Time Varying Risk Premia in Corporate Bond Markets

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First Version: April 2006
This Revision: March 17, 2008

Abstract

We study the link between corporate bond risk premia and equity returns in a large panel of corporate bond transaction data. In contrast to previous work, we find that a significant part of the time variation in bond risk premia can be explained by equity implied bond risk premium estimates. We also document a large time variation in the expected loss component of bond spreads. This component is related to total asset volatility, whereas the risk premium is related to systematic volatility. In addition, we show by means of linear regressions that augmenting the set of variables predicted by typical structural models with equity-implied bond default risk premia significantly increases explanatory power.

JEL Classification: G12

Keywords: corporate bonds; credit risk; structural model; volatility; default risk premia; idiosyncratic risk.

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1 Introduction

Yield spreads are an imperfect measure of the expected return on a corporate bond. For asset allocation or performance evaluation purposes, a key input is the risk premium earned by holding a particular security. However, bond spreads also contain significant compensation for expected losses - even in the absence of a risk premium. Two firms with the same default probability can earn very different spreads depending on their systematic risk. Two firms with the same spread can earn significantly different risk premia. This paper studies risk premia extracted from yield spreads.

The equity risk premium has received intense attention in the finance literature. In stark contrast, the literature on the risk return characteristics of corporate bonds is only now emerging.[1] One reason is likely the scarcity of corporate bond return data. Another is that disentangling the risk premium from spreads requires estimates of objective default probabilities, the measurement of which is in itself a complex task.[2] Using a large sample of US corporate bond transactions over a 10 year period, we measure corporate bond risk premia. Using a structural model, we first estimate actual default probabilities to compute the expected loss component of spreads. Second, we use this estimate to disentangle the risk premium component.[3]

Theoretically, a given firm's equity and bond returns should be closely related. Nevertheless, many papers have documented difficulties in relating equity factors and bond returns. Fama and French (1993) find that factors that explain the time series and the cross section of equity returns well are not that succesful in explaining corporate bond returns. Recent work has produced mixed results on the impact of financial distress on bond and stock returns. Some studies have documented that firms with higher default risk have low average stock returns.[4] Others have found that firms with a high likelihood of default experience high average stock returns.[5]

Our main contribution is to revisit the link between equity and bond risk premia. A visual inspection of time series of average equity risk premia and bond risk premia would likely suggest that they move largely independently of each other. In particular, between 1997 and 2002 equity premia trended slowly downwards, whereas the much more volatile bond premia increased irregularly. Our theoretical framework predicts a highly nonlinear relationship between these premia. This relationship depends on financial leverage, operating risk as well as bond specific characteristics. Empirically we find that equity returns are in fact useful in explaining bond risk premia when these are taken into account. Interestingly, we

[1] Notable exceptions, discussed further below, include Elton, Gruber, Agrawal & Mann (2001), Driessen (2005), Huang & Huang (2002), Berndt, Douglas, Duffie, Ferguson & Schranz (2004), Berndt, Lookman & Obreja (2006), Saita (2006) and Chen, Collin-Dufresne & Goldstein (2006).

[2] Recent work on estimating default probabilities includes Shumway (2001), Barath & Shumway (2004), Leland (2004) and Duffie et al. (2006).

[3] There is currently a debate in the literature regarding the choice of appropriate risk free rate. To address this, we carry out our analysis both using US Treasury and interest rate swap benchmark curves.

[4] See Dichev (1998), Campbell et al. (2008), and Garlappi et al. (2008).

[5] See Vassalou & Xing (2004).

show that the risk premium, as a function of the likelihood of distress is non-monotonic. For healthy firms risk premia will increase in risk, but for firms approaching distress, they can in fact decrease. As distress becomes more likely, uncertainty about the arrival of default decreases and then so do risk premia.

Previous studies have documented surprisingly volatile risk premia in default swap markets.[6] We find a very similar time series behavior and degree of time variation, which is interesting given that our study is based on different data, a different financial instrument and a different methodology.

Another contribution of our work is to document the charateristics of the expected loss components across firms and time. Previous work has shown that expected losses explain about a quarter of corporate bond spread levels.[7] Our overall average is of the same magnitude, however, the relative importance of expected loss component is (i) highly time varying and (ii) tends to be higher when spreads are high. For example, we find that across our sample of about 400 firms, it reaches a high of about 70% of the spread over government bonds in 2000, up from an average near a third the five preceeding years. Ignoring this time variation may lead to significant biases in estimating risk premium levels in spreads.

Risk premium and expected loss components of bond spreads behave quite differently from one another. As a proportion of the total spread, risk premia tend to be high in times of low spreads, whereas expected losses dominate during periods of high defaults. We are also able to shed light on recent events in credit markets. For example it appears that the spike in spreads after the LTCM episode in the late summer of 1998 is driven by increased risk premia rather than expected losses. On the other hand, the expected loss component is the dominant spread component during 2001, a period of unprecedented default losses in the US corporate bond markets.

Both spread components are intimately related to measures of volatility. The risk premium component appears to be closely tied to systematic volatility. The expected loss component on the other hand is closely tied to total risk. This is intuitive as idiosyncratic risk matters for default probabilities, while it should not influence risk premia. This allows us to comment on the results of Campbell & Taksler (2003). They document a period where corporate bond spreads increase in tandem with stock prices, an apparent contradiction. They attribute this to an increase in idiosyncratic equity volatility. We find support for this conclusion in that in our sample it is the expected loss component of spreads which increases, driven by an increase in total asset risk. In this period, market volatility exhibited no clear trend.

Implicitly, our study shows that structural credit risk models are useful tools in translating equity risk premia into the corporate bond specific counterparts. Although this casts the models in a favorable light, it begs the question why they have been relatively unsuccessful at explaining changes in corporate bond yield spreads, by relying on the variables implied by their specification. A candidate explanation that arises from our work is the strong degree of time variation that our measures of risk premia exhibit. Although structural models provide powerful cross-sectional predictions on the no-arbitrage relationship

[6] See Berndt et al. (2004), Berndt et al. (2006).
[7] See Elton et al. (2001).

between debt and equity, they tend to be silent on both the level and the time variation of the risk premium.

In an attempt to understand whether time varying risk premia can be part of the reason for the documented failure of structural models to explain credit spreads behavior, we carry out a regression analysis on credit spreads in the spirit of what has recently been done in the literature.[8] We take as a benchmark a regression motivated by the key drivers implied by structural models. We find that augmenting the regressions by our measures of equity-implied risk premia improves explanatory power considerably, in particular for high grade bonds.

The paper is organized as follows. The following section reviews the literature and relates our paper to existing work. Section 3 describes our methodology for measuring bond risk premia in corporate bond markets and compares it to alternative approaches used in the literature. Section 4 explains how we translate equity risk premia into bond risk premia. In sections 5 and 5.3 we present our estimation methods and data. In section 6 we discuss our main findings, while section 6.3 presents our regression results. In section 7 we examine the implications of a structural model for empirical work on risk premia. Section 8 closes our study.

2 Related literature

In what follows we will review work related to corporate bond risk premia. The first papers we discuss deal with the link between equity and corporate bond returns for varying levels of financial distress risk. Next, we consider papers that deal more directly with the risk premium in credit markets.

As mentioned above, Fama and French (1993) find that equity return factors have difficulties in explaining corporate bond returns. In theory, however, bond and stock returns should be closely related. For example, if finanical distress is imminent, bond and stock expected returns should both be high. Among others, Dichev (1998) and Campbell et al. (2008) have documented that stock returns for firms with high degrees of distress risk are surprisingly low. Garlappi et al. (2008) provide a justification for this finding based on the relative bargaining strenghts of the parties in financial distress. On the other hand some studies have have shown a common variation in the time-series of returns to portfolios of stocks and corporate bonds (see for example Keim & Stambaugh (1986) and Ferson & Harvey (1991)).

Elton, Gruber, Agrawal & Mann (2001) show that in addition to compensation for expected losses, corporate bond yield spreads appear to contain compensation for tax effects and that there is a non-trivial residual component, related to the Fama-French factors and thus interpreted as a risk premium. Our study differs from theirs in that we consider firm specific data, study the time series of both expected losses and risk premia; and most importantly we rely on a model which provides an exact non-linear

[8] See among others Collin-Dufresne, Goldstein & Martin (2001), Campbell & Taksler (2003), Cremers, Driessen, Maenhout & Weinbaum (2004).

relationship between risk premia in equity markets and those in bond specific yield spreads.

An interesting related paper by Huang & Huang (2002) measures how much of observed credit spreads over the Treasury curve can be explained by structural models. Their analytical approach allows for time varying risk premia but their study does not focus on measuring risk premium components in bond spreads. They find that it is difficult to reconcile observed and model spreads. Interestingly, Leland (2004) finds that a selection of structural models, faced with difficulty in explaining corporate bond prices, are in fact quite successful at predicting default probabilities consistent with historical levels.

Chen, Collin-Dufresne & Goldstein (2005) consider whether existing asset pricing models that have proven successful in explaining equity returns can, if reasonably calibrated, explain the levels and volatilities of credit spreads. They have some success with models that exhibit time varying risk premia, in particular if the default boundaries are permitted to be countercyclical. Perhaps the main conclusion of their paper is the necessity of time varying risk premia to explain credit spreads. Our study clearly illustrates the dramatic time variation of these premia in the marketplace and documents the explanatory power of equity-implied risk premium estimates for bond market spreads.

Using a reduced form credit risk model, Driessen (2005) decomposes corporate bond yield spreads into tax, liquidity, interest rate risk and risk premium components. He finds that the ratio of risk neutral to objective default intensities is greater than one, suggesting that default event risk is priced. He obtains cross-sectional estimates of spread components and risk premia but does not explore the time series variation of risk premia nor does he explore the link with equity markets.

