The Role of Bond Markets When Portfolio Choice is Constrained

—preliminary draft—

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Abstract

The impact of regulatory changes-such as tightening or loosening leverage restrictions on institutional investors-depends materially on the market environment at the time of implementation. Focusing on bond markets in a dynamic equilibrium setting, this paper shows that if regulation is tightened at times when investment opportunities are perceived to be good (thus being constrained is particularly onerous), the constrained investors tilt their portfolio towards few risky assets at the expense of diversification, leading to rising interest rates. If the tightening of regulatory constraints is undertaken when the leverage constraint binds less severely, the opposite occurs: risky substitute investments are less sought after, and the higher demand for safer bonds lowers the interest rate. This finding suggests that the timing of regulatory initiatives plays a crucial role in how constraints spill over into different markets. A related result on bond yield differentials across countries indicates that when investor-specific constraints lead groups of investors to specialize into certain asset markets, cross-country interest rate differentials are more volatile than they would be in the absence of constraints.

Keywords: bond markets, interest rates, leverage constraint, portfolio constraints, international finance
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Introduction

Many countries reacted to the recent financial crisis by establishing unilateral investment restrictions with the intent of supporting asset markets and some institutions. But with internationally connected financial markets, these constraints will also have an impact on the behavior of other markets as investors search for substitute assets. This paper provides a model about how constraints regulating investment into 'risky' assets such as stocks spills over into international bond markets and the relationship between countries' interest rates.

The empirical literature studying constraints (and their liberalization) in international capital markets has focused mainly on the effects on the world's stock markets, as most such restrictions applied to these markets; Investors have traditionally faced few if any restrictions on investing into government bonds across the world.

In this paper I study how such constraints affect government bond markets, and thus interest rates across countries. A constraint on any type of asset across investors will necessarily prevent optimal risk-sharing, and investors must look for alternative investment opportunities. Bond markets become one of the alternative asset classes used by investors to compensate for the constraints they face elsewhere in the capital market. The main results of the paper are as follows. First, it demonstrates that the role of bonds as alternative assets varies with the severity of the binding constraint. If investors are optimistic regarding the investment opportunities in the restricted assets, they will use the bond markets as a source of funds to load up on the desired risk factor via correlated assets that are more accessible. The portfolios become less diversified and interest rates rise. If perceptions about investment opportunities are not very optimistic, this severe tilt in portfolio will not happen, and bonds will rather serve as an alternative asset class to hold, depressing interest rates.

The nature of the constraints — for example whether one is looking at a constraint on a particular asset, e.g. restrictions on non-domestic stock, versus a limit on an investor's leverage — will have a different impact on the world's bond markets, as the set of substitute assets varies.

In this context, the model shows that in the debate about how a 'tightening' of constraints affects asset markets, one needs to clearly distinguish between two notions of tighter constraints: on one hand, a constraint, for example a leverage constraint, can become tighter when the fraction of wealth an investor is allowed to borrow is lowered. On the other hand, the same constraint can be said to become tighter as an investor comes closer to hitting the -unchanged- limit to his leverage, presumably at a time when access to additional funds is of high importance to him. The model shows that while the latter notion of 'tightening' will unambiguously lead to a decrease in real interest rates, the former notion of tightening a constraint has an ambiguous effect on interest rates.

A related second result in on the impact of investment constraints on interest rate differentials across countries. The effects described above lead to larger time variation in interest rate differentials as restrictions bind and investors' portfolios become more tilted, and less similar to one another.

The model is a simplified international model, studying an endowment economy with two countries, each of which produces a unique good and is populated by a single representative investor. The investors both choose to consume goods and invest into assets from either country, but one investor faces constraints on his portfolio choice. I consider in particular two scenarios of constraints that resemble constraints found in assets markets: a leverage constraint, limiting the extent of the investor's short positions in bond markets, as well as a constraint on holdings of the non-domestic risky asset. A key component of the model are the differences in investors' beliefs about growth rates in the two countries. While these differences among investors could potentially reflect imperfect information about future growth of the two economies, they are rather a tractable modeling technique to represent some form of heterogeneity between investors in their preferences for where to invest their money. Thereby, this parameter characterizes the desired investment, allowing me to distinguish in the model between the strictness and the severity of a constraint: how strict is the imposed level of a portfolio constraint, and how severely is the investor affected — is the deviation between his desired portfolio and his permitted portfolio large or small.

For example, when institutions are constrained in e.g. risky arbitrage positions due to capital requirements, there will be a negative correlation between the attractiveness of the investment opportunity and their restriction. In other words, without changing the leverage constraint itself, it may become more binding when a firm has just lost money on an arbitrage trade, but this is precisely when that arbitrage trade becomes more interesting. So the nature of their constraint has not changed, but they suffer more strongly from the same constraint. This relationship can be captured by the interaction of the constraint's stringency and an investor's belief about the attractiveness of the inaccessible investment opportunity.

The paper builds on previous work in the theoretical literature on constraints. Heaton and Lucas (1997) study how uninsurable risks affect portfolio allocation under various utility specifications. In a different setting, Basak (1996) studies the impact of segmentation on risk-free interest rate in a two-period model. There, partial segmentation is characterized by completely restricting one of the investors from any activity in a non-domestic market, rather than a qualified limit, as is the case in this model. Another critical difference to that earlier paper is the structure of bond markets. Basak (1996) sets up a 'world bond' that results in one interest rate, whereas in this paper both countries issue bonds, thus giving the opportunity to analyze interest rate differentials.

Other models of segmentation include Errunza and Losq (1989), He and Modest (1995), Heaton and Lucas (1996), Zapatero (1998), and more recently Pavlova and Rigobon (2008), who study how constraints that apply to a group of regional markets can induce excess comovement among them. Caballero and Krishnamurthy (2001) focus on the impact of collateral constraints during crises in emerging markets.

The empirical literature on the effects of international investment restrictions and the integration of markets has focused largely on international stock markets in recent decades. Edison and Warnock (2003) propose a measure of the intensity of capital controls, and the survey by Stulz (1995) as well as Karolyi and Stulz (2002) provide a good overview of the empirical findings on international integration and asset pricing.

Most recently, Warnock and Warnock (2009) find that international capital flowing into the US government bond markets has significantly lowered Treasury yields. Earlier studies on bond markets and their integration make use of a very different sample period, before broad international integration, e.g. Mishkin (1984), who studies the relationship between nominal interest rates and inflation expectations across countries. Barr and Priestley (2004) find evidence that international bond markets are not fully integrated, with local market risk having significant impact on returns. This is in line with findings of Harvey, Solnik, and Zhou (2002), who show that bond markets are sensitive to the same risk factors as equity markets. Ilmanen (1995) studies predictability in bond returns and finds excess bond returns highly correlated across countries.[1]

I Model

I.A The Economy and Investor Preferences

The pure exchange economy is comprised of two countries, *home* and *foreign*, each of which specializes in the production of one good, $j = h, f$. Each country is populated by one representative investor, $i = H, F$, who derive utility from consumption of both goods. The goods markets are assumed to be frictionless, there are no transportation costs or tariffs, so both investors face the same relative price for the two goods. The production of the two local production processes is given by the dividend processes

$$\begin{aligned} dY_t^h &= \mu_{Y_h} Y_t^h dt + \sigma_{Y_h} Y_t^h dW_{t,\mathsf{h}}, \\ dY_t^f &= \mu_{Y_f} Y_t^f dt + \sigma_{Y_f} Y_t^f dW_{t,\mathsf{f}}. \end{aligned} \tag{1}$$

The parameters μ_{Y_j} and σ_{Y_j} represent the expected economic growth rate of country j and the sensitivity of its output to fundamental shocks.[2] The Brownian Motions $dW_{t,\mathsf{h}}$ and $dW_{t,\mathsf{f}}$ represent the *home* and *foreign* countries' local production shocks, respectively, and are assumed to be uncorrelated. For simplicity, a country's output shocks are assumed to be determined exclusively by the local source of risk, there is no joint output risk.

Utility functions of both investors are separable and additive over the two goods in the economy,

[1]Jorion (1992) studies currency markets, which is related, albeit tenuously, to the model of this paper, where there is no nominal currency.

[2]Time-subscripts on parameters μ and σ above are suppressed, in the interest of parsimony of notation. As a special case, the production processes can be assumed to follow geometric Brownian motions, but the model goes through as long as the parameters are assumed to be adapted processes.

but investors differ in their consumption preferences: the *home* investor H has a preference for his local good Y^h, while the *foreign* investor F tilts his consumption toward his own local good Y^f.

$$u_H\left(C^{\mathsf{h}}_{Ht}, C^{\mathsf{f}}_{Ht}\right) = \alpha^H_t \log C^{\mathsf{h}}_{Ht} + (1-\alpha^H_t)\log C^{\mathsf{f}}_{Ht}, \tag{2}$$

$$u_F\left(C^{\mathsf{h}}_{Ft}, C^{\mathsf{f}}_{Ft}\right) = (1-\alpha^F)\log C^{\mathsf{h}}_{Ft} + \alpha^F \log C^{\mathsf{f}}_{Ft}. \tag{3}$$

Investor i maximizes expected utility $E\left[\int_0^T u_i\left(C^{\mathsf{h}}_{it}, C^{\mathsf{f}}_{it}\right)dt\right]$, subject to his budget constraint and potentially binding additional allocation constraints.

