Preliminary, not for quotation

Moral Hazard and the Structure of Debt Contracts

Robert R. Bliss
Calloway School of Business and Accountancy
Wake Forest University

and

Stephen D. Cauley
Richard S. Ziman Center
Anderson School of Management
University of California at Los Angeles

First draft: 9 March 2007

Abstract

Debt contracts use a number of devices to protect the creditor by giving them special rights over the debtor and other creditors. These include security interests in the debtor's property (e.g., mortgage loans) and seniority. Debt contracts may also include provisions to protect the creditor in the event of adverse changes in the debtor's financial condition, including covenants and associated acceleration clauses. However, several classes of debt contract apparently limit, rather than strengthen, the creditor's rights. These include non-recourse loans, which are common in commercial real estate; revenue bonds, which are used in municipal finance; and the use of subsidiaries to isolate the creditor from other assets of the parent company, as is common in project finance, securitization, and real estate development. We show that these latter structures are advantageous for both the creditor and the debtor once one accounts for the dynamic nature of business investment. Choice of future projects has the potential to expropriate early creditors, setting up a moral hazard problem. Inability to ring fence investments and associated creditor claims results in the moral hazard risk being priced into debt contracts, which in turn either limits future project choice or makes the initial project infeasible. However, ring fencing allows debtors to finance economically viable projects.

Moral Hazard and the Structure of Debt Contracts

It has long been understood that moral hazard arises in debt financing. Equity holders can expropriate bond holders by inducing them to lend at a lower rate of interest than the ex post risk of the firm deserves, either by misleading them as to true nature of the firm's risk (asymmetric information) or by changing the risk of the firm after the loan is made and before it is paid off (asset substitution). And yet long term debt financing is an important component of capital structure and debt financing is widely used. It follows that markets have evolved means of mitigation this problem.

Our thesis is that lenders are aware of the moral hazard problem and act rationally to price this into their debt contracts. The pricing of moral hazard risk can limit the future actions of managers, not by constraining them but by creating incentives to follow an anticipated (and ex ante priced) path. Different forms of debt contracts lead to different investment decisions for firms. The bankruptcy remote (legally separate projects) is the first best from the perspective of controlling moral hazard risk while giving investors the opportunity to invest in future opportunities. However, bankruptcy remote structuring of debt may not always be possible.

One approach to mitigating moral hazard risk is to attempt to prevent the equity holders from engaging in actions that would result in expropriation. Thus, bonds may include covenants that attempt to preclude risk-increasing corporate actions (e.g., mergers), or that ensure that the firm maintains certain financial ratios.[1] The efficacy of covenants in restraining managerial behavior depends of the ability to anticipate future risk-increasing possibilities and to enforce such restrictions. Both are problematic. It is practically impossible to anticipate all future states of the world which would have to include future financial innovations.[2] Enforcement of covenants depends firstly on (costly and imperfect) monitoring and secondly on the ability to enforce compliance when deviation from the covenants is observed. Realistically,

---

[1] It is commonplace to speak of "bond holders requiring" covenants, though in fact the choice of covenants is up the firm. Of course the presence or absence covenants may affect the cost of debt. Managers' choice of covenants is determined by cost of debt and the anticipated direct and opportunity costs of restrictions on their future actions that covenants impose. Since this cost benefit trade-off may change, bonds frequently include call provisions as much to provide a means of escaping covenants as for interest rate motivated refinancing. The recent innovation of "make whole" callable bonds, which are designed to protect bond holders from economic losses should their bonds be called, point to such a managerial motive.

[2] For instance, off balance sheet derivatives may vitiate the intent of covenants based on on-balance-sheet financial ratios.

covenants cannot prevent actions ex ante, they can only impose sanctions ex post. Usually, violations of covenants constitutes an "event of default" and result in a potential acceleration of repayment. However, such acceleration may result in costly bankruptcy, so creditors frequently agree to workouts that reduce their claims in the hopes of loss mitigation rather than enforcing their rights at the risk of incurring greater, and less predictable losses.

An alternative to contractually constraining managerial risk taking is to design contracts to reduce the incentives of managers to take unanticipated risks. It is this approach that we investigate in this paper.

Debt Contract Structures

Corporate and bankruptcy law and market practice provide a number of different frameworks in which debt contracting may take place. In the event of bankruptcy and liquidation, the majority of creditors are "general creditors" who share *pari passu* (on an equal basis) in the recovery value of the assets of the bankruptcy estate.[3] In this base case situation both creditors' claims and firm assets are **pooled**. However, several forms of contracting or corporate structuring can change the <u>effective</u> (if not *de jure*) priority of debt claims. **Secured** debt results in liens that take a portion of assets out of the bankruptcy estate. Thus, a mortgage results in a legal claim of the mortgage holder on the recovery value of the building up to the value of the mortgage debt, with the general creditors having a claim only on the residual (if any).[4] Creditors may accept contractually junior claims, creating **senior/junior** debt structures. Second mortgages and subordinated indentures are examples. *De facto* senior/junior structures can also be created by one creditor obtaining a "floating lien" on all the assets of the firm.[5] Finally, corporations can create legally separate entities through a holding company structure that legally partitions assets and associated claims against those assets into "**bankruptcy remote**" subsidiaries.

---

[3] There are several classes of creditors who have legal priority over general creditors. However, for most corporations, except for the government (taxes) and legal cost of administering the bankruptcy itself, the higher priority creditors are usually unimportant. Preferred stockholders and equity (common stock) are legally junior claimants. Under US banking law depositors are senior to non-deposit creditors, creating a major *de jure* senior/junior structure.

[4] Technically, the borrower only has a claim on an asset net of any liens so in bankruptcy only the borrowers claim goes into the bankruptcy estate, while the lien holders claim remains out side it. The lien holder is usually a general creditor for any unsatisfied portion of their claim.

[5] This results in *de facto* subordination of the remaining general creditors.

All four of these debt structures are found in financial markets. Pooling is the *de jure* base case and is common where there are large numbers of financial market creditors. Examples include uninsured depositors at banks and large corporations that rely primarily on bonds and/or commercial paper, rather than bank lending, for debt financing. Secured lending is common in many industries such as airlines and real estate. Leasing is a variation on the secured lending structure. Bank debt frequently involves a floating lien thus creating a de facto senior/junior structure. Depositor preference creates a senior/junior structure for bank creditors as do requirements that some banks have outstanding subordinated debt (creating three tiers of creditors). Senior/junior structures also arise when covenants are used which require any additional debt financing to be junior to existing debt. Bankruptcy remote structures are common in commercial real estate, project finance, and securitization. Sometimes, as is the case with commercial real estate development, each subsidiary may have multiple creditors and within-subsidiary differentiation through mortgage liens and subordination.

Literature Review

To be added

II. Motivation and Model

The thesis of this paper is that potential moral hazard issues associated with financing such a sequence of investments can have important implications for the design of loan contracts. The model developed in this section illustrates this point. Specifically, in the context of a sequential move game we show that potential lenders' response moral hazard risks under to a pooling debt structure (in comparison to non-recourse or bankruptcy remote) can bias investments toward more risky projects and limit the number of otherwise desirable investments undertaken.[6] As we will see, this stems from the inability of creditors to fully specify and investors to fully commit to the nature of future investments.

