)^2 + 4 \exp \left[ -t_{10}^* \right] dr_j^* + \exp \left[ -2t_{20}^* \right].
\]

It then follows from $t_{10}^* < t_{20}^*$ that $\partial Q_j/\partial \sigma_{0\xi_j}^2 < 0$. ■

**Proof of Lemma 7:** It follows from (15) that

\[
\iota_{21}(\bar{q}_o) = -\ln \left[ \frac{E \left[ u_2(c_2^1) | y_{\xi e}, \bar{q}_o \right]}{u_1(c_1^1)} \right] = \delta - \ln \left[ \frac{E \left[ c_2^1 \right]^{-\gamma} | y_{\xi e}, \bar{q}_o \right]}{c_1^1} \right] \]

\[
= \delta - \ln \left[ \frac{\exp \left[ \ln \left[ c_2^1 \right]^{-\gamma} \right] | y_{\xi e}, \bar{q}_o \right]}{\exp \left[ \ln \left[ c_1^1 \right]^{-\gamma} \right] \right] = \delta - \ln \left[ \frac{\exp \left[ -\gamma \ln \left[ c_2^1 \right] | y_{\xi e}, \bar{q}_o \right]}{\exp \left[ -\gamma \ln \left[ c_1^1 \right] \right] \right] \right] \]

\[
= \delta + \gamma \left( E \left[ \ln \left[ c_2^1 \right] | y_{\xi e}, \bar{q}_o \right] - \ln \left[ c_1^1 \right] \right) - \frac{1}{2} \gamma^2 \text{Var} \left[ \ln \left[ c_2^1 \right] | y_{\xi e}, \bar{q}_o \right] \]

\[
= \delta + \gamma \left( \pi + \beta y_{\xi e} + k \ln \bar{q}_o - \ln \left[ \bar{d}_1 - \bar{q}_o \right] \right) - \frac{1}{2} \gamma^2 \beta^2 \sigma_{1\xi e}^2. \]
Proof of Lemma 8: It follows from (15) that
\[ m_{21}^\dagger (\xi_e, y_{\xi_e}, \overline{q}_o) = \frac{u_2^\prime (c_2^\dagger (\xi_e, \overline{q}_o))}{E[u_2^\prime (c_2^\dagger ) | y_{\xi_e}, \overline{q}_o]} = \frac{\exp[\ln\left( c_2^\dagger (\xi_e, \overline{q}_o) \right)^{-\gamma}]}{E[\exp[\ln\left( c_2^\dagger \right)^{-\gamma}] | y_{\xi_e}, \overline{q}_o]} = \frac{\exp[-\gamma \ln[c_2^\dagger (\xi_e, \overline{q}_o)]]}{E[\exp[-\gamma \ln[c_2^\dagger ]] | y_{\xi_e}, \overline{q}_o]}
\]

\[ = \exp[-\gamma \left( E \left[ \ln[c_2^\dagger ] | y_{\xi_e}, \overline{q}_o \right] - \frac{1}{2} \gamma^2 \text{Var} \left[ \ln[c_2^\dagger ] | y_{\xi_e}, \overline{q}_o \right] \right) - \frac{1}{2} \gamma^2 \text{Var} \left[ \ln[c_2^\dagger ] | y_{\xi_e}, \overline{q}_o \right]}
\]

\[ = \exp[\gamma \beta (y_{\xi_e} - \xi_e) - \frac{1}{2} (\gamma \beta)^2 \sigma_{1\xi_e}^2].
\]

Proof of Lemma 9: (a): Since
\[ E \left[ d_{2j}^\dagger | y_{\theta jn}, y_{\xi jn}, \xi_e, q_j^\dagger (\cdot) \right] = a_j + c_j y_{\theta jn} + \exp \left[ \alpha_j + \psi_j (y_{\xi jn} + \frac{1}{2} \psi_j \sigma_{1\xi_j}^2) + \beta \xi_e \right] \left( q_j^\dagger (\cdot) \right)^k,
\]
and since \( m_{21}^\dagger \) only depends on the economy-wide productivity signal and state, it follows that
\[ RP_{21jn}^\dagger = -\text{Cov} \left[ m_{21}^\dagger, E \left[ d_{2j}^\dagger | y_{\theta jn}, y_{\xi jn}, \xi_e, q_j^\dagger (\cdot) \right] | y_{\theta jn}, y_{\xi jn}, y_{\xi_e}, q_j^\dagger (\cdot) \right]
\]
\[ = -\text{Cov} \left[ m_{21}^\dagger, \exp \left[ \alpha_j + \psi_j (y_{\xi jn} + \frac{1}{2} \psi_j \sigma_{1\xi_j}^2) + \beta \xi_e \right] \left( q_j^\dagger (\cdot) \right)^k | y_{\xi_e}, q_j^\dagger (\cdot) \right]
\]
\[ = -\text{Cov} \left[ m_{21}^\dagger, \exp \left[ \beta \xi_e \right] | y_{\xi_e}, q_j^\dagger (\cdot) \right] \exp \left[ \alpha_j + \psi_j (y_{\xi jn} + \frac{1}{2} \psi_j \sigma_{1\xi_j}^2) \right] \left( q_j^\dagger (\cdot) \right)^k.
\]
Since \( E \left[ m_{21}^\dagger | y_{\xi_e}, q_j^\dagger (\cdot) \right] = 1 \), it follows that
\[ RP_{21jn}^\dagger = - \left\{ E \left[ m_{21}^\dagger \exp \left[ \beta \xi_e \right] | y_{\xi_e}, q_j^\dagger (\cdot) \right] - E \left[ \exp \left[ \beta \xi_e \right] | y_{\xi_e}, q_j^\dagger (\cdot) \right] \right\}
\]
\[ \times \exp \left[ \alpha_j + \psi_j (y_{\xi jn} + \frac{1}{2} \psi_j \sigma_{1\xi_j}^2) \right] \left( q_j^\dagger (\cdot) \right)^k
\]
\[ = \left\{ \exp \left[ \beta (y_{\xi_e} + \frac{1}{2} \beta \sigma_{1\xi_e}^2) \right] - E \left[ m_{21}^\dagger \exp \left[ \beta \xi_e \right] | y_{\xi_e}, q_j^\dagger (\cdot) \right] \right\}
\]
\[ \times \exp \left[ \alpha_j + \psi_j (y_{\xi jn} + \frac{1}{2} \psi_j \sigma_{1\xi_j}^2) \right] \left( q_j^\dagger (\cdot) \right)^k.
\]
Inserting the expression in Lemma 8 in the second term in the brackets and simplifying yield