The preference parameters α^H_t and α^F are assumed to be greater than 0.5, implying the home bias in consumption. Orthogonal to production shocks to the economy are demand shocks driven by time variation in α^H_t, the extent of the *home* investor's preference for his local good. α^H_t follows a martingale, uncorrelated with production shocks:

$$d\alpha^H_t = \sigma_{t,\alpha} dW_{t,\alpha}. \tag{4}$$

To ensure that α^H_t remains larger than 0.5, $\sigma_{t,\alpha}$ will necessarily vary over time, taking on an appropriate form.[3] This approach to modeling demand shocks is consistent with Dornbusch, Fischer, and Samuelson (1977), who noted the importance of allowing for demand shifts in an international model of multiple-good economies. A home bias in consumption patterns is empirically persistent. Usually linked to reasons of inherent non-tradability of certain goods (notably services) and familiarity, in this model it is taken as exogenously given.

I.B Financial Markets

The capital markets are not segmented, and both investors can invest into the two stock markets, as well as into both countries' bonds. The risky claim to the future output of *home* good Y^h, stock S^h_t, will be referred to as the *home stock*. Within the model, the place of listing is immaterial — there are

[3] This form of demand shocks has been used previously in related models, e.g. Pavlova and Rigobon (2008) and Schornick (2009). One example of an admissible process is $\alpha^H_t = E[\alpha^H_T|\mathcal{F}_t]$, where the terminal value of the preference parameter is a fixed number between 0.5 and 1.

no differential transaction costs of trading stocks for the two agents. The 'geography' of the stocks is determined simply by the good they are a claim to. Accordingly, S_t^f is the stock to *foreign* good Y^f. The two stocks follow the dynamics

$$dS_t^h = \mu_t^{S_h} S_t^h dt + \vec{\sigma}_t^{S_h} S_t^h d\vec{W}_t, \tag{5}$$

$$dS_t^f = \mu_t^{S_f} S_t^f dt + \vec{\sigma}_t^{S_f} S_t^f d\vec{W}_t, \tag{6}$$

where expected return and volatility parameters $\mu_t^{S_j}$ and $\vec{\sigma}_t^{S_j}$ are determined in equilibrium. $\vec{\sigma}_t^{S_j}$ is a three-dimensional vector of stock S_t^j's sensitivities with respect to the mutually uncorrelated supply and demand shocks $d\vec{W}_t = \left(dW_{t,\mathsf{h}}, dW_{t,\mathsf{f}}, dW_{t,\alpha}\right)^\top$.

Zero net supply bonds are traded in both countries, creating 'locally' riskless assets. A country's bond effectively provides a forward contract on one unit of future local production — similar to the nature of sovereign bonds in reality, that represent borrowing against future economic output:

$$db_t^h = r_t^h b_t^h dt \qquad \text{in terms of good } Y_t^h, \tag{7}$$

$$db_t^f = r_t^f b_t^f dt \qquad \text{in terms of good } Y_t^f. \tag{8}$$

While default is ruled out here, even the guaranteed provision of the good has a risky payoff in terms of the value at the time. In a single-good economy the bond is instantaneously riskless, as the consumption good is the numeraire – whose relative price does not vary. In an economy with multiple goods (and heterogeneous preferences), the notion of a riskless bond depends on the choice of numeraire, as the equilibrium relative price of the two goods, $\bar{p}_t = p_t^f / p_t^h$ varies over time. Equilibrium goods prices p_t^j are determined by supply of and demand for each of the goods j at time t.[4] The time variation in equilibrium goods prices imposes a real exchange rate risk on the bonds. Taking this exchange rate risk into account, bond prices follow $dB_t^j = d(p_t^j b_t^j)$. As such, countries' bond yields in this multiple good economy will reflect the state of the local economy in relation to the world economy, even in absence of any default risk. Without loss of generality, I

[4]This price is the same for both investors, as goods markets are assumed to function without trading frictions.

set the *foreign* good to be the numeraire good, rendering $p_t^f = 1$ and thus B_t^f the instantaneously riskless asset.

I.C Constraints on Portfolio Choice

Investors' consumption and investment decision must satisfy their budget constraint

$$dX_t^i = X_t^i \left[\sum_{j=h}^{f} \pi_{it}^{S_j} (dS_t^j + p_t^j Y_t^j dt)/S_t^j + \sum_{j=h}^{f} \pi_{it}^{B_j} dB_t^j / B_t^j \right] - \sum_{j=h}^{f} p_t^j C_{it}^j dt \qquad \text{for } i = H, F \tag{9}$$

where $X_t^i \geq 0,$ is agent i's wealth and $\pi_{it}^{S_j}$ is the fraction of his wealth investor i chooses to invest in stock S_t^j, and $\pi_{it}^{B_j}$ the fraction invested into bond B_t^j.

This paper considers in particular how two types of investment restrictions on π_{it} affect return differentials in international bond markets: a leverage constraint and a limit on investors' position in non-domestic stock markets.

A number of large investors such as pension (and some) mutual funds face explicit restrictions on their investments — in terms of geographical regions or asset classes. Such restricitons within the industry have remained in place even as government-imposed investment barriers have been rescindend over the past decades. Leverage constraints are largely the result of the difficulties in contingent contracting: bankruptcy costs and agency costs make credit risk problematic, leading to explicit as well as implicit (through for example margin constraints) limitations on leverage even for large international investors. In the aftermath of the 2007-2008 financial crisis and its repercussions around the world, the debate about such regulation — and whether it must be in- or decreased — was reignited. While some countries have advocated limitations on cross-country investments to limit 'contagion', others have called for a broad-based cap on risk-taking by limiting leverage. This paper analyses how, in a world with otherwise intergrated asset markets, such constraints that generally focus on what are deemed 'risky' assets like stocks will affect international bond market behavior and interest rate spreads.

In the first scenario considered, investor F, domiciled in the *foreign* country, faces a constraint on the fraction of his portfolio invested into risky assets abroad, S_t^h. In the second scenario, the same investor instead faces the leverage constraint. He can borrow through the bond markets, but to a limited degree: his positions in risky assets overall cannot exceed a proportion $\eta > 1$ of his total wealth. In the remainder of the paper, the two scenarios are denoted as scenario **ND** — where F is constrained in the investment into his **N**on-**D**omestic Stock S_t^h — and scenario **LC** — where he faces the **L**everage **C**onstraint. The constraints on portfolio choice can be expressed as

$$\text{scenario } \mathbf{ND}: \quad \mathtt{I}_{ND}^\top \pi_{F,t} \leqslant \varphi, \tag{10}$$

$$\text{scenario } \mathbf{LC}: \quad \mathtt{I}_{LC}^\top \pi_{F,t} \leqslant \eta \tag{11}$$

where $\mathtt{I}_{ND} = [1,0,0]^\top$ and $\mathtt{I}_{LC} = [1,1,0]^\top$. $\pi_{i,t} = [\pi_{i,t}^{S_h}, \pi_{i,t}^{S_f}, \pi_{i,t}^{B_h}]^\top$ captures the portfolio decisions of investor i.[5]

Binding constraints prevent optimal risk sharing through trading, thereby changing the pricing of all assets. The constraints prevent that natural supply of liquidity to markets by the investor most willing to take on the risk, therefore other investors must be incentivized to supply this extra liquidity.

I.D Information Structure

The uncertainty in the economy is characterized by the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$. However, investors are not fully informed and potentially agree to disagree on the expected growth rates of the two economies, *home* and *foreign*. Despite observing the same public information, differences in initial prior beliefs about countries' expected growth rates imply a period of continued disagreement even if both investors learn rationally from observing changes in output over time.[6] The

[5] Satisfaction of the budget constraint implies $\pi_{i,t}^{B_f} = 1 - \mathbf{1}^\top \pi_{i,t}$.

[6] The volatility components of economic output is assumed to be perfectly known by investors. Quadratic variation allows them to draw exact inferences about the diffusion terms of dY_t^h and dY_t^f, as well as demand shocks $d\alpha_t^H$.

relation between the two rational investors' beliefs is determined by observational equivalence.

$$\begin{aligned} dY_t^j &= \mu_{Y_j} Y_t^j dt + \sigma_{Y_j} Y_t^j dW_{t,\mathrm{j}} \\ &= m^{(\mathrm{H})}_{Y_j,t} Y_t^j dt + \sigma_{Y_j} Y_t^j dW^{(\mathrm{H})}_{t,\mathrm{j}} \\ &= m^{(\mathrm{F})}_{Y_j,t} Y_t^j dt + \sigma_{Y_j} Y_t^j dW^{(\mathrm{F})}_{t,\mathrm{j}}. \;\text{ for j= h,f} \end{aligned} \tag{12}$$

Investors H and F attribute different portions of the observed output increase dY_t^h to 'expected' growth rates, $m^{(\mathrm{H})}_{Y_h,t}$ and $m^{(\mathrm{F})}_{Y_h,t}$, and their consumption and investment decisions will reflect these perceptions. The relationship between investors' perceptions about the three uncorrelated sources of risk in the economy is given by

$$d\vec{W}_t^{(\mathrm{F})} = d\vec{W}_t^{(\mathrm{H})} - \Delta\vec{m}_{t,Y} dt \qquad \text{where} \qquad \Delta\vec{m}_{t,Y} = \vec{\Sigma}^{-1}(\vec{m}_t^{(\mathrm{F})} - \vec{m}_t^{(\mathrm{H})}) \tag{13}$$

where Σ is the 3×3 diffusion matrix of the economy's fundamental processes, output and demand.

