

Learning From Stock Prices and Economic Growth

Joel Peress*

INSEAD

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JEL classification codes : O16, G11, G14

Keywords: Growth, financial development, stock market, capital allocation, learning, asymmetric information, noisy rational expectations equilibrium

*INSEAD, Department of Finance, Boulevard de Constance, 77305 Fontainebleau Cedex, France. Telephone: (33)-1 6072 4035. Fax: (33)-1 6074 5500. E-mail: joel.peress@insead.edu.

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Abstract

A competitive stock market is embedded into a neoclassical growth economy to analyze the interplay between the acquisition of information about firms, its partial revelation through stock prices, capital allocation and income. The stock market allows investors to share their costly private signals in an incentive-compatible way when the signals' precision is not contractible. It contributes to economic growth, but its impact is only transitory. Several predictions on the evolution of real and financial variables are derived, including capital efficiency, total factor productivity, industrial specialization, stock trading intensity and idiosyncratic return volatility.

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1 Introduction

Economic institutions are widely believed to play a crucial role for economic growth. In particular, there is now considerable evidence that financial institutions, once considered a “sideshow” (Robinson (1952)), promote economic growth by relaxing constraints undermining the efficiency of investments. In this paper, we analyze the role of one such institution, the stock market, in alleviating one such constraint, investors’ inability to perfectly communicate their private information. Economists have long argued that stock prices improve the allocation of capital by aggregating dispersed information and pointing to the most promising investment opportunities. While several aspects of the relation between the stock market and the real economy have been examined, “existing theories have not yet assembled the links in the chain from the functioning of stock markets, to information acquisition, and finally to aggregate long-run economic growth” (Levine (1997)).¹ This paper assembles these links.

We present a fully integrated model of information acquisition and dissemination through prices, capital allocation and economic growth. A competitive stock market in the spirit of Grossman and Stiglitz (1980) is embedded into a neoclassical growth economy. The economy is composed of firms that raise capital on the stock market, and overlapping generations of workers who invest their labor income in them. Firms’ productivity is unknown but agents can collect private signals about it at a cost. Specifically, they are endowed with one unit of free time which they can devote either to analyzing stocks or to enjoying leisure. Agents’ information is reflected in stock prices, but only partially so because of the presence of noise. Prices in turn guide investors in their portfolio allocations.

The only friction in the model stems from agents’ inability to contract on the precisions of their signals (in particular, there is no short-sales constraint, nor minimum investment requirement). If they could, then the first best outcome would be achieved: agents would commit to infinitesimal precisions (arbitrarily close but not equal to zero), pool their signals and discover firms’ productivity thanks to the Law of Large Numbers (signal errors are uncorrelated across agents and each generation consists of a continuum of agents).² Unfortunately, this outcome is not a Nash equilibrium when precisions are

¹Page 695. More recently, Levine (2005) confirms this assessment: “While some models hint at the links between efficient markets, information and steady-state growth, existing theories do not draw the connection between market liquidity, information production and economic growth very tightly” (page 9). See Levine (1997, 2005) for reviews of the empirical and theoretical literatures on finance and growth.

²Reaching the first best does not require all agents to select non-zero precisions. A randomly chosen subset suffices.

not contractible, as assumed here. Indeed, agents' best response is to set their precisions to zero and report noise, which results in no learning.

The stock market provides the means to share private information in an incentive-compatible way. For example, when agents receive optimistic signals about a firm, they buy its shares and bid up its stock price. The high stock price in turn indicates that investors collectively believe the firm to have good prospects. Thanks to stock prices, agents are better informed even though no new information is actually produced. Naturally, the effectiveness of the stock market is limited by the very existence of informative prices which undermines the incentive to collect costly information in the first place. Indeed, investors' cannot fully appropriate the benefit of their signals as they are leaked to competitors through prices (the Grossman-Stiglitz paradox). Thus, informative stock prices have an impact that is beneficial *ex post* but detrimental *ex ante* to capital efficiency. Noise trading provides the smoke screen behind which investors can conceal their informed trades and reap some benefit. We show that agents, though they reduce the precision of their private signals in response to a decline in the intensity of noise trading, are nevertheless better informed on the whole thanks to the increased accuracy of stock prices. That is, the information sharing benefit outweighs the disincentive cost. The allocation of capital improves, an illustration of the real effects of the stock market. Moreover, it converges to the first best as the intensity of noise trading approaches zero.

To a first approximation, income in the stock market economy is governed by a standard neoclassical law of motion similar to that which obtains under the first best. It grows at a decreasing rate until it reaches a steady-state in which it no longer grows.³ Hence, the process of learning about technologies cannot counter the diminishing returns to capital (whether or not stock prices reveal information). It does nevertheless contribute to long run consumption and welfare since they are ultimately determined by the steady-state *level* of income. In comparison to the neoclassical dynamics, income grows faster if investors' information improves with their income, and slower if information deteriorates with income. Indeed if wealthier agents are better informed for example, then they allocate their labor income more efficiently across the various firms. This enhances the marginal product of labor and makes the next generation of workers richer.

³There is no technological progress nor population growth in the model.

Whether or not information improves along the growth path depends on two competing forces. On the one hand, wealthier investors retire with more of the consumption good, which reduces its marginal utility. They prefer to consume more leisure and collect less information (the substitution effect). On the other hand, information generates increasing returns to scale – its benefit, unlike its cost, rises with the amount to be invested. Indeed, discriminating across firms is more valuable when one has more to invest. The substitution effect leads wealthier agents to learn less while the scale effect of information induces them to learn more. If the scale effect dominates, the precision of private information rises with income and income grows at an accelerated rate. If instead the substitution effect dominates, the precision decreases and the growth rate of income is reduced.⁴ In either case, the precision of information is more responsive to income (i.e., grows faster with income when the scale effect dominates, and declines faster with income when the substitution effect dominates) in the presence of informative stock prices. This is the case because the efficiency of the capital allocation, and therefore next period’s income, increase with the precision of investors’ private signals directly but also indirectly through the informativeness of stock prices (recall that the information sharing benefit dominates the disincentive cost). So the stock market strengthens the link between income and information production.

The implications of the model, when the scale effect of information dominates, are consistent with several patterns observed in the data. First of all, the stock market develops (e.g., as measured by the time spent analyzing stocks) in tandem with income, contributes to economic growth and its effect is only transitory. Empirically, Levine and Zervos (1998), Rousseau and Wachtel (2000) and Carlin and Mayer (2003) document that income grows faster in countries with better functioning stock markets. Atje and Jovanovic (1993) estimate that this growth effect is permanent, but Harris (1997) finds that it is only transitory after controlling for possible endogeneity problems. The model also implies that the stock market processes information only when income exceeds a threshold, again a consequence of the increasing returns to information. This is consistent with the casual observation that financial institutions only emerge once a critical level of income has been reached.

Second, the model implies first that capital is more efficiently allocated across firms as income grows. That is, more (less) capital is channeled to more (less) productive firms when agents are wealthier. This

⁴We derive conditions on preferences under which each effect dominates.

superior efficiency leads to higher total factor productivity (TFP), even though there is no technological progress.⁵ TFP is driven here by knowledge *about* technologies rather than by technological knowledge. Empirically, Wurgler (2000) documents that investments are more responsive to value added in more financially developed countries, and in particular in countries with a more informative stock market.⁶ Furthermore, Levine and Zervos (1998) show that stock markets promote TFP growth, rather than capital growth.⁷

Third, we show that the economy specializes as it grows. Indeed, agents invest more selectively, leading capital and profits to become more concentrated across firms. Empirically, Imbs and Wacziarg (2003) report that countries go through two stages of sectoral diversification. Diversification increases at first, but beyond a certain level of income, the process is reversed and economic activity starts concentrating. The pattern of specialization among advanced countries is consistent with our model as we show that private information is collected only once a critical level of income has been reached. Similarly, Kalemli-Ozcan, Sørensen and Yosha (2003) report that industrial specialization in a sample of developed countries is positively related to the share of the financial sector in GDP, a proxy for financial development.

Fourth, we establish that, as the economy grows, stocks' idiosyncratic and total volatility increase, while the market's volatility remains constant. Thus, individual stocks returns fluctuate more, but they fluctuate in a less synchronized manner. This pattern obtains because more information is incorporated into stock prices. Empirically, Morck, Yeung and Yu (2000) show that stock prices are less synchronous in richer economies. In line with this observation, Campbell, Lettau, Malkiel and Xu (2001) document a strong increase in idiosyncratic return volatility in the U.S. from 1962 to 1997, while the volatility of

⁵TFP, also known in the growth literature as the "Solow residual", is defined as the residual from a regression of income growth on factor growth. It encompasses any factor, beyond labor growth and the capital growth, that contributes to output growth. Empirically, most of the differences in income across countries and periods stem from differences in TFP (e.g. Jorgenson (1995, 2000), Prescott (1998), Hall and Jones (1999) and Harberger (1998)).

⁶Wurgler (2000) constructs cross-country estimates of the elasticity of investments to value added by regressing, for each country, growth in industry investment on growth in industry value added. As a proxy for stock market informativeness, he uses a measure developed by Morck, Yeung and Yu (2000) who estimate the extent to which stocks move together and argue (in line with our model) that prices move in a more unsynchronized manner when they incorporate more firm-specific information.

⁷The findings of Levine and Zervos (1998) are consistent with those of Caballero and Hammour (2000), Restuccia and Rogerson (2003) and Hsieh and Klenow (2006) who show that variations in the allocation of resources account for a large fraction of the cross-country differences in TFP. Moreover, Henry (2003) confirms that countries that liberalize their stock market experience a rise in TFP, and Bekaert, Harvey and Lundblad (2001, 2005), Bertrand, Schoar and Thesmar (2005), Galindo, Schiantarelli and Weiss (2005) and Chari and Henry (2006) that their allocative efficiency improves.

the market remained stable.

Finally, we characterize the dynamics of consumption inequality across agents and trading activity, measured by the share turnover – the ratio of the value of shares traded to the total capitalization of the market. Both decrease at first and then increase as the economy grows. Indeed, disagreement encourages agents to trade and leads them to more unequal consumption levels through more heterogeneous portfolios. Disagreement weakens as the economy grows because agents' private signals tend to be more precise and therefore more similar. But it tends to intensify beyond an income threshold because agents with more precise private signals rely more on them. Empirically, Levine and Zervos (1998) and Rousseau and Wachtel (2000) report that the share turnover on the stock market is positively related to output growth.

Our work relates to three main strands of theory. First and foremost, it contributes to the theoretical literature on finance and growth.⁸ Most closely related is the seminal paper by Greenwood and Jovanovic (1990). In their setup, investors choose whether to invest directly in their own project or through a financial intermediary in exchange for a fee. The intermediary pools numerous individual projects and discovers the state of the economy. Thanks to its superior information and its ability to eliminate project-specific risks, it offers a higher return and a lower risk on capital, thereby promoting growth. Greenwood and Jovanovic (1990) show that economic and financial development feed on each other, as in our model. Their dynamics are driven by the cost of financial intermediation which includes a fixed fee akin to our information cost. But they do not specify where investors' private signals (projects) come from nor how they are pooled. In particular, they do not study agents' incentives to produce and communicate private information. In contrast, we explicitly address these issues: we model how investors make their decisions to collect costly signals, and how the stock market aggregates and transmits these signals. Moreover, we can characterize the evolution of several observable features of the stock market as the economy grows, such as the volatility of stock returns and the trading intensity. Finally, we differ from Greenwood and Jovanovic (1990) in that they obtain a permanent growth effect while we do not.

⁸Many papers highlight the different functions fulfilled by financial institutions, such as monitoring managers, improving risk management, mobilizing savings and facilitating the exchange of goods and services. An important function consist in identifying the best investment opportunities, as in our paper. For example, King and Levine (1993), Acemoglu, Aghion and Zilibotti (2003) and Morales (2003) argue that financial intermediaries such as banks promote growth by selecting the best entrepreneurs. These papers do not deal specifically with stock markets and their information processing role.

But this is only because they assume that capital displays constant returns to scale while we assume that it is subject to diminishing returns.

Second, our work is connected to the endogenous growth literature (e.g. Romer (1986, 1990), Aghion and Howitt (1992), Grossman and Helpman (1991)). This literature models the discovery of technologies by profit-maximizing agents. In contrast to this literature, we *endow* the economy with technologies and focus instead on their selection by investors trading on the stock market. Similar issues arise nonetheless. In particular, technical innovations and information about stocks both give rise to increasing returns to scale, limited by the incomplete appropriability of the rents generated.⁹ Whether long-run growth is possible or not depends essentially on the law of motion postulated for technological progress rather than on the structure of the models.¹⁰ When technological progress is assumed away, we find that the information technology cannot generate any permanent growth effect. Finally, our work belongs to the body of research, too large to reference, on trading under endogenous and asymmetric information. A subset emphasizes the real benefits of informational efficiency. Our model contributes to this literature by developing a rational expectations framework in which income and learning interact dynamically.

The remaining of the paper is organized as follows. Section 2 describes the economy. Section 3 studies a benchmark economy in which the first best is achieved. Section 4 characterizes the equilibrium. Section 5 examines the dynamics of income. Section 6 discusses the role of the stock market. Section 7 derives predictions concerning the real and financial properties of the economy during its transition to the steady-state. Section 8 shows how the economy can emerge from or fall into a no-information regime. Section 9 concludes. Proofs are featured in the appendix

⁹Unlike standard goods, information is non-rival, i.e. it is costly to generate but costless to replicate. This property, which applies to financial information (information about stock returns) as well as to technological knowledge (such as the design for a new good), leads to increasing returns: the cost of information is fixed while its benefit rises with the scale of its applications (the number of shares traded or the number of goods sold). See Jones (2004) for an overview of the importance of this insight for endogenous growth theory. For applications to finance, see Acemoglu and Zilibotti (1999), Arrow (1987), Peress (2008) and Van Nieuwerburgh and Veldkamp (2006a, 2006b), Veldkamp (2005a, 2005b, 2006) and Zeira (1994). While models of endogenous growth and models of stock selection incorporate the scale effects of information, they differ in the way they preserve incentives to do research. The former grant some market power to innovators, while the latter introduce noise into the price system.