In a closely related paper, Berndt, Douglas, Duffie, Ferguson & Schranz (2004) (BDDFS) use expected default frequencies from Moody's KMV together with default swap prices to extract historical and risk neutral default intensities respectively. The ratio of these is interpreted as a measure of the risk premium observed in the marketplace. They document substantial time series variation in premia with a peak in the third quarter of 2002 and a subsequent dramatic drop. They show that their measure of the risk premium is strongly dependent on general stock market volatility after controlling for idiosyncratic equity volatility. They also find that their measure is increasing in credit quality. We document a similar behavior of the ratios of our risk neutral to objective default probabilities in our longer corporate bond sample, and show that this is in fact a prediction of the Leland & Toft (1996) structural credit risk model.

Berndt et al. (2006) (BLO) extract a factor representing the part of default swap returns, implied by a reduced form credit risk model, that does not relate to interest rate risk, expected default losses and the Fama-French factors. They find that this factor is priced in the corporate bond market but that they cannot establish with the same confidence that it is a factor for equity returns.

Saita (2006) studies the risk and return profiles of corporate bond portfolios using estimates of objective default probabilities obtained using a novel methodology.[9] He finds strikingly high levels of expected

[9] See Duffie et al. (2006).

excess returns that appear difficult to explain given the measured risks. For example, bond portfolio Sharpe ratios can be multiple times higher than the corresponding measure for the S&P 500 index.

In summary, although many studies have attempted to relate pricing in corporate credit and equity markets, the precise link between risk premia in the two markets is not yet well understood.

3 Measuring risk premia with corporate bond data

In equity markets, expected returns are most often proxied by average historical returns. Measuring expected returns and risk premia in corporate bond markets is a more daunting task due to the absence of long historical time series of regularly spaced data. Perhaps as a result, researchers tend to focus on bond yield spreads instead. This leaves us with another complication which is that the yield spread is an imperfect measure of the risk premium - it requires an adjustment as we shall explain in detail below.

Some recent related empirical work on risk premia in credit markets have relied on reduced from models. To permit a comparison of those results with ours, we briefly outline, in an appendix, the key theoretical results on risk premia in that literature. Our objective is also to estimate risk premia, but we rely on a contingent claims approach. In a first step, we seek to understand if this approach, based on using information on firm specific equities and bond specific contractual details, is able to explain the behavior of the risk premium reported by Berndt et al. (2004), Saita (2006) and Berndt et al. (2006).

To study the corporate bond risk premium, two different approaches avail themselves. First, one can measure the expected excess return directly and second, one can compute the part of the bond spread that represents a risk premium. The second approach has the advantage of also allowing us to learn about the expected loss component in the promised yield which has been central in many recent studies.[10]

3.1 Excess returns

In order to compute the return on a corporate discount bond that matures at time T, we would need to solve

$$R_B\left(t,T\right) - r(t,T) = \frac{E_t^P\left[\frac{B(v_T,T)}{B(v_t,T)} - 1\right]}{T-t} - r(t,T)$$

where $R_B\left(t,T\right)$ is the expected return and $r(t,T)$ is the risk free rate with maturity $T-t$.

However, when computing risk premia for holding coupon bearing bonds until maturity, we need to account for coupons as well as the rate at which these coupons are reinvested. There is no received solution to this problem, perhaps because most work on corporate bonds has focused on yield spreads instead of returns.

[10] See for example Elton et al. (2001).

We follow the methodology of Driessen & DeJong (2005) to estimate bond market implied risk premia. They show that

$$R_B(t,T) = [P_t(\tau < T)(1-l) + (1 - P_t(\tau < T))](1 + y(t,T))^T - 1 \tag{1}$$

where $R_B(t,T)$ is the expected return on a corporate discount bond that matures at time T, τ is the default time, l is the proportional loss given default, $r(t,T)$ the relevant benchmark risk free rate and $y(t,T)$ the corporate bond yield-to-maturity. The objective probability of a default prior to maturity is denoted $P_t(\tau < T)$, where τ represents the default time.

This expression is valid for discount bonds for which default losses are incurred at maturity. Maintaining the default timing assumption, this expression still holds for coupon bonds if we are willing to assume that coupons are reinvested at the initial yield.[11] The approach is also valid if we assume that coupons are reinvested at today's prevailing forward rates. Similar procedures for calculating expected corporate bond returns have been applied by Elton et al. (2001) and Campello et al. (2006).[12]

To estimate $P_t(\tau < T)$, we rely on the Leland & Toft (1996) (LT) model together with historical estimates of firm specific asset value risk premia.[13] Using aggregate data, Leland (2004) studies the ability of the LT model to predict default probabilities. He finds that the model is able to fit historical default experience for A, Baa and B rated debt reasonably well for horizons of 5 years and longer. The model underestimates shorter term default probabilities.

Recent work by Berndt et al. (2004) and Berndt et al. (2006) on risk premia relies on Moody's KMV expected one-year default frequencies.[14] These are based on using a structural credit risk model to compute a firm's distance to default. This metric is then mapped into historical probabilities using an extensive database of default experience. Conceptually, this approach is very similar to our method of using the LT model to predict default probabilities.[15] The key difference is that KMV only use the model to rank companies according to default riskiness, whereas we combine the model with firm risk premium estimates to arrive at default probabilities.

[11] An analoguous assumption is made when using yield-to-maturity as a measure of promised return in the government bond market.

[12] Campello et al. (2006) use an Ito's lemma approximation to compute the expected return from the duration and the convexities of bonds.

[13] Details of their estimation are provided below. The closed form solution for the default probablity is provided in the appendix.

[14] Saita (2006) uses a different approach, based on Duffie et al. (2006), where default probabilities are allowed to depend on both firm specific and macro-economic variables. One of the firm specific variables is distance-to-default.

[15] See Leland (2004) for an interesting discussion of the two approaches.

3.2 Yield spread components

3.2.1 An example

There is an important distinction between a bond risk premium and yield spread. Consider, for simplicity, a unit zero discount bond with zero recovery in default issued by a firm that can only default at time T. The value of that bond is $B(t) = e^{-r(t,T)T} E_t^Q [I_{\tau>T}]$, or $e^{-r(t,T)T} Q_t(\tau > T)$, the present value of the risk adjusted survival probability.

Assume for the time being that default is not a priced risk. Then there will be no distinction between historical and risk neutral survival probabilities. Suppose $P_t(\tau > T) = Q_t(\tau > T) = 80\%$, where P_t denotes the objective survival probability. Assume further that $r(t,T) = 10\%$, $T = 10$. Then the price of the bond is $B = e^{-0.10 \times 10} 0.8 = 0.2943$ and its continuously compounded yield is 12.23%. Thus the bond pays a spread of 223 basis points, while there is no risk premium.

In other words, the presence of a positive yield spread by no means implies that there is a risk premium for default, merely an actuarially fair compensation for expected losses – in this case the present value of expected default losses is $EL = e^{-0.1 \times 10} 0.2 = 0.073576$.

Now consider an economy where default is a priced risk, implying that $P_t(\tau > T) > Q_t(\tau > T)$. Assume the same parameters as above except that $Q_t(\tau > T) = 70\% < P_t(\tau > T) = 80\%$. Now the bond price is lower at $B = e^{-0.10 \times 10} 0.7 = 0.25752$ and accordingly, the yield spread has increased by 134 to 357 basis points. This increase (denoted π) reflects the risk premium for bearing default risk.

Another way of expressing this is that the value of a bond can be written as either (i) the present value (at the risk adjusted rate) of the expected payment at maturity or (ii) the present value of the full face value discounted at the risk free rate augmented by a spread s. This spread contains both a component γ which adjusts for expected losses (in this example 223 basis points) and a risk premium part π (134 basis points):

$$\begin{aligned} e^{-(r(t,T)+\pi)T} E[B(v_T)] &= e^{-(r(t,T)+s)T} 100 \\ \text{with } s &= \gamma + \pi \end{aligned}$$

Note that the π component corresponds to the excess return $R_B(t,T) - r(t,T)$ dicussed above.

3.2.2 Disentangling yield components in practice

In what follows we describe our methodology for measuring risk premia in yield spreads, π. The most common application of a structural model such as the Leland & Toft model is to use balance sheet information, together with estimates of asset value and volatility to infer term structures of risk neutral default probabilities, $\{Q_t(\tau < s); s \in (t, \infty)\}$. However, like Leland (2004), we use the model together with estimates of asset value risk premia to provide term structures of objective default probabilities,

$\{P_t(\tau < s); s \in (t, \infty)\}$.

In order to disentangle the risk premium component from market bond spreads, we use these objective probabilities. Given knowledge of $\{P_t(\tau < s); s \in (t, \infty)\}$, we obtain an estimate of the price that would prevail in a market without risk premia:

$$\begin{aligned} B_{t,T} &= \sum_{i=1}^{N} d_i \cdot c_i \cdot (1 - P_t(\tau < s_i)) + d_N \cdot p \cdot (1 - P_t(\tau < T)) \\ &\quad + R \cdot p \cdot \int_t^T d_s \cdot dP_t(s), \end{aligned} \tag{2}$$

where d_i are risk free discount factors, p the face value, c_i denote promised coupon payments and R represents the recovery rate in default.

In such a market the yield spread would only compensate for average losses. We call this spread the expected loss spread (c.f. the γ component above). The difference between the actual yield spread and the lower spread obtained using the price in (2) is our estimate of the risk premium components of corporate bond spreads.

In the next section we discuss our method for estimating bond risk premia without bond price data, using only equity and balance sheet data.

4 Estimating corporate bond risk premia with equity data

Since stocks and bonds are contingent claims on a firm's assets, it is natural to expect a large proportion of risk premia observed in the bond market to be explained by premia inferred from the equity market.

As noted above, common factors shown to explain equity returns have met with limited success in explaining corporate bond returns.[16] However bond returns cannot be viewed as linear functions of stock returns. They will depend on the issuing firm's characteristics such as leverage and business risk while also incorporating information about bond specific contractual features. In the remainder of this section we will delineate our methodology for extracting equity-implied bond specific risk premia, while providing a brief explanation of the model we use in the process.