Information asymmetries between developers and lenders are an important driver in many models. We wish to emphasize a different and we believe potentially

[6] A project is said to be desirable if its expected net present value of equity investment as a stand alone project is non negative.

more important uncertainty. Many lenders, such as banks, have considerable expertise in assessing the potential of projects they are considering financing and they are in a strong position at the point in the developer/creditor relation to obtain full disclosure and any information they consider relevant. What lenders cannot know at the time they make a loan however is the future project choices that the developer may chose to make. This is not so much information asymmetry as it is uncertainty of the future. The developer may face the same uncertainty, for while he may conceivably know his future projects choices, he cannot know their economic viability without knowing the future state of the economy.

Firms rarely invest in a single project with a single round of financing. In practice, many firms undertake a sequence of investment decisions utilizing the same or different sources for financing and these investments frequently overlap so that the outcome of a pervious project is not known at the time that a subsequent project is being financed. While, it is conceptually possible to write contracts to cover the future project choices and financing rounds, we believe that this is impractical and/or economically inefficient. Future states of the economy and types and risk structures of projects cannot be completely anticipated and enumerated in a contract covering all contingencies. The only practical contract to directly control future investment decisions is "no further projects." This, however, forces all investments to be conducted in separate firms. Such a restrictive contract would require forgoing any potential benefits of organizing multiple projects within a single firm.[7]

In the remainder of this section we describe the agents, investment possibilities, financing, and sequence of moves that make up the game. The model is designed to capture the sequential nature of investment decisions and the implications of the inability to fully specify the risk characteristics of future investment decisions that may be undertaken. To focus on the moral hazard problem with potential ongoing financing needs we use a simple model to capture the essential elements of the problem. The developer finances one or more projects sequentially and then projects payoffs are realized and loans paid off or defaulted on.

Agents

[7] These benefits might include efficiencies in back office processes, diversification and concomitant reduced expected bankruptcy costs, and greater access to capital markets.

Both lenders and developers are assumed to be risk-neutral. There is a competitive banking sector that makes loans whose expected rate of return equals the risk-free rate of interest. We consider a single developer in this environment who can make sequential investments (P_1 and P_2). However, the developer must finance each project in turn. They cannot be financed simultaneously.[8]

Project Types

There are two types of projects: safe (S) and risky (R).

- The safe projects may be risky, but is always less risky than the risky projects (i.e., $0 \le \sigma_S < \sigma_R$).
- The un-levered expected net present value, of the safe project is zero and the risky project type is non-negative.[9] We assume that**Error! Objects cannot be created from editing field codes.** or equivalently, $E\left(NPV_R\right) \ge E\left(NPV_S\right) = 0$. A positive expected net present value investment will be said to have a surplus.
- A project has at most two terminal values, a high value and a low value $(V_i^H, V_i^L,\ i = S \text{ or } R)$.[10]
- The outcomes are realized one period in the future.
- Projects' state-contingent outcomes and their probabilities (π_i^H and $\pi_i^L = 1 - \pi_i^H$, $i = S$ or R) are common knowledge.
- Outcomes for multiple projects, including projects of the same type, are uncorrelated.

[8] Allowing simultaneous financing would eliminate the "uncertain future".
[9] Because of the risk neutrality assumption, future payoffs are discounted at the risk free rate, r_f,
[10] For the safer project $V_S^h \ge V_S^l$ (i.e., it may be risk-less).

Financing

Projects are financed by both debt and equity. The project cost is denoted C_i and the loan-to-value ratio is denoted k_i, $i \in \{S, R\}$. Banks charge an interest rate $I_{\cdot}^{j}$, $j \in \{1,2\}$,[11] conditional on an observed first project, P_1, and a hypothetical second project, $\tilde{\mathrm{P}}_2$,[12] in the case of the first loan, $I^1_{P_1,\tilde{P}_2}$, or two observed projects, P_1, and P_2, and the first loan rate, in the case of the second loan set of projects, $I^2_{P_1,P_2,I^1_{P_1,\tilde{P}_2}}$, so that their expected rate of return on a loan is equal to the risk-free rate of interest.

Sequence of Decisions

The investor moves first, picking the type of the first project (S_1 or R_1) and solicits a loan in the amount of $D_0^1 = k_1 C_1$, to finance the investment. The initial project's type is common knowledge. The developer accepts or rejects the loan based upon his options for the second project. If the investor decides to do a second project, he then solicits financing from a (possibly the same) bank to finance the second project. The second loan is made with full knowledge of both project types and the rate of interest charged on the first loan. In one period following the financing round, the asset values are realized and payoffs are made according to the loan contracts. To summarize the moves in the game:

- The developer chooses the first project type S_1 or R_1 and solicits a loan to finance the investment.
- A bank finances the project at a contracted interest rate $I^1_{P_1,\tilde{P}_2}$ based upon the risk of the loan that is being solicited and the bank's anticipation as to the second project type (possibly none) that the developer will undertake.
- The developer chooses the second project type S_2 or R_2 and solicits a loan to finance the second investment or he decides not to do a second project, $P_2 = \Phi$.
- A second bank (possibly the same bank) finances the project at a contracted interest rate $I^2_{P_1,P_2,I^1}$ based on knowledge of the types of the first and second projects and the interest rate charged on the first loan.

[11] All rates are expressed as one plus the rate of interest.

[12] The symbol ~ will be used to indicate an anticipated or hypothetical future choice.

First Lender's Strategy

With all debt structures except the bankruptcy remote structure, the project type of the second investment can affect the expected payoff to the first loan. The first lender knows this. While the first bank can price it's loan along a continuum of interest rates, it prices the loan at an interest rate of $I^1_{.}$ such that the first loan earns an expected return, ***conditional upon the banks anticipation as to second project type***, equal to the risk free rate.[13] Since there are only 3 second project types $P_2 \in \{S, R, \Phi\}$, the bank need only choose between three loan rates, at most, The bank engages in the following analysis. For each possible second project, $P_2 \in \{S, R, \Phi\}$, the bank determines the rate it would need to lend at conditional on the known first project type and hypothetical second project type, $I^1_{P_1, \tilde{P}_2}$. The bank also computes the what the second loan rate will be under that scenario, under the assumption that the second loan will earn the risk free rate in expectation. Then, given that two hypothetical loan rates, the bank considers the possible responses of the developer since the developer need not do the hypothetical second project. The lender considers each possible alternative and first computes the second lender's loan rate under the known first project, hypothetical first loan rate, and alternative second project type. The first lender assumes that the developer will choose the second project to maximize his $E(NPV_E)$ of investment. If the investor's optimal second project choice, conditional on the first project and the hypothetical loan rate, results in a negative $E(NPV_1)$ to the bank, the bank will not lend based on that hypothetical second project or equivalently at the rate of interest he would have lent at if he was certain that that hypothetical project would in fact be done.[14] The first lender repeats this analysis for each possible second project type, including no second project, and determines the interest rate or rates that will ensure he earns an $E(NPV_1) = 0$, assuming that the investor will act in his own self-interest in choosing the second project.[15]

---

[13] The bank would like to earn a higher return, but the competitive banking environment makes attempting to do so impossible. Thus, the bank recognizing the inevitable prices the loan at $E(NPV_1) = 0$.