\[
E \left[ m_{21}^t \exp \left[ \beta \xi_e \right] | y_{\xi e}, q_j^t(\cdot) \right] = E \left[ \exp \left[ \gamma \beta (y_{\xi e} - \xi_e) - \frac{1}{2} (\gamma \beta)^2 \sigma_{\xi e}^2 \right] \exp \left[ \beta \xi_e \right] | y_{\xi e}, q_j^t(\cdot) \right] \\
= E \left[ \exp \left[ \beta \xi_e (1 - \gamma) \right] | y_{\xi e} \right] \exp \left[ \gamma \beta (y_{\xi e} - \frac{1}{2} \gamma \beta \sigma_{\xi e}^2) \right] \\
= \exp \left[ \beta (1 - \gamma) y_{\xi e} + \frac{1}{2} \beta^2 (1 - \gamma) \sigma_{\xi e}^2 \right] \exp \left[ \gamma \beta (y_{\xi e} - \frac{1}{2} \gamma \beta \sigma_{\xi e}^2) \right] \\
= \exp \left[ \beta (1 - \gamma) y_{\xi e} + \frac{1}{2} \beta^2 (1 + \gamma^2 - 2\gamma) \sigma_{\xi e}^2 + \gamma \beta (y_{\xi e} - \frac{1}{2} \gamma \beta \sigma_{\xi e}^2) \right] \\
= \exp \left[ \beta (y_{\xi e} + \frac{1}{2} \beta (1 - 2\gamma) \sigma_{\xi e}^2) \right].
\]

Hence,

\[
RP_{21jn}^t = \left\{ \exp \left[ \beta (y_{\xi e} + \frac{1}{2} \beta \sigma_{\xi e}^2) \right] - \exp \left[ \beta (y_{\xi e} + \frac{1}{2} \beta (1 - 2\gamma) \sigma_{\xi e}^2) \right] \right\} \\
\times \exp \left[ \alpha_j + \psi_j (y_{\xi jn} + \frac{1}{2} \psi_j \sigma_{\xi j}^2) \right] \left( q_j^t(\cdot) \right)^k \\
= \left\{ 1 - \exp \left[ -\beta^2 \gamma \sigma_{\xi e}^2 \right] \right\} \exp \left[ \beta (y_{\xi e} + \frac{1}{2} \beta \sigma_{\xi e}^2) \right] \\
\times \exp \left[ \alpha_j + \psi_j (y_{\xi jn} + \frac{1}{2} \psi_j \sigma_{\xi j}^2) \right] \left( q_j^t(\cdot) \right)^k \\
= \left\{ 1 - \exp \left[ -\beta^2 \gamma \sigma_{\xi e}^2 \right] \right\} E[h_j^t|y_{\xi jn}, y_{\xi e}, q_j^t(\cdot)],
\]

which demonstrates (a).

(b): It follows from (18) and (a) that

\[
v_{1jn}^t = \exp[-t_{21}^t] \left[ a_j + c_j y_{\theta jn} + \exp \left[ -\beta^2 \gamma \sigma_{\xi e}^2 \right] E[h_j^t|y_{\xi jn}, y_{\xi e}, q_j^t(\cdot)] \right].
\]

The one-period ahead expected rate of return of firm \( n \) of type \( j \) conditional on information at \( t = 1 \) is thus given as

\[
\mu_{21jn}^t = \ln \left[ \frac{a_j + c_j y_{\theta jn} + E[h_j^t|y_{\xi jn}, y_{\xi e}, q_j^t(\cdot)]}{\exp[-t_{21}^t] \left[ a_j + c_j y_{\theta jn} + \exp \left[ -\beta^2 \gamma \sigma_{\xi e}^2 \right] E[h_j^t|y_{\xi jn}, y_{\xi e}, q_j^t(\cdot)] \right]} \right] \\
= t_{21}^t + \ln \left[ \frac{a_j + c_j y_{\theta jn} + E[h_j^t|y_{\xi jn}, y_{\xi e}, q_j^t(\cdot)]}{a_j + c_j y_{\theta jn} + \exp \left[ -\beta^2 \gamma \sigma_{\xi e}^2 \right] E[h_j^t|y_{\xi jn}, y_{\xi e}, q_j^t(\cdot)]} \right].
\]

The conditional rate of return risk premium conditional on information at \( t = 1 \) is therefore
given as
\[
\omega_{21jn} = \ln \left[ \frac{a_j + c_j y_{\theta jn} + E[h_j^1|y_{\xi jn}, y_{\xi \epsilon}, q_j^1(\cdot)]}{a_j + c_j y_{\theta jn} + \exp \left[ -\beta^2 \gamma \sigma_{1j}^2 \right] E[h_j^1|y_{\xi jn}, y_{\epsilon j}, q_j^1(\cdot)]} \right],
\]
which demonstrates (b).

(c): Repeating the steps in (a) and (b) with \( \bar{h}(\xi, \bar{q}_o) \) instead of \( E \left[ d_{ij}^1|y_{\theta jn}, y_{\xi jn}, \xi_j, q_j^1(\cdot) \right] \) demonstrates (c). Alternatively recognize that if for an asset it holds that \( a_j + c_j y_{\theta jn} = 0 \), then \( \omega_{21jn}^* = \beta^2 \gamma \sigma_{1j}^2 \). The market portfolio is such an asset which establishes (c).