To estimate excess returns and bond spread components, we need a model that give us the price of the equity and the bond as well as the sensitivity of the equity and the bond with respect to the value of the asset at any time t. We will base our discussion on the LT model, although this is not crucial to the implications. Leland & Toft (1996) assume that the value of a firm's assets evolves as a geometric

[16] For example, Fama & French (1993) show that common factors in the equity market have some explanatory power for the bond market but mainly when augmenting the set of equity factors with bond market factors (term structure and default premium).

Brownian motion:

$$dv_t = (\mu_v - \beta)\, v_t dt + \sigma v_t dW_t$$

where β is the payout ratio, σ is the volatility of the asset value return and W_t is a Brownian motion.

Default is triggered by the shareholders' endogenous decision to stop servicing debt. The value of the firm differs from the value of the assets by the values of the tax shield and the expected bankruptcy costs. Coupon payments are tax deductible at a rate τ and the realized costs of financial distress amount to a fraction α of the value of the assets in default (i.e. L). The firm continuously issues debt of maturity M, while retiring older vintages. Hence, at any given time, the firm has many overlapping debt contracts outstanding. The LT framework allows closed form solutions for the value of the firm's equity and liabilities.[17] In addition, it allows us to derive straightforward closed form solutions for a bond's price as well as its sensitivity to changes in the asset value. These will prove useful in the next step as we turn to computing excess returns.

4.1 Equity-implied bond excess returns

Following Campello et al. (2006), we use the Euler equation together with Ito's lemma and explicitly link the risk premia for stocks and bonds. The key to this approach is that it allows estimates of instantaneous expected equity return $R_S(t)$ to be translated into bond specific instantaneous expected return $R_B(t)$. Note that this relation requires only the existence of a state price density and that the mean rate and the volatility of the asset return are functions of time and the value of the asset itself only. More precisely

$$\begin{aligned} (R_B(t) - r) &= \Delta_{B/S} \cdot (R_S(t) - r), \\ \text{where } \Delta_{B/S} &= \left(\frac{\frac{\partial B(v_t,t)}{\partial v_t}}{\frac{\partial S(v_t,t)}{\partial v_t}} \frac{S(v_t,t)}{B(v_t,t)} \right), \end{aligned} \tag{3}$$

$$\begin{aligned} R_S(t)\, dt &= E_t \left[\frac{dS(v_t,t)}{S(v_t,t)} \right] \text{ and} \\ R_B(t)\, dt &= E_t \left[\frac{dB(v_t,t)}{B(v_t,t)} \right] \end{aligned}$$

where S and B denote stock and bond prices respectively. We use the Leland & Toft (1996) model for the senstitivies $\frac{\partial B(v_t,t)}{\partial v_t}$ and $\frac{\partial S(v_t,t)}{\partial v_t}$ and the term structure of risk-adjusted default probabilities used in pricing the bond.

The key determinants of the bond risk premium in (3) are (i) the premium in the equity market, (ii)

[17] The relevant expressions are reproduced in the appendix.

the characteristics of the firm and (iii) the contractual features of the bond.

For example, two bonds issued by the same firm may have different expected excess returns simply due to differences in maturity and cash flow structure. Identical bonds issued by two different firms with the same objective default probability (as measured by the credit rating) may be different depending on the degree of systematic risk at the firm level (e.g as measured by beta). To date most empirical work has ignored bond characteristics and intra rating category differences in systematic risk.

We now turn to our second risk premium metric: the part of a bond's yield spread that compensates for systematic risk.

4.2 Equity-implied yield spread components

We have already discussed how to compute the bond price that would result in a market without systematic risk. In a market with risk premia on the other hand the bond price is given by

$$\begin{aligned} B_t &= \sum_{i=1}^{N} d_i \cdot c_i \cdot \left(1 - Q_t\left(\tau < s_i\right)\right) + d_N \cdot p \cdot \left(1 - Q_t\left(\tau < T\right)\right) \\ &\quad + R \cdot p \cdot \int_t^T d_s \cdot dQ_t\left(s\right), \end{aligned} \tag{4}$$

where d_i are risk free discount factors, p the face value, c_i promised coupon payments and R the recovery rate, respectively. Risk adjusted probabilities are denoted $Q\left(\cdot\right)$.

The difference between the model yield spread obtained using the price in (4) and the lower spread obtained using the price in (2) defines our equity implied measure of the risk premium component of a corporate bond spread.

We now turn to the empirical implementation of our framework.

5 Empirical implementation

In this section we will describe our estimation methodology for equity-implied and bond market measured risk premia.

Since we do not observe government bond yields or swap rates for all relevant maturities, we estimate the term structure of default free zero coupon interest rates using the extended Nelson & Siegel form due to Svensson (1995):

$$r(t,T) = \delta_{1,t} + \delta_{2,t}\frac{1 - e^{-\delta_{3,t}(T-t)}}{\delta_{3,t}(T-t)} + \delta_{4,t}e^{-\delta_{3,t}(T-t)} + \delta_{5,t}\frac{1 - e^{-\delta_{6,t}(T-t)}}{\delta_{6,t}(T-t)}$$

Each day from 1990 to 2004 we estimate the parameters $\delta_{1,t}, \delta_{2,t}, \delta_{3,t}, \delta_{4,t}, \delta_{5,t}, \delta_{6,t}$ by minimizing the sum of squared bond pricing errors for constant maturity treasury yields and interest rate swap yields.[18]

5.1 Estimating structural credit risk models

From equation (3), it is clear that to estimate equity implied bond risk premia, we require estimates of equity risk premia as well as the price of the bonds and the sensitivities of the bond and the equity with respect to the asset value. This is equally valid when disentangling risk premia from bond yield spreads.

5.1.1 Bond prices and sensitivities

In addition to benchmark term structures, the following inputs are needed to price bonds, and to compute the sensitivity of the stock and the bond with respect to the value of the assets

- the bond's principal amount, p, the coupons c , maturity T and the coupon dates, t_i;
- the recovery rate of the bond, ψ;
- the total nominal amount of debt, N, coupon C and maturity M
- the costs of financial distress, α
- the tax rate, τ
- the rate, β, at which earnings are generated by the assets, and finally
- the current value, v, and volatility of assets, σ

The bond's principal amount, p, the coupons c , maturity T and the coupon dates are readily observable. The recovery rate of the bond in financial distress is not. We set it equal to 40%, roughly consistent with average defaulted debt recovery rate estimates for US industrial entities between 1985-2003.

The nominal amount of debt is measured by the total liabilities as reported in COMPUSTAT. Since book values are only available at the quarterly level, we linearly interpolate in order to obtain daily figures. For simplicity, we assume that the average coupon paid out to all the firm's debt holders equals the risk-free rate: $c = r \cdot N$.[19] We set the maturity of newly issued debt equal to 6.76 years, consistent with empirical evidence reported in Stohs & Mauer (1994).

Finally, we assume that 15% of the firm's assets are lost in financial distress before being paid out to debtholders and fix the tax rate at 20%.The choice of 15% distress costs lies within the range estimated

[18] For robustness, alternative term structure specifications have been used - including cubic splines with smoothness conditions ruling out negative forward rates. The specification has negligible results for our study.

[19] This assumption is made for convenience. We checked this assumption by considering randomly selected firms' actual interest expense ratios. We found that our approximation performs well.

by Andrade & Kaplan (1998). The choice of 20% for the effective tax rate is consistent with the previous literature (see e.g. Leland (1998)) and is intentionally lower than the corporate tax rate to reflect personal tax benefits to equity returns, thus reducing the tax advantage of debt.

The payout rate β is an important parameter. We compute β as the weighted average of net of tax interest expenses (relative to total liabilities (TL)) and the equity dividend yield (DY):

$$\beta = \frac{IE}{TL} \times lev \times (1 - TR) + DY \times (1 - lev) \tag{5}$$

where

$$lev = \frac{TL}{TL + MC}$$

where MC denotes the firm's equity market capitalization and TR is the effective tax rate. The average net debt payout rate in our sample is 2.9%.[20]

We then require estimates of asset value and volatility. The methodology utilized was first proposed by Duan (1994). The maximum likelihood estimation relies on a time series of stock prices, $E^{obs} = \left\{\mathcal{E}_i^{obs} : i = 1...n\right\}$. If we let $w\left(\mathcal{E}_i^{obs}, t_i; \sigma\right) \equiv E^{-1}\left(\mathcal{E}_i^{obs}, t_i; \sigma\right)$ be the inverse of the equity function, the likelihood function for equity can be expressed as

$$\begin{aligned} L_{\mathcal{E}}\left(\mathcal{E}^{obs}; \sigma, \mu\right) &= L_{\ln v}\left(\ln w\left(\mathcal{E}_i^{obs}, t_i; \sigma, \mu\right) : i = 2...n; \sigma\right) \\ &\quad - \sum_{i=2}^{n} \ln v_i \left.\frac{\partial\,\mathcal{E}\left(v_i, t_i; \sigma\right)}{\partial\, v_i}\right|_{v_i = w\left(\mathcal{E}_i^{obs}, t_i; \sigma\right)} \end{aligned} \tag{6}$$

$L_{\ln v}$ is the standard likelihood function for a normally distributed variable, the log of the asset value, and $\frac{\partial\,\mathcal{E}_i}{\partial v_i}$ is the "delta" of the equity formula. An estimate of the asset values is computed using the inverse equity function: $v_t = w\left(\mathcal{E}_n^{obs}, t_n; \widehat{\sigma}\right)$. Once we have obtained the pair $(\widehat{v}_t, \widehat{\sigma})$ it is straightforward to compute equity and debt values as well as sensitivities. See appendix.

5.2 Estimating default probabilities

Above, we describe our methodology for extracting observed risk premia. To apply equation (1), we need to estimate bond spreads, loss rates and have a proxy for the default probabilities for different horizons. This is also necessary for disaggregating spreads into risk premia and expected losses.