[14] This is the essence of the moral hazard problem. The investors optimal choice of a second project depends on the first lender's interest rate, and that second project choice could result in an $E(NPV)<0$ for the first lender.

[15] This "investor profit maximization assumption" is stronger than having the first lender pick an interest rate that ensures they will have an $E(NPV) \geq 0$ regardless of the second project choice. As we will show, it is possible to construct examples where the first lender would be expropriated, $E(NPV) < 0$, even if the investor chooses a second project type that yields a positive, but not the highest E(NPV).

Investor's Strategy

The investor is aware that the first lender will do the forgoing analysis, and does the same analysis for each first project type. The investor then chooses the first project type to maximize the investor's E(NPV_E) given the assumption that the first lender will choose the first loan rate to ensure that the first lender earns E(NPV_1) ≥ 0.

Second Lender

In a two project world, the second bank has full information and cannot be expropriated. The pricing of the second loan ($I_{.}^{2}$) is fully determined by the previous decisions.

Model Intuition

The intuition for these results is that to avoid expropriation the first lender prices their loan on a worst-case—that is, most risky—basis. This in turn forces the developer to pursue that same strategy in choosing the second project. To do otherwise would be to adopt a less risky series of projects without the benefit of a lower interest rate on the first loan. This would result in a wealth transfer from the developer to the first lender who would be receiving an above market rate of interest. The combination of two or more risky projects is, within this model, less risky than a single risky project, do to diversification effects. Similarly, adding a safe project to a risky project reduces total risk. Thus, the investor will either follow a safe first project with a single risky project, or a risky first project with no second project.

Appendix A works out the model analytically for the pooling debt structure. While limited in scope, this model illustrates the importance of contract design when sequential projects are typical. In the next section we consider a number of numerical examples that investigate the relationship between the characteristics of the projects, loan contract design, and the optimal response of investors.

III. Numerical Examples

Our examples are based on a riskless rate of 4%, and uncorrelated projects. Both project have an initial cost of $1 and are have a loan-to-value ratio of 80%. *No project surpluses*

Initially, we consider the following two project types:

	Type	S (safe)	R (risky)
	π^u	.8	.8
Payoffs in state…	Good	\$1.100	\$1.175
	Bad	\$0.800	\$0.500

With these parameterizations, un-levered, each project earns a zero expected NPV discounting at the riskless rate. Thus, neither project has an expected surplus (or expected deficit).

Pooling debt structure: Table 1a shows the game for the pooling debt structure. The E(NPV) matrix, if the developer chooses a safe first project (**S_1**), is shown in the top panel of Table 1a. The E(NPV) to Bank-1 and the developer are shown in each cell. Because Bank-2 observes both project choices and **I_1**, Bank-2's E(NPV) = \$0 by construction and is omitted from the payoff matrix.

If the developer behaves as the bank anticipates in pricing the loan, both Bank-1 and the developer will expect to earn E(NPV) = \$0 (diagonal of the E(NPV) matrix). However, pricing the loan below 5.94% creates the opportunity for the developer to expropriate the bank. If the bank prices the first loan at 5.00%, anticipating the developer will do S_1 only ($S_1 \cap \Phi$), the developer can do a second risky project (R_2) resulting in a positive E(NPV_E) = \$0.007, while Bank-1 earns E(NPV_1) = −\$0.007. If the bank assumes the developer will do $S_1 \cap S_2$ and prices the first loan at 4.17% the developer can obtain a positive E(NPV_E) at the expense of bank 1 either by doing $S_1 \cap \Phi$ or $S_1 \cap R_2$ earning E(NPV_E) = \$0.005 or \$0.013 respectively. The developer would thus choose to do $S_1 \cap R_2$ and Bank-1 would earn E(NPV_1) = −\$0.013. Anticipating these possible expropriations, Bank-1 would price the first loan at 5.94%. However, doing so means that the developer will earn a negative E(NPV_E) unless he does $S_1 \cap R_2$.

If Bank-1 attempts to charge more than 5.94%, the developer will earn a negative E(NPV_E) and so will do no projects at all. However, attempting to charge above 5.94% is not possible in a competitive banking market. If the Bank-1 charges anything less than 5.94%, the developer can earn a positive E(NPV_E) at the bank's expense by doing $S_1 \cap R_2$. In other words, the bank's optimal choice of interest rate on the first loan determines the developer's second project choice. The equilibrium combination of I_1 and second project choice is highlighted in Table 1.

Now consider what happens if the developer chooses Project-1 to be risky (R_1). The E(NPV) matrix for this game is shown in the bottom panel of Table 1a. In this case Bank-1 can avoid expropriation only by pricing the first loan at 14.38%. However, at this interest rate it is not in the developer's interest to do a second project. Either S_2 or R_2 would result in $E(NPV_E) < 0$ for the developer and a wealth transfer to Bank-1. Again Bank-1's optimal choice of interest rate determines the developer's second project choice (highlighted in Table 1b). The developer would be indifferent between choosing a safe or risky first project, unless there is some reason outside of the model for prefer more projects to fewer.[16]

Senior/junior and secured debt structures: Tables 1b and 1c show the "no surplus" games for senior/junior and secured debt structures respectively. The two sub-games (conditional on the developer's choice of Project-1) are shown in the upper and lower panels of each sub-table. With no surpluses both debt structures produce identical results. The developer will do a single project only and is indifferent it being risky or safe.

Bankruptcy remote: Tables 1d shows the "no surplus" games for the bankruptcy remote debt structures respectively. The two sub-games (conditional on the developer's choice of Project-1) are shown in the upper and lower panels. Under the bankruptcy remote structure, with no surpluses, any combination of projects is equally optimal. This is because under this debt structure the developer cannot earn a positive $E(NPV_E)$ by expropriating Bank-1. In other words, there is no moral hazard problem under the bankruptcy remote structure.

<u>Surpluses</u>

The introduction of surpluses, produces another dimension to the problem. With no surpluses the developer can only earn excess returns by expropriating the first lender causing the first lenders $E(NPV_1)$ to become negative. With surpluses that possibility remains, but must be traded off against maximizing the realization of the surpluses. Since banks price loans to earn zero E(NPV) they do not "compete for the

[16] If the developer wishes to engage in additional projects, he can still do so in one of two ways. He can form an entirely separate corporation to do the projects, or he can go back bank to Bank-1 for funding. In the latter case, Bank-1 will negotiate the second loan rate to ensure that it earns the risk free return in expectation over both loans. Neither option introduces new moral hazard problems for Bank-1.

surplus. Whether the gains from expropriation exceed the gains from the surplus depends on the size of the surplus and the type of debt structure as the following numerical examples will show.