**Proof of Proposition 12:** (a): It follows from (23) that

\[
\iota_{21}^*(y_{\xi \epsilon}) = \ln \left[ k \pi (\bar{q}_o^*(y_{\xi \epsilon}))^{k-1} \right] + \ln \left[ \exp \left[ \beta \left( y_{\xi \epsilon} + \frac{1}{2} \beta \sigma_{1\xi \epsilon}^2 \right) \right] \right] + \text{Cov} \left[ m_{21}^*, \exp \left[ \beta \xi \epsilon | y_{\xi \epsilon} \right] \right].
\]

It follows from the proof of Lemma 9 that

\[
\text{Cov} \left[ m_{21}^*, \exp \left[ \beta \xi \epsilon | y_{\xi \epsilon} \right] \right] = \exp \left[ \beta \left( y_{\xi \epsilon} + \frac{1}{2} \beta \sigma_{1\xi \epsilon}^2 \right) \right] \left\{ \exp \left[ -\beta^2 \gamma \sigma_{1\xi \epsilon}^2 \right] - 1 \right\}
\]

and, hence,

\[
\iota_{21}^*(y_{\xi \epsilon}) = \ln \left[ k \pi (\bar{q}_o^*(y_{\xi \epsilon}))^{k-1} \right] + \ln \left[ \exp \left[ \beta \left( y_{\xi \epsilon} + \frac{1}{2} \beta \sigma_{1\xi \epsilon}^2 \right) \right] \exp \left[ -\beta^2 \gamma \sigma_{1\xi \epsilon}^2 \right] \right] \]

\[= \ln \left[ k \pi (\bar{q}_o^*(y_{\xi \epsilon}))^{k-1} \right] + \beta \left( y_{\xi \epsilon} + \frac{1}{2} \beta \sigma_{1\xi \epsilon}^2 \right) - \beta^2 \gamma \sigma_{1\xi \epsilon}^2 \]

\[= \ln \left[ k \pi (\bar{q}_o^*(y_{\xi \epsilon}))^{k-1} \right] + \beta \left( y_{\xi \epsilon} + \frac{1}{2} \beta \left( 1 - 2 \gamma \right) \sigma_{1\xi \epsilon}^2 \right).\]

It now follows from the proof of Proposition ??(a) that \( \partial \iota_{21}^*/\partial \sigma_{0\xi j}^2 > 0 \), since the last additive term does not depend on \( \sigma_{0\xi j}^2 \).

Furthermore, it follows from (16) and the above that

\[
\frac{\partial}{\partial y_{\xi \epsilon}} \iota_{21}^* = \gamma \left( \beta + k \frac{1}{\bar{q}_o(y_{\xi \epsilon})} \frac{\partial \bar{q}_o^*}{\partial y_{\xi \epsilon}} + \frac{1}{\bar{d}_1 - \bar{q}_o(y_{\xi \epsilon})} \frac{\partial \bar{q}_o^*}{\partial y_{\xi \epsilon}} \right)
\]

\[= (k - 1) \frac{1}{\bar{q}_o(y_{\xi \epsilon})} \frac{\partial \bar{q}_o^*}{\partial y_{\xi \epsilon}} + \beta.\]

Suppose \( \partial \iota_{21}^*/\partial y_{\xi \epsilon} < 0 \). Then

\[(k - 1) \frac{1}{\bar{q}_o(y_{\xi \epsilon})} \frac{\partial \bar{q}_o^*}{\partial y_{\xi \epsilon}} + \beta < 0 \iff \frac{\partial \bar{q}_o^*}{\partial y_{\xi \epsilon}} > \beta \bar{q}_o^*(y_{\xi \epsilon}) / (1 - k).\]

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Substituting this into the above, we get
\[
\frac{\partial \nu_{21}^*}{\partial y_{\xi e}} = \gamma \left( \beta + k \frac{1}{\widetilde{q}_o(y_{\xi e})} \frac{\partial \eta_o^*}{\partial y_{\xi e}} + \frac{1}{d_1 - \widetilde{q}_o(y_{\xi e})} \frac{\partial \eta_o^*}{\partial y_{\xi e}} \right) 
\]
\[
> \gamma \left( \beta + \beta k / (1 - k) + \frac{\widetilde{q}_o(y_{\xi e})}{d_1 - \widetilde{q}_o(y_{\xi e})} \beta / (1 - k) \right). 
\]

Since all the terms on the right-hand side are positive, we obtain a contradiction to the assumption that \( \frac{\partial \nu_{21}^*}{\partial y_{\xi e}} < 0 \) and, thus, \( \frac{\partial \nu_{21}^*}{\partial y_{\xi e}} > 0 \). To show the last claim in (a) take the derivative of \( \nu_{21}^* (y_{\xi e}) \) wrt. \( \sigma_{0\xi e}^2 \) recognizing that \( \sigma_{0\xi e}^2 = 1 - \sigma_{1\xi e}^2 \).

\[
\frac{\partial \nu_{21}^*}{\partial \sigma_{0\xi e}^2} = -(1 - k) \frac{1}{\widetilde{q}_o(y_{\xi e})} \frac{\partial \eta_o^*}{\partial \sigma_{0\xi e}^2} - \beta^2 \left( \frac{1}{2} - \gamma \right). 
\]

In (c) we establish that \( \frac{\partial \eta_o^*}{\partial \sigma_{0\xi e}^2} < 0 \). Hence, the first term is positive. If \( \gamma > 1/2 \), the second term is positive as well. This establishes (a).

(b): It follows from Lemma 9 that the rate of return risk premium of the market portfolio \( \widetilde{\mu}_{21}^* = \gamma \beta^2 \sigma_{1\xi e}^2 \) is independent of \( \sigma_{0\xi e}^2, y_{\xi e}, \) and \( \widetilde{q}_o \). Since \( \sigma_{1\xi e}^2 = 1 - \sigma_{0\xi e}^2 \) the last claim follows easily. This establishes (b).