We set the loss rate l equals to 60%, roughly consistent with average defaulted debt recovery rate

[20] An alternative method for estimating the cash flow rate is to use bond coupons as a proxy for the firm's proportional interest expenses. Our estimates of the cash flow rate will be lower than if we had used this approach. First, coupons are pre-tax and second corporate bonds are long term instruments. While the bond coupon may proxy well for the interest expense on long term liabilities, we find that in our sample it overestimates the interest expenses paid on short term debt. Our average net of tax interest expense ratio is about 3% which is just less than half the average bond coupon of 7.2% in our sample.

estimates for US entities between 1985-2003. Like much of previous work, our paper is limited by not considering stochastic recovery rates.

Previous studies on the default risk premium Berndt et al. (2004), Saita (2006) and Berndt et al. (2006) used Expected Default frequencies (EDFs) provided by Moody's KMV as their estimate of the historical default probabilities. In this paper, we estimate company specific default probabilities using the Leland & Toft (1996) model. This methodology yields estimates conceptually similar to EDFs.

The default probabilities $P_t(\tau > T_i)$ are provided in closed form in the appendix. The only parameter that still needs to be estimated at this point is the expected return of the asset value under the objective measure denoted μ_v.

Previous studies such as Leland (2004) and Huang & Huang (2002) have used the CAPM beta of the firm multiplied with an average market risk premium figure to provide an estimate of the expected asset return. In contrast, we use the same methodology that we applied above to link the bond risk premium and the equity risk premium. Equity is a contingent claim on the asset value and we can write

$$\begin{aligned} \mu_v - r &= (R_v(t) - r) = \Delta_E \cdot (R_E(t) - r) \\ \text{where } \Delta_E &= \left(\frac{\frac{\partial E(v_t,t)}{\partial v_t} v}{E(v_t,t)} \right)^{-1} \end{aligned}$$

where $\frac{\partial E(v_t,t)}{\partial v_t}$ is computed using the LT model and $(R_E(t) - r)$ is the estimated equity risk premium. Equity risk premia are constructed using average realized returns. For every transaction in our bond data, we check if 1200 daily returns prior to the transaction date are available. Otherwise a bond is dropped.

5.3 Data

We use the following data for our estimation: firm market equity values, balance sheet information, and term structures of swap rates. Daily equity values are obtained from CRSP. Quarterly firm balance sheet data are taken from COMPUSTAT. Since balance sheet information is only available at quarterly level, we transform it into daily data through linear interpolation. Swap rates are acquired from DataStream. We take the US constant maturity Treasury rates from the Federal Reserve Board data archive.

Our bond transaction data are sourced from the National Association of Insurance Commissioners (NAIC). Bond issue- and issuer-related descriptive data are obtained from the Fixed Investment Securities Database (FISD). The majority of transactions in the NAIC database take place between 1994 and 2004. Cleaning of the raw NAIC database was carried out in four steps.

1. Bond transactions with counterpart names other than clearly recognizable financial institutions were removed. Transactions without a clearly defined counterparty were deemed unreliable.

2. We restricted our sample to fixed coupon rate USD denominated bonds with issuers in the industrial sector. Furthermore, we eliminated bond issues with option features, such as callables, putables, and convertibles. Asset-backed issues, bonds with sinking funds or credit enhancements were also removed to ensure bond prices in the sample truly reflect the underlying credit quality of issuers.

3. We eliminated bonds issued by Municipal, provincial,and any special agencies to ensure that the bond prices in the sample are cleaned from any special tax treatment inherent the issuer type.

4. The last step involves selecting those bonds for which we have their issuers' complete and reliable market capitalizations as well as accounting information about liabilities. Then we eliminated those bonds for which there is less than 1200 previous daily equity returns available, for the purpose of computing the equity risk premia at that transaction date.

6 Empirical results

We begin, in Table 1, by describing, on an aggregate basis, the inputs to our estimation (panel A) and the intermediate firm specific outputs (panel B). The data covers a wide variety of firms and bonds. Firm sizes vary between just over 130 million dollars to just less than half a trillion. Leverage ratios range from naught to almost 100%. The bonds vary widely in maturity (between a few months and 100 years), in yield spreads (1 to 1050 basis points relative to the Treasury curve) and credit rating (AAA to defaulted). Our MLE estimation yields estimates of firm asset values of on average 49 billion dollars, asset volatilities of 20% on average. Our estimates of asset volatility are consistent with previous work by Schaefer & Strebulaev (2004). Figure 1 plots their estimates and ours by rating categories. Our across sample average is somewhat lower but the pattern across rating categories is strikingly similar with the exception of the lowest category.[21]

[21] We only have about 130 transactions, most of which relate to one firm in our CCC category, whereas they have more than 1600. This suggests that the noticeable difference between our estimates in this particular category may be outlier driven.

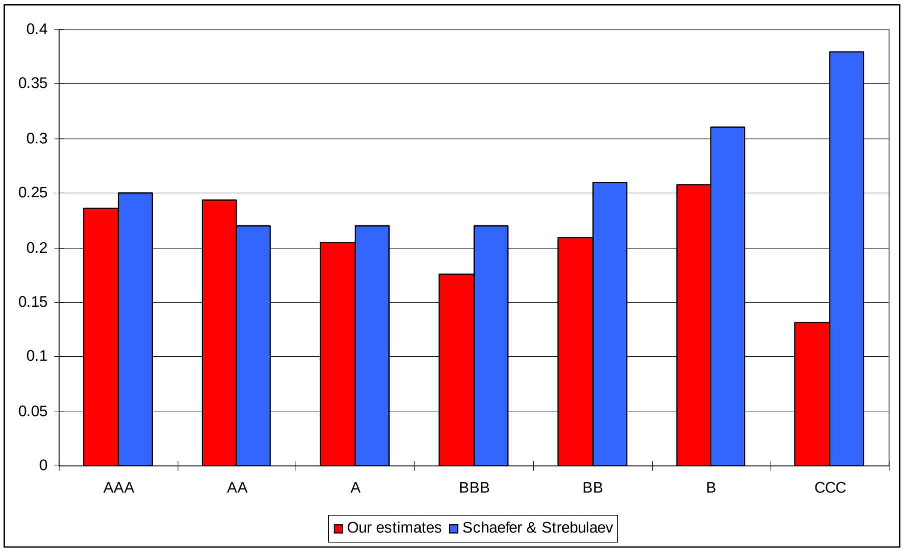

Figure 1

Table 1: Summary statistics

Panel A. Summary statistics of firm specific variables.

	N	Mean	Std. Dev.	Min	Max
Firm size (USD billion)	34,414	51,610	78.88	0.13	471.35
Leverage (%)	34,414	45.48	20.28	1.31	96.83
# of transactions / firm	34,414	216	285	1	1947
Bond maturity (T)	34,414	11.80	9.77	0.01	100
Bond yield spread (bps)	34,414	146	131	1	1055
S&P rating	34,414	8.3	4.4	1	27
Historical equity volatility (%)	34,414	22.26	6.31	9.94	45.74
Firm equity β	34,414	0.86	0.24	-0.12	1.92

Panel B. Summary statistics of estimated firm specific characteristics

	N	Mean	Std. Dev.	Min	Max
Model yield spread		97	151	0	1077
asset value (v_t) (USD million)		48,815	74,552	119.72	442,031
Asset volatility (σ) (%)		20.24	10.45	2.19	89.93
Default threshold (L)		16,163	36,209	0.69	391,350

Before moving to our estimates of risk premia in corporate bond markets, we present intermediate results on our estimation of objective default probabilities. For our risk premium estimates to be quantitatively reasonable, the employed default probabilities need to be as well. Note that most work prior to Berndt et al. (2004) rely on historical rating based default probabilities.For example Elton et al. (2001) use a constant rating transition matrix for a ten year period and across all firms within a rating category. Recent work by Campello et al. (2006) relies on time varying rating based default probabilities. Our approach permits us to capture simultaneously the variation in default probabilities across time and across firms within rating categories.

To begin, we benchmark our estimates of default probabilities across horizons and rating categories. Figure 2 plots our estimates together with historical averages provided by Moody's cumulative default rates for the period 1920-2004. Given that the sample periods are very different, it is not clear what to expect. It is interesting to note that for the largest rating category in our sample, BBB, our average estimates across the 20 different years are quite similar to historical averages.

Table 2 provides a more detailed overview of the relationship between our estimated default probabilities and historical averages per rating category and horizon.

	Mod AA	Actual AA	Mod A	Actual A	Mod BBB	Actual BBB	Mod BB	Actual BB	Mod B	Actual B
1	0.00%	0.06%	0.12%	0.08%	0.42%	0.31%	0.32%	1.39%	1.08%	4.56%
2	0.00%	0.19%	0.34%	0.25%	1.29%	0.93%	1.63%	3.36%	3.96%	9.97%
3	0.02%	0.32%	0.71%	0.54%	2.32%	1.69%	3.34%	5.48%	7.06%	15.24%
4	0.07%	0.49%	1.21%	0.87%	3.41%	2.55%	5.11%	7.71%	9.93%	19.85%
5	0.14%	0.78%	1.76%	1.22%	4.50%	3.40%	6.80%	9.93%	12.48%	23.80%
6	0.23%	1.11%	2.34%	1.58%	5.54%	4.28%	8.38%	12.01%	14.74%	27.13%
7	0.34%	1.48%	2.93%	1.98%	6.53%	5.12%	9.83%	13.84%	16.74%	30.16%
8	0.45%	1.85%	3.51%	2.34%	7.45%	5.95%	11.16%	15.65%	18.51%	32.62%
9	0.57%	2.20%	4.07%	2.76%	8.32%	6.83%	12.37%	17.25%	20.08%	34.74%
10	0.70%	2.57%	4.61%	3.22%	9.12%	7.63%	13.48%	19.00%	21.49%	36.51%
11	0.83%	3.01%	5.13%	3.71%	9.87%	8.42%	14.50%	20.60%	22.77%	38.24%
12	0.95%	3.50%	5.63%	4.21%	10.56%	9.22%	15.43%	22.16%	23.92%	39.80%
13	1.07%	3.98%	6.10%	4.65%	11.21%	10.00%	16.29%	23.72%	24.96%	41.23%
14	1.19%	4.48%	6.55%	5.09%	11.82%	10.70%	17.08%	25.10%	25.92%	42.67%
15	1.31%	4.87%	6.97%	5.56%	12.38%	11.32%	17.82%	26.31%	26.80%	43.92%
16	1.42%	5.13%	7.38%	6.02%	12.92%	11.91%	18.50%	27.44%	27.60%	45.21%
17	1.53%	5.35%	7.76%	6.30%	13.41%	12.51%	19.14%	28.59%	28.35%	46.15%
18	1.64%	5.57%	8.13%	6.60%	13.88%	13.04%	19.73%	29.70%	29.04%	46.89%
19	1.74%	5.87%	8.47%	6.89%	14.32%	13.49%	20.28%	30.58%	29.69%	47.52%
20	1.84%	6.09%	8.80%	7.19%	14.74%	13.95%	20.80%	31.48%	30.29%	47.79%

Table 2. Historical and model predicted default probabilities by rating categories and horizon. The historical probabilities represent Moody's cumulative default rates for 1920- 2004.