Risky project with small surplus

We begin with a small surplus using the following project parameterizations. x

	Type	S (safe)	R (risky)
	π^u	.8	.8
Payoffs in state…	Good	$1.100	$1.229
	Bad	$0.800	$0.554
E(NPV)		$0.000	$0.054

With a 5 cent per risky project payoff, the developer can potentially achieve an $E(NPV_E)$ = $0.108 without expropriating the lenders by doing two risky projects. The results of the sequential game in this situation for the four debt structure types are shown in Table 2. With the pooling and secured debt structures, the developer can achieve his first best result by leading with a risky first project. The bank will offer an intermediate interest rate of 5.45% or 4.73% depending on the structure.[17] The developer could then expropriate the bank, but doing so would result in a smaller $E(NPV_E)$ < $0.108, so he will not. The bankruptcy remote debt structure also produces the first best outcome for the developer while leaving the banks earning their required E(NPV) = $0.000.

With the senior/junior debt structure however, the first bank will not lend at the $R_1 \cap R_2$ rate of 4.00%, because the developer could respond by doing $R_1 \cap \Phi$ earning $E(NPV_E)$ = $0.110 at the expense of the bank earning a negative $E(NPV_1)$ = – $0.056. Instead, the first bank will adopt a high interest rate strategy forcing the developer to do only the risky project and earn $E(NPV_E)$ = $0.054. This being the case, the developer will be indifferent between a risky first project and a safe first project that will lead to an equilibrium of $S_1 \cap R_2$, in either case earning $E(NPV_E)$ = $0.050.

[17] If the bank could enforce a higher interest rate, for instance 12.81%, the project surplus would be shared between the developer and Bank 1. However, by assumption the banking market is competitive and the banks would compete by offering lower interest rates until their zero E(NPV) constraint was met.

Risky project with larger surplus

We next consider a larger surplus using the following project parameterizations.

	Type	S (safe)	R (risky)
	π^u	.8	.8
Payoffs in state…	Good	$1.100	$1.235
	Bad	$0.800	$0.560
E(NPV)		$0.000	$0.060

With a 6 cent per risky project payoff, the developer can potentially achieve an $E(NPV_E)$ = $0.12 without expropriating the lenders by doing two risky projects. The results of the sequential game in this situation for the four debt structure types are shown in Table 3. With all debt structures, the developer can achieve his first best result by leading with a risky first project. The bank will offer a low interest rate. Again, the developer could expropriate the bank, but doing so would result in a smaller $E(NPV_E)$ < $0.120, so he will not.

In summary, holding project risk constant, as the surplus on the risky project increases from zero, at first only the bankruptcy remote structure allows the developer to achieve his first best investment of $R_1 \cap R_2$. As the surplus increases the relative benefits of expropriate decline, first the pooling (not shown), then the secured, and finally the senior/junior structures also permit the developer's first best to be realized.[18]

IV. Discussion and Conclusions

We have constructed a model of rational lending to help explain the observed variety of debt structures. The model captures the dynamic and ongoing nature of business financing, instead of relying on information asymmetries in a single period framework. The model does not require that lenders be mislead. Rather it assumes that both lenders and investors are rational and informed agents facing uncertainty as to the future and moral hazard. Instead of attempting to solve the moral hazard problem with implausible or economically inefficient ex ante contracts, we show that rational lenders can price their loans so as to create incentives for developers to

[18] Because the safe and risky projects of similar in size and leverage the range of surpluses over which the pooling, secured, and senior junior allow the developer's first best to be obtained, is quite narrow ($0.048, $0.052, and $0.056 respectively).

behave in a predictable way in the future, thus circumventing the moral hazard problem inherent in sequential financing of projects. This solution comes at a cost of possibly leading to forgoing potentially economically sensible projects, depending on how debt contracts are structured.

We find that the bankruptcy remote debt structure permits the investor to invest in any economically feasible project, because that structure is free of moral hazard that underlies the strategic loan pricing that constrains future investor choices in other structures. However, the bankruptcy remote structure requires strong conditions of asset separability, freedom from asset substitution, and legally enforceable separation of assets and creditors. We observe these conditions where we observe bankruptcy remote structures being used, for instance in securitizations or commercial real estate. But these conditions do not normally obtain.

With small or no surpluses the developer either gets priced out of making future investments or must forgo the competitive banking market and return to his first lender to obtain additional financing thus effectively renegotiating his existing loans with each additional project. This creates market power for the first lender. With sufficiently large surpluses and an appropriate choice of debt structure the investor can escape this dilemma.

Our model has abstracted from a potentially important consideration that may further illuminate the choice of debt structure. We have assumed that the investor can choose his projects at will. If the future project choices are uncertain, the investor may not undertake first projects on terms that compel him to do a specific second project in order to break even.[19] This uncertainty about second projects would also result in a different strategy for the first lender as they could no longer rely on the investor forgoing expropriating the lender in order to make a higher return.

While this would alter the details and specific parameter-dependent outcomes of the game it would not fundamentally alter its structure and usefulness. A model of bank lending and loan pricing needs to take into account the potential moral hazard problem, and recognize that rational economic agents anticipate future actions, rather than just looking at the current information set, thus setting up a sequential game in

[19] This would not be an issue under the bankruptcy remote structure.

which decisions are made. This has potential adverse consequences, but the structure in which lending and investment take place can mitigate this problem.

Table 1: Expected net present values for Bank-1 and developer given **no surpluses**, for various interest rates and Project-1 and Project-2 choices.

Table 1a: **Pooling** debt structures

Developer Chooses P_1 =	**$Bank_1$ Anticipates P_2 =**	**Sets I_1 =**	**Developer Does $S_1 \cap \Phi$**	**$S_1 \cap S_2$**	**$S_1 \cap R_2$**
S_1	**Φ**	5.00%	\$0.000,\$0.000	\$0.007,-\$0.007	-\$0.007,\$0.007
	S_2	4.17%	-\$0.005,\$0.005	\$0.000,\$0.000	-\$0.012,\$0.013
	R_2	5.94%	\$0.006,-\$0.006	\$0.014,-\$0.014	\$0.000,\$0.000
			$R_1 \cap \Phi$	**$R_1 \cap S_2$**	**$R_1 \cap R_2$**
R_1	**Φ**	14.38%	\$0.000,\$0.000	\$0.059,-\$0.059	\$0.050,-\$0.050
	S_2	5.94%	-\$0.054,\$0.054	\$0.000,\$0.000	-\$0.002,\$0.002
	R_2	6.25%	-\$0.052,\$0.052	\$0.002,-\$0.002	\$0.000,\$0.000