(c): As \( \widetilde{\mu}_{21}^* = \nu_{21}^* + \widetilde{\nu}_{21}^* = \nu_{21}^* + \beta^2 \gamma \sigma_{1\xi e}^2 \), it follows from (a) and (b) that the one-period ahead expected rate of return of the market portfolio is increasing in \( \sigma_{0\xi e}^2 \) and \( y_{\xi e} \).

From above it follows that
\[
\frac{\partial \nu_{21}^*}{\partial \sigma_{0\xi e}^2} \frac{\partial \nu_{21}^*}{\partial \sigma_{0\xi e}^2} = \frac{\partial \nu_{21}^*}{\partial \sigma_{0\xi e}^2} + \frac{\partial \nu_{21}^*}{\partial \sigma_{0\xi e}^2} = -(1 - k) \frac{1}{\widetilde{q}_o(y_{\xi e})} \frac{\partial \eta_o^*}{\partial \sigma_{0\xi e}^2} - \beta^2 \left( \frac{1}{2} - \gamma \right) - \gamma \beta^2. 
\]

Now, from (16) and the above it follows that in equilibrium
\[
\delta + \gamma (\ln(\bar{\pi}) + k \ln(\bar{q}_o^*) + \beta y_{\xi e} - \ln(d_1 - \bar{q}_o^*)) = \frac{1}{2} \gamma \beta^2 \sigma_{1\xi e}^2 
\]
\[
= \ln(\bar{\pi}k(\bar{q}_o^*)^{k-1}) + \beta(y_{\xi e} + \frac{1}{2} \beta \sigma_{1\xi e}^2) - \beta^2 \gamma \sigma_{1\xi e}^2 
\]
\[
= \ln(\bar{\pi}k(\bar{q}_o^*)^{k-1}) + \beta(y_{\xi e} + \frac{1}{2} \beta(1 - 2\gamma) \sigma_{1\xi e}^2). 
\]

Define the function \( g(\cdot) \) from the equation
\[
g(\sigma_{0\xi e}^2, \bar{q}_o^*) = \delta + \gamma \left[ \ln(\bar{\pi}) + k \ln(\bar{q}_o^*) + \beta y_{\xi e} - \ln(d_1 - \bar{q}_o^*) - \frac{1}{2} \gamma \beta^2 \sigma_{1\xi e}^2 \right] 
\]
\[
- \left\{ \ln(\bar{\pi}k(\bar{q}_o^*)^{k-1}) + \beta(y_{\xi e} + \frac{1}{2} \beta(1 - 2\gamma) \sigma_{1\xi e}^2) \right\} = 0. 
\]
The implicit function theorem then says

\[
\frac{dq_o^*}{d\sigma_{0e}^2} = -\frac{\partial g/\partial \sigma_{0e}^2}{\partial g/\partial q_o^*}.
\]

Now,

\[
\frac{\partial g/\partial \sigma_{0e}^2}{\partial g/\partial q_o^*} = \frac{1}{2} \gamma^2 \beta^2 + \frac{1}{2} \beta^2 - \beta^2 \gamma = \frac{1}{2} \beta^2 (\gamma^2 - 2\gamma + 1) = \frac{1}{2} \beta^2 (1 - \gamma)^2,
\]

and

\[
\frac{\partial g/\partial q_o^*}{\partial g/\partial q_o^*} = \gamma \left[ \frac{1}{q_o^*}k + \frac{1}{q_o^*} \right] - (k - 1) \frac{1}{q_o^*}.
\]

Thus,

\[
\frac{dq_o^*}{d\sigma_{0e}^2} = -\frac{\gamma \left[ \frac{1}{q_o^*}k + \frac{1}{q_o^*} \right] - (k - 1) \frac{1}{q_o^*}}{\frac{1}{2} \beta^2 (1 - \gamma)^2} < 0.
\]

Inserting this into the expression for \( \partial \mu_{21}^*/\partial \sigma_{0e}^2 \) yields

\[
\frac{\partial \mu_{21}^*}{\partial \sigma_{0e}^2} = \frac{\partial t_{21}^*}{\partial \sigma_{0e}^2} + \frac{\partial \omega_{21}^*}{\partial \sigma_{0e}^2} = (1 - k) \frac{1}{q_o^*} \gamma \left[ \frac{1}{q_o^*}k + \frac{1}{q_o^*} \right] - (k - 1) \frac{1}{q_o^*} - \beta^2 (\frac{1}{2} - \gamma) - \gamma \beta^2
\]

\[= -\frac{1}{2} \beta^2 \frac{\gamma}{q_o^*} \left( d_1 + (1 - k)(1 - \gamma)(d_1 - q_o^*) \right) \gamma (k + \frac{q_o^*}{d_1 - q_o^*}) + (1 - k).
\]

This implies

\[
\text{sign} \left\{ \frac{\partial \mu_{21}^*}{\partial \sigma_{0e}^2} \right\} = -\text{sign} \left\{ d_1 + (1 - k)(1 - \gamma)(d_1 - q_o^*) \right\}.
\]

Now,

\[
\text{sign} \left\{ \frac{\partial \mu_{21}^*}{\partial \sigma_{0e}^2} \right\} < 0 \quad \text{for} \quad \gamma - 1 < \frac{1}{1-k} \iff (1 - \gamma)(1 - k) > -1,
\]

\[
\text{sign} \left\{ \frac{\partial \mu_{21}^*}{\partial \sigma_{0e}^2} \right\} > 0 \quad \text{for} \quad \gamma < 1,
\]

This implies

\[
\frac{\partial \mu_{21}^*}{\partial \sigma_{0e}^2} < 0 \quad \text{for} \quad \gamma - 1 < \frac{1}{1-k}.
\]