¹⁰For example, if the rate of growth of technological knowledge, dA/dt , increases linearly with the level of technological knowledge, A , as in Romer (1990), then the economy grows without bound. Otherwise, growth is only transitory. As Romer (1990, page 84) puts it, “linearity in A (in the equation for dA/dt) is what makes unbounded growth possible, and, in this sense, unbounded growth is more like an assumption than a result of the model”.

2 Economic Environment

We embed a competitive stock market *à la* Grossman and Stiglitz (1980) into Diamond's (1965) neo-classical growth economy. The economy is composed of two sectors – a final and an intermediate goods sector, and overlapping generations of agents. Firms in the intermediate goods sector raise capital on the stock market by issuing claims to their future profits. Young agents save by purchasing these claims.

2.1 Agents

The economy is populated by overlapping generations of agents who live for two periods. There is no population growth. Each generation consists of a continuum of agents with mass L indexed by $l \in [0, L]$. Young agents are each endowed with one unit of labor time and one unit of free time. Utility, derived from the consumption of the final good g and leisure j , is represented by a function $U(g, j)$, increasing and concave in each argument and with a positive cross-derivative, $\partial^2 U / \partial g \partial l$. Two aspects of preferences are of particular relevance to our analysis: risk aversion and the degree of substitutability between final goods and leisure. We define the following functions:

$$\tau(g) \equiv -\frac{\frac{\partial U}{\partial g}(g, 1)}{\frac{\partial^2 U}{\partial g^2}(g, 1)} \quad \text{and} \quad \rho(g) \equiv \frac{\frac{\partial U}{\partial j}(g, 1)}{\frac{\partial U}{\partial g}(g, 1)}.$$

$\tau(g)$ measures the absolute risk tolerance of an agent consuming g units of the final good and one unit of leisure. Attitudes toward risk are entirely determined by the curvature of the utility function with respect to the consumption of the final good, because leisure consumption is not uncertain in our setting. We assume that τ is increasing in g , an assumption that is supported by virtually all empirical studies. The function ρ measures the marginal rate of substitution between final goods and leisure, again for an agent consuming g units of the final good and one unit of leisure. Naturally, ρ is increasing in g because the marginal utility of the final good declines while that of leisure rises when more final goods are consumed.

For example, $U(g, j) \equiv (\varpi g^\sigma + (1 - \varpi)j^\sigma)^{1/\sigma}$, where ϖ is in $(0, 1)$ and $\sigma < 1$, displays a constant elasticity of substitution (CES). The case $\sigma = 0$ corresponds to Cobb-Douglas utility ($U(g, j) \equiv g^\varpi j^{1-\varpi}$). Under these preferences, $\tau(g) = g(\varpi g^\sigma + 1 - \varpi)/(1 - \sigma)/(1 - \varpi)$ and $\rho(g) = g^{1-\sigma}(1 - \varpi)/\varpi$ – the elasticity of substitution between goods and leisure equals $1/(1 - \sigma)$.

Young agents are employed in the final good sector, to which they supply their unit of labor time inelastically for a competitive wage w_t , so aggregate labor supply equals L . They save their entire labor income by investing in the stock market to consume in the next period when they are old.¹¹ They divide their unit of free time between enjoying leisure and analyzing stocks. There are no short-sales constraints.

2.2 Technologies

2.2.1 Final Good Sector

The final good is produced according to a riskless technology that employs labor and intermediate goods:

$$G_t \equiv L^{1-\beta} \sum_{m=1}^M (Y_t^m)^\beta,$$

where G_t is final output, L is labor, M is the number of types of intermediate goods, Y_t^m is the employment of the m 'th type and $0 < \beta < 1$ is the factor share of intermediate goods in the production of the final good. The production function follows Spence (1976), Dixit and Stiglitz (1977) and Romer (1987, 1990) among others. The final good is used as the numeraire. Many identical firms compete in the final good sector and aggregate to one representative firm.

2.2.2 Intermediate Good Sector

M firms operate in the intermediate goods sector. Firm m is the exclusive producer of good m . Its production is determined by a risky technology that displays constant returns to capital:

$$\tilde{Y}_{t+1}^m \equiv \tilde{A}_t^m K_t^m \quad \text{for } m = 1, \dots, M$$

where \tilde{Y}_{t+1}^m is the quantity of goods produced in period $t+1$ by firm m net of capital depreciation, \tilde{A}_t^m is its random productivity and K_t^m is the amount of capital it raises in period t . Tildes denote random variables not yet realized. Firms are liquidated immediately after production.¹²

¹¹Thus the saving rate is exogenously set to one. We make this assumption not only to simplify the model but also because the evidence suggests that financial development enhances growth through higher productivity rather than through higher saving rates (Levine and Zervos (1998), Beck, Levine and Loayza (2000)).

¹²Assuming firms are liquidated just after production simplifies the dynamics of the economy and allows to focus on the early stage of a firm's development. It is well known that young firms, because they have little retained earnings, are more dependent on external financing than mature firms. Several empirical studies confirm that financial development fosters growth mainly through the former (Rajan and Zingales (1998), Kumar, Rajan and Zingales (1999), Demirgüç-Kunt and Maksimovic (1998), Beck, Demirgüç-Kunt and Maksimovic (2001), Love (2003), Brown, Fazzari and Petersen (2008)).

The productivity shocks \tilde{A}_t^m are assumed to be log-normally distributed and independent from one another and over time. Because there is no closed-form solution to investors' portfolio choice under general preferences, we resort to a small-risk expansion to solve the model. We assume that productivity shocks are small and log-linearize the return on investors' portfolio (e.g. Campbell and Viceira (2002)). Specifically, we assume that $\ln \tilde{A}_t^m \equiv \tilde{a}_t^m z$ where $\tilde{a}_t^m z$ is normally distributed with mean $\tilde{\alpha}_t^m z$ and variance $\sigma_a^2 z$, $\tilde{\alpha}_t^m$ is normally distributed with mean 0 and variance σ_α^2 and z is a scaling factor. The model is solved in closed-form by driving z toward zero. Throughout the paper, we assume that z is small enough for the approximation to be valid.¹³

Firms raise capital in the stock market. Firm m issues one perfectly divisible share – a claim to its entire future profit, for a price P_t^m . The productivity shock \tilde{a}_t^m is not observed at the time agents invest but they can learn about its average $\tilde{\alpha}_t^m$ as we describe next.

2.2.3 Information Technology

At the time they invest, agents do not observe intermediate firms' productivity. Instead, they receive private signals about its average. The private signal $s_{l,t}^m$ received by agent l in period t about firm m 's average productivity shock is given by:

$$s_{l,t}^m = \beta \tilde{\alpha}_t^m + \tilde{\varepsilon}_{l,t}^m,$$

where $\tilde{\varepsilon}_{l,t}^m$ is an agent-specific disturbance independent of \tilde{a}_t^m , $\tilde{\alpha}_t^m$, across firms and time. $\tilde{\varepsilon}_{l,t}^m$ is normally distributed with mean 0 and variance $1/x_{l,t}^m$ (precision $x_{l,t}^m$). Investors choose the precision of their signals before the stock market opens. Observing a signal of precision $x_{l,t}^m$ costs $C(x_{l,t}^m)z$ units of free time, where C is continuous, increasing, convex and $C(0) = C'(0) = 0$. We emphasize that the information technology does not lead to the discovery of new physical technologies nor improve existing ones. Instead, it allows to allocate capital more efficiently to the physical technologies.

¹³Rational expectations models of competitive stock trading under asymmetric information typically conjecture that equilibrium stock prices are linear functions of random variables. This conjecture is not valid in a neoclassical framework because productivity and capital interact multiplicatively in the production of goods, and capital itself is a function of stock prices.

2.2.4 Noise Trading

Agents know that stock prices reflect other investors' private information in equilibrium, and they learn from them. Some noise is needed to blur price signals and avoid the Grossman-Stiglitz paradox, that is, preserve incentives to collect costly information. We assume that a fraction q of agents form their portfolio guided by exogenous shocks. The source of these shocks is not specified but they could stem from liquidity needs, preference shifts, random stock endowments, private risky investment opportunities, or some form of irrationality. Specifically, noise traders believe that the expected return on stock m equals $\tilde{\theta}_t^m$, where $\tilde{\theta}_t^m$ is normally distributed with mean 0 and variance σ_θ^2 , and is independent of \tilde{a}_t^m , $\tilde{\varepsilon}_{l,t}^m$, across firms and time.¹⁴

2.3 Timing

The timeline is summarized in figure 1. An agent lives one period as a young agent (as a worker, then as an investor) and one period as an old agent (as a consumer). After earning a wage and before the stock market opens, workers choose how to divide their free time between stock analysis and leisure, by setting the precision of their signals. Then, they invest their wage across the different stocks, guided by stock prices and their private signals. In the following period, the young become old, productivity shocks are revealed, final goods are produced and old agents consume their share of profits.

2.4 Notation

For any firm-specific variable ψ_t^m , $\bar{\psi}_t$ denotes its average across firms and $\Delta\psi_t^m$ its deviation from the average:

$$\bar{\psi}_t \equiv \frac{1}{M} \sum_{m=1}^M \psi_t^m \quad \text{and} \quad \Delta\psi_t^m \equiv \psi_t^m - \bar{\psi}_t.$$

The variable enclosed in brackets, $\{\psi_t^m\}$, represents the vector of stacked variables for $m = 1$ to M .

Finally, we adopt the following notation to keep track of the quality of the approximation: $o(1)$, $o(z)$

¹⁴Several comments on the formulation of noise traders' beliefs may be useful. First, their accuracy arbitrary and does not affect our findings. Second, including an agent-specific component to these beliefs has no incidence on the equilibrium. Third, the intensity of noise trading remains commensurate with that of rational trading as the economy grows. As equation 8 below shows, portfolio holdings are scaled by a function of income, $\tau(\varphi(w))/\varphi(w)$. If for example this function increases with income (e.g. $\sigma > 0$ under CES utility), then trades, both rational and noise-motivated, grow with the economy. If we assumed instead that noise trades equal an exogenous constant, then they would shrink relative to rational trades. This would mechanically make stock prices more informative and the allocation of capital more efficient, and reinforce the usefulness of the stock market.

and $o(z^2)$ capture respectively terms of an order of magnitude smaller than 1, z and z^2 .

2.5 Equilibrium Concept

We describe the equilibrium concept working backwards from production in period $t + 1$, to capital allocation and information acquisition in period t . The gains from trade depend on how much information is collected in aggregate and revealed through prices. We denote $X_t^m \equiv \int_l x_{l,t}^m/L$ the average precision of private information about firm m . A rational expectations equilibrium satisfies the following conditions.

1. Market clearing in the intermediate goods sector

Final goods producers maximize their profit. Since labor and intermediate goods trade in competitive markets and aggregate labor supply equals L , the following equilibrium factor prices obtain in period $t + 1$:

$$\tilde{w}_{t+1} = (1 - \beta) \sum_{m=1}^M (\tilde{Y}_{t+1}^m/L)^\beta \quad \text{and} \quad \tilde{\rho}_{t+1}^m = \beta(L/\tilde{Y}_{t+1}^m)^{1-\beta},$$

where $\tilde{\rho}_{t+1}^m$ denotes the price of intermediate good m in period $t + 1$ and $\tilde{\Pi}_{t+1}^m = \tilde{\rho}_{t+1}^m \tilde{Y}_{t+1}^m$ is firm m 's profit.

2. Capital allocation

Let $f_{l,t}^m$ denote the fraction of her wage that agent l invests in stock m in period t or her ‘portfolio weights’. She sets $\{f_{l,t}^m\}$ to maximize her expected utility, guided by stock prices and private signals, and taking as given her income w_t , her leisure time j_t , the precision of her signals $\{x_{l,t}^m\}$, the average precisions $\{X_t^m\}$, share prices and capital stocks:

$$\max_{\{f_{l,t}^m\}} E[U(\tilde{g}_{l,t+1}, j_t) \mid \mathcal{F}_{l,t}] \quad \text{subject to} \quad \begin{cases} \tilde{g}_{l,t+1} = w_t \tilde{R}_{l,t+1} \\ \tilde{R}_{l,t+1} = \sum_{m=1}^M f_{l,t}^m \tilde{R}_{t+1}^m, \\ \sum_{m=1}^M f_{l,t+1}^m = 1 \end{cases} \quad (1)$$

where $\mathcal{F}_{l,t} \equiv \{s_{l,t}^m, P_t^m \text{ for } m = 1 \text{ to } M\}$, $\tilde{g}_{l,t+1}$, $\tilde{R}_{l,t+1}$ and $\tilde{R}_{t+1}^m = \tilde{\Pi}_{t+1}^m/P_t^m$ denote respectively agent l 's information set, her consumption of the final good, the return on her portfolio and the return on stock m . We call $U_0(\{x_t^m, X_t^m\}, j_t, w_t)$ the value function for this problem.

In equilibrium, prices clear the stock market. Since each firm issues one share, its capital stock

coincides with its stock price: Formally,

$$\int_l w_t f_{l,t}^m = K_t^m = P_t^m \quad \text{for } m = 1, \dots, M,$$

where the integral sums up the demand emanating from rational and noise traders.