Table 1b: **Senior/junior** debt structures

Developer Chooses P_1 =	**$Bank_1$ Anticipates P_2 =**	**Sets I_1 =**	**Developer Does $S_1 \cap \Phi$**	**$S_1 \cap S_2$**	**$S_1 \cap R_2$**
S_1	**Φ**	5.00%	\$0.000,\$0.000	\$0.008,-\$0.008	\$0.008,-\$0.008
	S_2	4.00%	-\$0.006,\$0.006	\$0.000,\$0.000	\$0.000,\$0.000
	R_2	4.00%	-\$0.006,\$0.006	\$0.000,\$0.000	\$0.000,\$0.000
			$R_1 \cap \Phi$	**$R_1 \cap S_2$**	**$R_1 \cap R_2$**
R_1	**Φ**	14.38%	\$0.000,\$0.000	\$0.083,-\$0.083	\$0.083,-\$0.083
	S_2	4.00%	-\$0.066,\$0.066	\$0.000,\$0.000	\$0.000,\$0.000
	R_2	4.00%	-\$0.066,\$0.066	\$0.000,\$0.000	\$0.000,\$0.000

Table 1c: **Secured** debt structures

Developer Chooses P_1 =	**$Bank_1$ Anticipates P_2 =**	**Sets I_1 =**	**Developer Does $S_1 \cap \Phi$**	**$S_1 \cap S_2$**	**$S_1 \cap R_2$**
S_1	**Φ**	5.00%	\$0.000,\$0.000	\$0.008,-\$0.008	\$0.007,-\$0.007
	S_2	4.01%	-\$0.006,\$0.006	\$0.000,\$0.000	\$0.000,\$0.000
	R_2	4.07%	-\$0.006,\$0.006	\$0.000,-\$0.001	\$0.000,\$0.000
			$R_1 \cap \Phi$	**$R_1 \cap S_2$**	**$R_1 \cap R_2$**
R_1	**Φ**	14.38%	\$0.000,\$0.000	\$0.067,-\$0.067	\$0.066,-\$0.066
	S_2	5.11%	-\$0.059,\$0.059	\$0.000,\$0.000	\$0.000,\$0.000
	R_2	5.14%	-\$0.059,\$0.059	\$0.000,\$0.000	\$0.000,\$0.000

Table 1d: **Bankruptcy remote** debt structures

Developer Chooses P_1 =	**$Bank_1$ Anticipates P_2 =**	**Sets I_1 =**	**Developer Does $S_1 \cap \Phi$**	**$S_1 \cap S_2$**	**$S_1 \cap R_2$**
S_1	**Φ**	5.00%	\$0.000,\$0.000	\$0.000,\$0.000	\$0.000,\$0.000
	S_2	5.00%	\$0.000,\$0.000	\$0.000,\$0.000	\$0.000,\$0.000
	R_2	5.00%	\$0.000,\$0.000	\$0.000,\$0.000	\$0.000,\$0.000
			$R_1 \cap \Phi$	**$R_1 \cap S_2$**	**$R_1 \cap R_2$**
R_1	**Φ**	14.38%	\$0.000,\$0.000	\$0.000,\$0.000	\$0.000,\$0.000
	S_2	14.38%	\$0.000,\$0.000	\$0.000,\$0.000	\$0.000,\$0.000
	R_2	14.38%	\$0.000,\$0.000	\$0.000,\$0.000	\$0.000,\$0.000

Table 2: Expected net present values for Bank-1 and developer given: risky projects have expected $0.054 **surplus** and for various debt structures, interest rates, and Project-1 and Project-2 choices.

Table 2a: **Pooling** debt structures

Developer Chooses P_1 =	Bank$_1$ Anticipates P_2 =	Bank$_1$ Sets I_1 =	Developer Does		
			$S_1 \cap \Phi$	$S_1 \cap S_2$	$S_1 \cap R_2$
S_1	Φ	5.00%	$0.000,$0.000	$0.007,-$0.007	-$0.001,$0.055
	S_2	4.17%	-$0.005,$0.005	$0.000,$0.000	-$0.007,$0.061
	R_2	5.09%	$0.001,-$0.001	$0.007,-$0.007	$0.000,$0.054
			$R_1 \cap \Phi$	$R_1 \cap S_2$	$R_1 \cap R_2$
R_1	Φ	12.69%	$0.000,$0.054	$0.054,$0.000	$0.056,$0.052
	S_2	5.09%	-$0.049,$0.103	$0.000,$0.054	-$0.003,$0.111
	R_2	5.45%	-$0.046,$0.100	$0.003,$0.052	$0.000,$0.108

Table 2b: **Senior/junior** debt structures

Developer Chooses P_1 =	Bank$_1$ Anticipates P_2 =	Bank$_1$ Sets I_1 =	Developer Does		
			$S_1 \cap \Phi$	$S_1 \cap S_2$	$S_1 \cap R_2$
S_1	Φ	5.00%	$0.000,$0.000	$0.008,-$0.008	$0.008,$0.046
	S_2	4.00%	-$0.006,$0.006	$0.000,$0.000	$0.000,$0.054
	R_2	4.00%	-$0.006,$0.006	$0.000,$0.000	$0.000,$0.054
			$R_1 \cap \Phi$	$R_1 \cap S_2$	$R_1 \cap R_2$
R_1	Φ	12.69%	$0.000,$0.054	$0.070,-$0.015	$0.070,$0.039
	S_2	4.00%	-$0.056,$0.110	$0.000,$0.054	$0.000,$0.108
	R_2	4.00%	-$0.056,$0.110	$0.000,$0.054	$0.000,$0.108

Table 2c: **Secured** debt structures

Developer Chooses P_1 =	Bank$_1$ Anticipates P_2 =	Bank$_1$ Sets I_1 =	Developer Does		
			$S_1 \cap \Phi$	$S_1 \cap S_2$	$S_1 \cap R_2$
S_1	Φ	5.00%	$0.000,$0.000	$0.008,-$0.008	$0.007,$0.047
	S_2	4.01%	-$0.006,$0.006	$0.000,$0.000	$0.000,$0.054
	R_2	4.06%	-$0.006,$0.006	$0.000,$0.000	$0.000,$0.054
			$R_1 \cap \Phi$	$R_1 \cap S_2$	$R_1 \cap R_2$
R_1	Φ	12.69%	$0.000,$0.054	$0.060,-$0.006	$0.062,$0.046
	S_2	4.55%	-$0.052,$0.106	$0.000,$0.054	-$0.001,$0.109
	R_2	4.73%	-$0.051,$0.105	$0.001,$0.053	$0.000,$0.108

Table 2d: **Bankruptcy remote** debt structures

Developer Chooses P_1 =	Bank$_1$ Anticipates P_2 =	Bank$_1$ Sets I_1 =	Developer Does		
			$S_1 \cap \Phi$	$S_1 \cap S_2$	$S_1 \cap R_2$
S_1	Φ	5.00%	$0.000,$0.000	$0.000,$0.000	$0.000,$0.054
	S_2	5.00%	$0.000,$0.000	$0.000,$0.000	$0.000,$0.054
	R_2	5.00%	$0.000,$0.000	$0.000,$0.000	$0.000,$0.054
			$R_1 \cap \Phi$	$R_1 \cap S_2$	$R_1 \cap R_2$
R_1	Φ	12.69%	$0.000,$0.054	$0.000,$0.054	$0.000,$0.108
	S_2	12.69%	$0.000,$0.054	$0.000,$0.054	$0.000,$0.108
	R_2	12.69%	$0.000,$0.054	$0.000,$0.054	$0.000,$0.108

Table 3: Expected net present values for Bank-1 and developer given: risk projects have expected \$0.06 **surplus** and for various debt structures, interest rates, and Project-1 and Project-2 choices.