For a fixed \( k \) one may suspect that \( \partial \mu_{21}^*/\partial \sigma_{0e}^2 > 0 \) for \( \gamma \) sufficiently large, and in our numerical examples this turns out to be the case. This establishes (c).
(d): Inserting the expression for the interest rate (16) into (23) and simplifying yield
\[
\delta + \gamma (\ln [\pi] + \beta y_{t\kappa} + k \ln [\overline{\rho}_0 (y_{t\kappa})] - \ln [d_1 - \overline{\rho}_0 (y_{t\kappa})]) - \frac{1}{2} \gamma^2 \beta^2 \sigma_{t\kappa}^2 \\
= \ln \left[ k \pi (\overline{\rho}_0 (y_{t\kappa}))^{k-1} \right] + \beta (y_{t\kappa} + \frac{1}{2} \beta (1 - 2\gamma) \sigma_{t\kappa}^2) \\
\delta + \gamma (\ln [\pi] + k \ln [\overline{\rho}_0 (y_{t\kappa})] - \ln [d_1 - \overline{\rho}_0 (y_{t\kappa})]) \\
= \ln \left[ k \pi (\overline{\rho}_0 (y_{t\kappa}))^{k-1} \right] + \beta (1 - \gamma) y_{t\kappa} + \frac{1}{2} \beta^2 (1 + \gamma^2 - 2\gamma) \sigma_{t\kappa}^2 \\
\delta + \gamma (\ln [\pi] + k \ln [\overline{\rho}_0 (y_{t\kappa})] - \ln [d_1 - \overline{\rho}_0 (y_{t\kappa})]) \\
= \ln \left[ k \pi (\overline{\rho}_0 (y_{t\kappa}))^{k-1} \right] + \beta (1 - \gamma) y_{t\kappa} + \frac{1}{2} \beta^2 (1 - \gamma)^2 \sigma_{t\kappa}^2.
\]

Since the two last additive terms on the right-hand side does not depend on \(\sigma_{t\kappa}^2\), and the other terms are the same as in the setting with no economy-wide productivity risks, it follows from the proof of Proposition ??(b) that \(\partial \overline{\rho}_0 (y_{t\kappa}) / \partial \sigma_{t\kappa}^2 > (>) 0\) if, and only if, \(\gamma < (>) 1\).

It follows from the above that
\[
\frac{\partial}{\partial y_{t\kappa}} \left[ \delta + \gamma (\ln [\pi] + k \ln [\overline{\rho}_0 (y_{t\kappa})] - \ln [d_1 - \overline{\rho}_0 (y_{t\kappa})]) \right] \\
= \frac{\partial}{\partial y_{t\kappa}} \left[ \ln \left[ k \pi (\overline{\rho}_0 (y_{t\kappa}))^{k-1} \right] + \beta (1 - \gamma) y_{t\kappa} + \frac{1}{2} \beta^2 (1 - \gamma)^2 \sigma_{t\kappa}^2 \right] \\
\left[ \gamma k \frac{1}{\overline{\rho}_0 (y_{t\kappa})} + \frac{1}{d_1 - \overline{\rho}_0 (y_{t\kappa})} \right] \frac{\partial \overline{\rho}_0}{\partial y_{t\kappa}} = (k - 1) \left[ \frac{1}{\overline{\rho}_0 (y_{t\kappa})} \frac{\partial \overline{\rho}_0}{\partial y_{t\kappa}} + \beta (1 - \gamma) \right] \\
\left[ \gamma k \frac{1}{\overline{\rho}_0 (y_{t\kappa})} + \frac{1}{d_1 - \overline{\rho}_0 (y_{t\kappa})} + (1 - k) \frac{1}{\overline{\rho}_0 (y_{t\kappa})} \right] \frac{\partial \overline{\rho}_0}{\partial y_{t\kappa}} = \beta (1 - \gamma).
\]

Since all terms in the brackets are positive, we get that \(\partial \overline{\rho}_0 / \partial y_{t\kappa} > (>) 0\) if, and only if \(\gamma < (>) 1\). That \(\overline{\rho}_0 (y_{t\kappa})\) is decreasing in \(\sigma_{t\kappa}^2\) for each \(y_{t\kappa}\) was demonstrated in the proof of (c). This establishes (d).

**Proof of Proposition 14:** The cum-dividend value of the market portfolio at \(t = 1\) is (see equation (3)):
\[
\bar{V}_1 (y_{t\kappa}) = \bar{d}_1 - \bar{\overline{\rho}}_0 (y_{t\kappa}) + \bar{v}_1 (y_{t\kappa}) = \bar{d}_1 - \bar{\overline{\rho}}_0 (y_{t\kappa}) \\
+ \exp [-t_2^*(y_{t\kappa})] \left[ \mathbb{E} \left[ \hat{h} (\xi_t, \overline{\rho}_0 (y_{t\kappa})) \right] | y_{t\kappa} \right] + \text{Cov}(m_{t2}, \hat{h} (\xi_t, \overline{\rho}_0 (y_{t\kappa})) | y_{t\kappa}) \\
= \bar{d}_1 - \bar{\overline{\rho}}_0 (y_{t\kappa}) + \exp [-t_2^*(y_{t\kappa})] \left[ \mathbb{E} \left[ m_{t2} \hat{h} (\xi_t, \overline{\rho}_0 (y_{t\kappa})) \right] | y_{t\kappa} \right].
\]
Using Lemma 8 and simplifying, we get