$$\begin{pmatrix} dW^{(\mathrm{F})}_{t,\mathrm{h}} \\ dW^{(\mathrm{F})}_{t,\mathrm{f}} \\ dW^{(\mathrm{F})}_{t,\alpha} \end{pmatrix} = \begin{pmatrix} dW^{(\mathrm{H})}_{t,\mathrm{h}} \\ dW^{(\mathrm{H})}_{t,\mathrm{f}} \\ dW^{(\mathrm{H})}_{t,\alpha} \end{pmatrix} - \begin{pmatrix} \sigma_{Y_h} & 0 & 0 \\ 0 & \sigma_{Y_f} & 0 \\ 0 & 0 & \sigma_{t,\alpha} \end{pmatrix}^{-1} \begin{pmatrix} m^{(\mathrm{F})}_{Y_h,t} - m^{(\mathrm{H})}_{Y_h,t} \\ m^{(\mathrm{F})}_{Y_f,t} - m^{(\mathrm{H})}_{Y_f,t} \\ 0 \end{pmatrix} dt. \tag{14}$$

Because both investors know that demand shocks follow a martingale, the last element of $\Delta\vec{m}_{t,Y}$ is equal to zero. F is the more 'optimistic' of the two investors regarding the growth of *home*'s economy when the first element of this vector is positive, and accordingly more optimistic about domestic, i.e. *foreign* growth when the second element is positive. A home bias in beliefs about investment opportunities would therefore be reflected by the first element of $\Delta\vec{m}_{t,Y}$ being negative, the second one positive.[7]

The time-subscripts in $m^{(i)}_{Y_h,t}$ and $m^{(i)}_{Y_f,t}$ reflect potential time variation in investors' beliefs through learning. In a typical Bayesian setup, investors would use the information in observed output to

[7] This type of economy is discussed in more detail in various papers like Basak (2005) in a single-good economy, and in Schornick (2009) in a multi-good economy.

update their priors of the economies' growth rates: the precision of an investor's prior on a country's growth rate determines how strongly he adjusts his belief after observing unusually high or low levels of output. Though there has been empirical support for the notion that local investors have more precise priors about the state of their own economy than foreigners, I do not impose a fully paramterized learning model in this paper. This is mainly for reasons of parsimony, as it would introduce a number of additional parameters into the model, for which we have less intuition: for example, about the cross-country correlation of priors, i.e. how a high realization of local output would affect investors' expectation of growth rates abroad. The correlation of priors is not adequately described by looking at correlations of economic fundamental correlations. Different precisions of priors let investors' beliefs diverge in the interim, even though rational updating ultimately leads to beliefs converging in the long run. Particular assumptions regarding learning would affect implications about the behavior of bond markets in the time-series. But testable implications about the cross section, which is what this paper focuses on, do not rely on a particular assumption about *how* investors learn, as long as the process of disagreement $\Delta\vec{m}_t^Y$ remains bounded.

The assumption of differences in beliefs is a technically tractable way to allow for constraints to bind with different degrees of severity — without changing the levels of the restrictions, η and φ. How strict the constraint is and how severely investors find themselves constrained by it are two separate, albeit related, issues. How the constrained investor adjusts his portfolio to compensate for the imposed restriction will depend on his beliefs about the alternative investment opportunities. If the level of restriction remains the same but beliefs change, equilibrium market rates will change, despite the fact that portfolio positions in the restricted asset cannot change.

II Equilibrium

The competitive equilibrium is established by aggregating the two investors into one representative agent — taking into account how the two investors' different consumption preferences and beliefs must be weighted.

$$U(C_H, C_F) = u_H\left(C_{Ht}^h, C_{Ht}^f\right) + \lambda_t u_F\left(C_{Ft}^h, C_{Ft}^f\right), \tag{15}$$

In this constructed representative agent, the state variable λ_t represents the weight of investor F in the market equilibrium and plays a central role in characterizing the market equilibrium. It is the ratio of investors' state price densities, and can be interpreted as the importance a social planner would give to F, or alternatively the impact he has on the competitive equilibrium in consumption and financial markets.

$$\begin{aligned} C^h_{Ft} &= \frac{\lambda_t\left(1-\alpha^F\right)}{\alpha^H_t + \left(1-\alpha^F\right)\lambda_t} Y^h_t = s^F_h Y^h_t; \qquad C^h_{Ht} = (1-s^F_h)Y^h_t \\ C^f_{Ft} &= \frac{\lambda_t \alpha^F}{1-\alpha^H_t + \alpha^F \lambda_t} Y^f_t = s^F_f Y^f_t; \qquad C^f_{Ht} = (1-s^F_f)Y^f_t \end{aligned} \tag{16}$$

The investors' consumption shares, here characterized by s^F_i for good i, are determined by their respective prefences for the goods, and their relative weight in the economy, λ_t. Due to investors' log-utility, investors' savings motive is not affected by the time variation in their investment opportunity set, so λ_t is equal to the ratio of investors' respective wealth levels, X^F_t / X^H_t.

If investors' state prices coincide at all times, λ_t would be constant, reflecting only the distribution of initial endowments as both investors would choose to hold identical portfolios – the market. Differences in beliefs and portfolio constraints contribute to the stochastic properties of λ_t. The former reflect the desire to hold different portfolios according to beliefs. These beliefs can reflect sentiment, differences in (hedgeable) labor income risk, or similar sources of differences between investors. Constraints distort the desired portfolio choice — as the affected investor seeks alternative assets, in a manner that allows him to replicate his desired portfolio as closely as possible.

The constraints as given by (11) prevent the investors from holding portfolio positions that reflect their true beliefs about investment opportunities. Cvitanic and Karatzas (1992) first introduced the methodology used in this paper to incorporate investment constraints on portfolio choice.[8] The optimal adjustment to constrained portfolios, and the resulting effects on equilibrium prices, can be determined by considering an appropriately 'distorted' state price density ξ^F_t that reflects the

[8] Other related papers are e.g. He and Pearson (1991) and Cuoco (1997).

unattainability of certain consumption paths under the imposed constraint.

$$d\xi_t^H = -r_t \xi_t^H dt - \vec{\kappa}_t^{H^\top} \xi_t^H d\vec{W}_t^{(H)}, \tag{17}$$

$$d\xi_t^F = -\left(r_t + \delta(\upsilon_t)\right) \xi_t^F dt - \vec{\kappa}_t^{F^\top} \xi_t^F d\vec{W}_t^{(F)}. \tag{18}$$

where $\vec{\kappa}_t^H = \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(H)} - r_t \mathbf{1}\right)$ is investor H's market price of risk, while $\vec{\kappa}_t^F$ reflects F's constraint in the distortion of his perceived risk-return tradeoff:

$$\vec{\kappa}_t^F = \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right) + \vec{\sigma}_{S,t}^{-1} \upsilon_t^{case} \mathtt{I}_{case} \qquad \text{for } \textit{case}\text{=}\textit{ND,LC}. \tag{19}$$

υ_t^{case} and $\delta(\upsilon_t^{case})$ are scalar parameters that capture the effect of the constraint in *case ND* or *LC* on investor F's investment decisions: the constraint changes the relative attractiveness of all assets, including the riskfree bond. Therefore, his portfolio will be consistent with that of a unconstrained investor that faces a different interest rate than his counterparty, investor H. Under these appropriately augmented parameters, F's portfolio choices are ensured to be admissible under the constraint.

The two possible cases of different constraints are considered in isolation. Investor H always remains unconstrained in his portfolio choice, and thus functions as the marginal investor, even when F is affected by a constraint. This simplifies the assessment of equilibrium prices of stocks and bonds.[9] Some degree of cross-sectional variation in stocks' endogenous behavior within endowment economies is both desirable in terms of resembling real financial markets, as well as critical from a modeling point of view, for market completeness. Investors' distinct preferences over different goods play an important role for this. The fact that Y_t^h and Y_t^f's consumption markets clear separately prevents stock price indeterminacy. If the goods were perfectly substitutable, investors would be indifferent between consuming output from either 'tree', resulting in perfect stock market correlation — effectively just one stock.

[9] A related paper, Schornick (2009) looks at stock market volatilities in the more complicated case where *both* investors face constraints, which somewhat complicates the functional form of equilibrium stock prices.

Proposition 1. *Taking good Y_t^f to be the numeraire, equilibrium stock and bond prices in the home country are functions of the relative price of the local good, $\bar{p}_t = p_t^h / p_t^f = p_t^h/1$.*

$$
\begin{aligned}
S_t^h &= \bar{p}_t Y_t^h (T-t), && (20)\\
B_t^h &= \bar{p}_t b_t^h, && (21)\\
S_t^f &= Y_t^f (T-t), && (22)\\
B_t^f &= b_t^f \qquad \forall t && (23)\\
where \quad \bar{p}_t &= \frac{\alpha_t^H + (1-\alpha^F)\lambda_t}{(1-\alpha_t^H) + \alpha^F \lambda_t} \frac{Y_t^f}{Y_t^h}. && (24)
\end{aligned}
$$

λ_t follows dynamics $d\lambda_t = \lambda_t \delta(\upsilon_t^{case})dt + \lambda_t \Delta\vec{\kappa}_t^\top d\vec{W}_t^{(H)}$, where $\Delta\vec{\kappa}_t^\top = \left[\Delta\kappa_t^h, \Delta\kappa_t^f, \Delta\kappa_t^\alpha\right]$ is the difference in investors' market prices of home, foreign, and demand risk, which depends on the binding of the constraint: $\Delta\vec{\kappa}_t = \Delta\vec{m}_t^Y + \vec{\sigma}_{S,t}^{-1} (\upsilon_t^{case} \mathtt{I}_{case})$. Adjustment terms υ_t^{case} are non-positive in both case ND and LC iff the constraint is binding, and zero otherwise.