3. Precision choice

An agent's optimal precisions $x_{l,t}^m = x(w_t, \{X_t^m\})$ maximize her *ex ante* expected utility subject to her free time budget constraint, taking her income w_t and the average precisions $\{X_t^m\}$ as given:

$$\max_{\{j_t \geq 0, x_{l,t}^m \geq 0\}} E[U_0(\{x_{l,t}^m, X_t^m\}, j_t, w_t)] \quad \text{subject to} \quad \sum_{m=1}^M C(x_{l,t}^m)z + j_t = 1,$$

where $C(x_{l,t}^m)z$ is the time spent investigating stock m and $1 - \sum_{m=1}^M C(x_{l,t}^m)z$ is the time left for leisure.

In equilibrium, the average and optimal precisions must be consistent:

$$X_t^m = x(w_t, \{X_t^m\}) \quad \text{for } m = 1, \dots, M.$$

3 First Best

Before we proceed to the general case, we describe the first-best outcome, in which agents perfectly share their private information. It will serve as a benchmark when we examine the role of the stock market. The first-best is achieved when signal precisions are contractible. In that case, agents all commit to infinitesimal precisions – very close but not equal to zero, and reveal their private signals to a central planner who invests on their behalf. The central planner can perfectly infer productivity shocks thanks to the Law of Large Numbers because there is a continuum of signals with finite variances and uncorrelated errors ($\int_l \varepsilon_{l,t+1}^m = 0$). The central planner chooses capital allocations $\{K_t^{mFB}\}$ to maximize agents' expected utility subject to an economy-wide resource constraint, taking as given their income w_t :

$$\max_{\{K_t^{mFB}\}} E[U(\tilde{g}_{l,t+1}, 1) | \{\tilde{\alpha}_t^m\}] \quad \text{subject to} \quad \begin{cases} \tilde{g}_{l,t+1} = \sum_{m=1}^M \tilde{\Pi}_{t+1}^{mFB} / L \\ \sum_{m=1}^M K_t^{mFB} = Lw_t \end{cases}, \quad (2)$$

where $\tilde{\Pi}_{t+1}^{mFB} = \beta L^{1-\beta} (\tilde{A}_t^m K_t^{mFB})^\beta$ denotes the profit generated by firm m , to be divided equally between agents. The following lemma describes the capital allocation in this economy.

Lemma 1

In the first-best outcome, firm m 's capital stock equals $K_t^{mFB} = \frac{Lw_t}{M} \exp(k_t^{mFB} z) + o(z)$ where

$$k_t^{mFB} = \frac{1}{1-\beta} \Delta \beta \tilde{\alpha}_t^m. \quad (3)$$

When z , the factor that scales shocks, equals zero, the firms are perfectly identical so capital is equally distributed across them, each firm receiving Lw_t/M units of goods.¹⁵ When $z > 0$, the allocation depends on firms' productivity relative to one another. The more productive firms (higher $\Delta \tilde{\alpha}_t^m \equiv \tilde{\alpha}_t^m - \overline{\tilde{\alpha}_t}$) receive more capital. The elasticity of investments to productivity shocks, $\partial(\ln K_t^{mFB})/\partial(\ln \tilde{A}_t^m) = 1/(1-\beta)$, captures the efficiency of the capital allocation. It increases with β , the factor share of capital because a higher β indicates that firms' marginal profits decline with their stock of capital at a slower rate, so more capital can be invested in the better firms without immediately damaging their return.

Given its capital stock, firm m produces $\tilde{Y}_{t+1}^m = \tilde{A}_t^m K_t^{mFB}$ intermediate goods. As a result, the number of final goods produced is:

$$\tilde{G}_{t+1} = Lw_t^\beta M^{1-\beta} \exp(\beta(\tilde{\alpha}_t^m z + k_t^{mFB} z)),$$

and the wage equals:

$$\tilde{w}_{t+1} = (1-\beta)\tilde{G}_{t+1}/L = (1-\beta)w_t^\beta M^{1-\beta} \exp(\beta(\tilde{\alpha}_t^m z + k_t^{mFB} z)).$$

The wage is random as it depends on the realizations of the productivity shocks. The following lemma characterizes the dynamics of the economy along its average path, i.e. assuming that the wage realized in any period equals its mean. This is a good description of the economy if the number of firms is large.

Lemma 2

In the first-best outcome, average income evolves according to the following equation:

$$E(\tilde{w}_{t+1}) = \Lambda \exp(\lambda^{FB} z^2) w_t^\beta, \quad (4)$$

where Λ and λ^{FB} are two positive constants given by:

$$\Lambda \equiv (1-\beta)M^{1-\beta} \exp\left(\frac{1}{2}\beta^2(\sigma_a^2 z + \sigma_\alpha^2 z^2)\right), \quad (5)$$

and

$$\lambda^{FB} \equiv \frac{M-1}{M} \frac{\beta^3}{(1-\beta)^2} \left(1 - \frac{\beta}{2}\right) \sigma_\alpha^2 + o(1). \quad (6)$$

¹⁵Firm m 's marginal profit, $\partial \Pi_{t+1}^{mFB} / \partial K_t^{FBm} = \partial[\beta L^{1-\beta} (A_t^m K_t^{FBm})^\beta] / \partial K_t^m = \beta^2 L^{1-\beta} A_t^{m\beta} K_t^{FBm\beta-1}$, is a decreasing function of K_t^{FBm} . Hence, if firms are identical, the central planner distributes capital equally across the M firms.

Average income converges to a steady-state, w^{FB} , given by:

$$w^{FB} = \Lambda^{1/(1-\beta)} \exp\left(\frac{\lambda^{FB}}{1-\beta} z^2\right). \quad (7)$$

The average wage evolves according to a standard neoclassical law of motion. The marginal product of labor increases with current income (assuming income is initially below its steady-state value) but at a decreasing rate, until it reaches a steady-state in which it no longer grows. The growth rate of income is given by $\Gamma^{FB}(w_t) \equiv E(\tilde{w}_{t+1})/w_t = \Lambda w_t^{-(1-\beta)} \exp(\lambda^{FB} z^2)$. It declines at the rate $-(1-\beta)$, i.e. $d \ln \Gamma^{FB}(w_t)/d \ln w_t = -(1-\beta)$. The steady-state level of income w^{FB} solves $w^{FB} = \Lambda w^{FB\beta} \exp(\lambda^{FB} z^2)$, which leads to equation 7. The dashed curves in figures 6 and 7 illustrate the dynamics of income in the first best. Steady-state income increases with the number of intermediate goods M as the production possibility set expands, and with the variance of productivity shocks $\sigma_a^2 z + \sigma_\alpha^2 z^2$ because output is a convex function of these shocks – a positive shock increases \tilde{G}_{t+1} more than a negative shock decreases it. It decreases with the factor share of intermediate goods β as the marginal product of labor is reduced.

The first best obtains in particular in Greenwood and Jovanovic (1990). In their model, a financial intermediary pools numerous projects (signals) supplied by individuals and discovers the state of the economy. The reason the first best is achieved in their equilibrium is that agents are endowed with a project rather than produce it at a cost. Here in contrast, the first-best is not achievable because agents cannot commit to strictly positive signal precisions. Indeed, suppose all investors do agree to acquire some information about a stock, however imprecise, and to report it to the central planner. This will allow the planner to learn the stock's productivity shock. Given that the cost of information is not zero, the optimal strategy for an agent is to deviate from the agreement, i.e. to not collect any information and make a random announcement to the central planner. But if all agents make random announcements, then the productivity shock cannot be learned. Thus, the first-best outcome cannot be reached if signal precisions are not contractible.

4 Equilibrium Characterization

The remainder of the paper assumes that signal precisions are not contractible and that some trades are motivated by noise. In that case, the stock market offers a way to share information, albeit imperfectly.

We characterize first investors' portfolios and the allocation of capital, and then information acquisition decisions. Throughout this section, we take as given investors' income w_t which we endogenize in the next section.

4.1 Capital Allocation

We follow the usual method for solving a noisy rational expectations equilibrium: We guess that capital is a log-linear function of shocks, solve for portfolio, derive the equilibrium capital allocation, and check that the guess is valid. The following lemma displays investors' portfolio composition for the conjectured capital allocation.

Lemma 3

Assume that firm m 's capital stock takes the form $K_t^m = \frac{Lw_t}{M} \exp(k_t^m z)$ where $k_t^m \equiv k_{\alpha t}^m (\beta \Delta \tilde{\alpha}_t^m + \mu_t^m \Delta \tilde{\theta}_t^m) + o(z)$ and μ_t^m is a deterministic scalar. Agent l 's portfolio weights are given by:

$$f_{l,t}^m = \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2 z} E(\Delta \ln R_{t+1}^m | \mathcal{F}_{l,t}) + o(1), \quad (8)$$

$$\text{where } \varphi(w) \equiv \beta M^{1-\beta} w^\beta. \quad (9)$$

- The portfolio weights for a rational agent l who receives private signals of precision $\{x_{l,t}^m\}$ are given by:

$$f_{l,t}^m = \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \left\{ \frac{x_{l,t}^m}{H(\mu_t^m) + x_{l,t}^m} \Delta s_{l,t}^m - \left(\frac{1}{(H(\mu_t^m) + x_{l,t}^m)\mu_t^{m2}\sigma_\theta^2 k_{\alpha t}^m} - (1-\beta) \right) k_t^m \right\} + o(1). \quad (10)$$

$$\text{where } H(\mu) \equiv \frac{1}{\beta^2\sigma_a^2} + \frac{1}{\mu^2\sigma_\theta^2}. \quad (11)$$

- The portfolio weights for a noise trader are given by:

$$f_t^m = \frac{1}{M} + \frac{\tau(\varphi(w_t))}{\varphi(w_t)\beta^2\sigma_a^2} \Delta \tilde{\theta}_t^m + o(1). \quad (12)$$

Stock m 's portfolio weight equals the weight it would receive if firms were identical, $1/M$, tilted by a measure of the stock's expected excess performance relative to the market, $E(\Delta \ln \tilde{R}_{t+1}^m | \mathcal{F}_{l,t}) \equiv E(\ln \tilde{R}_{t+1}^m - \overline{\ln R_{t+1}} | \mathcal{F}_{l,t})$. The deviation from equal portfolio shares is more pronounced when stocks are less risky (lower β or σ_a^2), or when agents are relatively more risk tolerant. $\tau(\varphi(w_t))$ measures investors' absolute risk tolerance in a neighborhood of their consumption – to a first approximation (at the order 0 in z), they consume $\varphi(w_t)$ units of the final good. Relative risk tolerance, the ratio

of absolute risk tolerance to consumption, $\tau(\varphi(w_t))/\varphi(w_t)$, determines how aggressively investors trade on their information. Though absolute risk tolerance $\tau(\varphi(w))$ rises with income by assumption, this need not be the case for relative risk tolerance, $\tau(\varphi(w))/\varphi(w)$. For example, under CES preferences $\tau(\varphi(w))/\varphi(w) = (\varpi\beta^\sigma M^{\sigma(1-\beta)} w^{\sigma\beta} + 1 - \varpi)/(1 - \sigma)/(1 - \varpi)$. If $\sigma > 0$, then $\tau(\varphi(w))/\varphi(w)$ increases with income, and wealthier investors' portfolio weights deviate more from equal shares. If instead $\sigma < 0$, then $\tau(\varphi(w))/\varphi(w)$ decreases with income and wealthier investors' portfolio weights deviate less from equal shares. If $\sigma = 0$ (Cobb-Douglas utility), then $\tau(\varphi(w))/\varphi(w)$ is a constant, $1 - \varpi$, so portfolio weights are independent of wealth as in the case of constant relative risk aversion.

Equation 10 expresses portfolio weights as a combination of the stock price (the k_t^m term) and the relative private signal (the $\Delta s_{t,t}^m$ term). In this expression, the stock price plays a dual role: it clears the stock market and provides information about the firm's productivity. Given our conjecture, observing stock prices is equivalent to observing $\beta\Delta\alpha_t^m + \mu_t^m\Delta\theta_t^m$ for each firm, a signal about $\beta\Delta\alpha_t^m$ with error $\mu_t^m\Delta\theta_t^m$. Thus, μ_t^m represents the *noisiness of stock m 's price*. The function $H(\mu_t^m) + x_{t,t}^m = 1/\text{Var}(\beta\alpha_t^m | \mathcal{F}_{t,t})$ measures the total precision of an investor's information about a stock. She receives information from three sources: her priors (the $1/(\beta^2\sigma_\alpha^2)$ term), the price (the $1/(\mu_t^{m2}\sigma_\theta^2)$ term) and her private signal (the $x_{t,t}^m$ term), and their precisions simply add up.

The next proposition describes the equilibrium allocation of capital, taking as given the information environment. It can be characterized equivalently in terms of the average precisions about stocks X_t^m , or in terms of their price noisiness μ_t^m , as a one-for-one mapping connects the two variables. The proposition presents the equilibrium capital allocation for an arbitrary level of noisiness μ_t^m (we will solve for the equilibrium value of μ_t^m in Proposition 5).

Proposition 4

Let $\mu_t^m (> \frac{q}{1-q})$ be the noisiness of stock m 's price. There exists a log-linear rational expectations equilibrium in which firm m 's capital stock and its share price equal $K_t^m = P_t^m = \frac{Lw_t}{M} \exp(k_t^m z) + o(z)$ where:

$$k_t^m \equiv k_\alpha(\mu_t^m)(\beta\Delta\tilde{\alpha}_t^m + \mu_t^m\Delta\tilde{\theta}_t^m), \tag{13}$$

$$k_\alpha(\mu) \equiv \frac{1}{1-\beta} \left(1 - \frac{1}{\beta^2\sigma_\alpha^2(H(\mu) + X(\mu))} \right) > 0, \tag{14}$$

$$\text{and } X(\mu) \equiv \frac{H(\mu)}{\frac{1-q}{q}\mu - 1}. \tag{15}$$

The proposition establishes that capital and stock prices are approximately log-linear functions of productivity and noise shocks. As in the first best, they equal those that would obtain if firms were identical (Lw_t/M), disturbed by an order- z function of shocks. Productivity shocks appear directly in the price function though they are not known by any agent, because individual signals, $\tilde{s}_{t,t}^m$, once aggregated, collapse to their mean, $\beta\tilde{\alpha}_t^m$. Noise traders' introduce noise $\tilde{\theta}_t^m$ into the price system through their trades. Stock prices are defined up to a period t -measurable multiplicative constant of order z . We choose a normalization that preserves symmetry.¹⁶ As mentioned, the average precision X_t^m and stock price noisiness μ_t^m are related one for one through equation 15. A higher noisiness μ_t^m corresponds to a lower average precision X_t^m , as figure 2 illustrates.