\[
\mathbb{E} \left[ m_{21}^* \tilde{h}(\xi_e, \tilde{q}_o^* (y_{\xi_e})) \mid y_{\xi_e}, \tilde{q}_o^* (\cdot) \right] \\
= \mathbb{E} \left[ \exp \left[ \gamma \beta (y_{\xi_e} - \xi_e) - \frac{1}{2} \gamma^2 \beta^2 \sigma_{1 \xi_e}^2 \right] \frac{\pi}{\tilde{q}_o^* (y_{\xi_e})} \right] \mathbb{E} \left[ \beta (y_{\xi_e})^k \right] \\
= \exp \left[ \gamma \beta y_{\xi_e} - \frac{1}{2} \gamma^2 \beta^2 \sigma_{1 \xi_e}^2 \right] \frac{\pi}{\tilde{q}_o^* (y_{\xi_e})} \mathbb{E} \left[ \beta (1 - \gamma) \xi_e \right] \\
= \exp \left[ \gamma \beta y_{\xi_e} - \frac{1}{2} \gamma^2 \beta^2 \sigma_{1 \xi_e}^2 \right] \frac{\pi}{\tilde{q}_o^* (y_{\xi_e})} \mathbb{E} \left[ \beta (1 - \gamma) y_{\xi_e} + \frac{1}{2} \beta^2 (1 - \gamma)^2 \sigma_{1 \xi_e}^2 \right] \\
= \exp \left[ \gamma \beta y_{\xi_e} + \frac{1}{2} \beta^2 (1 - 2\gamma) \sigma_{1 \xi_e}^2 \right] \frac{\pi}{\tilde{q}_o^* (y_{\xi_e})} \mathbb{E} \left[ \beta (1 - \gamma) y_{\xi_e} + \frac{1}{2} \beta^2 (1 - \gamma)^2 \sigma_{1 \xi_e}^2 \right]
\]

and, thus,

\[
\tilde{V}_1^* (y_{\xi_e}) = \tilde{d}_1 - \tilde{q}_o^* (y_{\xi_e}) + \exp \left[ -\iota_{21}^* (y_{\xi_e}) \right] \mathbb{E} \left[ \beta y_{\xi_e} + \frac{1}{2} \beta^2 (1 - 2\gamma) \sigma_{1 \xi_e}^2 \right] \frac{\pi}{\tilde{q}_o^* (y_{\xi_e})} \mathbb{E} \left[ \beta (1 - \gamma) y_{\xi_e} + \frac{1}{2} \beta^2 (1 - \gamma)^2 \sigma_{1 \xi_e}^2 \right].
\]

Taking the total derivative with respect to \( \sigma_{0 \xi_j}^2 \) yields

\[
\frac{d\tilde{V}_1^* (y_{\xi_e})}{d\sigma_{0 \xi_j}^2} = \left[ -1 + \exp \left[ -\iota_{21}^* (y_{\xi_e}) \right] \frac{\pi}{\tilde{q}_o^* (y_{\xi_e})} \mathbb{E} \left[ \beta y_{\xi_e} + \frac{1}{2} \beta^2 (1 - 2\gamma) \sigma_{1 \xi_e}^2 \right] \frac{\pi}{\tilde{q}_o^* (y_{\xi_e})} \mathbb{E} \left[ \beta (1 - \gamma) y_{\xi_e} + \frac{1}{2} \beta^2 (1 - \gamma)^2 \sigma_{1 \xi_e}^2 \right] \frac{d\tilde{q}_o^* (y_{\xi_e})}{d\sigma_{0 \xi_j}^2} \\
- \exp \left[ -\iota_{21}^* (y_{\xi_e}) \right] \frac{\pi}{\tilde{q}_o^* (y_{\xi_e})} \mathbb{E} \left[ \beta y_{\xi_e} + \frac{1}{2} \beta^2 (1 - 2\gamma) \sigma_{1 \xi_e}^2 \right] \frac{\pi}{\tilde{q}_o^* (y_{\xi_e})} \mathbb{E} \left[ \beta (1 - \gamma) y_{\xi_e} + \frac{1}{2} \beta^2 (1 - \gamma)^2 \sigma_{1 \xi_e}^2 \right] \frac{d\tilde{q}_o^* (y_{\xi_e})}{d\sigma_{0 \xi_j}^2} \\
+ \exp \left[ -\iota_{21}^* (y_{\xi_e}) \right] \frac{\pi}{\tilde{q}_o^* (y_{\xi_e})} \mathbb{E} \left[ \beta y_{\xi_e} + \frac{1}{2} \beta^2 (1 - 2\gamma) \sigma_{1 \xi_e}^2 \right] \frac{\pi}{\tilde{q}_o^* (y_{\xi_e})} \mathbb{E} \left[ \beta (1 - \gamma) y_{\xi_e} + \frac{1}{2} \beta^2 (1 - \gamma)^2 \sigma_{1 \xi_e}^2 \right] \frac{d\tilde{q}_o^* (y_{\xi_e})}{d\sigma_{0 \xi_j}^2}.
\]

The first term on the right-hand side is equal to zero, since \( \tilde{q}_o^* (y_{\xi_e}) \) by Lemma 5 maximizes the cum-dividend value of the market portfolio at \( t = 1 \). The last two terms can be rewritten in terms of the ex-dividend value of the market portfolio \( \tilde{V}_1^* (y_{\xi_e}) \) such that

\[
\frac{d\tilde{V}_1^* (y_{\xi_e})}{d\sigma_{0 \xi_j}^2} = \tilde{V}_1^* (y_{\xi_e}) \left\{ \frac{d\pi}{d\sigma_{0 \xi_j}^2} \frac{1}{\tilde{V}_1^* (y_{\xi_e})} \frac{d\iota_{21}^* (y_{\xi_e})}{d\sigma_{0 \xi_j}^2} \right\} = \tilde{V}_1^* (y_{\xi_e}) \frac{d}{d\sigma_{0 \xi_j}^2} \left( \ln(\pi) - \iota_{21}^* (y_{\xi_e}) \right).
\]

It follows from the proof of Proposition 12(a) that

\[
\ln(\pi) - \iota_{21}^* (y_{\xi_e}) = -\ln \left[ k \left( \tilde{q}_o^* (y_{\xi_e}) \right)^{k-1} \right] - \beta \left( y_{\xi_e} + \frac{1}{2} \beta (1 - 2\gamma) \sigma_{1 \xi_e}^2 \right)
\]

and, thus,

\[
\frac{d\tilde{V}_1^* (y_{\xi_e})}{d\sigma_{0 \xi_j}^2} = \tilde{V}_1^* (y_{\xi_e}) (1 - k) \frac{1}{\tilde{q}_o^* (y_{\xi_e})} \frac{d\tilde{q}_o^* (y_{\xi_e})}{d\sigma_{0 \xi_j}^2}. 
\]