case ND: investor F faces a constraint on holdings of S_t^h

$$
\pi_{Ht} = (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \left(\vec{m}_{S,t}^{(H)} - r_t \mathbf{1}\right), \tag{25}
$$

$$
\pi_{Ft} = \begin{cases} (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right) + (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \upsilon_t^{ND} \mathtt{I}_{ND} & \text{if } \upsilon_t^{ND} < 0 \\ (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right) & \text{otherwise} \end{cases} \tag{26}
$$

where $\upsilon_t^{ND} = \min\left(\frac{\varphi - \mathtt{I}_{ND}^\top (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right)}{\mathtt{I}_{ND}^\top (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \mathtt{I}_{ND}}, 0\right)$*, ie.*

$$
\upsilon_t^{ND} = \begin{cases} \frac{-\sigma_{Y^h}\sigma_\alpha^2\left[\Delta m_t^{Y_h} - \left(\varphi(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_{Y^h}\right]}{\left(\varphi\lambda_t(\alpha_t^H + \alpha^F - 1) + (1-\alpha_t^H)(\alpha_t^H + (1-\alpha^F)\lambda_t)\right)^2 \sigma_{Y^h}^2 + \sigma_\alpha^2} & \text{if } \Delta m_t^{Y_h} > \left(\varphi(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_{Y_h}, \\ 0 & \text{otherwise.} \end{cases} \tag{27}
$$

The collateral adjustment in this case is $\delta(\upsilon_t^{ND}) = -\varphi \upsilon_t^{ND}$.

case LC: investor F faces a leverage constraint

$$\pi_{Ht} = (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \left(\vec{m}_{S,t}^{(H)} - r_t \mathbf{1}\right), \tag{28}$$

$$\pi_{Ft} = \begin{cases} (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right) + (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \upsilon_t^{LC} \mathtt{I}_{LC} & \text{if } \upsilon_t^{LC} < 0 \\ (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right) & \text{otherwise} \end{cases} \tag{29}$$

where $\upsilon_t^{LC} = \min\left(\frac{\eta - \mathtt{I}_{LC}^\top (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right)}{\mathtt{I}_{LC}^\top (\vec{\sigma}_{,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \mathtt{I}_{LC}}, 0\right)$, *ie.*

$$\upsilon_t^{LC} = \begin{cases} \frac{-\sigma_{Yh}\sigma_{Yf}\sigma_\alpha^2\left[\Delta m_t^{Y_h}\sigma_{Yf} + \Delta m_t^{Y_f}\sigma_{Yh} - (\eta-1)(1+\lambda)\sigma_{Yh}\sigma_{Yf}\right]}{(\eta-1)^2\lambda_t^2(\alpha_t^H + \alpha^F - 1)^2\sigma_{Yh}^2\sigma_{Yf}^2 + (\sigma_{Yh}^2 + \sigma_{Yf}^2)\sigma_\alpha^2} & \text{if } \Delta m_t^{Y_h}\sigma_{Y_f} + \Delta m_t^{Y_f}\sigma_{Y_h} > (\eta-1)(1+\lambda_t)\sigma_{Y_f}\sigma_{Y_h}, \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

The collateral adjustment in this case is $\delta(\upsilon_t^{LC}) = -\eta\upsilon_t^{LC}$.

υ_t^{ND} and υ_t^{LC} are negative whenever the related constraint binds, as both represent a limitation on F's long portfolio position—when the constraint binds, his portfolio will reflect a less optimistic perception of investment opportunities than in a benchmark economy in which he is unconstrained. The investor seeks to compensate for his binding constraint by investing into a combination of the remaining assets that is highly correlated with the desired, but inaccessible, asset. The role of assets' covariance structure is captured in the terms $(\vec{\sigma}_{S,t}^{-1})^\top(\vec{\sigma}_{S,t}^{-1}\upsilon_t^{case}\mathtt{I}_{case})$ in (26) and (29).

In the absence of constraints, the investor that is more optimistic about economic growth will tend to borrow money to leverage his investments into the risky assets. The other investor, who may be more pessimistic or has stronger hedging needs, has a higher demand for safe investments, thus providing a cheap borrowing rate to the leveraged investor. When a constraint binds, investors cannot choose their portfolios according to their true beliefs about investment opportunities, which drives a wedge between true expectations about growth rates and the expectations reflected in asset prices via portfolio choice. This shift however, will not be symmetric, rather the *relative* attractiveness of risky and riskfree investments will shift in an environment of constrained markets. How significant this distortion is will depend on two characteristics: how strict the level of the constraint is, ie. the level of φ and η, as well as how severely investor F is constrained, ie. by how much does a given level of constraint distort his portfolio choice. The latter will depend on expectations or sentiment about investment opportunities.

While output of the *home* and *foreign* economies are uncorrelated, stock and bond markets are related through equilibrium prices coming from investors' consumption and investment choices. The equilibrium relative price of consumption goods, $\bar{p}_t = p_t^h / p_t^f$ as stated in (24), is determined by the relation of either investor's marginal utilities with respect to the two goods, $\bar{p}_t = u^i_{C^h}(\cdot) / u^i_{C^f}(\cdot)$, and reflects relative demand and supply of the two goods. These terms of trade take on the role of real exchange rates between the two countries, and are the conductor of fundamental shocks to propagate from the goods to the financial markets.

III Bond Market Dynamics

III.A The Riskfree Interest Rate

When investor F's constraint is binding, his portfolio holdings seem to reflect a different 'riskfree rate' than that used by the other investor: while the portfolio of investor H is consistent with rate r_t, F instead seems to be basing his decision on an augmented rate $r_t + \delta(\upsilon_t)$. For any outside observer including the econometrician, the only visible rate is the stated r_t, thus making the decisions of the investor F seem irrational; while the level of which is indeed affected by the presence of F's constraint and its implications for consumption and savings decisions, the constrained investor treats the riskfree asset differently than a log-investor without such constraints would. Optimal equilibrium consumption decisions determine volatility and growth rates of H's consumption of the *home* good, $\sigma_{C_{Hh}}$ and $\mu_{C_{Hh}}$:

$$\begin{aligned} d\xi_t^H &= -\left[\mu_{C_{Hh}} + \mu_{p^h} - \sigma^2_{p^h} - \sigma^2_{C_{Hh}} - \mathsf{cov}_{C_{Hh},p^h} + \frac{1}{\alpha_t^H}\mathsf{cov}_{\alpha,p^h} + \frac{1}{\alpha_t^H}\mathsf{cov}_{C_{Hh},\alpha}\right]\xi_t^H dt - \\ &\quad - \left[\sigma_{p^h} + \sigma_{C_{Hh}} - \frac{1}{\alpha_t^H}\sigma_{t,\alpha}\right]\xi_t^H d\vec{W}_t^{(H)}. \end{aligned} \tag{31}$$

The notion of what constitutes a 'riskfree' asset depends on the definition of the numeraire good. Either of the two goods can be used as such in this model, and without loss of generality, proposition 1 gives the results taking the *foreign* good Y_t^f to be the numeraire. A consumption basket composed of fraction β of the *home* good Y_t^h and fraction $(1-\beta)$ of Y_t^f could likewise be used, such that relative goods prices are defined as $\beta p_t^h + (1-\beta)p_t^f = 1$. Redefining the numeraire is simply a normalization and does not fundamentally affect the implications of the model. In particular, the *relative* pricing of assets, e.g. the relative interest rates across the two countries, remains identical. However, it is useful to keep in mind that this 'riskfree' rate guarantees one unit of this numeraire consumption basket over the next instant, which does not necessarily coincide with either investor's personal choice of consumption basket. In that sense, it is similar to the type of riskfree asset provided by the existing government bond markets.

The properties of equilibrium interest rates in economies shaped by general DARA utility functions has been well studied in the literature, so here I focus on the characteristics that heterogeneous beliefs and investment constraints add to the behavior of the bond markets.[10] As is typical for equilibrium in endowment economies, r_t is positively related to aggregate consumption growth and negatively related to aggregate consumption risk, due to the precautionary savings motive. Heterogeneity in beliefs introduces further factors that depend on the level of disagreement. Abstracting for a moment from any investment constraints, the equilibrium interest rate in the unconstrained economy, r_t^U, would be as follows, again taking Y_t^f to be the numeraire:

$$r_t^U = s_f^F m_{Yf,t}^{(F)} + (1 - s_f^F) m_{Yf,t}^{(H)} - \sigma_{Yf,t}^2, \tag{32}$$

where $s_f^F = \frac{\lambda_t \alpha^F}{1-\alpha_t^H + \alpha^F \lambda_t}$ is the fraction of total *foreign* good consumption that falls to investor F, as given in (16). The interest rate is determined by a weighted average of investors' beliefs about economic growth rates of the numeraire good, where the weight depends on inherent consumption preferences α and how wealth is distributed in the economy, λ_t. Heterogeneity of investor

[10]Xiong and Yan (2009) also study the effect of heterogeneity on the term structure of interest rates, but their work differs from the model considered here in that they exogenously impose goods and stock price dynamics.

expectations affect interest rates because investors' 'average' expected consumption growth rate—i.e. the wealth weighted sum of individuals' expectations—exceeds the true aggregate growth rate: $E^H[dC_{Hf,t}] + E^F[dC_{Ff,t}] = \mu_{Y_f} + s_f^F(1 - s_f^F)(\Delta m_t^{Y_h\,2} + \Delta m_t^{Y_f\,2})$. Each of the investors believes he is making the better decisions, based on their disagreement about fundamentals. This leads to inflated measures of aggregate expected consumption growth reflected in the market interest rate. This puts upward pressure on the interest rate, which is dampened by the effect of disagreement risk—how variable disagreement is over time. These two effects exactly offset one another for log utility investors, and are therefore not visible in r_t^U above. While investors' expectations about *home* growth rates do not impact r_t directly, they will affect its dynamics: s_f^F is a function of λ_t, the stochastic behavior of which depends on investors' disagreement regarding growth rates in all economies, not just the numeraire economy.