Some previous work has relied on KMV expected default frequencies (EDFs) as measures of objective default probabilities. As noted above, KMV use a structural model to estimate firm specific default metrics which are mapped into default probabilities using historical default experience. Given the relative similarity of historical default experience as reported by Moody's and our estimates of default probabilities we don't expect a systematic bias to be induced by our methodology relative to using EDFs. Hence we expect that our results can be related to those reported in BLO and BDDFS. In what follows, we turn to a discussion of our estimated risk premium metrics.

6.1 Risk premia measured in bond markets

In table 3 we report the two measures of bond risk premia discussed above: the expected excess return and the risk premium component of bond yield spreads.

Consider first our measure of risk premia based on equation (1). Figure 3 plots the time series of average excess return imputed from bond market spreads using the methodology in section 3.

Table 3: Market measured bond risk premium metrics

	N	Mean	Std. Dev.	Min	Max
Bond Excess return $E[R_B(t) - r]$ measured in bond markets (bps)	34414	77	110	-821	950
Spread RP component measured in bond markets	34414	91	123	-822	1219

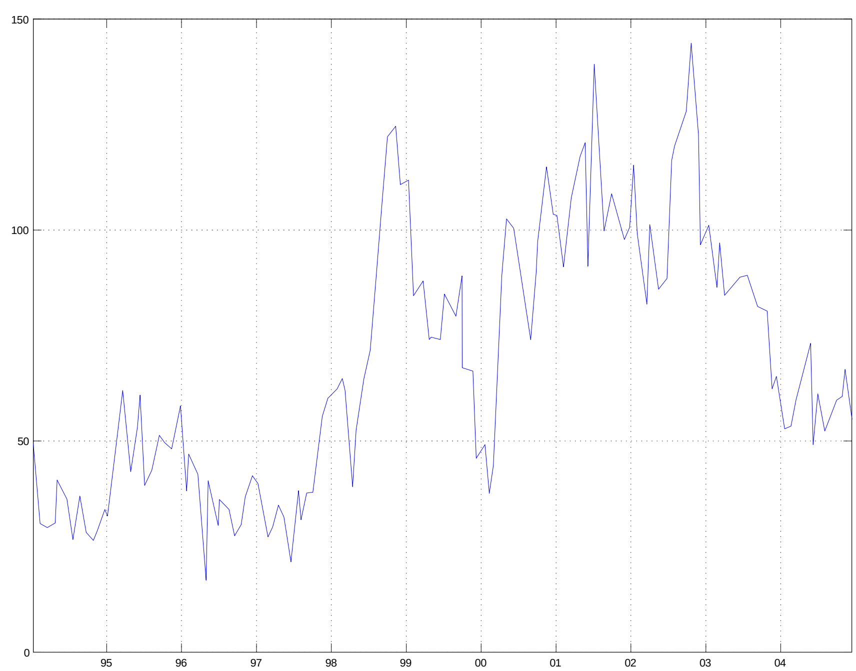

Figure 3: Monthly average bond market measured excess returns 1994-2004. For each bond the excess return is computed using equation (1).

Figure 4: Monthly average excess returns for the sample period in Berndt et al. (2004).

Clearly this measure of the bond risk premium is highly time varying. There is a peak after the LTCM crisis in late 1998 followed by a sharp drop until early 2000. After late 2000, the overall level seems to have shifted up to a higher level of about 100 basis points, then decreasing until the end of the sample.

BDDFS study the period 2001-2004. For ease of comparison, Figure 4 shows the same risk premium metric as Figure 3 for the same sample period. They document a peak in the third quarter of 2002 with a steady drop until the end of 2003, in particular for the broadcasting industry. We find the same pattern, although for the whole of our sample: the risk premium peaks at 140 basis points decreasing to levels of about 50 basis points by the end of that year. The observed similarity in patterns is all the more striking when one considers that we are using a dataset with a different cross section of firms, a different financial instrument (bonds rather than credit swaps) and employ a different methodology. We interpret this as an implicit robustness test of our default probability estimates, which we argued above should be similar to the EDFs used by BDDFS.

We are also able to identify two earlier peaks in risk premia: one after the LTCM period in 1998 and another lesser in mid 2001. The post LTCM peak is followed by a drop in premia of the same magnitude as the 2002 episode. All three peaks appear to be short-lived, lasting no longer than 2-3 months.

Next we turn to a decomposition of bond yield spreads. We separate the risk premium component from the total spread. This will provide a robustness check on our results and express them in an easily interpretable unit. Another important motivation is that this allows us to analyze the impact of expected losses on bond yield spreads. As we shall see the two components behave quite differently.

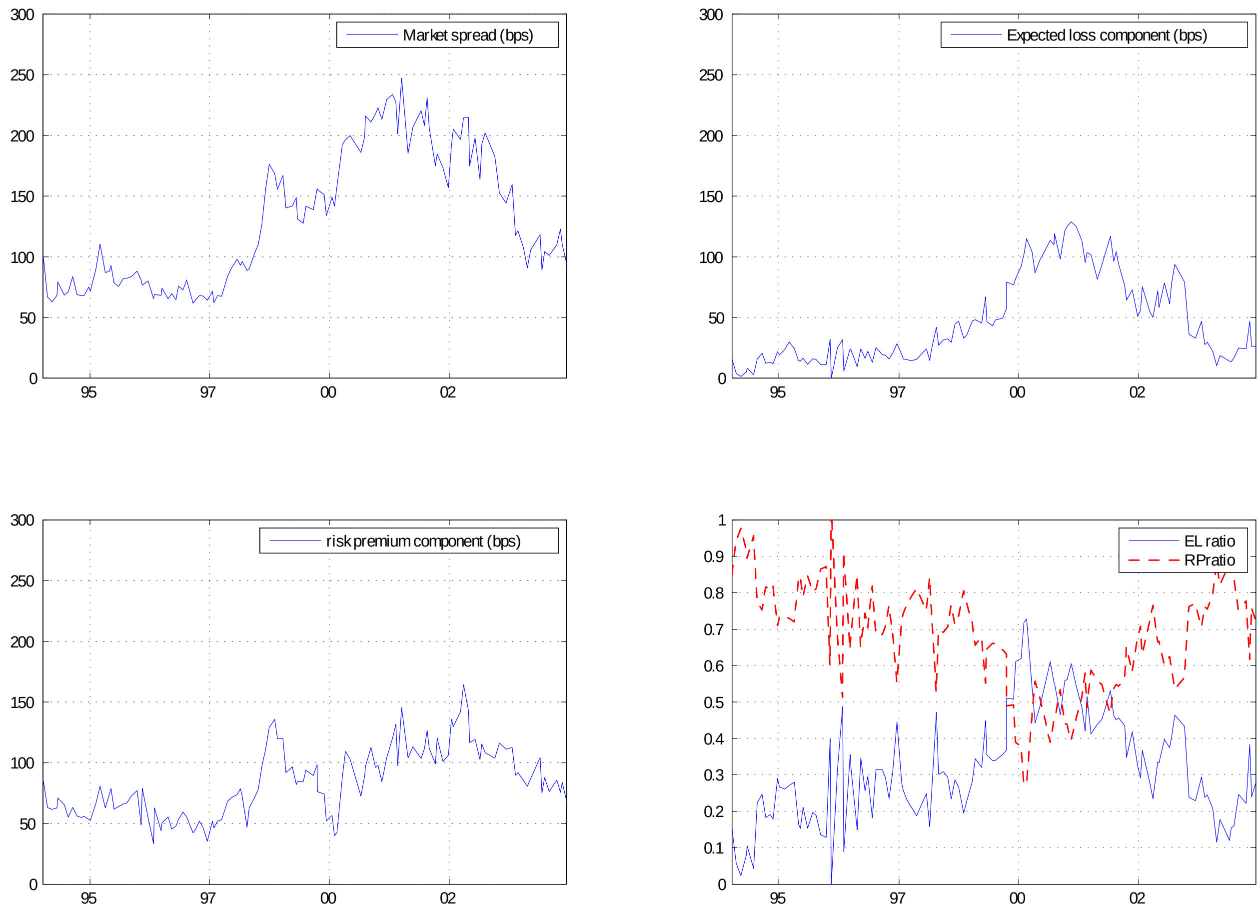

Figure 5. Overview of estimated market spread components. Panel A plots the monthly average market spread relative to the CMT curve for our full sample. Panels B and C plots the expected loss and risk premium components respectively. Panel D plots the ratio of the two components to the total spread.

Panel A of figure 5 plots the average monthly market bond spread in our sample. Spreads were stable during the first part of our sample, whereas the post LTCM period is marked by a dramatic increase in spread levels and volatility. Considering the expected loss spreads in panel B, we find that these peak quite a bit later - towards the beginning of 2000. Panel C reports the risk premium component of the spreads in panel A. The risk premium component does not simply mirror the behavior of the spread. For example it appears that the spike in spreads at the end of 1998 is driven by increased risk premia rather than expected losses. Post LTCM, risk premia decline to reach a low of just less than 50 basis points in early 2000, whereas the expected loss component on the other hand is reaching a high plateau which persists during 2001.