Since the first three terms on the right-hand side are all positive, \( d\tilde{V}_1^* (y_{\xi_e})/d\sigma_{0 \xi_j}^2 \) has the same sign as \( d\tilde{q}_o^* (y_{\xi_e})/d\sigma_{0 \xi_j}^2 \).
The \textit{ex ante} value of the market portfolio is determined by the truncated version of equation (3) as
\[
\bar{v}_0^* = E_{y_{\xi e}} \left[ \frac{u'_i(\bar{d}_1 - \bar{\eta}_o(y_{\xi e}))}{u'_o(c_0)} \bar{V}_1^*(y_{\xi e}) \right].
\]
Since \(d\bar{v}_0^*/d\sigma_{0,\xi j}^2\) has the same sign as \(d\sigma_o^*(y_{\xi e})/d\sigma_{0,\xi j}^2\), we get by the concavity of the utility function that
\[
\text{sign} \left\{ \frac{d}{d\sigma_{0,\xi j}^2} \left( \frac{u'_i(\bar{d}_1 - \bar{\eta}_o(y_{\xi e}))}{u'_o(c_0)} \bar{V}_1^*(y_{\xi e}) \right) \right\} = \text{sign} \left\{ \frac{d\sigma_o^*(y_{\xi e})}{d\sigma_{0,\xi j}^2} \right\}
\]
for all economy-wide productivity signals \(y_{\xi e}\). Since the informativeness of the firm-specific productivity information for firm type \(j\) does not affect the distribution of the economy-wide productivity signals, it then follows from Proposition 12(d) that \(d\bar{v}_0^*/d\sigma_{0,\xi j}^2 < (>) 0\) if, and only if, \(\gamma > (<) 1\). ■

\textbf{Proof of Proposition 15:} From Proposition 14, \(\bar{v}_0^*\) is decreasing in \(\sigma_{0,\xi j}^2\) when \(\gamma > 1\). Further, Proposition 12(a) yields that \(v_{21}^*(y_{\xi e})\) is increasing in \(\sigma_{0,\xi j}^2\). As
\[
v_{21}^*(y_{\xi e}) = \delta + \gamma \ln \frac{\pi(\bar{\eta}_o(y_{\xi e}))}{\bar{d}_1 - \bar{\eta}_o(y_{\xi e})} \exp \left[ \beta y_{\xi e} \right] - \frac{1}{2} \gamma^2 \beta^2 \sigma_{1,\xi e}^2
\]
\[
= \delta + \gamma \ln \left[ \frac{\pi(\bar{\eta}_o(y_{\xi e}))}{\bar{d}_1 - \bar{\eta}_o(y_{\xi e})} \exp \left[ \beta (y_{\xi e} + \frac{1}{2} \beta \sigma_{1,\xi e}^2) \right] \right] - \frac{1}{2} \beta^2 \sigma_{1,\xi e}^2 (\gamma + \gamma^2)
\]
is increasing in \(\sigma_{0,\xi j}^2\), it follows that expected consumption growth, \(\pi(\bar{\eta}_o(y_{\xi e})) \exp[\beta(y_{\xi e} + \frac{1}{2} \beta \sigma_{1,\xi e}^2)]\), is increasing in \(\sigma_{0,\xi j}^2\). Now, as \(\bar{v}_0^*(y_{\xi e})\) is decreasing in \(\sigma_{0,\xi j}^2\) when \(\gamma > 1\), it follows that \(\pi(\bar{\eta}_o(y_{\xi e})) \exp[\beta(y_{\xi e} + \frac{1}{2} \beta \sigma_{1,\xi e}^2)]\) and \(\bar{d}_1 - \bar{\eta}_o(y_{\xi e})\) are both increasing and thus are \(E_{y_{\xi e}}[\pi(\bar{\eta}_o(y_{\xi e})) \exp[\beta(y_{\xi e} + \frac{1}{2} \beta \sigma_{1,\xi e}^2)]\] and \(E_{y_{\xi e}}[\bar{d}_1 - \bar{\eta}_o(y_{\xi e})]\). The implied cost of capital on the market portfolio, \(\bar{\pi}^*\), is determined as the discount rate solving the equation
\[
\bar{\pi}^* = E_{y_{\xi e}} [\bar{d}_1 - \bar{\eta}_o(y_{\xi e})] \exp[-\bar{\pi}^*] + E_{y_{\xi e}} [\pi(\bar{\eta}_o(y_{\xi e})) \exp[\beta(y_{\xi e} + \frac{1}{2} \beta \sigma_{1,\xi e}^2)] \exp[-2\bar{\pi}^*].
\]
Thus, as \(\bar{v}_0^*\) is decreasing while the RHS is increasing in \(\sigma_{0,\xi j}^2\) for fixed \(\bar{\pi}^*\), it follows that \(\bar{\pi}^*\) is increasing in \(\sigma_{0,\xi j}^2\). ■

\textbf{Proof of Proposition 16:} Using (2), the equilibrium ex-dividend value of the market portfolio at \(t = 0\) can be written as
\[
\bar{v}_0^* = \sum_{\tau=1}^2 B_{\tau 0}^* E_0 \left[ m_{\tau 0}^* d_{\tau}^* \right],
\]
\[ B_{r0}^* = \frac{E_0[u'_t(c_t^*)]}{u'_0(c_0^*)}, \quad m_{r0}^* = \frac{u'_t(c_t^*)}{E_0[u'_t(c_t^*)]} \]