Potentially binding constraints on investor F's investments introduce additional effects on the interest rate. In the equation above, the relationship between investors' beliefs is as characterized in section II.D: $m_{Yh,t}^{(F)} = m_{Yh,t}^{(H)} + \Delta m_t^{Y_h}\sigma_{Yh,t}$ and $m_{Yf,t}^{(F)} = m_{Yf,t}^{(H)} + \Delta m_t^{Y_f}\sigma_{Yf,t}$. In the presence of constraints however, interest rates are not only affected by the extent of investors' dispersion in beliefs, but also by the extent to which the constraint distorts portfolio choice: $\Delta\vec{\kappa}_t = \Delta\vec{m}_{Y,t} + \vec{\sigma}_{S,t}^{-1}\upsilon_t^{case}$.

$$r_t^{case} = m_{Yf,t}^{(H)} - \sigma_{Yf}^2 + s_f^F \Delta\kappa_t^f \sigma_{Yf} - s_f^F \delta(\upsilon_t^{case}) \tag{33}$$

The type of constraint, as indicated by the superscript *case* in (33), determines the effect on the riskfree rate. A constraint on non-domestic stock ownership would not only bind under different circumstances than would a leverage constraint, but also leaves a different set of remaining assets for the investor to compensate for the respective constraint in his portfolio choice. The role of bond markets as an 'asset class' is different.

scenario ND:

$$r_t^{ND} = r_t^U + s_f^F \sigma_{Yf}(\vec{\sigma}_{S,t}^{-1}\upsilon_t^{ND})_{\text{el.2}} - s_f^F \delta(\upsilon_t^{ND}) \tag{34}$$

where $(\cdot)_{\text{el.i}}$ denotes the i'th element of the vector $(\cdot)$.

There are two effects on the interest rate compared to the unconstrained case. The first term in (34)is a risk-sharing effect, while the second term reflects a change in the precautionary savings motive of the restricted investor. The risk-sharing effect arises if the constrained investor adjusts his portfolio—relative to his desired 'unconstrained' portfolio—such that the two investors share local foreign production risk differently. If a constraint on F's holdings in the *home* affects his conditionally optimal exposure to *foreign* economic risk, the two investors' wealth will react differently to economic shocks, changing their respective impact on the aggregate expected growth rate in equilibrium. The second term is akin to an endowment effect. The constraint prevents the *foreign* investor from accessing certain consumption paths, which he compensates for by altering his consumption–savings decision.

In scenario ND, $(\vec{\sigma}_{S,t}^{-1} \upsilon_t^{ND})_{\text{el.2}} = 0$. Because the fundamentals of the two economies are unrelated, F's constraint on *home* stock holdings does not lead to a suboptimal exposure to *foreign* (i.e. numeraire) production risk. Note that this does not mean holdings of the local stock S_t^f remain the same; but the additional bond markets provide alternative assets that his conditionally optimal portfolio does not require him to compromise on his desired exposure to his local source of fundamental risk, that is optimally shared by both investors. When F's holdings of S_t^h are limited, he reallocates these funds into a combination of the *home* bond B_t^h, whose exposure to the terms of trade are the same as his desired S_t^h, and the remaining risky asset, S_t^f, which allows him to sustain his level of overall risk exposure to some degree.

However, the collateral adjustment $\delta(\upsilon_t^{ND}) = \frac{\varphi \sigma_{Y^h} \sigma_\alpha^2 \left[\Delta m_t^{Y_h} - \left(\varphi(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_{Y^h}\right]}{\left(\varphi \lambda_t (\alpha_t^H + \alpha^F - 1) + (1-\alpha_t^H)(\alpha_t^H + (1-\alpha^F)\lambda_t)\right)^2 \sigma_{Y^h}^2 + \sigma_\alpha^2} > 0$ affects the precautionary savings motive. A constraint on long positions implies that this (rather optimistic) investor would like to invest more and thus feels precluded from participating in future growth, which he partly compensates for by investing more of his wealth, albeit into the riskfree asset; this lowers the interest rate.

In studying the effects of constraints on financial markets, the main questions revolve around how asset prices and their behavior changes in response to an easing or a tightening of restrictions. In this model, there are two distinct ways to interpret 'tightening' of a constraint, and they have very different implications for interest rates. Firstly, a constraint is tighter when the imposed investment limit, here φ, is lowered: when the constraint binds, F is forced to invest even less into S_t^h and liquidate part of his holdings. Secondly, a given constraint is tighter when it—endogenously—binds more severely. This is the case when, keeping φ fixed, investor F becomes more interested in holding stock S_t^h, so that the permitted level of holdings imply a more severe deviation from the true desired holdings. Investor beliefs and changes therein over time capture this latter effect. As investor F becomes more optimistic about the growth rate of the *home* economy relative to investor H, $\Delta m_t^{Y_h}$ becomes more positive, and the constraint binds more severely: investor F is more bullish about investment opportunities in *home* country, but cannot purchase more of the stock. How these two distinct notions of 'tightening' investment restrictions differ in their effect on the risk free rate is discussed in the remainder of this section.

First, consider a tightening of constraints in the latter sense: how is the interest rate affected when a constraint that is in place starts binding more severely. Recall from (33) that r_t^U is independent of beliefs regarding Y_t^h's growth rate; the riskfree asset provides one unit of the numeraire consumption good in the future, and is therefore independent of any other, unrelated, goods.Substituting equilibrium terms from proposition 1 into (34) shows that this independence no longer holds when the *foreign* investor is bound by his constraint:

$$\frac{\partial r_t^{ND}}{\partial \Delta m_t^{Y_h}} = \frac{-\varphi s_f^F \sigma_{Yh} \sigma_\alpha}{\left(\varphi \lambda_t (\alpha_t^H + \alpha^F - 1) + (1 - \alpha_t^H)(\alpha_t^H + (1 - \alpha^F)\lambda_t)\right)^2 \sigma_{Yh}^2 + \sigma_\alpha^2} < 0.$$

This result indicates that in an economy with investment frictions, interest rates can seem 'excessively' sensitive to seemingly unrelated fundamentals. In an international context, this means that local interest rates of relatively closed economies will be more sensitive to the fundamentals abroad. Furthermore, they will be sensitive to changes in 'sentiment' about these unrelated economies. For the constraint discussed here, the interest rate r_t^{ND} is lower the more severely the *foreign* investor is constrained in his holdings of non-domestic risky assets. His conditionally optimal portfolio reallocation increases demand for the riskfree bond, lowering interest rates.

As discussed above, the correlation structure of fundamentals allows F to retain his desired exposure to *foreign* production risk, so the interest rate will retain the same sensitivity to the beliefs as in the unconstrained environment, reflecting Y_t^f's role as numeraire. $\frac{\partial r_t^{ND}}{\partial \Delta m_t^{Yf}} = \frac{\partial r_t^{U}}{\partial \Delta m_t^{Yf}} = s_f^F \sigma_{Yf} > 0$: as disagreement about the *foreign* growth rate increases, the aggregate expected growth rate of consumption increases, putting upwards pressure on interest rate.

Now consider the other notion of 'tightening' constraints: lowering the level φ. In contrast to the previous issue, where the investor was simply more affected by the restriction, this change leads to trade in the restricted asset. Investor F, already constrained, now has to sell some of his holdings in the *home* stock. The net effect of a small change to φ is ambiguous:

$$\frac{\partial r_t^{ND}}{\partial \varphi} \begin{cases} > 0 & \text{if } \varphi > \sqrt{\frac{(1-\alpha_t^H)^2(\alpha_t^H+(1-\alpha^F)\lambda_t)^2\sigma_{Yf}^2+\sigma_\alpha^2}{\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Yf}^2}} \\ < 0 & \text{if } \varphi < \sqrt{\frac{(1-\alpha_t^H)^2(\alpha_t^H+(1-\alpha^F)\lambda_t)^2\sigma_{Yf}^2+\sigma_\alpha^2}{\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Yf}^2}} \ \& \\ & \Delta m_t^{Y_h} > \left(\varphi\left(1+\lambda_t\right) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_{Yh} + A(\varphi) \end{cases}$$

where $A(\varphi)$ is a positive function, the details of which can be found in the appendix.

Consider the first of these two cases above, where the interest rate falls as the constraint becomes more strict. This happens only if φ is initially not too severe, and the intuition is as follows. As the constraint is tightened and F is forced to liquidate some of his holdings of S_t^h, he reallocates these freed funds into substitute assets—bond markets and local stock market S_t^f. The increased demand for bonds means markets will clear at lower interest rates.