It is interesting and reassuring to note that the spread risk premium component in panel C behaves similarly to the measured excess return in figure 3. A striking way of depicting the relative importance of risk premia and expected losses is provided in panel D. It plots the respective percentage of the total spread explained by risk premia and expected losses. In the earlier part of the sample, the dominant component of the spread is the risk premium. Its importance trends downwards until the beginning of 2000 when it begins to recover and eventually reach a level of about 75% towards the end of the sample. The period when the expected loss component is the most important coincides with a period of unprecedented default losses in the US corporate bond markets.

In summary, we have measured risk premia in corporate bond markets. Our results are consistent with previous findings based on reduced form models in credit derivative markets for those subperiods when our samples overlap. So far we have used a structural model to estimate default probabilities. We now wish to see if a structural model taken together with estimates of risk premia in equity markets can explain the observed risk premia in credit markets.

6.2 Risk premia inferred from equity markets

We now discuss the results for our bond risk premia estimated from equity risk premia. In brief, bond risk premia can be thought of as a non-linear translation of equity risk premia. In a general contingent claims model of a firm's security prices, we can derive a relationship between risk premia on bonds, stocks and firm values (see equation (3) above). To operationalize this relationship we rely on the Leland & Toft (1996) model as a link between equity prices, asset value and volatility.

Most work in asset pricing measures excess returns relative to Treasury rates. In fixed income markets, the choice of benchmark rate is an important and more subtle issue. In fixed income derivatives markets, practitioners typically rely on interest rate swap rates to construct a reference curve. In the cash market, corporate bond spreads are also often measured against this curve. One reason for this is the arbitrage relationship between credit default swaps and corporate bonds. The argument links the basis point price of default protection with the spread of a corporate bond over a floating rate benchmark, in practice the swap curve.[22] Recent work suggests that structural models are able to predict the level of default protection prices and that of bond spreads over the swap curve, while underestimating spreads relative to the government curve. [23] For our purposes in this paper, we need an unbiased model of the bond spread

[22] For details see e.g. Rajan et al. (2007).

[23] See Ericsson et al. (2006). Choudhry (2006) provides an interesting discussion of the default swap basis - the differential pricing of credit risk in bond and credit derivative markets.

in order to avoid a bias in the decomposition of spreads into risk premia and expected loss components respectively. This leads us to rely on the swap curve as the key benchmark, but initially we also report results for the government benchmark curve for completeness.

As an input, we require an estimate of the risk premium in equity markets, which we obtain as described above. Table 4 summarizes the inputs as well as key outputs of this exercise. The average equity risk premium during our sample is 10.3%. This translates into an implied bond excess return of 80 basis points for the government curve and 63 basis points for the swap curve. When yield spreads are measured against the Treasury curve, 41 basis points of that spread represents compensation for default risk as implied by the premium in equity markets. When the swap curve is used, the risk premium spread is 34 basis points. Table 5 provides a summary of our bond market measured risk premia based on the swap curve.

The equity implied excess returns appear able to capture the excess returns in the bond market on average, regardless of the benchmark employed. For the spread components, the choice of benchmark will be a key determinant of the average levels.

Table 4: Equity implied bond risk premium metrics

	N	Mean	Std. Dev.	Min	Max
$\Delta_{B/S}$		0.078	0.071	0	0.283
Equity risk premium (%) $(R_S(t) - r)$	34414	10.30	2.93	-1.47	23.08
Bond Excess return $E[R_B(t) - r]$ estimated from equity data (CMT)	34414	80	78	-6	486
Spread RP component estimated from equity data (CMT)	34414	41	49	-89	1158
Bond Excess return $E[R_B(t) - r]$ estimated from equity data (Swap curve)	34414	66	70	-6	473
Spread RP component estimated from equity data (Swap curve)	34414	34	43	-86	342

Table 5: Market measured bond risk premium metrics (Swap curve)

	N	Mean	Std. Dev.	Min	Max
Bond Excess return $E[R_B(t) - r]$ measured in bond markets (bps)	34414	63	115	-788	985
Spread RP component measured in bond markets	34414	51	121	-760	1031

Figure 6 plots our measured excess returns in bond markets and our equity implied bond excess returns. Clearly, on average the fit is rather good.[24] There are period of divergence, such as the period 1999-2000 where our model overpredicts the risk premium and in late 2002 when it underestimates the premium. The latter time period coincides with the period during which BDDFS document, like us, a sharp increase in the risk premium. This figure suggests that across firms on average, there is a clear relationship between equity and corporate bond risk premia.

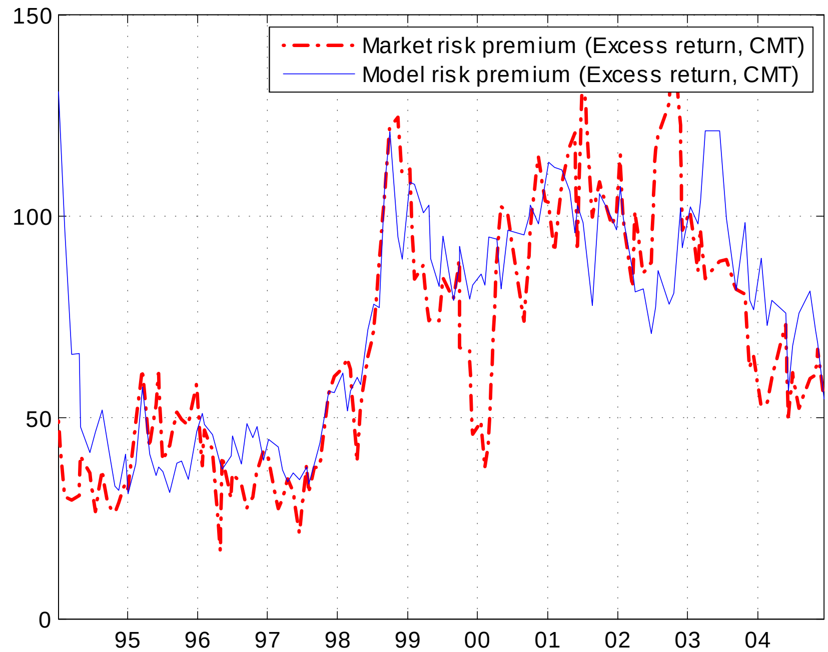

Figure 6: bond market measured and equity implied bond excess returns. Based on using the CMT curve as risk free benchmark.

Figure 7 plots the average raw equity risk premia that are used as inputs in equation (3) together with the average bond risk premia. It is clear that bond risk premia are highly non-linear translations of risk premia for the corresponding stocks. Note for example the period 1998-2003. During this period equity risk premia trended slowly downwards, while bond risk premia in fact did the opposite in a less regular fashion. In addition the volatility of the bond risk premium seems more variable than that of the equity premia. In a structural model, keeping leverage and volatilities constant the relationship between the two should be positive, just like for spreads - when stock prices increase bond prices should increase as well. Thus, the explanation must lie in time variation in either leverage or volatility. We return to a more detailed discussion of this below, but note that this result is related to findings in Campbell &

[24] Although not reported, a similar pattern is found when the swap curve is used.

Taksler (2003). They find that during the late nineties, bond spreads increased as stock markets were performing well, an apparent contradiction.

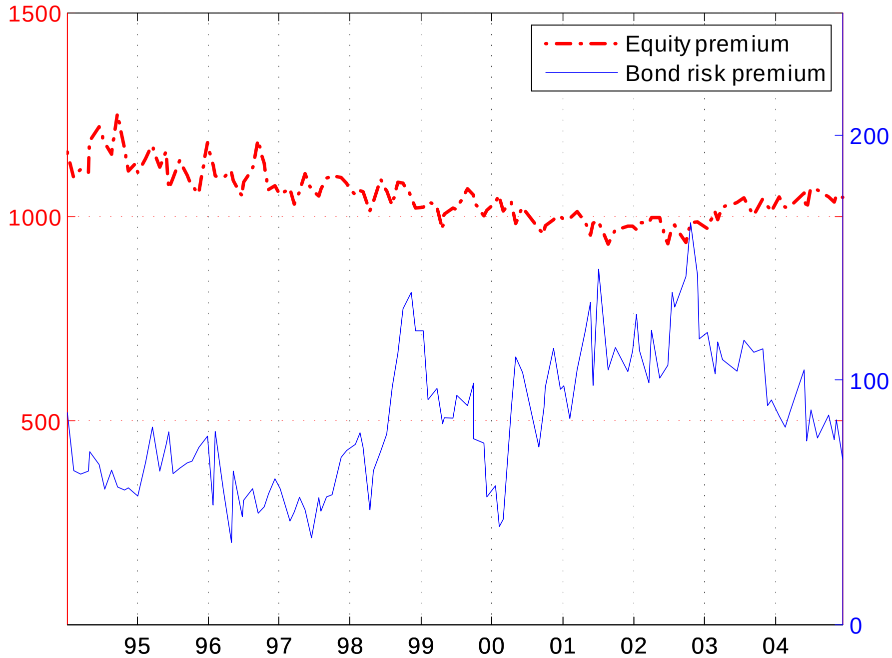

Figure 7: Equity and corporate bond risk premia. Equity risk premia are measured using the Fama & MacBeth (1973) methodology and bond risk premia using equation (1).

In order to provide a more disaggregated view of risk premia, Figure 8 reports a breakdown of risk market and model premia across 8 industries: manufacturing, media, oil and gas, rail, retail, services, transportation and telephone. The equity implied bond excess returns track their bond market measured counterparts quite well for those industries that represent a large fraction of our dataset, in particular manufacturing.