Table 3a: **Pooling** debt structures

Developer Chooses P_1 =	$Bank_1$ Anticipates P_2 =	$Bank_1$ Sets I_1 =	Developer Does $S_1 \cap \Phi$	Developer Does $S_1 \cap S_2$	Developer Does $S_1 \cap R_2$
S_1	Φ	5.00%	\$0.000,\$0.000	\$0.007,-\$0.007	\$0.000,\$0.060
	S_2	4.17%	-\$0.005,\$0.005	\$0.000,\$0.000	-\$0.006,\$0.066
	R_2	5.00%	\$0.000,\$0.000	\$0.007,-\$0.007	\$0.000,\$0.060
			$R_1 \cap \Phi$	$R_1 \cap S_2$	$R_1 \cap R_2$
R_1	Φ	12.50%	\$0.000,\$0.060	\$0.053,\$0.007	\$0.055,\$0.065
	S_2	5.00%	-\$0.048,\$0.108	\$0.000,\$0.060	-\$0.003,\$0.123
	R_2	5.42%	-\$0.045,\$0.105	\$0.003,\$0.057	\$0.000,\$0.120

Table 3b: **Senior/junior** debt structures

Developer Chooses P_1 =	$Bank_1$ Anticipates P_2 =	$Bank_1$ Sets I_1 =	Developer Does $S_1 \cap \Phi$	Developer Does $S_1 \cap S_2$	Developer Does $S_1 \cap R_2$
S_1	Φ	5.00%	\$0.000,\$0.000	\$0.008,-\$0.008	\$0.008,\$0.052
	S_2	4.00%	-\$0.006,\$0.006	\$0.000,\$0.000	\$0.000,\$0.060
	R_2	4.00%	-\$0.006,\$0.006	\$0.000,\$0.000	\$0.000,\$0.060
			$R_1 \cap \Phi$	$R_1 \cap S_2$	$R_1 \cap R_2$
R_1	Φ	12.50%	\$0.000,\$0.060	\$0.068,-\$0.008	\$0.068,\$0.052
	S_2	4.00%	-\$0.054,\$0.114	\$0.000,\$0.060	\$0.000,\$0.120
	R_2	4.00%	-\$0.054,\$0.114	\$0.000,\$0.060	\$0.000,\$0.120

Table 3c: **Secured** debt structures

Developer Chooses P_1 =	$Bank_1$ Anticipates P_2 =	$Bank_1$ Sets I_1 =	Developer Does $S_1 \cap \Phi$	Developer Does $S_1 \cap S_2$	Developer Does $S_1 \cap R_2$
S_1	Φ	5.00%	\$0.000,\$0.000	\$0.008,-\$0.008	\$0.007,\$0.053
	S_2	4.01%	-\$0.006,\$0.006	\$0.000,\$0.000	\$0.000,\$0.060
	R_2	4.06%	-\$0.006,\$0.006	\$0.000,\$0.000	\$0.000,\$0.060
			$R_1 \cap \Phi$	$R_1 \cap S_2$	$R_1 \cap R_2$
R_1	Φ	12.50%	\$0.000,\$0.060	\$0.059,\$0.001	\$0.061,\$0.059
	S_2	4.49%	-\$0.051,\$0.111	\$0.000,\$0.060	-\$0.002,\$0.122
	R_2	4.70%	-\$0.050,\$0.110	\$0.002,\$0.058	\$0.000,\$0.120

Table 3d: B**ankruptcy remote** debt structures

Developer Chooses P_1 =	$Bank_1$ Anticipates P_2 =	$Bank_1$ Sets I_1 =	Developer Does $S_1 \cap \Phi$	Developer Does $S_1 \cap S_2$	Developer Does $S_1 \cap R_2$
S_1	Φ	5.00%	\$0.000,\$0.000	\$0.000,\$0.000	\$0.000,\$0.060
	S_2	5.00%	\$0.000,\$0.000	\$0.000,\$0.000	\$0.000,\$0.060
	R_2	5.00%	\$0.000,\$0.000	\$0.000,\$0.000	\$0.000,\$0.060
			$R_1 \cap \Phi$	$R_1 \cap S_2$	$R_1 \cap R_2$
R_1	Φ	12.50%	\$0.000,\$0.060	\$0.000,\$0.060	\$0.000,\$0.120
	S_2	12.50%	\$0.000,\$0.060	\$0.000,\$0.060	\$0.000,\$0.120
	R_2	12.50%	\$0.000,\$0.060	\$0.000,\$0.060	\$0.000,\$0.120

Appendix A

Analytical Model

We begin our analysis by solving theoretically for a simple case. This is sufficient to demonstrate on major hypothesis that sequential financing of overlapping projects sets up a moral hazard problem and that the bank's solution to the problem can restrict the developer's subsequent investment decisions. Following that we explore through numerical simulations how these effects depend on the structure of the debt contract the risk of the projects and the size of any surpluses.

For our analytic analysis we further assume:

1. Both project types have expected returns on assets equal to the risk-free rate: $E[ROA_R] = E[ROA_S] = r_f$.
2. Each project is assumed to cost $C_j = \$1,\ j \in \{S, R\}$.
3. Loan to value ratios for both project types are identical: $k_S = k_R = k$.
4. The safe project is risk-less, that is $V_S^H = V_S^L = (1 + r_f) \equiv R$.
5. The payoffs V_S^H and V_S^L, to the risky project are such that

$$R + V_R^L < 2kR.^{20} \tag{1}$$

6. The debt structure is pooling.

We start by considering the case when the first project is safe (S_1).