The second case follows generally the same logic but gives additional insight into how constrained investors choose appropriate substitute assets, and how this distorts equilibrium. The optimal portfolio adjustment is a function of both the level of the restriction itself and the investor's perception about investment opportunities. Assume that φ is stringent and at the same time it binds severely — $\Delta m_t^{Y_h}$ is large and positive. The constraint requires that F put only a very small proportion of his wealth into the *home* stock, deviating significantly from his desired portfolio. The *home* bond gives him similar exposure to the real 'exchange rate' risk (the relative price $\bar{p}_t$) as the restricted *home* stock, but also skews the relative exposure to fundamental economic risk of *home* and *foreign*'s economic shocks. A sudden change in regulation which further limits F's holdings of

S_t^h will thus cause him to tilt his portfolio more towards the alternative *risky* asset, the correlated stock S_t^f, to retain sufficient exposure to economic risk. The associated drop in demand for bonds raises interest rates.

The previous case discussed can also be framed in this light: if the constraint is lenient enough, it does, regardless of investors' belief dispersion, allow sufficient portfolio holdings in S_t^h that F will not need to lever up holdings in other risky assets, but will rather look to the country's bond markets as an alternative asset.

scenario LC:

$$r_t^{LC} = r_t^U + s_f^F \sigma_{Yf} (\vec{\sigma}_{S,t}^{-1} \upsilon_t^{LC})_{\text{el.2}} - s_f^F \delta(\upsilon_t^{LC}), \tag{35}$$

Again, $(\cdot)_{\text{el.i}}$ denotes the i'th element of the vector $(\cdot)$. Structurally the effects are as described in the case *ND*, but the broader nature of the constraint changes the quantitative impact: case *LC* is broader in that it does not prevent F from taking on a large risky position in a particular type of assets, but rather only restricts the overall position in risky assets, leaving F to decide how to allocate the funds among the two stocks. This means that, as opposed to case *ND* earlier, his holdings of S_t^f directly affect the investments he can make into S_t^h, so in equilibrium his holdings of both stocks will be affected. $(\vec{\sigma}_{S,t}^{-1} \upsilon_t^{LC})_{\text{el.2}} < 0$ captures this effect—a binding leverage constraint necessarily results in the investor's optimism regarding economic growth rates to be only partially reflected in all financial markets. r_t^{LC} is thus lower than it would be in an economy with identical fundamentals but unconstrained investment. The effect of the collateral adjustment $\delta(\upsilon_t^{LC}) > 0$ due to the binding constraint has already been discussed: to compensate for unattainable consumption paths, the restricted investor saves more, putting downwards pressure on interest rates.

Recall from (32) that r_t^U will be independent of $\Delta m_t^{Y_h}$, and react positively to an increase in $\Delta m_t^{Y_f}$: as the aggregate expected growth rate of the numeraire good increases, the interest rate rises. As a leverage constraint binds more tightly, due to an increase in either parameter, the sensitivity of the interest rate changes in the following ways.

$$\frac{\partial r_t^{LC}}{\partial \Delta m_t^{Y_h}} = s_f^F \sigma_{Yf} \frac{\partial(\vec{\sigma}_{S,t}^{-1} \upsilon_t^{LC})_{\text{el.2}}}{\partial \Delta m_t^{Yh}} - s_f^F \frac{\partial \delta(\upsilon_t^{LC})}{\partial \Delta m_t^{Yh}} = \frac{-(1+\eta) s_f^F \sigma_{Yh} \sigma_{Yf}^2 \sigma_\alpha^2}{\left((\eta-1)^2 \lambda_t^2 (\alpha_t^H + \alpha^F - 1)^2 \sigma_{Yh}^2 \sigma_{Yf}^2 + (\sigma_{Yh}^2 + \sigma_{Yf}^2) \sigma_\alpha^2\right)} < 0,$$

$$\frac{\partial r_t^{LC}}{\partial \Delta m_t^{Yf}} = \frac{\partial r_t^U}{\partial \Delta m_t^{Yf}} + s_f^F \sigma_{Yf} \frac{\partial(\vec{\sigma}_{S,t}^{-1} \upsilon_t^{LC})_{\text{el.2}}}{\partial \Delta m_t^{Yf}} - s_f^F \frac{\partial \delta(\upsilon_t^{LC})}{\partial \Delta m_t^{Yf}} = s_f^F \sigma_{Yf} \left(1 - \frac{(1+\eta) \sigma_{Yh}^2 \sigma_\alpha^2}{(\eta-1)^2 \lambda_t^2 (\alpha_t^H + \alpha^F - 1)^2 \sigma_{Yh}^2 \sigma_{Yf}^2 + (\sigma_{Yh}^2 + \sigma_{Yf}^2) \sigma_\alpha^2}\right) \lesseqgtr 0.$$

The effect of $\frac{\partial r_t^{LC}}{\partial \Delta m_t^{Y_h}}$ is unambiguous. When F's leverage constraint binds, he cannot hold as much in stocks as would be optimal. As his optimism increases, captured by a rise in belief dispersion about the *home* growth rate, this deviation from the optimal risk-sharing becomes more severe, and investor H must be further incentivized to hold the 'excess' supply of stocks. The risk-free alternative must become less attractive, in order to clear markets and convince H to invest into riskier assets. For the case of beliefs regarding the *foreign* economy, the same effect is partially offset by the impact of increased optimism on saving behavior: Higher belief dispersion about *foreign* economic growth rates puts upward pressure on the interest rate—both in case U as well as case *LC*. But in the latter case, there is an offsetting effect coming from the binding constraint, which prevents the particularly optimistic investor, F, from providing the bond market with liquidity. Which of the two effects dominates depends both on the exogenous level of the leverage constraint, η, as well as the weight the constrained investor has in the economy, λ_t.

The effect of a change in regulation, as opposed to an endogenous change in how severely an existing constraint binds, is structurally similar to the case *ND* discussed earlier.

$$\frac{\partial r_t^{LC}}{\partial \eta} \begin{cases} > 0 & \text{if } \eta > 1 + \frac{(\sigma_{Yh}^2 + \sigma_{Yf}^2)\sigma_\alpha^2}{2\lambda_t^2 (\alpha_t^H + \alpha^F - 1)^2 \sigma_{Yh}^2 \sigma_{Yf}^2} \\ < 0 & \text{if } \eta < 1 + \frac{(\sigma_{Yh}^2 + \sigma_{Yf}^2)\sigma_\alpha^2}{2\lambda_t^2 (\alpha_t^H + \alpha^F - 1)^2 \sigma_{Yh}^2 \sigma_{Yh}^2} \;\& \\ & \Delta m_t^{Y_h} \sigma_{Y_f} + \Delta m_t^{Y_f} \sigma_{Y_h} > (\eta - 1)(1 + \lambda_t) \sigma_{Y_f} \sigma_{Y_h} + B(\eta) \end{cases}$$

where $B(\eta)$ is a positive function. If η is lenient, the effect of tightening the restriction follows straightforward intuition: as F is forced to liquidate his stock positions, demand for the risk-free bond rises, lowering interest rates. However, when leverage limitations that are in place are already strict, and they bind severely, then a further tightening of the limit has the opposite effect. In the context of this economy, where non-local bonds exist, the restricted investor has some degree of flexibility in how he allocates the wealth he can no longer hold in stocks. The constraint at any level changes the shadow state prices for the affected investor, thus altering his investments. But when his shadow prices are already severely distorted, further changes to the limit η lead him to not invest more into the risk-free bond, but rather shift more money into the non-domestic, *risky* stock B_t^h—perhaps even taking a larger short position in the risk-free bond. His exposure to economic risk was already far below what he desires as the constraint was binding severely. So instead of further lowering the exposure in response to a tightened leverage constraint, he opts for higher—albeit tilted towards 'risky bonds'— exposure. The interest rate will rise. This could be one explanation for the market behavior we have seen in the aftermath of the crisis. The large price drops provided interesting investment opportunities for those investors with access to funds. Many were already constrained by lenders getting into financial difficulties, and when regulation (or public investor sentiment) became more concerned about leverage levels, companies were even further restricted. But because investment opportunities in risky asset classes were perceived as high, these constraints did not lead to these investors putting their money into safer assets, but rather into other assets that were not part of the restricted asset class. Indeed, looking in particular at financial institutions, some received equity injections with the purpose of alleviating the binding leverage constraints, in order to allow for lending. But the amount of loans—the equivalent of 'buying bonds'— did not rise as much as expected, the institutions seemed to be putting the additional resources into other asset classes that promised a higher return.[11]

These findings suggest that in the debate about introducing or reducing regulation in order to manage the level of interest rates, one needs to carefully take into account how the market's perception about investment opportunities affect investors' substitution across different asset markets. There is an interaction between the stringency of constraints, how severely they bind, and how regulatory changes will affect asset holdings.

[11]The particular issues related to the leverage restrictions of financial institutions, e.g. the Basel requirements, are obviously not modeled here which should be taken as a caveat, but an equity injection can be seen as a similar action than relaxing the limit η, taking the bank's initial capital as given.