Proposition 4 outlines the allocative function performed by the stock market. Equation 13 implies that capital and technology shocks are positively correlated. The key parameter is $k_\alpha \equiv \partial(\ln K_t^m)/\partial(\ln \tilde{A}_t^m)$, the elasticity of investments to productivity shocks. A positive k_α means that funds tend to flow to the most productive firms. Moreover, it increases with the quality of information. When there is no information (μ_t^m is infinite and $X_t^m = 0$), $k_\alpha = 0$ so capital is allocated independently from productivity shocks. k_α increases as information improves (μ_t^m falls) until it reaches $1/(1 - \beta)$ under perfect information ($\mu_t^m = q/(1 - q)$ and X_t^m is infinite). In this limiting case, the elasticity of investments to productivity shocks coincides with that of the first best. To summarize, better-informed economies allocate capital more efficiently.

Proposition 4 makes clear the information processing role of the stock market. This can best be understood by comparison to a fictitious economy in which agents collect the same private signals but stock prices do not reveal any of their content. In such an economy, the average precision $X(\mu_t^m)$ is the same as in the 'normal' economy, but an investor's total precision is lower because the precision of the price signal, $1/(\mu_t^{m2}\sigma_\theta^2)$, is lost – the total precision equals $1/(\beta^2\sigma_\alpha^2) + X(\mu_t^m) < H(\mu_t^m) + X(\mu_t^m)$. Accordingly, the elasticity of investments to productivity shocks falls to $(1 - 1/(1 + \beta^2\sigma_\alpha^2 X(\mu_t^m)))/(1 - \beta)$ which is below $k_\alpha(\mu_t^m)$. The allocation of capital is not as efficient though the same private signals were produced. Thanks to the stock market, private signals do not only serve the agents who observe them

¹⁶If $\{K_t^m\}$ clears the market for the M stocks, so does $\{K_t^m \times \exp(v_t z)\}$ for any t -measurable scalar v_t . Indeed as lemma 3 makes clear, stock demands do not depend on absolute returns but on returns relative to the market. We normalize stock prices such that the geometric average stock price is independent of the realized shocks as in the first best, i.e. by setting $(\prod_{m=1}^M K_t^m)^{1/M} = Lw_t/M$.

but benefit all through prices. Investors who collect private signals of precision $X(\mu_t^m)$ actually receive signals of precision $X(\mu_t^m) + 1/(\mu_t^{m2}\sigma_\theta^2)$. In short, the stock market allows investors to share their information (the *ex post* information sharing effect). Moreover, this effect is more powerful when noise trading weakens, i.e. when q or σ_θ^2 decline. Importantly, since investors communicate their private information through their trades, its transmission is incentive-compatible. This is an essential quality for an information sharing mechanism when signals are costly to acquire and privately observed.¹⁷

4.2 Information Acquisition

We turn to the information acquisition decisions. The following lemma characterizes how much free time an investor devotes to learning about productivity shocks for an arbitrary level of stock price noisiness μ_t^m , and given her income w_t .

Lemma 5

Let μ_t^m ($> \frac{q}{1-q}$) be the noisiness of stock m 's price. Investors set the precision of their private signal about stock m , x_t^m , such that

$$\rho(\varphi(w_t))C'(x_t^m) = \tau(\varphi(w_t))\frac{M-1}{2M\beta^2\sigma_a^2} \left(\frac{1}{H(\mu_t^m) + x_t^m} \right)^2. \quad (16)$$

Investors choose a signal precision that equates the marginal benefit of information to its marginal cost, taking into account how much is revealed through stock prices. The left hand side of equation 16 represents the marginal cost and can be interpreted as follows. Increasing the precision of a signal from x to $x + \delta$ requires cutting leisure time by $C'(x)\delta$ units and suffering a utility loss of $\frac{\partial U}{\partial l}C'(x)\delta$. This loss is equivalent to a reduction in the consumption of the final good of $\frac{\partial U}{\partial l}C'(x)\delta/\frac{\partial U}{\partial g}$ units. Thus, the left hand side of equation 16 measures the utility cost, denominated in units of the final good, of a marginal increase in the signal precision. This cost depends on income through the coefficient $\rho(\varphi(w_t))$, which measures the marginal rate of substitution between goods and leisure in a neighborhood of consumption. This coefficient, and therefore the cost of information, increase with income because of a *substitution effect*: wealthier agents invest more, hence consume more of the final good, which decreases its marginal utility and makes leisure more enjoyable.

¹⁷Again, there is no incentive problem in Greenwood and Jovanovic (1990) because agents are endowed with a private signal about the state of the economy (a project).

The right hand side of equation 16 represents the utility benefit from a marginal increase in precision, again denominated in units of the final good. This benefit has the following properties. First, it rises when public information is less accurate – so private information acts as a substitute for public information. This happens when priors are less precise (σ_α^2 larger) or when stock prices are less informative (μ_t^m or σ_θ^2 larger). Indeed, stock prices reveal private signals, albeit partially, thereby limiting investors' ability to appropriate the full benefit from their information expenditures (the *ex ante* disincentive effect). Private information is more valuable when it is easier to conceal, i.e. when the price system is more noisy. Second, the benefit of private information decreases with the conditional variance of productivity shocks σ_a^2 because agents tilt less their portfolio weights away from equal shares. Third, it rises with investors' income through their absolute risk tolerance, τ . Indeed, discriminating across firms is more valuable when one has more to invest. Thanks to its non-rival nature, information can be applied to every dollar of investment without requiring its cost to be incurred repeatedly. Putting it differently, information generates increasing returns with respect to the scale of investments, captured by $\tau(\varphi(w))$.

Equation 16 admits a unique solution because its left hand side is monotonically increasing in x_t^m and spans the entire positive real line ($C'(0) = 0$ by assumption), while its right hand side is monotonically decreasing. Moreover, it implies that signal precisions are identical across agents for any stock m ($x_{l,t}^m = X_t^m$ for all l). The properties of x_t^m follow from those of the marginal cost and benefit of information. x_t^m rises when σ_α^2 , μ_t^m and σ_θ^2 are larger, and when σ_a^2 and C' are lower. Most of these properties obtain in the usual framework with exponential utility, normally distributed random variables and a riskless asset (e.g. Verrecchia (1982)).¹⁸

The influence of income depends on which of the marginal rate of substitution or risk tolerance is the more sensitive to income. It will be discussed in detail in the next section. Finally, the impact on x_t^m of the factor share of intermediate goods, β , is complex. First, a lower β reduces investors' share of GDP and their consumption (the $\varphi(w_t)$ term), which enhances the marginal utility of final goods so

¹⁸In an economy similar to ours except that i) preferences display constant absolute risk aversion with a coefficient of absolute risk tolerance τ , ii) stocks have normally distributed payoffs with variance σ_Π^2 and iii) a riskless asset with gross return R^f is available, the equilibrium precision of private signals solves $2R^f C'(x_t) = \tau/(H_t + x_t)$ where $H_t \equiv 1/\sigma_\Pi^2 + 1/(\mu_t^2 \sigma_\Theta^2)$ and σ_Θ^2 is the variance of noise trading. From this equation, x_t rises when σ_Π^2 , τ or $\mu_t^2 \sigma_\Theta^2$ increase or when C decreases.

both ρ and τ increase. Second, a lower β implies that stocks are less sensitive to productivity shocks. These shocks have a component that can be learnt (its average) and one that cannot, so the implications are twofold. On the one hand, a lower β means that the average productivity shock $\tilde{\alpha}_t^m$ has a smaller impact on a firm's profit so learning about it is less valuable (the term $1/\beta^2\sigma_\alpha^2$ embedded in $H(\mu_t^m)$ on the right hand side of equation 16). On the other hand, it implies that stocks are less risky so investors trade them more aggressively, which makes information more valuable (the $\beta^2\sigma_a^2$ on the right hand side of the equation).

The following proposition characterizes the degree of noisiness in equilibrium, μ_t^m , for a given level of income w_t .

Proposition 6

In equilibrium, the noisiness of stock prices, μ_t , is the unique solution to:

$$\rho(\varphi(w_t))C' \left(\frac{H(\mu_t)}{\frac{1-q}{q}\mu_t - 1} \right) = \tau(\varphi(w_t)) \frac{M-1}{2M\beta^2\sigma_a^2} \left(\frac{1 - \frac{q}{(1-q)\mu_t}}{H(\mu_t)} \right)^2. \quad (17)$$

The noisiness of prices in equilibrium is determined by observing that the individual and average precisions, x_t^m and X_t^m , coincide since agents choose identical precisions, and by substituting equation 15 which relates X_t^m to μ_t^m into the first-order condition 16 (this procedure amounts to searching for a fixed point to the system of equations, $X_t^m = x(w_t, \{X_t^m\})$ for $m = 1$ to M). The resulting noisiness and average precisions are identical across stocks so we drop the superscript m from now on ($X_t^m \equiv X_t$ and $\mu_t^m = \mu_t$ for all m). This implies further that individual precisions are identical across stocks ($x_t^m = x_t$ for all m). Equation 17 admits a unique solution μ_t for any level of income w_t , because its left hand side is monotonically decreasing in μ_t and spans the entire positive real line, while its right hand side is monotonically increasing. It is illustrated in figure 3.

The properties of the average precision X_t are identical to those of individual precisions x_t , discussed above. Those of the equilibrium noisiness μ_t follow. It decreases (i.e. stock prices are more informative) when when priors are more accurate (σ_α^2 smaller), when the variance of noise trades σ_θ^2 is larger, when the conditional variance of productivity shocks σ_a^2 or the marginal cost of information C' are lower. In contrast, μ_t increases with the fraction of noise traders q . This is because q has a direct effect on μ_t in equilibrium that dominates its indirect effect through X_t .

5 Dynamics of Income

In this section, we tie together investments, learning and income, and analyze the evolution of income along the economy's average path. We describe first qualitatively the interplay between learning and income. We start with the impact of a generation's noisiness on the next generation's income.

Lemma 7

Income is larger on average in the next period when the noisiness of stock prices is lower, for a given level of current income:

$$\frac{\partial E(w_{t+1})}{\partial \mu_t} \Big|_{w_t \text{ fixed}} < 0.$$

More accurate information leads to more efficient investments and a larger supply of intermediate goods on average. This in turn increases the marginal product of labor and the next generation's average income. The following lemma considers the reverse relationship, from income to noisiness.

Lemma 8

If τ/ρ is an increasing function of consumption, then the noisiness of stock prices falls with income:

$$\frac{d\mu_t}{dw_t} < 0.$$

If τ/ρ is a decreasing function of consumption, then the noisiness of stock prices rises with income:

$$\frac{d\mu_t}{dw_t} > 0.$$

We observed above that current income increases both the marginal cost of information (through a substitution effect) and its marginal benefit (through a scale effect). The impact of income on the equilibrium precision of information depends on which of these two effects dominates. If the scale effect dominates, i.e. the marginal benefit rises with income faster than the marginal cost does (τ/ρ increasing in consumption), then agents collect more information as they grow wealthier. If instead the substitution effect dominates (τ/ρ decreasing in consumption), then agents collect less information. Under CES utility for example, information improves with income if $\sigma > 0$, but deteriorates if $\sigma < 0$. The substitution and scale effects offset each other exactly under Cobb-Douglas utility ($\sigma = 0$, or constant relative risk aversion). In that case, income has no impact on the quality of information. Under constant absolute risk aversion – preferences that are commonly assumed in rational expectations models of trading under asymmetric information (e.g. $U(g, j) = (-\exp(-\tau g))v(j)$ or $U(g, j) = -\exp(-\tau g) +$

$v(j)$), the substitution effect works alone so the precision of information is a decreasing function of income. Figure 4 illustrates lemma 8. The following proposition ties together lemmas 7 and 8 to describe the dynamics of income.

Proposition 9

- *Average income evolves according to the following equation:*

$$E(\tilde{w}_{t+1}) = \Lambda \exp(\lambda(w_t)z^2) w_t^\beta, \quad (18)$$

where

$$\lambda(w_t) \equiv \frac{M-1}{M} \beta^2 \left(k_\alpha(\mu_t) \beta \sigma_\alpha^2 + \frac{k_\alpha(\mu_t)^2}{2} (\beta^2 \sigma_\alpha^2 + \mu_t^2 \sigma_\theta^2) \right) + o(1) > 0, \quad (19)$$

and Λ , k_α and $\mu_t = \mu(w_t)$ are defined respectively in equations 5, 14, and 17.