If the first bank ***hypothesizes*** that there will be no second project, (i.e., $\tilde{\Phi}_2$), or if the first bank hypothesizes that the second project is to be riskless (i.e., $\tilde{S}_2$), the corresponding interest rate on the first loan would be

[20] This ensures the combination of a riskless and a risky project under a pooled structure would be risky. It also implies that a loan to a single risky project would be risky. Combined with assumption 1 this also implies that $V_R^H > k\, I^R$, so that it is possible to make a loan of amount k to the risky project with an expected return equal to the risk free rate, at rate I_R, so that $k R = \pi^H k\, I^R + \left(1 - \pi^H\right) V_R^L$.

$$I^1_{S_1 \cap \tilde{\Phi}_2} = I^1_{S_1 \cap \tilde{S}_2} = R. \qquad \textbf{(2)}$$

If the first bank hypothesizes that the second project is to be risky (i.e., $\tilde{R}_2$), the corresponding first loan interest rate would be:

$$I^1_{S_1 \cap \tilde{R}_2} = \frac{1}{\pi^H_R}\left[R - (1-\pi^H_R)\frac{R+V^L_R}{2k}\right] > I^1_{S_1 \cap \tilde{\Phi}_2}.$$

Moral Hazard

In this model the developer's objective is to choose the sequence of projects that maximizes the expected net present value of his equity investments. Because the interest rate on the first loan depends upon the creditor's anticipations as to the nature of the second project, the expected net present value of equity for the sequence of projects will also depend upon the first creditor's anticipation as to the choice of second project. First we consider the cases where the loans used to finance the investments are priced in accord with the investor's choice of projects (i.e., anticipation equals choice). For example, a developer whose first project is S decides to undertake a risky second project. Here the expected net present value of their equity investment is:

$$E[NPV\,equity \mid S_1 \cap R_2) = \frac{\pi^H_R[V^H_R + R - kI^c_1 - kI^c_2]}{R} - 2(1-k) \qquad (3)$$

When the bank anticipates the investor will do S_1 and R_2, and the developer in fact wants to do S_1 and R_2, then $I^c_1 = I^c_2 = I^c{}_{S_1 \cap \widetilde{R_2}}$. Substituting for I^c_1 and I^c_2 in equation (3) and we find that the expected net present value will be:

$$E[NPV\,equity \mid S_1 \cap R_2) = \frac{\pi^h_R[(V^h_R - I^c{}_{S_1 \cap R_2}k) + (R^f - I^c{}_{S_1 \cap R_2}k)]}{R^f} - 2(1-k) = 0 \qquad (4)$$

To illustrate the moral hazard issue, consider the situation if the bank prices the loan for the first project as if both projects will be safe (i.e., $I^C_1 = R^f$). If the developer then decides to do R_2 rather than S_2 Bank 2 will finance the second project with an interest rate $I^c_2 < I^c_{S \cap \tilde{R}}$ and it follows that:

$$E[NPV\ equity \mid S_1 \cap R_2] = \frac{\pi_R^h[(V_R^h - I_2^c k) + (R^f - R_1^f k)]}{R^f} - 2(1-k)$$

and (5)

$$E[NPV\ equity \mid S_1 \cap R_2] > 0$$

Now the institution that financed the first project will have an expected rate of return on the loan of:

$$\frac{E[D_1 \mid S_1 \cap R_2]}{k} = \frac{\pi_R^h R^f k + (1-\pi_R^h)\left(\dfrac{R^f + V_R^l}{2}\right)}{k} < R^f \quad (6)$$

What has happened is that the financial institution that provided the loan to finance the second project, priced it so that the bank would earn the risk-free rate on the second loan, given the interest rate charged on the first loan ($I_1^C = R^f$) this in turn results in a transfer of wealth from the creditor for the first project to the developer. To ameliorate this situation the first bank must charge an interest rate on the first loan greater than $I_1^c = I^c{}_{S_1 \cap \widetilde{S}_2}$. The problem associated with charging an interest rate on the first loan of $I_1^c > I^c{}_{S_1 \cap \widetilde{S}_2} > R^f$ is that it makes it economically unprofitable to undertake a safe second project.

In summary we considered investment projects both of which would be undertaken in isolation (i.e., with non-recourse or bankruptcy remote, financing). A developer is considering a sequence of two of these project both of which will be financed with loans obtained from a competitive banking industry.[21] We conclude that with pooling loan contracts, the first lender's response to the potential moral hazard problem is to price their loan so that the developer's only non-negative expected present value choice is to do a risky second project. Absent surplus the second project in a sequence of project will never be a safe project. This contrasts with the situation where loans are non-recourse. Here the price of a loan is independent of future project selections and any investment with non-negative net present value, in isolation, will be undertaken. If we consider the possibility of doing more than two projects we find that the moral hazard issues will limit the investor who starts with a safe project to two projects, specifically a safe and risky project. In Appendix A we consider the case when the develops' first project is risky, here we

[21] The projects do not have a surplus.

show that the moral hazard problem forces the bank to price the first loan so that it is only optimal for the developer to do one risky project. The model can be extended to consider the possibility of the investor undertaking additional projects, here we find that pooling limits the number of projects undertaken.

We next investigate the optimal behavior or investors, and banks when the investor has decided that the first project will be risky. We start by considering the case where the financial institution knows the first project will be risky and it anticipates that the investor will ***not*** undertake a second project. Next we consider interest rate determination when the first bank knows that the first project will be risky and anticipates that a second risky project will also be undertaken. We then conclude that the first bank will charge an interest rate equal to the one project risky interest rate. At any rate less than this the investor will only do one project, but wealth will be transferred from the creditor to the developer (i.e., the expected net present value of the investments will be positive). Thus even when the investor can do two risky projects only one will be undertaken and a competitive bank will charge the corresponding interest rate.

Next we consider the case where the investor solicits a loan for a risky project and the financial institution ***anticipates*** that the investor will not do a second project.

Then the expected payment to the creditor is

$$\begin{aligned} E[D_1 \mid R_1 \cap \widetilde{\varnothing_2}] &= \pi_R^h Min(I^R_{R_1 \cap \widetilde{\varnothing_2}} k, V_R^h) + \pi_R^l Min(I^R_{R_1 \cap \widetilde{\varnothing_2}} k, V_R^l) \\ &= \pi_R^h I^R_{R_1 \cap \widetilde{\varnothing_2}} k + \pi_R^l V_R^l \end{aligned}$$

Competitive loan pricing implies

$$\frac{E[D_1 \mid R_1 \cap \widetilde{\varnothing_2}]}{k} = R^f \Rightarrow I^R_{R_1 \cap \widetilde{\varnothing_2}} = \frac{R^f}{\pi_R^h} - \frac{\pi_R^l V_R^l}{k \pi_R^h}. \tag{7}$$

When the first bank anticipates the investor will not undertake a second project and realizations are the same as expectations, the expected net present value of the developer's investment is:

$$E[NPV\ equity \mid R_1 \cap \varnothing) = \frac{\pi_R^h[(V_R^h - I^c_{R_1 \cap \widetilde{\varnothing}_2} k)]}{R^f} - 2(1-k) = 0 \qquad \textbf{(8)}$$

From this relationship we see that a contracted rate less than $I^c_{R_1 \cap \widetilde{\varnothing}_2}$ will result in a positive E[NPV] when only one risky project is undertaken.