III.B The Behavior of International Bond Markets

The previous section analyzed the effect of constraints on the riskfree interest rate, earned on the *foreign* bond. The *home* country's bond offers a risky return in terms of the numeraire: while db_t^h is a deterministic process offering an guaranteed instantaneous rate of 'return' of r_t^h in the *home* good, in terms of the numeraire the bond follows the process $dB_t^h = d(p_t^h b_t^h)$. This risk can be considered an exchange rate risk of investing into non-domestic bonds for an investor. It does not reflect a sovereign 'political' risk sometimes associated with countries' bond markets. Looking at the relative bond prices and controlling for the 'exchange rate' $\bar{p}_t$, the ratio of the *home* and *foreign* bonds gives the difference between the interest rates in the two countries and allows us to analyze the time variation in interest rate differentials.

$$\frac{d(B_t^f/B_t^h)}{B_t^f/B_t^h} = E[g_t^{\Delta r}]dt + \begin{pmatrix} \sigma_{Y_h} + \frac{\lambda_t^2(\alpha_t^H+\alpha^F-1)\Delta\kappa_{t,h}}{(1-\alpha_t^H+\lambda_t\alpha^F)(\alpha_t^H+(1-\alpha^F)\lambda_t)} \\ -\sigma_{Y_f} + \frac{\lambda_t(\alpha_t^H+\alpha^F-1)\Delta\kappa_{t,f}}{(1-\alpha_t^H+\lambda_t\alpha^F)(\alpha_t^H+(1-\alpha^F)\lambda_t)} \\ \frac{\lambda_t(\alpha_t^H+\alpha^F-1)\Delta\kappa_{t,\alpha}-(1+\lambda_t)\sigma_\alpha}{(1-\alpha_t^H+\lambda_t\alpha^F)(\alpha_t^H+(1-\alpha^F)\lambda_t)} \end{pmatrix}^\top d\vec{W}_t$$

where $E[g_t^{\Delta r}]$ is the expected instantaneous growth rate of the interest rate differential between the *home* and *foreign* countries.[12]

The effects of the two constraints considered in scenario ND and LC will have, through $\Delta\vec{\kappa}_t$, an effect on the relative bond returns as well as the bonds' relative sensitivity to economic shocks. As discussed in the previous section, a constrained investor will choose to invest in other, available, assets that are as highly correlated as possible, thus altering the relationship between the two countries' interest rates. The expected change in interest rates is determined by the countries' terms of trade. A binding constraint affects these terms of trade through putting restrictions on the wealth transfers across countries as economic fundamentals shift. Similarly for the sensitivity to fundamental economic shocks. If the constraint in place serves to mitigate portfolio differences, and $|\Delta\vec{\kappa}_t| < |\Delta\vec{m}_{Y,t}|$, the difference between the two countries' interest rates will fluctuate less around the mean. However, if constraints exacerbate differences in portfolio holdings and wealth transfers

[12]Note that the relative bond prices are independent of the numeraire.

between countries are significant in response to fundamental shocks, the difference between the two rates will likewise become more volatile.

IV Conclusion

The paper analyzes in a two-country setup how restrictions on international capital flows interact to affect bond market returns across countries and the risk-free rate of interest. In particular, the model shows that the effect of loosening a binding constraint on some investor's portfolios can have different effects on interest rates, depending on how stringent the constraint was previously, and also how severely it had been binding.

Secondly, comparing bond yields across countries, the model shows that constraints can lead to more volatile interest rate differentials across countries when constraints lead portfolios to diverge and asset ownership becomes more concentrated.

These findings, while preliminary, give some insight into testable implications for studying the effect of constrained investment choices on aggregate market behavior.

Appendix

A Optimal Consumption

Investors H and F maximize their respective expected utility, subject to budget constraints. Equilibrium is established by maximizing the aggregated utility function

$$U(C_H, C_F) = u_H\left(C^h_{H,t}, C^f_{H,t}\right) + \lambda_t u_F\left(C^h_{F,t}, C^f_{F,t}\right)$$

where

$$u_H\left(C^h_{H,t}, C^f_{Ht}\right) = \alpha^H_t \log C^h_{Ht} + (1-\alpha^H_t)\log C^f_{H,t},$$
$$u_F\left(C^h_{F,t}, C^f_{Ft}\right) = (1-\alpha^F)\log C^h_{Ft} + \alpha^F \log C^f_{F,t},$$

and $\lambda_t = \frac{y_H \xi^H_t}{y_F \xi^F_t}$, the ratio of investors' state price densities.

FOC of optimal consumption of goods $j = h, f$, of investors $i = H, F$: $u^i_{C^j}(\cdot) = \frac{\partial u_i\left(C^i_{it}, C^j_{it}\right)}{\partial C^j_{it}} = y_i p^j_t \xi^i_t$, where p^j_t is the relative price of good j, ξ^i_t is investor i's state price density and y_i the associated Lagrange multiplier, reflecting initial endowment.

	investor H:	*investor F:*
good h:	$\frac{\alpha^H_t}{C^h_{Ht}} = y_H p^h_t \xi^H_t$	$\frac{1-\alpha^F}{C^h_{Ft}} = y_F p^h_t \xi^F_t$
good f:	$\frac{1-\alpha^H_t}{C^f_{Ht}} = y_H p^f_t \xi^H_t$	$\frac{\alpha^F}{C^f_{Ft}} = y_F p^f_t \xi^F_t$

Market clearing requires $\sum_i C^j_i = Y^j$ for both goods $j = h, f$, giving equilibrium total consumption in section 4.

B Optimal Wealth

Current wealth is an appropriately discounted value of all future consumption levels. Log utility in a finite horizon economy implies that both investors will consume a fixed portion of their wealth each period, as a function of the time remaining. The below is described for investor H, analogous values for investor F follow directly.

$$X_t^H = \frac{1}{\xi_t^H} E\left[\int_t^T \left(\xi_s^H p_s^h C_{Hs}^h + \xi_s^H p_s^f C_{Hs}^f\right) ds\right]$$

From FOC above, $\frac{\alpha_t^H}{y_H} = C_{Ht}^h p_t^h \xi_t^H$ and $\frac{1-\alpha_t^H}{y_H} = C_{Ht}^f p_t^f \xi_t^H$ holds, therefore:

$$X_t^H = \frac{1}{\xi_t^H} E\left[\int_t^T \left(\frac{\alpha_s^H}{y_H} + \frac{1-\alpha_s^H}{y_H}\right) ds\right] = \frac{1}{y_H \xi_t^H}(T-t).$$

Linking wealth X_t^i back to consumption above gives

$$\begin{aligned} X_t^H &= C_{Ht}^h \cdot \frac{p_t^h}{\alpha_t^H}(T-t) = C_{Ht}^f \cdot \frac{p_t^f}{1-\alpha_t^H}(T-t), \\ X_t^F &= C_{Ft}^h \cdot \frac{p_t^h}{1-\alpha^F}(T-t) = C_{Ft}^f \cdot \frac{p_t^f}{\alpha^F}(T-t). \end{aligned}$$

C Relative Goods Prices

The relative price of the two goods is determined by their relative marginal utilities, which must be equal across the two agents, since both are faced with identical prices for goods, there are no frictions in goods markets: $\bar{p}_t = \frac{p_t^f}{p_t^h} = \frac{u_{C^f}^i(\cdot)}{u_{C^h}^i(\cdot)}$. The basket of goods $\beta p_t^h + (1-\beta)\, p_t^f = 1$ defines the numeraire. $\beta \in [0,1]$ and represents the weight of the *home* good in the basket. This weight does not represent either agent's de facto consumed basket. The levels of stock prices will be affected by the chosen β, but the relation between the two stocks will not be. Interesting special cases include $\beta = 0$, $\beta = 1$ or $\beta = \alpha^F$, denoting Y_t^f, Y_t^h or F's true consumption basket as the numeraire, respectively. The main insights from the paper are not sensitive to the choice of β.

Using the equilibrium marginal utilities from market clearing restrictions $\sum_i C_i^j = Y^j$ for goods $j = h, f$ gives:

$$\bar{p}_t = \frac{p_t^f}{p_t^h} = \frac{u_{C^f}^H(\cdot)}{u_{C^h}^H(\cdot)} = \frac{y_H p_t^f \xi_t^H}{y_H p_t^h \xi_t^H} = \frac{(1-\alpha_t^H) + \alpha^F \lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{Y_t^h}{Y_t^f}.$$

The dynamics of relative goods prices $\bar{p}_t$ follow

$$\begin{aligned} d\bar{p}_t &= (\cdot)dt + \frac{1-\alpha_t^H+\alpha^F\lambda_t}{\alpha_t^H+(1-\alpha^F)\lambda_t}\frac{1}{Y_t^f}dY_t^h - \frac{1-\alpha_t^H+\alpha^F\lambda_t}{\alpha_t^H+(1-\alpha^F)\lambda_t}\frac{Y_t^h}{(Y_t^f)^2}dY_t^f - \\ &\quad - \frac{\lambda_t+1}{(\alpha_t^H+(1-\alpha^F)\lambda_t)^2}\frac{Y_t^h}{Y_t^f}d\alpha_t^H + \frac{2\alpha_t^H-1}{(\alpha_t^H+(1-\alpha^F)\lambda_t)^2}\frac{Y_t^h}{Y_t^f}d\lambda_t. \end{aligned}$$

D Auxiliary Market: Portfolio Choice in Constrained Markets

The constraints studied are limitations on the fraction of wealth $\pi_{i,t}^j$ that investor i places into one or more assets j. I assume that portfolio positions $\pi_{i,t}^j$ in assets $j = S_t^h, S_t^f, B_t^h, B_t^f$ are constrained to lie in a closed, convex, non-empty set K that contains the origin. The analysis here is based on the methodology developed in Cvitanic and Karatzas (1992).

The martingale analysis of incomplete markets requires the construction of a fictitious market that fictitiously augments the market parameters of the original constrained market. Under these augmented market parameters, the constrained investor will optimally choose a portfolio permissible within the constraints. This is then the optimal portfolio also under the original, constrained market.[13]

The set of admissible trading strategies is defined by the set K, the support function is $\delta(\upsilon_t^i) \equiv \delta(\upsilon_t^i|K) \equiv \sup\left(-\pi_{i,t}^\top \upsilon_t^i : \pi_{i,t} \in K\right)$ and the barrier cone of the set $-K$ is defined as $\bar{K} \equiv \left\{\upsilon_t^i \in \mathbb{R}^2 | \delta(\upsilon_t^i) < \infty\right\}$. υ_t^i is a square-integrable, progressively measurable process taking values in $\bar{K}$ to ensure boundedness.