- *The economy converges to a steady-state in which it no longer grows. The steady-state level of income w^* is given by:*

$$w^* = w^{FB} \exp\left(-\frac{\lambda^{FB} - \lambda((1-\beta)^{1/(1-\beta)}M)}{1-\beta} z^2\right) < w^{FB}. \quad (20)$$

- *If τ/ρ is an increasing function of consumption, then λ increases with income from $\lim_{w_t \rightarrow 0} \lambda(w_t) = \frac{M-1}{2M} \left(\frac{\beta}{1-\beta} \frac{q}{1-q}\right)^2 \sigma_\theta^2 \equiv \lambda_0$ to $\lim_{w_t \rightarrow \infty} \lambda(w_t) = \lambda^{FB}$. If τ/ρ is a decreasing function of consumption, it decreases with income from $\lim_{w_t \rightarrow 0} \lambda(w_t) = \lambda^{FB}$ to $\lim_{w_t \rightarrow \infty} \lambda(w_t) = \lambda_0$.*

To a first approximation (at the order 0 in z), the dynamics of income are similar to those obtained when under the first-best: income grows at a declining rate until it reaches a steady-state w^* (assuming the wage is initially below w^*). Thus, the dynamics of income continue to be dominated by the neoclassical force of diminishing returns to capital – learning only generates a deviation of order z^2 from the neoclassical path. This is the case by construction in our model. Indeed, learning about productivity shocks generates benefits that are small since we *assume* these shocks to be small. We conjecture that this property extends to large shocks since income admits the first-best as an upper bound – that is, starting from the same arbitrary level of income, income in the next period is lower than in the first-best in which capital is more efficiently allocated – and income in the first-best eventually reaches a steady-state.

Proposition 9 is illustrated in figure 6 which displays the law of motion of income along the economy’s average path under CES utility (equation 18). The solid (dotted) curve corresponds to $\sigma = 0.5$ ($\sigma =$

−0.5), in which case information improves (deteriorates) with income. The steady-state is located at their intersection with the 45° line (solid line). If initial income w_0 is below (above) w^* , then the wage increases (decreases) until it reaches w^* .

The effect of learning on income is captured by the function λ , illustrated in figure ???. The steady-state level of income is lower than in the first-best. Its growth rate during the transition to the steady-state, $\Gamma(w_t) \equiv E(\tilde{w}_{t+1})/w_t$, is lower too than in the first best, by a factor $\exp [-(\lambda^{FB} - \lambda(w_t))z^2]$. Figure 7 depicts $\Gamma(w_t)$ for various utility functions as well as in the first-best economy. When the scale effect of information dominates the substitution effect (e.g., when $\sigma > 0$ under CES utility), investors collect more information as the economy grows, which contributes to growth further. As a result, the growth rate of income declines less quickly than in the first best:

$$\frac{d \ln \Gamma(w_t)}{d \ln w_t} = -(1 - \beta) + \frac{d\lambda(w_t)}{d \ln w_t} z^2 > -(1 - \beta),$$

where $-(1 - \beta) = d \ln \Gamma^{FB}(w_t)/d \ln w_t$ is the change in the growth rate of income in the first-best. Thus in this case, learning has a transitory beneficial effect on growth, that mitigates the negative neoclassical force. When the scale effect of information dominates the substitution effect (e.g., when $\sigma < 0$ under CES utility), investors collect less information as the economy grows, which slows down growth. So, the growth rate of income falls at a faster rate than in the first best:

$$\frac{d \ln \Gamma(w_t)}{d \ln w_t} = -(1 - \beta) + \frac{d\lambda(w_t)}{d \ln w_t} z^2 < -(1 - \beta).$$

6 The Role of the Stock Market

This section discusses the impact of the stock market in more detail. A simple approach to assessing its informational role is to vary the fraction of noise traders q . The following lemma describes the consequences of a reduction in noise trading.

Lemma 10

When the fraction of noise traders q decreases, less information is produced but more is shared through stock prices. The net effect is an improvement in total information, $H_t + X_t$, and in the efficiency of investments, captured by a higher $k_{\alpha t}$.

On the one hand, for a given precision of private signals, more information is conveyed through prices as noise trading weakens (the *ex post* information sharing effect). As a result, capital is more efficiently

deployed. Formally, $\partial\mu_t/\partial q > 0$, $\partial H(\mu_t)/\partial q < 0$ and $\partial k_\alpha(\mu_t)/\partial q < 0$ holding the average precision X_t fixed, and using respectively equations 15, 11, 14 and 19. On the other hand, agents collect less private information (the *ex ante* disincentive effect). This dampens the beneficial influence that information sharing has on capital efficiency, but does not reverse it. Formally, $d\mu_t/dq > 0$, $d(H(\mu_t) + X(\mu_t))/dq < 0$ and $dk_\alpha(\mu_t)/dq < 0$. Consider for example, the net effect on investors' total precision, $H(\mu_t) + X(\mu_t)$:

$$\frac{d(H(\mu_t) + X(\mu_t))}{dq} = \underbrace{\frac{\partial H_t}{\partial \mu_t}_{X_t \text{ fixed}} * \frac{\partial \mu_t}{\partial q}_{X_t \text{ fixed}}}_{< 0} + \underbrace{\frac{\partial H_t}{\partial \mu_t}_{X_t \text{ fixed}} * \frac{\partial \mu_t}{\partial X_t}_{q \text{ fixed}} * \frac{dX_t}{dq} + \frac{dX_t}{dq}}_{> 0}.$$

Ex post information sharing
Ex ante disincentive

The *ex post* information sharing effect more than compensates for the *ex ante* disincentive effect. Only under a linear information cost do these two effects exactly balance out. In that case, the left-hand side of equation 17 is constant, so must be the right-hand side, which implies that the total precision $H(\mu_t) + X(\mu_t)$ is constant regardless of q .

It should be noted that a fall in q does not always lead to an increase in the steady-state level of income and in its transitory growth rate. That is, higher noise trading can actually be beneficial to income in spite of harming the capital allocation. This is because it increases the variability of the capital allocation and therefore the average income, a convex function thereof, through a Jensen inequality effect (positive noise shocks increase output more than negative shocks decrease it). We do not elaborate on this effect (reflected in the term $\mu_t^2 \sigma_\theta^2$ in equation 19) because it is a direct consequence of the presence of noise in agents' beliefs, rather than the result of the information processing role of the stock market.

The following lemma assesses the efficiency of the stock market, by comparing the allocation of capital achieved through the stock market to the first best. Since we had to introduce noise into the stock market economy to avoid the Grossman-Stiglitz paradox, we make the comparison in the limiting situation in which noise vanishes, i.e. as the fraction of noise traders goes to zero.

Lemma 11

The allocation of capital achieved through the stock market converges to the first best allocation as the fraction of noise traders goes to zero:

$$\lim_{\substack{q \rightarrow 0 \\ q > 0}} k_t^m = k_t^{mFB} \quad \text{for } m = 1, \dots, M.$$

Moreover, the steady-state level of income and its transitory growth rate converge to those achieved in the first best:

$$\lim_{\substack{q \rightarrow 0 \\ q > 0}} w^* = w^{FB} \quad \text{and} \quad \lim_{\substack{q \rightarrow 0 \\ q > 0}} \Gamma(w_t) = \Gamma(w_t)^{FB}$$

The lemma establishes that the capital allocation achieved through the stock market can be made arbitrarily close to the first best allocation by making the fraction of noise traders q sufficiently small.¹⁹ It follows that the dynamics of income (the steady-state level of income and the growth rate during the transition) too can be made arbitrarily close to those obtained in the first best economy. Lemmas 10 and 11 are illustrated in figure 5 which displays μ_t , X_t , $H_t + X_t$, $k_{\alpha t}$, λ_t and w^* as a function q under CES utility.

Overall, the stock market, by aggregating and transmitting private information, contributes to the level of income in the long-run and to its growth rate during the transition. Empirically, Levine and Zervos (1998), Rousseau and Wachtel (2000) and Carlin and Mayer (2003) document that income grows faster in countries with better functioning stock markets.²⁰ Atje and Jovanovic (1993) estimate that this growth effect is permanent, but Harris (1997) finds that it is only transitory after controlling for possible endogeneity problems.²¹

7 Properties of the Growth Path

In this section, we derive various observable properties of the economy during its transition to the steady-state (for an initial wage below its steady-state level). Throughout, we assume that the scale effect of information dominates the substitution effect (e.g. $\sigma > 0$ under CES utility) so information about firms improves (μ_t declines) as the economy grows, in line with the evidence discussed below.

¹⁹However, q cannot exactly equal zero, else there is no equilibrium (the Grossman-Stiglitz paradox). The beneficial impact of higher noise trading on income resulting from a Jensen inequality effect vanishes as q approaches zero.

²⁰Levine and Zervos (1998) and Rousseau and Wachtel (2000) proxy for stock market development using measures of market capitalization trading volume, while Carlin and Mayer (2003) use accounting standards.

²¹Aghion, Howitt and Mayer-Foulkes (2005) also document that financial development only has a transitory growth effect for sufficiently advanced economies using measures of financial intermediation such as private credit rather than measures of stock market development. They propose a model of agency problems and credit constraints to explain their findings.

We start with the real side of the economy and then proceed to the financial side. The following two propositions characterize the efficiency and concentration of the capital allocation.

Proposition 12

The elasticity of investments to productivity shocks, $\frac{\partial \ln K_t^m}{\partial \ln \tilde{A}_t^m}$, and TFP increase as the economy grows.

Better-informed agents distribute capital more efficiently across firms, leading to a higher elasticity of investments to productivity shocks. This superior efficiency translates into higher TFP. We define TFP from the following economy-wide production function:

$$E(\tilde{G}_{t+1}) = ML^{1-\beta} E[(\tilde{A}_t^m K_t^m)^\beta] = L^{1-\beta} E(\tilde{A}_t^{m\beta}) E(K_t^m)^\beta \exp[Cov(\beta \tilde{a}_t^m z, \beta k_t^m z) - \beta(1-\beta)Var(k_t^m)/2], \tag{21}$$

where we interpret the term $\exp[Cov(\beta \tilde{a}_t^m z, \beta k_t^m z) - \beta(1-\beta)Var(k_t^m)/2]$ as TFP. It captures the additional output obtained from distributing capital in relation to productivity shocks, in comparison to an economy in which capital is arbitrarily allocated. From equation 13, TFP equals $\exp\{[k_\alpha \beta^3 \sigma_\alpha^2 - \beta(1-\beta)k_\alpha^2(\beta^2 \sigma_\alpha^2 + \mu_t^2 \sigma_\theta^2)/2]z^2(M-1)/M\}$ and increases when μ_t declines. We stress that technological progress is not required to generate growth in TFP. TFP grows thanks to a more efficient allocation of capital, keeping stationary the distribution of productivity shocks and the cost of information.

The empirical evidence is consistent with proposition 12. Wurgler (2000) constructs cross-country estimates of the elasticity of investments to value added, our parameter k_α . He finds that this elasticity increases with the country's degree of financial development, and in particular with the informativeness of its stock market. That is, countries with more informative stock markets increase investments more in their growing industries, and decrease investments more in their declining industries, than countries with less informative stock markets.²² These countries also tend to display higher TFP. Indeed, Levine and Zervos (1998) show that stock markets promote growth in total factor productivity.²³

²²Wurgler (2000) uses a proxy for informativeness developed by Morck, Yeung and Yu (2000). They measure the extent to which stocks move together and argue that prices move in a more unsynchronized manner when they incorporate more firm-specific information. This is indeed the case in the present model (see proposition 12). Durnev, Morck and Yeung (2004) and Durnev, Morck, Yeung and Zarowin (2003) report that the synchronicity measure is related to accounting estimates of stock price informativeness as well as to the efficiency of corporate investments captured by Tobin's q .

²³Levine and Zervos (1998) measure stock market development using the ratio of market capitalization to GDP, the ratio of the value of trades to GDP and the ratio of the value of trades to market capitalization. Their finding is consistent with those of Caballero and Hammour (2000), Restuccia and Rogerson (2003) and Hsieh and Klenow (2006) who show

We examine next the concentration of economic activity, measured using Herfindhal indices, $Her(K_t^m) \equiv E(K_t^{m2})/[E(K_t^m)]^2$ and $Her(\tilde{\Pi}_{t+1}^m) \equiv E(\tilde{\Pi}_{t+1}^{m2})/[E(\tilde{\Pi}_{t+1}^m)]^2$.

Proposition 13

Capital and profits are more concentrated across firms as the economy grows. Formally, $dHer(K_t^m)/dw_t > 0$ and $dHer(\tilde{\Pi}_{t+1}^m)/dw_t > 0$.

Agents become more selective in their investments as their income grows. They channel increasingly more (less) capital to the more (less) productive firms, so fewer firms account for a larger fraction of the economy’s stock of capital. Profits tend to be even more concentrated than capital because they compound the effect of a high productivity shock with that of a large capital stock. Thus, the economy grows more specialized by both measures of economic activity.

Empirically, Imbs and Wacziarg (2003) report that countries go through two stages of sectoral diversification. Diversification increases at first, but beyond a certain level of income, the process is reversed and economic activity starts concentrating. This pattern is consistent with our model to the extent that the model applies to more advanced economies.²⁴ Similarly, Kalemli-Ozcan, Sørensen and Yosha (2003) report that industrial specialization in a sample of developed countries is positively related to the share of the financial sector in GDP. This fact is consistent with proposition 13 to the extent that this share is positively related with information expenditures about public companies.

We discuss next the evolution of inequality. Given that agents are *ex ante* identical, we consider here the distribution of final wealth or consumption, $\tilde{g}_{l,t+1}$.

Proposition 14

As the economy grows, income inequality narrows at first and then widens.

Final wealth is unequal because agents, guided by their private signals, choose different portfolios. Two forces work in opposite directions when the precision of private signals rises. On one hand, agents put more weight on private information relative to public information, which increases disagreement. On the other hand, idiosyncratic errors shrink so private signals are more similar. This tends to reduce

that variations in the allocation of resources account for a large fraction of the cross-country differences in total factor productivity. Moreover, Henry (2003) confirms that countries that liberalize their stock market experience a rise in total factor productivity, and Bertrand, Schoar and Thesmar (2005), Galindo, Schiantarelli and Weiss (2005) and Chari and Henry (2006) that their allocative efficiency improves.