In equilibrium, consumption for the representative investor equals average dividends at each consumption date, i.e., \( c_t^* = \bar{d}_t, t = 0, 1, 2 \). Hence,

\[ \tau_0^* = \sum_{\tau=1}^{2} \frac{E_0 \left[ u'_t(c_t^*) \bar{d}_t \right]}{u'_0(c_0^*)} = \sum_{\tau=1}^{2} \frac{E_0 \left[ u'_t(c_t^*)c_0^* \right]}{u'_0(c_0^*)} \]

Inserting the power utility function \( u_t(c_t) = \exp[-\delta t] \frac{1}{1-\gamma} c_t^{1-\gamma} \), yields

\[ \tau_0^* = (c_0^*)^\gamma \sum_{\tau=1}^{2} E_0 \left[ \exp[-\delta \tau] (c_t^*)^{-\gamma} c_t^* \right] \]

\[ = (c_0^*)^\gamma (1 - \gamma) \sum_{\tau=1}^{2} E_0 \left[ u_t(c_t^*) \right] \]

\[ = (c_0^*)^\gamma (1 - \gamma) \{ EU^* - u_0(c_0^*) \} \).

Since \( t = 0 \) consumption \( c_0^* > 0 \) does not depend on the information system at \( t = 1 \), the derivative of the \textit{ex ante} value of the market portfolio with respect to economy-wide productivity informativeness \( \sigma_{0\xi e}^2 \) is

\[ \frac{d}{d\sigma_{0\xi e}^2} \tau_0^* = (c_0^*)^\gamma (1 - \gamma) \frac{d}{d\sigma_{0\xi e}^2} EU^*. \]

Since the investors’ equilibrium expected utility is strictly increasing in economy-wide productivity informativeness when optimal aggregate capital investments vary with the economy-wide productivity signals and their informativeness (see Propositions 6(d) and 12(d)), the equilibrium \textit{ex ante} value of the market portfolio decreases (increases) in \( \sigma_{0\xi e}^2 \) if \( \gamma > 1 \) \((\gamma < 1)\).

**Proof of Proposition 17**: The rate of return risk premium on the market portfolio between \( t = \hat{\tau} \) and \( t = \hat{\tau} \) is

\[ \omega_{2\hat{\tau}} = \ln \left[ \frac{\exp \left[ \beta (\hat{y}_{\xi e} + \frac{1}{2} \beta \hat{\sigma}_{1\xi e}^2) \right] \pi(\bar{q}_0^*(y_{\xi e}))^k}{\exp \left[ -\gamma \beta \sigma_{1\xi e}^2 \right] \exp \left[ \beta (\hat{y}_{\xi e} + \frac{1}{2} \beta \hat{\sigma}_{1\xi e}^2) \right] \pi(\bar{q}_0^*(y_{\xi e}))^k} \right] \]

\[ = \gamma \beta^2 \sigma_{1\xi e}^2. \]

Similarly, the rate of return risk premium on the market portfolio between \( t = 1 \) and \( t = \hat{\tau} \)
\[
\omega_{11} = \ln \left[ \frac{E_{\hat{y}_{t_e}} \left[ \exp \left( -\gamma \beta^2 \sigma^2_{t_e} \right) \exp \left( \beta(y_{t_e} + y_{t_e}^k + \frac{1}{2} \beta \sigma^2_{t_e}) \right) \pi(y_{t_e}) \right] } {\exp \left( -\gamma \beta^2 \sigma^2_{t_e} \right) \exp \left( \beta(y_{t_e} + \frac{1}{2} \beta \sigma^2_{t_e}) \right) \pi(y_{t_e})} \right]
\]

\[
= \ln \left[ \frac{\exp \left( -\gamma \beta^2 \sigma^2_{t_e} \right) \exp \left( \beta(y_{t_e} + \frac{1}{2} \beta \sigma^2_{t_e}) \right) \pi(y_{t_e})^k } {\exp \left( -\gamma \beta^2 \sigma^2_{t_e} \right) \exp \left( \beta(y_{t_e} + \frac{1}{2} \beta \sigma^2_{t_e}) \right) \pi(y_{t_e})^k} \right] = \gamma \beta^2 (\sigma^2_{t_e} - \hat{\sigma}^2_{t_e}).
\]

(c) follows trivially from (a) and (b). To prove (d) notice the dollar risk premium between \( t = 1 \) and \( t = \hat{1} \) is

\[
RP_{11} = \exp \left( -\gamma \beta^2 \sigma^2_{t_e} \right) \exp \left( \beta(y_{t_e} + \frac{1}{2} \beta \sigma^2_{t_e}) \right) \pi(y_{t_e})^k \left[ \exp \left( \gamma \beta^2 (\sigma^2_{t_e} - \hat{\sigma}^2_{t_e}) \right) - 1 \right] - 1
\]

\[
= \exp \left( \beta(y_{t_e} + \frac{1}{2} \beta \sigma^2_{t_e}) \right) \pi(y_{t_e})^k \left[ \exp \left( -\gamma \beta^2 \sigma^2_{t_e} \right) - \exp \left( -\gamma \beta^2 \sigma^2_{t_e} \right) \right],
\]

and the dollar risk premium between \( t = \hat{1} \) and \( t = 2 \) is

\[
RP_{2\hat{1}} = \exp \left( \beta(y_{t_e} + \frac{1}{2} \beta \sigma^2_{t_e}) \right) \pi(y_{t_e})^k \left[ 1 - \exp \left( -\gamma \beta^2 \sigma^2_{t_e} \right) \right].
\]

Taking expectations over \( RP_{2\hat{1}} \) yields

\[
E_{\hat{y}_{t_e}} \left[ RP_{2\hat{1}} | y_{t_e} \right] = \exp \left( \beta(y_{t_e} + \frac{1}{2} \beta \sigma^2_{t_e}) \right) \pi(y_{t_e})^k \left[ 1 - \exp \left( -\gamma \beta^2 \sigma^2_{t_e} \right) \right].
\]

Summing \( RP_{11} \) and \( E_{\hat{y}_{t_e}} \left[ RP_{2\hat{1}} | y_{t_e} \right] \) yields the desired result. □

References