Investor F's state price density adjust to reflect these augmented market perceptions due to the constraints:

$$d\xi_t^F = -\left(r_t + \delta(\upsilon_t^F)\right)\xi_t^F dt - \kappa_t^{F\top}\xi_t^F d\vec{W}_t^{(F)}, \tag{36}$$

where investor F's adjusted market price of risk is $\vec{\kappa}_t^F = (\sigma_{S,t}^{-1})\left(m_{S,t}^{(F)} + \upsilon_t^F \iota_F - r_t\mathbf{1}\right) = \kappa_{o,t}^F + \sigma_{S,t}^{-1}\upsilon_t^F$. $\kappa_{o,t}^F$ represents the market price of risk that the investor would base his portfolio decisions on, i.e. those reflecting his true beliefs. The second term, $+\sigma_{S,t}^{-1}\upsilon_t^F$, adjusts the market price of risk s.t. the

[13] This setting is a straightforward application of that in Cvitanic and Karatzas (1992), and it can be easily shown that their convex duality approach for convex constraint sets holds here.

investor does not violate his constraint, and at the same time captures the market price of risk that will be reflected in portfolio choice and thus equilibrium market prices.

E State Price Density

Investor H consumes a fraction $\frac{\alpha_t^H}{\alpha_t^H+(1-\alpha^F)\lambda_t}$ of good Y_t^h and a fraction $\frac{1-\alpha_t^H}{1-\alpha_t^H+\alpha^F\lambda_t}$ of good Y_t^f. This and equilibrium relative prices $\bar{p}_t$ gives

$$\xi_t^H = \beta\frac{\alpha_t^H + (1-\alpha^F)\lambda_t}{y_H Y_t^h} + (1-\beta)\,\frac{1-\alpha_t^H + \alpha^F\lambda_t}{y_H Y_t^f}. \tag{37}$$

Analogously, investor F consumes a fraction $\frac{\lambda_t\left(1-\alpha^F\right)}{\alpha_t^H+(1-\alpha^F)\lambda_t}$ of good Y_t^h and a fraction $\frac{\lambda_t\alpha^F}{1-\alpha_t^H+\alpha^F\lambda_t}$ of good Y_t^f:

$$\xi_t^F = \beta\frac{\alpha_t^H + (1-\alpha^F)\lambda_t}{\lambda_t y_F Y_t^h} + (1-\beta)\,\frac{1-\alpha_t^H + \alpha^F\lambda_t}{\lambda_t y_F Y_t^f}. \tag{38}$$

F Asset Valuation

Proof of Proposition 1: The proof follows closely that in Schornick (2009), under the simpler situation that H does not face a constraint.

Market clearing in asset markets requires

$$S_t^h + S_t^f = X_t^H + X_t^F = p_t^h Y_t^h (T-t) + p_t^f Y_t^f (T-t). \tag{39}$$

Each asset $j = h, f$ is valued as the sum of discounted dividends, taking into account the effects of future binding constraints — the second integral in the equation below.

$$S_t^j = \frac{1}{\xi_t^H} E_t\left[\int_t^T \xi_s^H p_s^j Y_s^j\, ds\right] \qquad j = h, f.$$

Using $\frac{1}{p_t^h\xi_t^H} = \frac{Y_t^h y_H}{\alpha_t^H+(1-\alpha^F)\lambda_t}$ and goods market clearing, as well as $\lambda_t = \frac{y_H\xi_t^H}{y_F\xi_t^F}$ in the pricing function of S_t^h:

$$S_t^h = p_t^h Y_t^h (T-t) + \frac{p_t^h Y_t^h}{\alpha_t^H + (1-\alpha^F)\lambda_t}(1-\alpha^F)\left[E_t \int_t^T \lambda_s ds - \lambda_t (T-t)\right] \tag{40}$$

$$S_t^f = p_t^f Y_t^f (T-t) + \frac{p_t^f Y_t^f}{1-\alpha_t^1 + \alpha^2 \lambda_t}\alpha^F \left[E_t \int_t^T \lambda_s ds - \lambda_t (T-t)\right] \tag{41}$$

Under the constraints on investor F $d\lambda_t$ is a supermartingale under all possible equilibria. Therefore,

$$\begin{aligned} S_t^h &= p_t^h Y_t^h (T-t), \\ S_t^f &= p_t^f Y_t^f (T-t), \end{aligned} \tag{42}$$

where p_t^h and p_t^f can be rewritten in terms of $\bar{p}_t$.

G Interest Rate Effects

In section III.A the sensitivity of the interest rate in scenario ND with respect to restriction parameter φ is detailed. $A(\varphi) = \frac{(1+\lambda_t)\varphi\sigma_{Y_h}\left(\left(\varphi\lambda_t(\alpha_t^H+\alpha^F-1)+(1-\alpha_t^H)(\alpha_t^H+(1-\alpha^F)\lambda_t)\right)^2\sigma_{Y_h}^2+\sigma_\alpha^2\right)}{-\varphi^2\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Y_h}^2+(1-\alpha_t^H)^2(\alpha_t^H+(1-\alpha^F)\lambda_t)^2\sigma_{Y_h}^2+\sigma_\alpha^2} > 0$

In scenario LC, the function that determines the sign of $\frac{\partial r_t^{ND}}{\partial \Delta m_t^{Y_f}}$ is $B(\cdot) = \frac{\sigma_\alpha^2\sigma_{Y_h}^2 \pm \sqrt{\left(\sigma_\alpha^2\sigma_{Y_h}^2+4\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Y_f}^2(\sigma_{Y_h}^2-\sigma_{Y_f}^2)\right)\sigma_\alpha^2\sigma_{Y_h}^2}}{2\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Y_f}^2\sigma_{Y_h}^2}$.

References

Barr, D. G., R. Priestley, 2004. Expected returns, risk and the integration of international bond markets. Journal of International Money and Finance 23, 71–97.

Basak, S., 1996. An Intertemporal Model of International Capital Market Segmentation. Journal of Financial and Quantitative Analysis 31(2), 161 – 188.

——— , 2005. Asset Pricing with Heterogeneous Beliefs. Journal of Banking and Finance 29(11), 2849–2881.

Caballero, R. J., A. Krishnamurthy, 2001. International and domestic collateral constraints in a model of emerging market crises. Journal of Monetary Economics 48(3), 513–548.

Cuoco, D., 1997. Optimal Consumption and Equilibrium Prices with Portfolio Constraints and Stochastic Income. Journal of Economic Theory 72, 33–73.

Cvitanic, J., I. Karatzas, 1992. Convex Duality in Constrained Portfolio Optimization. The Annals of Applied Probability 2, 767–818.

Dornbusch, R., S. Fischer, P. A. Samuelson, 1977. Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods. American Economic Review 67(5), 823–39.

Edison, H. J., F. E. Warnock, 2003. A Simple Measure of the Intensity of Capital Controls. Journal of Empirical Finance 10(1), 81 – 103.

Errunza, V., E. Losq, 1989. Capital Flow Controls, International Asset Pricing, and Investors' Welfare: A Multi-Country Framework. Journal of Finance 44(4), 1025–1037.

Harvey, C. R., B. Solnik, G. Zhou, 2002. What Determines Expected International Asset Returns?. ANNALS OF ECONOMICS AND FINANCE 3, 249298.

He, H., D. M. Modest, 1995. Market Frictions and Consumption-Based Asset Pricing. Journal of Political Economy 103(1), 94–117.

He, H., N. Pearson, 1991. Consumption and Portfolio Policies with Incomplete Markets and Short Sale Constraints: The Infinite Dimensional Case. Journal of Economic Theory 54, 259–304.

Heaton, J., D. Lucas, 1996. Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Prices. Journal of Political Economy 104(3), 443–487.

———, 1997. Market frictions, savings behavior, and portfolio choice. Macroeconomic Dynamics pp. 76–101.

Ilmanen, A., 1995. Time-Varying Expected Returns in International Bond Markets. Journal of Finance 50(2), 481–506.

Jorion, P., 1992. Term premiums and the integration of the Eurocurency markets. Journal of International Money and Finance 11, 1739.

Karolyi, G. A., R. M. Stulz, 2002. Are Financial Assets Priced Locally or Globally?. NBER Working Paper.

Mishkin, F. S., 1984. The Real Interest Rate: A Multi-Country Empirical Study. The Canadian Journal of Economics 17(3), 283–311.

Pavlova, A., R. Rigobon, 2008. The Role of Portfolio Constraints in the International Propagation of Shocks. Review of Economic Studies 75, 1215–1256.

Schornick, A. V., 2009. International Capital Constraints and Stock Market Dynamics. Working Paper, INSEAD.

Stulz, R. M., 1995. International Portfolio Choice and Asset Pricing: An Integrative Survey. Handbook of Modern Finance (ed. R. Jarrow, M. Maximovich, and W. Ziemba) pp. 201–228.

Warnock, F. E., V. C. Warnock, 2009. International capital flows and U.S. interest rates. Journal of International Money and Finance 29, 903919.

Xiong, W., H. Yan, 2009. Heterogeneous Expectations and Bond Markets. Review of Financial Studies forthcoming.

Zapatero, F., 1998. Effects of financial innovations on market volatility when beliefs are heterogeneous. Journal of Economic Dynamics and Control 22(4), 597–626.