²⁴An extension of the model presented in the next section shows that the economy produces more information as it grows, only if income is above a threshold.

portfolio heterogeneity.²⁵ The second effect tends to dominate for low private precision X_t and the first for high private precision, so inequality narrows first and then widens.

We conclude with two financial variables, the volatility of stock returns and the trading intensity. We assume a “pre-opening trading session” takes place before private signals are observed. A small number of shares is issued during this session but no information is revealed so agents, including noise traders, assign the same portfolio weight to all stocks, $1/M$. Prices (P_t^0) that equate the supply of shares to their demand emerge but trades do not actually take place. Trade occurs during the second round once agents receive their private signals – they set their portfolio weights according to equation 10 (substituting X_t for x_t^m to obtain equilibrium portfolio weights). Stock returns are computed by dividing stock prices in the first and second rounds ($\ln(P_t^0/P_t^m)$). Trades are based on the difference between portfolio weights in the first and second rounds. The value of shares traded equals $\sum_{m=1}^M \int_l |(f_{l,t}^m - 1/M)w_t|/2$ where the factor 2 avoids double counting matching buys and sells. We measure the trading intensity as the share turnover, defined as the ratio of the value of shares traded to the total capitalization of the market, $\sum_{m=1}^M K_t^m$.

Proposition 15

As the economy grows, stocks’ idiosyncratic and total volatility increase, while the market volatility remains constant. Formally, $dVar(\Delta\tilde{r}_{t+1}^m)/dw_t > 0$, $dVar(\tilde{r}_{t+1}^m)/dw_t > 0$ and $dVar(\bar{r}_{t+1})/dw_t = 0$.

The proposition establishes that individual stocks returns fluctuate more – whether fluctuations are measured as total or idiosyncratic volatility – as the economy grows and prices incorporate more information. Since the market in contrast does not, the decline in the cross-correlation of returns offsets the rise in individual stock volatility. Thus, stock prices vary in a less synchronized manner. Empirically, Morck, Yeung and Yu (2000) show that stock prices are less synchronous in richer economies. In line with this observation, Campbell, Lettau, Malkiel and Xu (2001) document a strong increase in idiosyncratic return volatility in the U.S. from 1962 to 1997, while the volatility of the market remained stable.²⁶ The following proposition describes the trading activity.

Proposition 16

²⁵Formally, according to equation 10 (substituting X_t for x_t^m to obtain equilibrium portfolio weights), an agent’s portfolio weights are a function of $(X_t/h(X_t))\Delta s_{l,t}^m = (X_t/h(X_t))\Delta\tilde{\varepsilon}_{l,t}^m$ +other terms. When X_t grows, on one hand the ratio of the precision of the private signal to the total precision, $X_t/h(X_t)$, rises, but on the other hand $var(\tilde{\varepsilon}_{l,t}^m) \equiv 1/X_t$ falls.

²⁶Explanations, other than information-based, have been suggested for these volatility patterns. See for example Thesmar and Thoenig (2004) for an alternative view based on firms’ changing risk profile.

As the economy grows, trading on the equity market weakens at first and then intensifies.

The logic of Proposition 16 is identical to that of Proposition 14 on wealth inequality. Agents trade because they disagree. On one hand, disagreement rises with the precision of private signals because agents use them more aggressively, but on the other hand, it declines because idiosyncratic signal errors shrink. The trading intensity declines first and then rises beyond a threshold. Empirically, Levine and Zervos (1998) and Rousseau and Wachtel (2000) report that the share turnover on the stock market is positively related to output growth.

8 No-Information Trap

In the model, agents always collect private signals. This is because the cost of learning is assumed to satisfy $C'(0) = 0$, i.e. an infinitesimal amount of private information is costless. Empirically however, financial institutions only emerge once a critical level of income has been reached. In this section, we assume that $C'(0) > 0$ and show that information production only takes place for sufficiently developed economies. The following proposition describes how investors' learning decisions are altered.

Proposition 17

Suppose that $C'(0) > 0$. Investors collect information if and only if $\frac{\tau(\varphi(w_t))}{\rho(\varphi(w_t))} > \frac{2M\sigma_a^2(\sigma_\alpha^2)^2}{(M-1)\beta^2}C'(0)$. In that case, its precision is the unique solution to equation 17.

If $C'(0) > 0$, then equation 17 that determines the equilibrium precision may admit no solution. For example, when $\rho(\varphi(w_t))$ is large relative to $\tau(\varphi(w_t))$, the marginal cost of information (the left-hand side of equation 17) may exceed its marginal benefit (the right-hand side) for all levels of noisiness. In that case, no information is collected in equilibrium as it is too costly to be profitable. The condition on τ/ρ for learning to take place leads to a condition on income. This can easily be seen in the case of CES utility, as the following lemma shows.

Lemma 18

Suppose that $C'(0) > 0$ and that utility is CES. Let

$$\underline{w} \equiv \left(\frac{1-\varpi}{2\varpi} \left(\sqrt{1 + \frac{8\varpi(1-\sigma)M\sigma_a^2(\sigma_\alpha^2)^2}{(1-\varpi)(M-1)\beta^2}C'(0)} - 1 \right) \right)^{1/\sigma}.$$

When $\sigma > 0$, investors collect information if and only if their income exceeds the threshold \underline{w} .

When instead $\sigma < 0$, they collect information if and only if their income is below the threshold \underline{w} .

The threshold \underline{w} is the unique income level such that $\tau(\varphi(\underline{w}))/\rho(\varphi(\underline{w})) = C'(0)2M\sigma_a^2(\sigma_\alpha^2)^2/(M - 1)/\beta^2$. When $\sigma > 0$, the scale effect of information dominates so wealthier investors collect information only if their income w_t is large enough. When $\sigma < 0$, the substitution effect dominates so investors stop collecting information when their income exceeds \underline{w} . The properties of \underline{w} mirror those of the equilibrium precision X_t : the factors that increase (decrease) X_t tend to decrease (increase) \underline{w} . Assuming that $\sigma > 0$ and that $w^* > \underline{w} > w_0$ where w_0 is the initial level of income, the economy goes through two stages of development. At first, it behaves as the standard neoclassical economy with no information. Once income reaches a threshold, agents start collecting private signals and growth accelerates by a factor $\exp(\lambda(w_t)z^2)$. Thus in this case, the stock market only operates as an information processor if the economy is sufficiently developed. If instead $w_0 < w^* < \underline{w}$, then no information is ever collected.

9 Conclusion

A competitive stock market with partially revealing prices is embedded into a neoclassical growth economy to analyze the interplay between information acquisition and dissemination through stock prices, capital allocation and income. The stock market contributes to growth by allowing investors to share their costly private signals in an incentive-compatible way when the signals' precision is not contractible, but its impact is only transitory. Several predictions on the evolution of real and financial variables are derived, including capital efficiency, total factor productivity, industrial specialization, stock trading intensity and idiosyncratic return volatility.

A Proof of Lemma 3

Given the conjectured capital allocation, observing the M stock prices (or the M capital stocks) is equivalent to observing $\Delta\xi_t^m$ for every firm m where $\xi_t^m \equiv \beta a_t^m + \mu_t^m \theta_t^m$.²⁷ The first step is to relate stock returns to productivity shocks and capital.

- Stock returns

Given its capital stock K_t^m , firm m sells $\tilde{Y}_{t+1}^m = \tilde{A}_t^m K_t^m$ intermediate goods for a profit $\tilde{\Pi}_{t+1}^m = \tilde{\rho}_{t+1}^m \tilde{Y}_{t+1}^m = \beta L^{1-\beta} \tilde{Y}_{t+1}^{m\beta} = \beta L^{1-\beta} (\tilde{A}_t^m K_t^m)^\beta$. The gross return on stock m is then $\tilde{R}_{t+1}^m = \tilde{\Pi}_{t+1}^m / K_t^m = \beta L^{1-\beta} K_t^{0\beta-1} \exp[\beta \tilde{a}_t^m z - (1-\beta)k_t^m z]$ where $K_t^0 \equiv Lw_t/M$ denotes the firm's capital stock when $z = 0$. (Indeed when $z = 0$, firms, since they are identical to one another, offer the same return, which implies that they have identical capital stocks.) The log return on stock m is $\ln \tilde{R}_{t+1}^m = \ln R_t^0 + \tilde{r}_{t+1}^m z$ where $R_t^0 = \beta L^{1-\beta} K_t^{0\beta-1} = \beta M^{1-\beta} w_t^{\beta-1} = \varphi(w_t)/w_t$ and $\tilde{r}_{t+1}^m z = \beta \tilde{a}_t^m z - (1-\beta)k_t^m z$. We show below that investors' portfolio weights depend on expected relative returns $E(\Delta r_{t+1}^m z | \mathcal{F}_{l,t})$ and on the variance of returns $Var(r_{t+1}^m z | \mathcal{F}_{l,t})$. These are given by:

$$E(\Delta r_{t+1}^m z | \mathcal{F}_{l,t}) = E(\beta \Delta \tilde{a}_t^m z | \mathcal{F}_{l,t}) - (1-\beta) \Delta k_t^m z = E(\beta \Delta \tilde{\alpha}_t^m z | \mathcal{F}_{l,t}) - (1-\beta) k_t^m z, \quad (22)$$

$$\text{and } Var(r_{t+1}^m z | \mathcal{F}_{l,t}) = Var(\beta \tilde{a}_t^m z | \mathcal{F}_{l,t}) = \beta^2 \sigma_a^2 z + Var(\beta \tilde{\alpha}_t^m z | \mathcal{F}_{l,t}) = \beta^2 \sigma_a^2 z + o(z) \quad (23)$$

where we use the normalization $\bar{k}_t = 0$ in equation 22 (see footnote 21).

We note that the variance of returns is constant at the order z . The next step is to estimate the expectation of $\Delta \tilde{\alpha}_t^m$ using the conjectured prices (or equivalently the $\Delta \xi_t^m$'s) and private signals $s_{l,t}^m$.

- Signal extraction

The noisiness of stock prices is $\{\mu_t^m\}$ and the precision of agent l 's signal is $\{x_{l,t}^m\}$. For the capital allocation given in equation 13 (k_t^m linear in $\Delta \tilde{\alpha}_t^m$ and $\Delta \tilde{\theta}_t^m$), the conditional mean and variance of $\Delta \tilde{\alpha}_t^m$ are for agent l :

$$E(\beta \Delta \tilde{\alpha}_t^m z | \mathcal{F}_{l,t}) = c_{\xi_t}^m \Delta \xi_t^m z + c_{s_t}^m \Delta s_{l,t}^m z \quad (24)$$

$$\text{where } \hat{h}_t^m \equiv H(\mu_t^m) + x_{l,t}^m, \quad c_{\xi_t}^m \hat{h}_t^m \equiv \frac{1}{\mu_t^{m2} \sigma_\theta^2} \quad \text{and} \quad c_{s_t}^m \hat{h}_t^m \equiv x_{l,t}^m.$$

$E(\beta \Delta \tilde{\alpha}_t^m z | \mathcal{F}_{l,t})$ is a weighted average of priors (which equal 0), public and private signals where the weight on the private signal is increasing in $x_{l,t}^m$.

- Portfolio weights

We now solve for the optimal portfolio of an investor. An agent with a wage w_t and precisions $\{x_t^m\}$ maximizes $E[U(\tilde{g}_{l,t+1}, j_t) | \mathcal{F}_{l,t}]$, where $\tilde{g}_{l,t+1} = w_t \tilde{R}_{l,t+1}$ and $j_t = 1 - \sum_{m=1}^M C(x_t^m) z$ are her consumption of final goods and leisure, and $r_{l,t+1} z \equiv \ln R_{l,t+1} - \ln R_t^0$ captures terms of order z and

²⁷We assume that agents extract information only from the leading terms in the capital allocation equation 11, i.e. they do not learn from k_2^m . This assumption is consistent with the notion that each stock price provides only one signal.

smaller in her log portfolio return. This log portfolio return can be related to individual stock returns and weights as follows:

$$\begin{aligned}
r_{l,t+1}z &= \ln \left(\sum_{m=1}^M f_{l,t}^m \exp(r_{t+1}^m z) \right) \\
&= \ln \left(\sum_{m=1}^M f_{l,t}^m (1 + r_{t+1}^m z + \text{Var}(r_{t+1}^m z | \mathcal{F}_{l,t})/2) + o(z) \right) \\
&= \sum_{m=1}^M \left(f_{l,t}^m r_{t+1}^m + \frac{1}{2} f_{l,t}^m (1 - f_{l,t}^m) \text{Var}(r_{t+1}^m | \mathcal{F}_{l,t}) \right) + o(z) \\
&= \sum_{m=1}^M \left(f_{l,t}^m r_{t+1}^m + \frac{1}{2} f_{l,t}^m (1 - f_{l,t}^m) \beta^2 \sigma_a^2 z \right) + o(z)
\end{aligned}$$

where we use $\sum_{m=1}^M f_{l,t}^m = 1$ and equation 23. Thus, the log portfolio return is approximately normal when z is small (e.g. Campbell and Viceira (2002)) and its moments are given by:

$$\begin{aligned}
E(r_{l,t+1}z | \mathcal{F}_{l,t}) &= \sum_{m=1}^M \left\{ f_{l,t}^m e_{l,t}^m z + \frac{1}{2} f_{l,t}^m (1 - f_{l,t}^m) \beta^2 \sigma_a^2 z \right\} + o(z) \text{ where } e_{l,t}^m \equiv E(r_{t+1}^m | \mathcal{F}_{l,t}) \\
\text{and } \text{Var}(r_{l,t+1}z | \mathcal{F}_{l,t}) &= \sum_{m=1}^M f_{l,t}^m \text{Var}(r_{t+1}^m z | \mathcal{F}_{l,t}) + o(z) = \sum_{m=1}^M f_{l,t}^m \beta^2 \sigma_a^2 z + o(z).
\end{aligned} \tag{25}$$