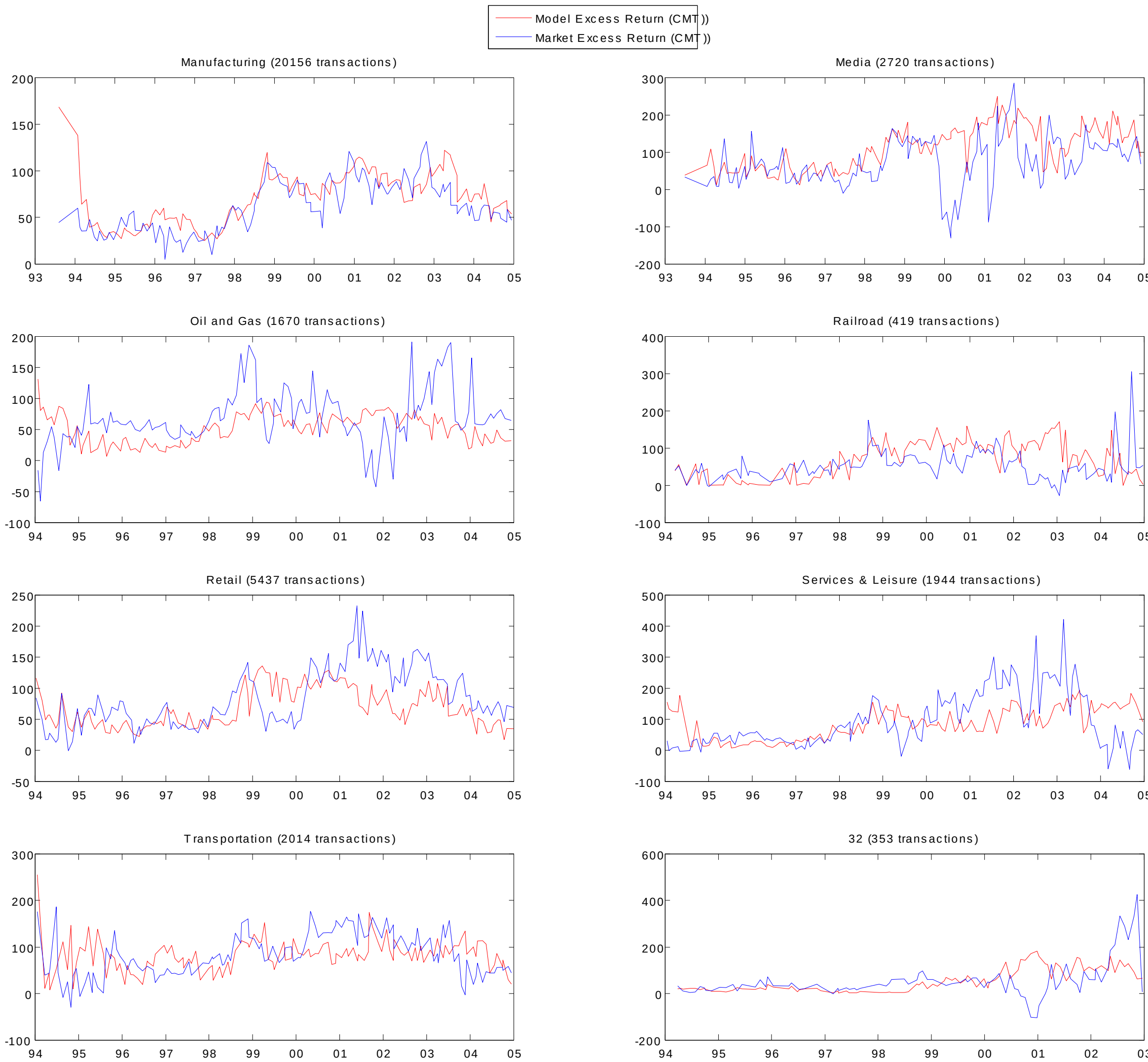

Figure 8: market and model risk excess return across industries. Monthly averages. The CMT curve is used as benchmark. Results based on excess returns using the swap curve as benchmark are, although not reported, very similar.

As discussed above, an alternative way of measuring the risk premium is achieved by decomposing bond yield spreads into an expected loss component and a risk premium component. Much of the academic work on bond spreads has relied on Treasury securities as risk free assets. In contrast, most practitioners will argue that a more informative measure of yield spreads is obtained when using the swap curve as a benchmark. In addition it has recently been shown in the literature on structural models that the oft documented underprediction appears to be related to the choice of benchmark curve.[25]

We find the same result - when the Leland & Toft model is used to predict bond yield spreads, it fits well on average when the swap curve is used and underestimates when the Treasury curve is used. Figures 9 & 10 clearly illustrate this result. There is no systematic gap between average market and model spreads relative to the swap curve. That is not to say that the model cannot at times under or overpredict as it does interchangeably as of 2000. The spread underestimation as of mid 2001 can at least partially be explained by recent findings in Feldhütter & Lando (2007). They decompose swap spreads - the difference between fixed rates on interest rate swaps and corresponding Treasury yields - into three components: a convenience yield for holding Treasuries, a credit risk component and a swap market specific factor. The period of extremes in spread underprediction in our findings coincide with a period when they document an unusually negative swap factor which yields low swap rates. They provide an explanation based on hedging activity in the mortgage backed securities market. Thus the observed spreads in our sample appear higher than they should be, had the swap rate been a better proxy for the risk free rate.

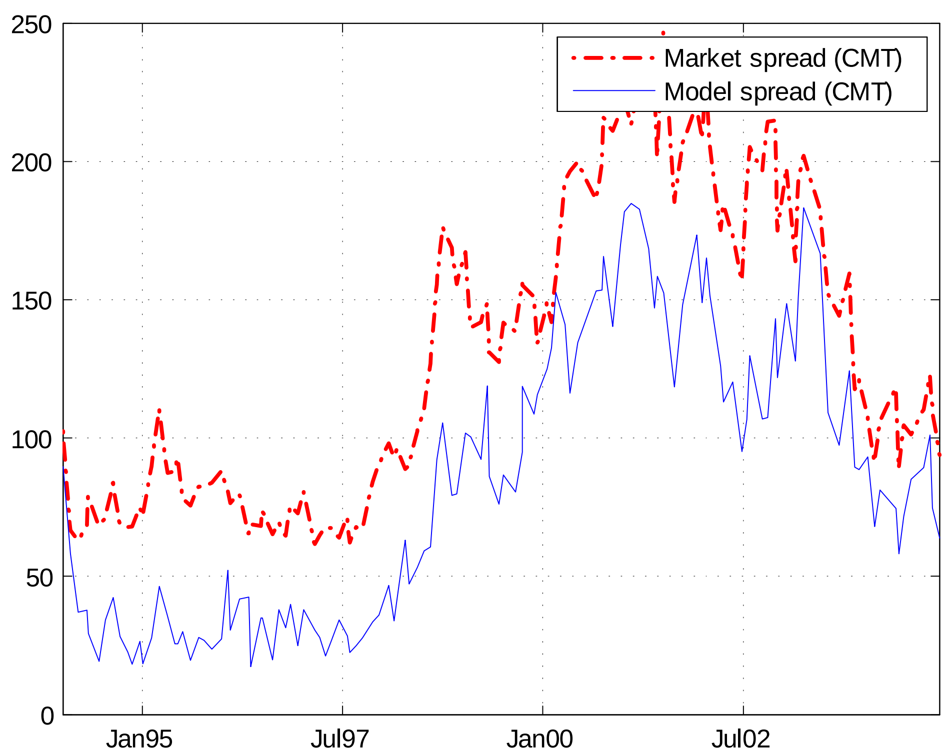

Figure 9: Monthly averages of model and market bond spreads relative to the CMT curve.

Although imperfect, the swap curve appears to be the better choice for our purposes. In addition to

[25] See Ericsson et al. (2006).

mitigating the influence of Treasury market liquidity effects it removes the need to correct for differential taxation between corporate bonds and treasury bonds.[26] We will from now on report only results based on this curve.

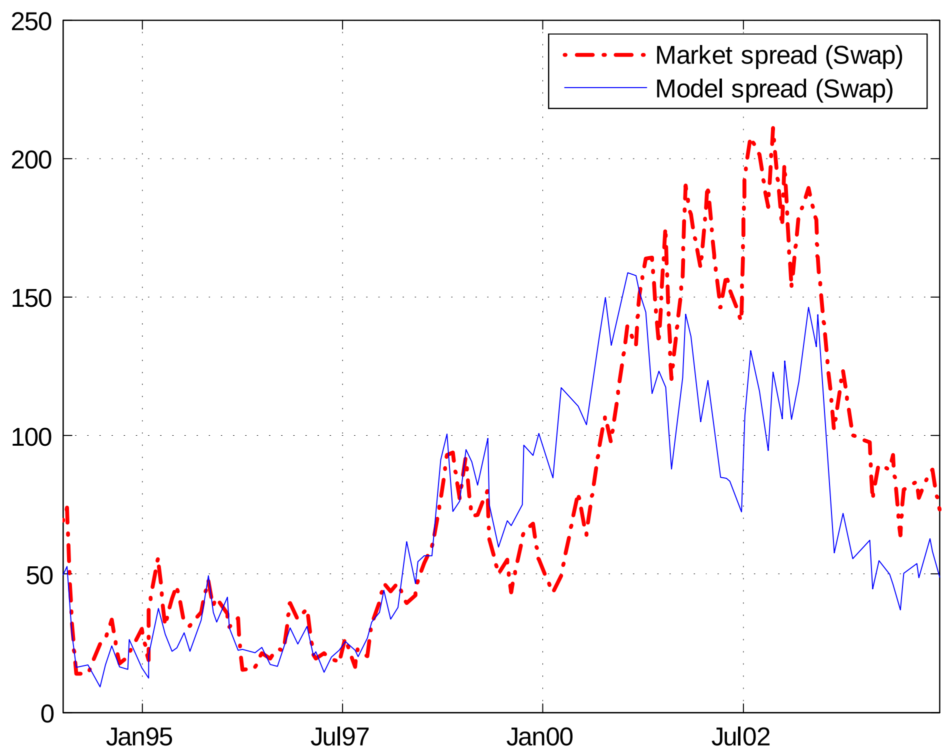

Figure 11: Monthly averages of model and market bond spreads relative to the swap curve.