Two Risky Projects

We now consider the situation where an investor chooses a risky first project and the bank anticipates that he will also choose a risky second project. Here the expected payment on the first loan is:

$$E[D_1 \mid R_1 \cap \widetilde{R_2}] =$$
$$= (\pi_R^h)^2 Min(I_1^R k(1), \frac{V_R^h + V_R^h}{2}) + 2\pi_R^h \pi_R^l Min(I_1^R k(1), \frac{V_R^h + V_R^l}{2}) + (\pi_R^l)^2 Min(I_1^R k(1), \frac{V_R^l + V_R^l}{2})$$

Valuation of the first and last term of the above expression follows from the underlying assumptions about the nature of the investments and we have:

$$E[D_1 \mid R_1 \cap \widetilde{R_2}] = (\pi_R^h)^2 I_1^R k(1) + 2\pi_R^h \pi_R^l Min(I_1^R k(1), \frac{V_R^h + V_R^l}{2}) + (\pi_R^l)^2 (V_R^l)$$

The above expression can be simplified based upon the fact that the risky investment has zero surplus, i.e., $\pi_R^h V_R^h + \pi_R^l V_R^l = R^f$ and we have

$$E[D_1 \mid R_1 \cap \widetilde{R_2}] = (\pi_R^h)^2 I_1^R k + 2\pi_R^h \pi_R^l Min(I_1^R k, \frac{R^f + V^h 2\pi_R^l - 1}{2\pi_R^l}) + (\pi_R^l)^2 (V_R^l)$$

The value of the above expression is determined by which is smaller $I_1^R k$ or $\frac{V_R^h + V_R^l}{2}$, which in turn is determined by the characteristics of the risky investment and the loan to value ratio (k).[22] What we do is derive the contracted interest rate in both cases and show that in both cases it is optimal for the first bank to price the first loan in such a way that the investor undertakes only one risky project. First we consider the case where $I_1^R k \le \frac{V_R^h + V_R^l}{2}$ here the expected rate of return on debt is, given competitive markets, is:

---

[22] Numerical investigation of the problem indicate that $I_1^R k > \frac{V_R^h + V_R^l}{2}$ only for very high LTVs (e.g., $k > .95$).

$$\frac{E[D_1 \,|\, R_1 \cap \widetilde{R_2}]}{k} = \frac{(\pi_R^h)^2 I_1^R k + 2\pi_R^h \pi_R^l I_1^R k + (\pi_R^l)^2 (V_R^l)}{k} = R^f$$, which in turn

implies the contracted interest rate on the first loan will equal:

$$\begin{aligned} I^R_{R_1 \cap \widetilde{R_2}} &= \frac{R^f}{[(\pi_R^h)^2 + 2\pi_R^l \pi_R^h]} - \frac{(\pi_R^l)^2 V_R^l}{k[(\pi_R^h)^2 + 2\pi_R^l \pi_R^h]} \\ &= \frac{1}{(2-\pi_R^h)} \left[\frac{R^f}{\pi_R^h} - \frac{(1-\pi_R^h)^2 V_R^l}{k\pi_R^h} \right] \end{aligned} \tag{9}$$

Thus we find that the when the bank anticipate the second project will be risky the contracted interest rate will be less than the rate when no second project is anticipate, that is

$$I^R_{R_1 \cap \widetilde{\varnothing_2}} = \frac{R^f}{\pi_R^h} - \frac{\pi_R^l V_R^l}{k\pi_R^h} > I^R_{R_1 \cap \widetilde{R_2}} = \frac{1}{(2-\pi_R^h)} \left[\frac{R^f}{\pi_R^h} - \frac{(1-\pi_R^h)^2 V_R^l}{k\pi_R^h} \right]$$

When the first bank anticipates the investor undertaking two risky projects and the investor actually undertakes two risky projects then $I_1^c = I_2^c = I^c_{R_1 \cap \widetilde{R_2}}$. Here the investor's net present value of their equity investment is investments will be:

$$\begin{aligned} E[NPV\, equity \,|\, R_1 \cap R_2) &= \frac{(\pi_R^h)^2 [2(V_R^h - I^c_{R_1 \cap \widetilde{R_2}} k)] + 2\pi_R^h \pi_R^l \left[(V_R^h - I^c_{R_1 \cap \widetilde{R_2}} k) + (V_R^l - I^c_{R_1 \cap \widetilde{R_2}} k) \right]}{R^f} - 2(1-k) \\ &= \frac{(\pi_R^h)^2 [2(V_R^h - I^c_{R_1 \cap \widetilde{R_2}} k)] + 2\pi_R^h \pi_R^l \left[(V_R^h + V_R^l) - 2 I^c_{R_1 \cap \widetilde{R_2}} k \right]}{R^f} - 2(1-k) \end{aligned} \tag{10}$$

Substituting (9) for $I^c_{R_1 \cap \widetilde{R_2}}$ in equation (10) we find that the expected net present value of the developer's investments will be zero when the first loan is priced in anticipation that the second project will be risky and the second loan is priced give knowledge that both projects are risky.

Next we consider the case where $I_1^R k > \frac{V_R^h + V_R^l}{2}$ here the expected rate of return on debt is, given competitive markets is:

$$\frac{E[D_1 \mid R_1 \cap \widetilde{R_2}]}{k} = \frac{(\pi_R^h)^2 I_1^R k + 2\pi_R^h \pi_R^l \frac{V_R^h + V_R^l}{2} + (\pi_R^l)^2 (V_R^l)}{k} = R^f ,$$

which in turn

$$I_1^R = \frac{R^f}{(\pi_R^h)^2} - \frac{\pi_R^l (V_R^l + V_R^h)}{k\pi_R^h} - \frac{(\pi_R^l)^2 V_R^l}{k(\pi_R^h)^2}$$

we know that $\pi_R^l = 1 - \pi_R^l$ and it follows that

$$I_1^R = \frac{R^f}{(\pi_R^h)^2} - \frac{1}{k\pi_R^h}\left[\pi_R^l (V_R^l + V_R^h) + \frac{(\pi_R^l)^2 V_R^l}{\pi_R^h}\right]$$

To understand the investor's insensitive we again compare the competitive interest rate when the first bank anticipates no second project with the rate when the bank anticipates the developer will undertake a risky second project (i.e., when $I_1^R k > \frac{V_R^h + V_R^l}{2}$):

$$I_{R_1 \cap \widetilde{\varnothing_2}}^R = \frac{R^f}{\pi_R^h} - \frac{\pi_R^l V_R^l}{k\pi_R^h} > I_{R_1 \cap \widetilde{R_2}}^R = \frac{R^f}{(\pi_R^h)^2} - \frac{1}{k\pi_R^h}\left[\pi_R^l (V_R^l + V_R^h) + \frac{(\pi_R^l)^2 V_R^l}{\pi_R^h}\right]$$

In this case as was the first the competitive interest rate when a second risky project is anticipated is below the rate when no second project is anticipated. Again when realizations are the same as expectations the expected net present value of equity investment are zero.

Moral Hazard and Project Choice

We would expect the financial institution that provides the first loan to know what the developer's optimal response to the interest rate charged on its loan. At a contracted interest rate of $I_{R_1 \cap \widetilde{R_2}}^c$ doing two risky projects result in zero expected net value for the investors. In contrast doing a single risky project at the same interest will result in a positive expected net present value for investors which results from a wealth transfer from the creditor to the developer. The conclusion is that it is optimal for the first financial institution to charge a rate of $I^{c}_{R_1 \cap \widetilde{\varnothing}_2}$ and for the developer to do only one risky project.