The agent's utility can be expanded in a Taylor series in a neighborhood of $z = 0$, i.e. for $\tilde{g}_{l,t+1}$ and j_t respectively around $\varphi(w_t) = w_t R_t^0$ and 1. We denote the pair $(\varphi(w_t), 1)$ with a $*$:

$$U(\tilde{g}_{l,t+1}, j_t) = U(*) + \frac{\partial U}{\partial g}(*).(\tilde{g}_{l,t+1} - \varphi(w_t)) + \frac{\partial U}{\partial l}(*).(j_t - 1) + \frac{1}{2} \frac{\partial^2 U}{\partial g^2}(*).(\tilde{g}_{l,t+1} - \varphi(w_t))^2 + o(z).$$

Noting that $j_t - 1 = -\sum_{m=1}^M C(x_{l,t}^m)z$ and that $\tilde{g}_{l,t+1} - \varphi(w_t) = \varphi(w_t)(w_t \tilde{R}_{l,t+1} / \varphi(w_t) - 1) = \varphi(w_t)(\tilde{R}_{l,t+1} / R_t^0 - 1) = \varphi(w_t)(\exp(r_{l,t+1}z) - 1)$ allows to write the above expression as:

$$\begin{aligned}
U(\tilde{g}_{l,t+1}, j_t) &= U(*) + \frac{\partial U}{\partial g}(*).\varphi(w_t)(\exp(r_{l,t+1}z) - 1) - \frac{\partial U}{\partial l}(*).\sum_{m=1}^M C(x_{l,t}^m)z \\
&\quad + \frac{1}{2} \frac{\partial^2 U}{\partial g^2}(*).(\tilde{g}_{l,t+1} - \varphi(w_t))^2 + o(z).
\end{aligned}$$

Taking expectations yields:

$$\begin{aligned}
E(U(\tilde{g}_{l,t+1}, j_t) | \mathcal{F}_{l,t}) &= U(*) + \frac{\partial U}{\partial g}(*).\varphi(w_t)(E(r_{l,t+1}z | \mathcal{F}_{l,t}) + \text{Var}(r_{l,t+1}z | \mathcal{F}_{l,t})/2) \\
&\quad - \frac{\partial U}{\partial l}(*).\sum_{m=1}^M C(x_{l,t}^m)z + \frac{1}{2} \frac{\partial^2 U}{\partial g^2}(*).\varphi(w_t)^2 \text{Var}(r_{l,t+1}z | \mathcal{F}_{l,t}) + o(z).
\end{aligned} \tag{26}$$

Maximizing this expression with respect to $f_{l,t}^m$, subject to $\sum_{m=1}^M f_{l,t}^m = 1$, leads to the first-order conditions:

$$\frac{\partial U}{\partial g}(*).(e_{l,t}^m + \frac{1}{2} \beta^2 \sigma_a^2) + \beta^2 \sigma_a^2 \varphi(w_t) \frac{\partial^2 U}{\partial g^2}(*).f_{l,t}^m + o(1) = \varsigma_{l,t} \text{ for } m = 1, \dots, M \tag{27}$$

in which $\varsigma_{l,t}$ denotes the Lagrange multiplier on the constraint. Averaging equation 27 across stocks yields:

$$\frac{\partial U}{\partial g}(*).(\bar{e}_{l,t} + \frac{1}{2} \beta^2 \sigma_a^2) + \beta^2 \sigma_a^2 \varphi(w_t) \frac{\partial^2 U}{\partial g^2}(*).\frac{1}{M} + o(1) = \varsigma_{l,t}. \tag{28}$$

Subtracting equation 28 from equation 27 leads to the formula for portfolio weights presented in equation 8:

$$f_{l,t}^m = \frac{1}{M} + \frac{1}{\gamma(w_t)\beta^2\sigma_a^2}\Delta e_{l,t}^m + o(1), \quad (29)$$

where γ is defined in equation 9. Substituting in the expression for $\Delta e_{l,t}^m = E(\Delta r_{t+1}^m z \mid \mathcal{F}_{l,t})$ using equation 22 leads to equation 10.

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	<i>Young</i>	<i>Old</i>
<i>Generation t</i>	<ul style="list-style-type: none"> • Earn wage w_t • Choose leisure j_t and precisions x_t^m • Observe signals $s_{l,t}^m$ and P_t^m choose portfolio weights $f_{l,t}^m$ 	<ul style="list-style-type: none"> • Consume proceeds from investments $g_{l,t+1}$

	<i>Young</i>	<i>Old</i>
<i>Generation t+1</i>	<ul style="list-style-type: none"> • Earn wage w_{t+1} • Choose leisure j_{t+1} and precisions x_{t+1}^m • Observe signals $s_{l,t+1}^m$ and P_{t+1}^m choose portfolio weights $f_{l,t+1}^m$ 	<ul style="list-style-type: none"> • Consume proceeds from investments $g_{l,t+2}$

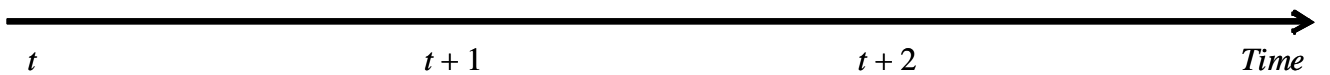


Figure 1: Timing.

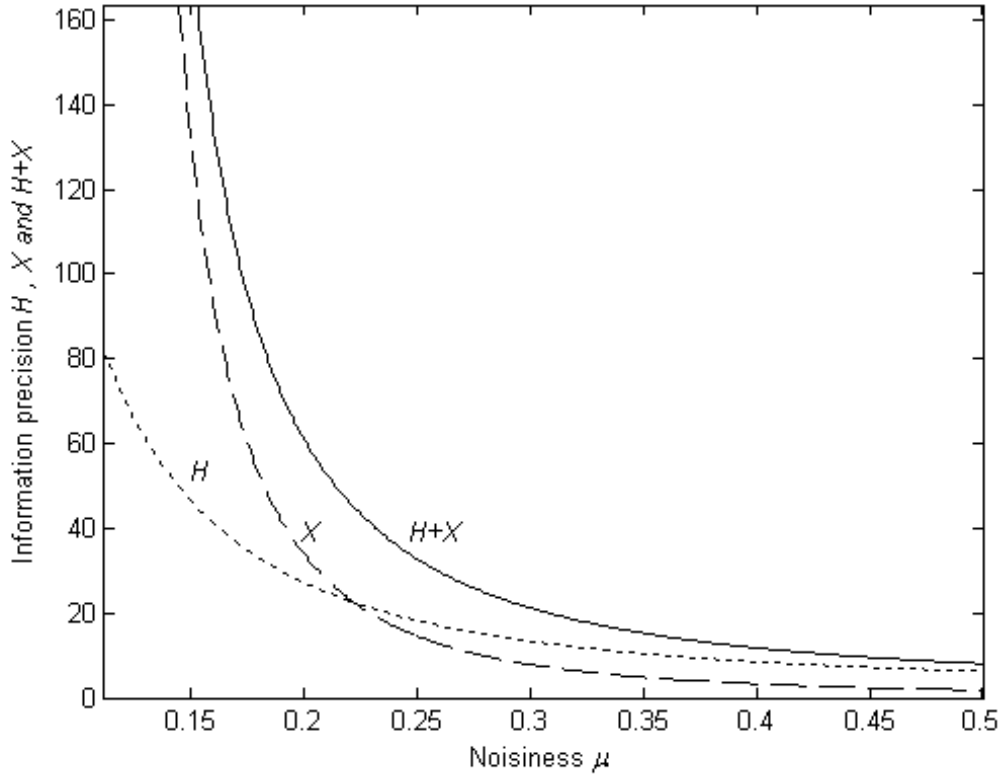


Figure 2: Signal precisions and the noisiness of the price system. The picture depicts the precision of the stock price H (dotted curve), the precision of an investor's private signal X (dashed curve) and her total precision $H + X$ (solid curve) as a function of the stock price noisiness μ . Utility is CES ($U(g, j) \equiv (\varpi g^\sigma + (1 - \varpi)j^\sigma)^{1/\sigma}$). The parameters are $\beta = 2/3$, $C(x) = x^2$, $q = 0.1$, $\sigma_a^2 = 0.01$, $\sigma_\theta^2 = \sigma_\alpha^2 = 1$, $\omega = 0.5$, $M = 50$ and $z = 0.5$.

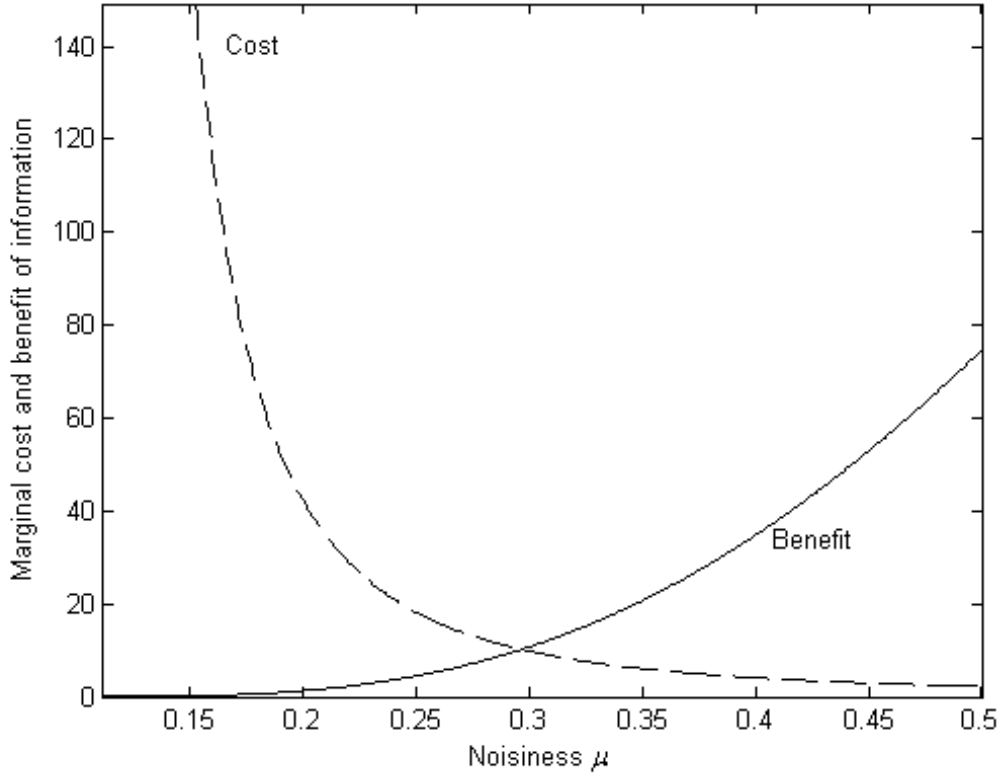


Figure 3: The benefit and cost of information in equilibrium. The picture depicts the marginal benefit of private information (solid curve) and its marginal cost (dashed curve) in equation 17 as a function of the equilibrium noisiness μ . Utility is CES ($U(g, j) \equiv (\varpi g^\sigma + (1 - \varpi)j^\sigma)^{1/\sigma}$) with $\sigma = 0.5$. The other parameters are $\beta = 2/3$, $C(x) = x^2$, $q = 0.1$, $\sigma_a^2 = 0.01$, $\sigma_\theta^2 = \sigma_\alpha^2 = 1$, $\omega = 0.5$, $M = 50$ and $z = 0.5$.

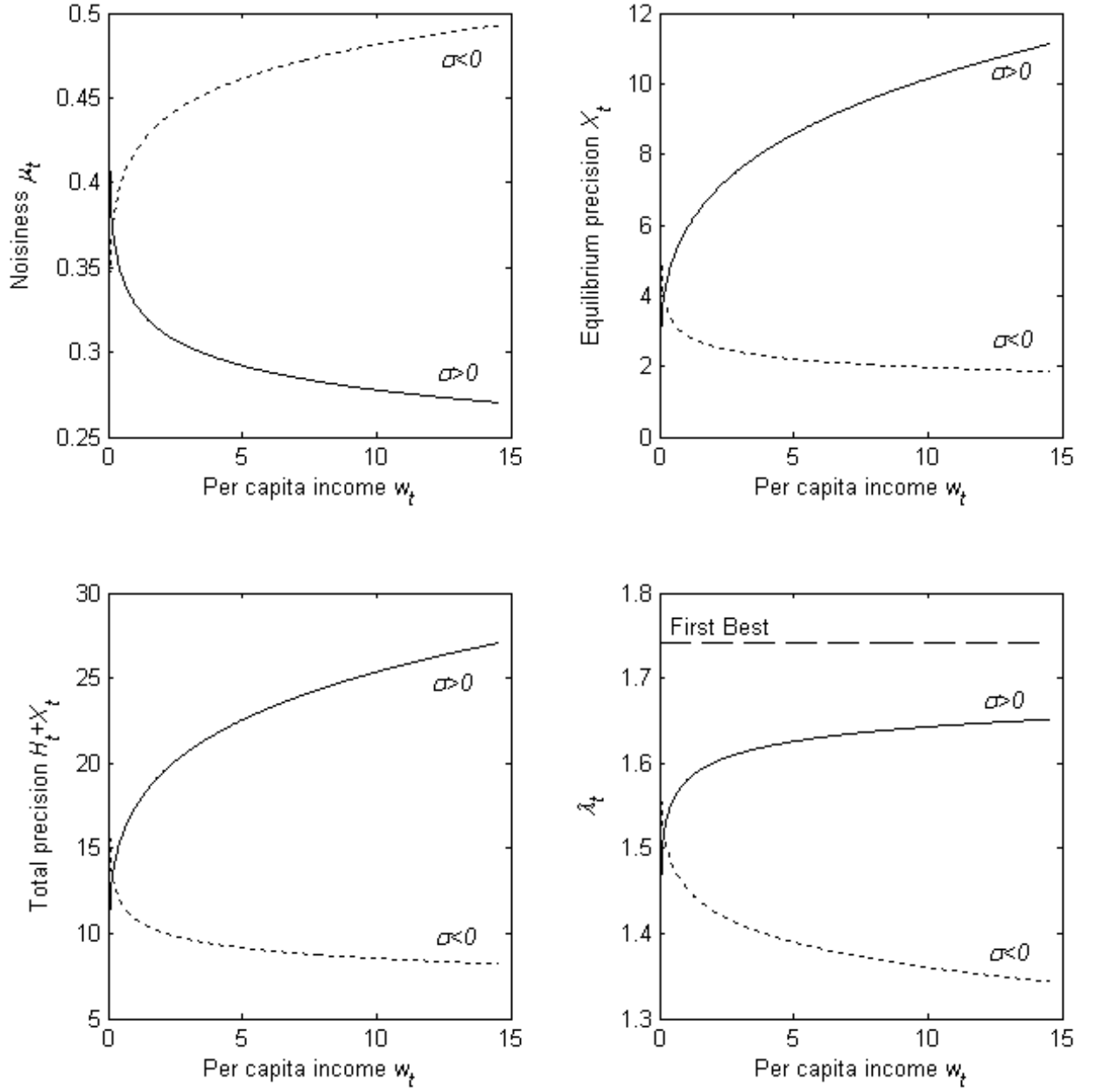


Figure 4: The impact of income on the equilibrium. The picture depicts the equilibrium noisiness μ_t (top left panel), the precision of private information X_t (top right panel), the total precision $H_t + X_t$ (bottom left panel) and λ_t which captures the effect of learning on income (bottom right panel) as a function of current income w_t . Utility is CES ($U(g, j) \equiv (\varpi g^\sigma + (1 - \varpi)j^\sigma)^{1/\sigma}$). The solid curves correspond to $\sigma = 0.5$ and the dotted curves to $\sigma = -0.5$. The other parameters are $\beta = 2/3$, $C(x) = x^2$, $q = 0.1$, $\sigma_a^2 = 0.01$, $\sigma_\theta^2 = \sigma_\alpha^2 = 1$, $\omega = 0.5$, $M = 50$ and $z = 0.5$.

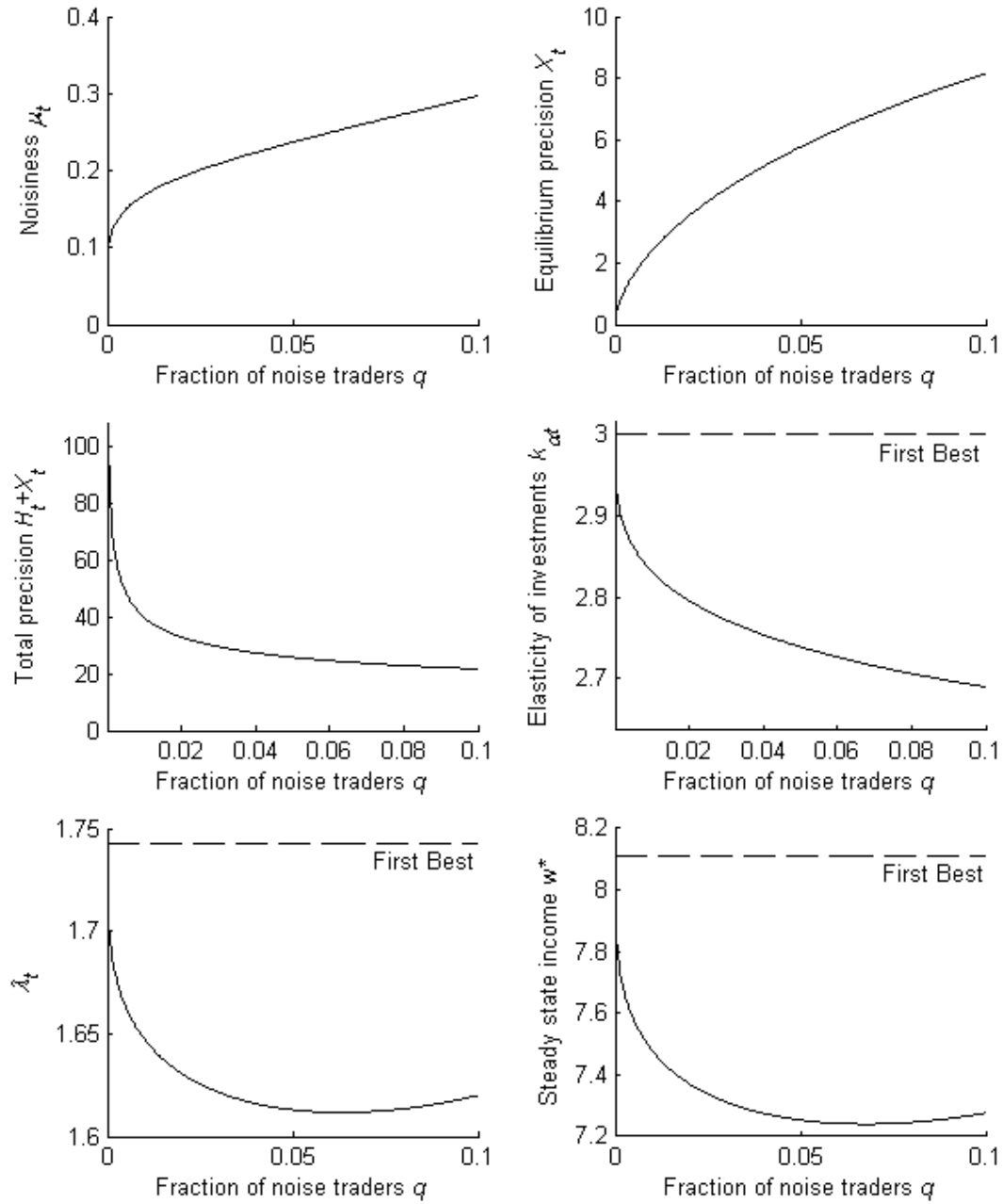


Figure 5: The impact of the fraction of noise traders on the equilibrium. The picture depicts the equilibrium noisiness μ_t (top left panel), the precision of private information X_t (top right panel), the total precision $H_t + X_t$ (middle left panel), the elasticity of investments to productivity shocks $k_{\alpha t}$ (middle right panel), λ_t which captures the effect of learning on income (bottom left panel) and the steady state level of income w^* (bottom right panel) as a function of the fraction of noise traders q . Utility is CES ($U(g, j) \equiv (\varpi g^\sigma + (1 - \varpi)j^\sigma)^{1/\sigma}$) with $\sigma = 0.5$. The other parameters are $\beta = 2/3$, $C(x) = x^2$, $q = 0.1$, $\sigma_a^2 = 0.01$, $\sigma_\theta^2 = \sigma_\alpha^2 = 1$, $\omega = 0.5$, $M = 50$ and $z = 0.5$.

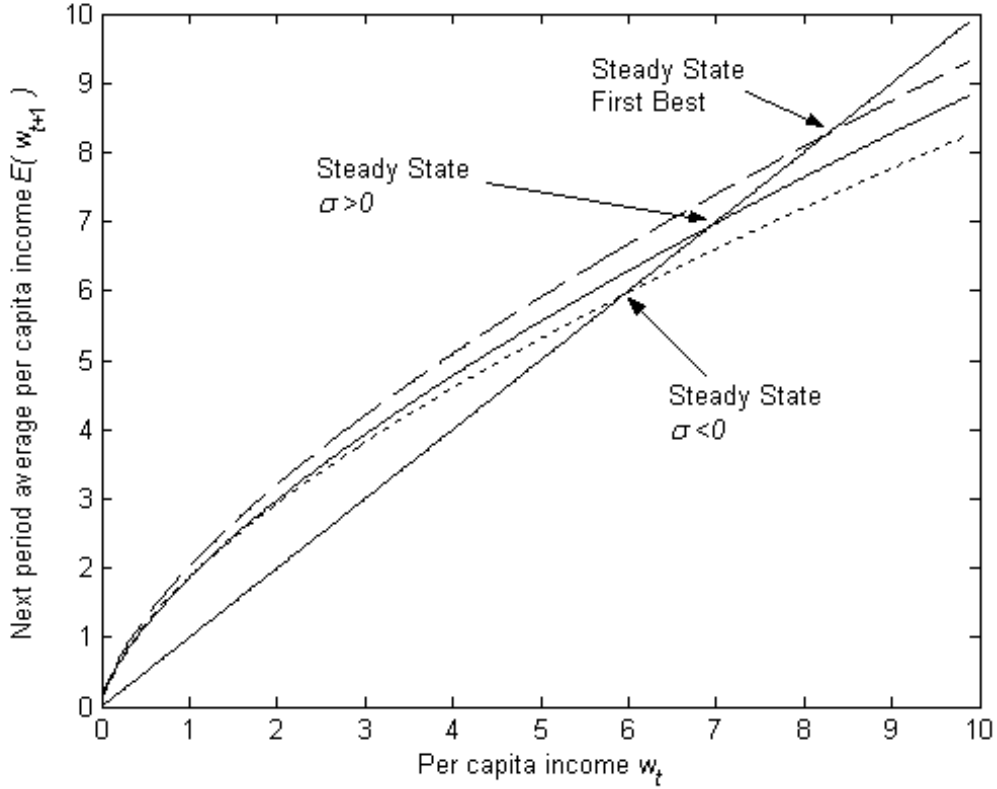


Figure 6: The dynamics of income in an economy along its average path. The curves represent the average income in period $t+1$, $E(w_{t+1})$, as a function of income in period t , w_t . Utility is CES ($U(g, j) \equiv (\varpi g^\sigma + (1 - \varpi)j^\sigma)^{1/\sigma}$). The solid curve corresponds to $\sigma = 0.5$ and the dotted curve to $\sigma = -0.5$. The dashed curve corresponds to the first-best economy. The economies' steady-states are located at the intersections of these curves with the 45° line, represented as a solid line. The other parameters are $\beta = 2/3$, $C(x) = x^2$, $q = 0.1$, $\sigma_a^2 = 0.01$, $\sigma_\theta^2 = \sigma_\alpha^2 = 1$, $\omega = 0.5$, $M = 50$ and $z = 0.5$.

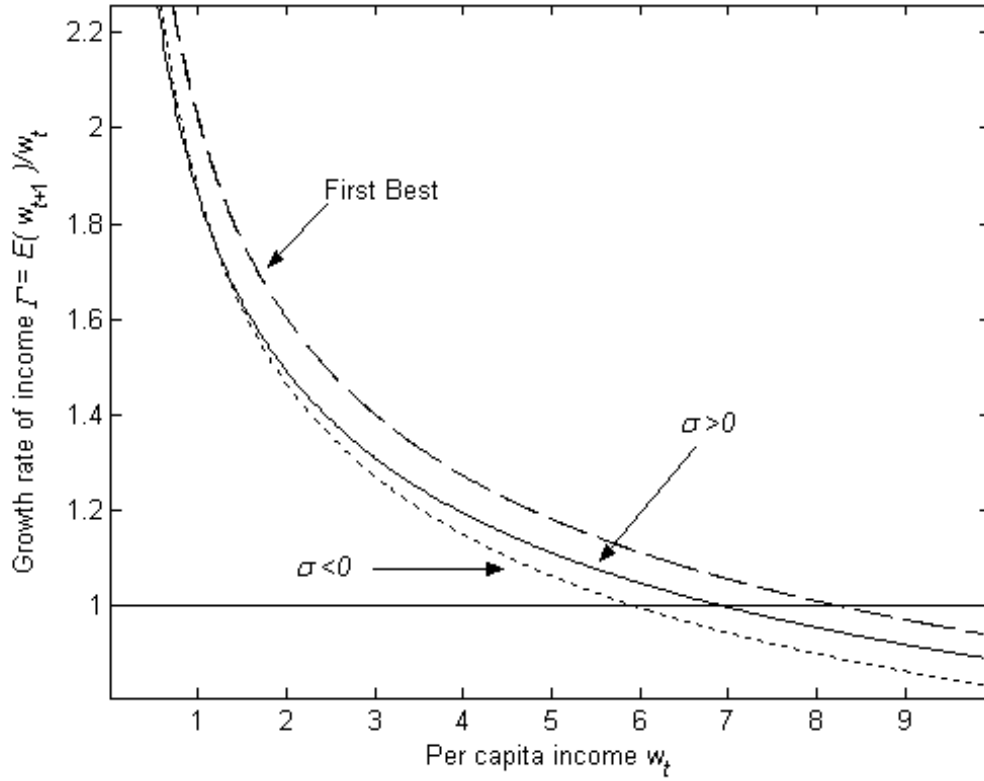


Figure 7: The growth rate of income. The picture depicts the growth rate of income, $\Gamma(w_t) \equiv E(\tilde{w}_{t+1})/w_t$, during the transition to the steady-state. Utility is CES ($U(g, j) \equiv (\varpi g^\sigma + (1 - \varpi)j^\sigma)^{1/\sigma}$). The solid curve corresponds to $\sigma = 0.5$ and the dotted curve to $\sigma = -0.5$. The dashed curve corresponds to the first-best economy. The other parameters are $\beta = 2/3$, $C(x) = x^2$, $q = 0.1$, $\sigma_a^2 = 0.01$, $\sigma_\theta^2 = \sigma_\alpha^2 = 1$, $\omega = 0.5$, $M = 50$ and $z = 0.5$.