Figure 12 plots the equity implied risk premium in spreads vs. the premium inferred from bond spreads. The model predicted risk premium component tracks the market measured quite well to begin with, but under and overestimates in the same way as the spread in the second part of the sample. For example, if hedging demand in MBS biases spreads over the swap curve, then this bias will be inherited by the risk premium component. Note that the effects are mitigated when the excess return measure is used (see Figure 6). At this stage, we conclude that our equity implied risk premia are partially successful in explaining the time series variation of average risk premia as measured in corporate bond markets. Nevertheless there appears to be an important unexplained component to market measured risk premia, unrelated to equity risk premia. This direction is pursued in interesting work by Berndt et al. (2006) and Saita (2006).

[26] See Elton et al. (2001).

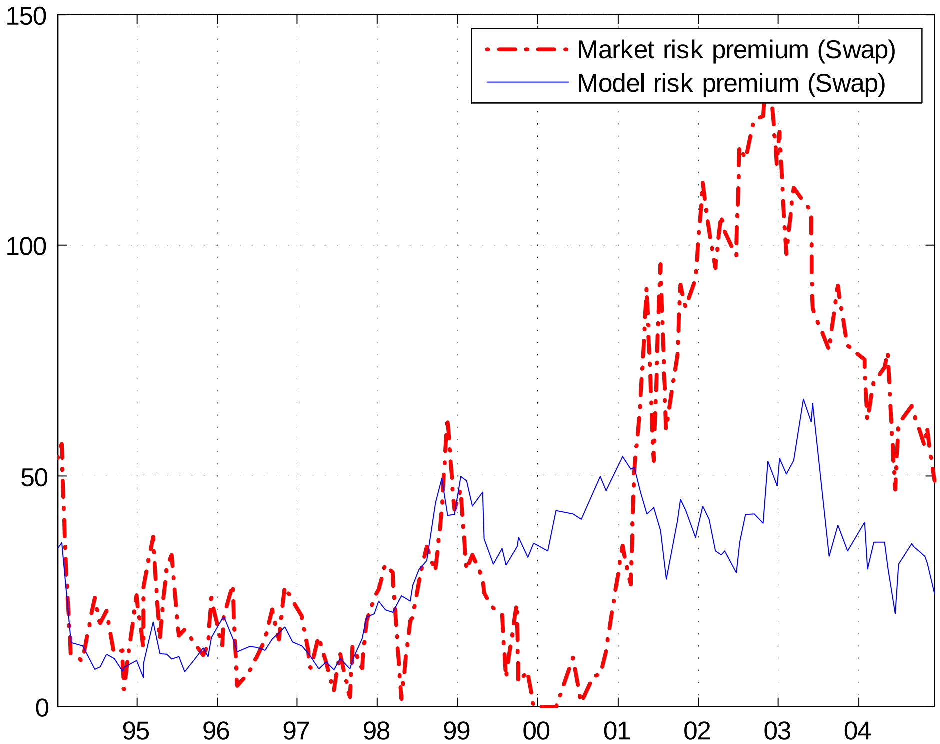

Figure 12: equity implied and bond market measured risk premium components in basis points.

It is interesting to observe the time series behavior of spread components over time. A number of interesting observations can be made from Figure 13, which plots the time series of the average model risk premium and expected loss component. First the expected loss component of spreads is more volatile than the risk premium. Second, although most of the time they appear to move together there are notable exceptions. For example during 2002, risk premia increased steadily while expected losses moved around without clear trend. Between 1999 and mid 2002 on the other hand there was no clear trend in risk premia while expected losses first increased to a peak in early 2001 and then decreased, although irregularly until the end of the period. The expected loss component is relatively larger in periods of high spreads and defaults (for example in 2001 - a year with spectacular defaults) and is similar or lower than the risk premium in lower spread periods.

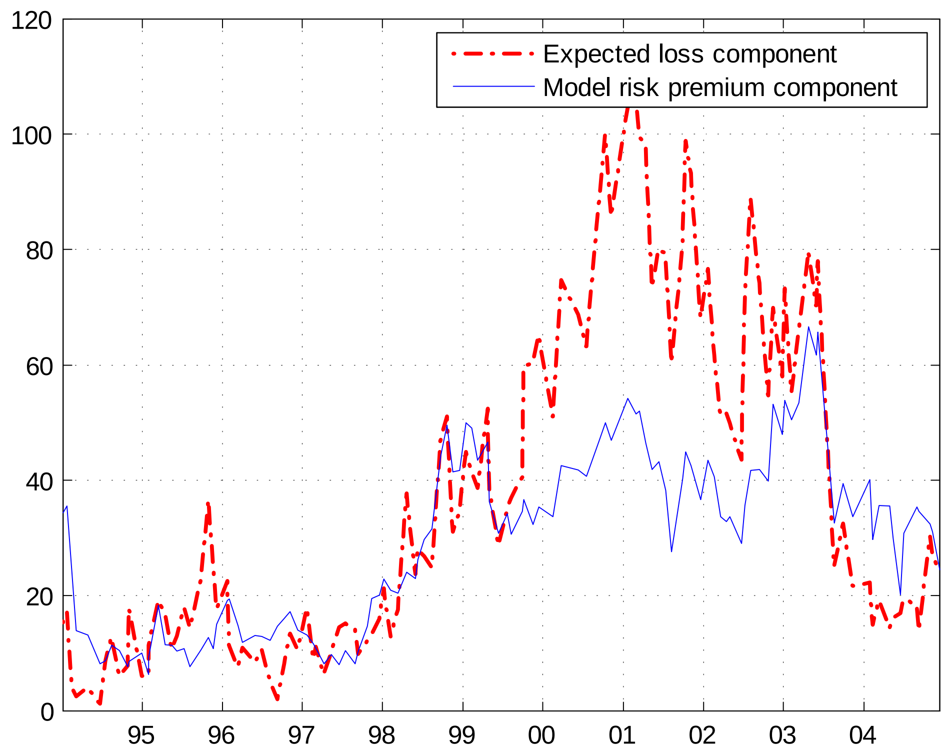

Figure 13: expected losses vs. the risk premium component in bond spreads.

Campbell & Taksler (2003) point out that during the late nineties, bond yield spreads increased while stock prices were rising. They argue that this puzzling pattern can be explained by an increase in idiosyncratic volatility over the same period. Our analysis allows us to make a related observation. Note that during 1999-2001, the increase in spreads was largely driven by an increase in the average expected loss component. The risk premium component should depend critically on systematic risk, whereas the expected loss should derive from a firm's total risk. Figure 14 plots the average trailing 250 day S&P500 return volatility and the average asset volatility as proxies for these two risk sources respectively.

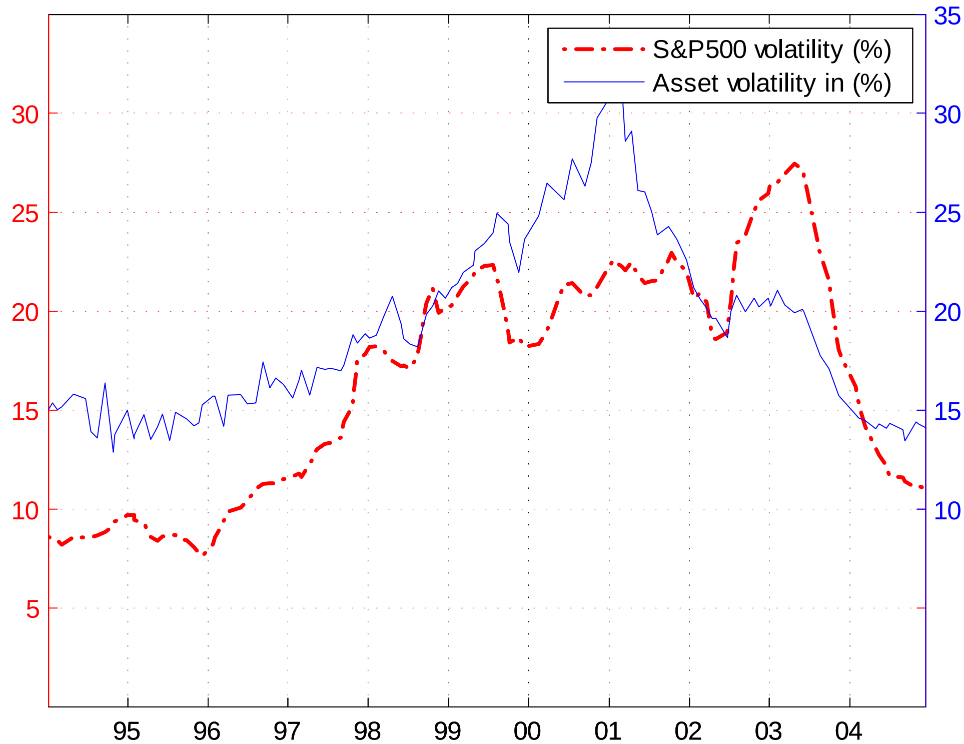

Figure 14: volatility measures. The solid line represents the average asset volatility across firms in our sample. The dashed line draws the 250 day historical volatility on the S&P 500 index. The latter is intended as a proxy for market volatility, whereas the asset volatility measures total risk, including idiosyncratic volatility.

Although these two metrics are not directly comparable, one being an average of asset volatilities and the other an equity volatility, the pattern is suggestive. Between 1999 and 2001, there is no clear trend in the market equity volatility, while average asset volatilities increased steadily. Absent a trend in firms' average leverage, this suggests that idiosyncratic risk increased during this period whereas systematic risk did not.[27] This is consistent with our observed pattern in the expected loss and risk premium spread components respectively.

[27] There is no clear trend in leverage during our sample. The average leverage oscillates around 45% between 1994 and 2004.

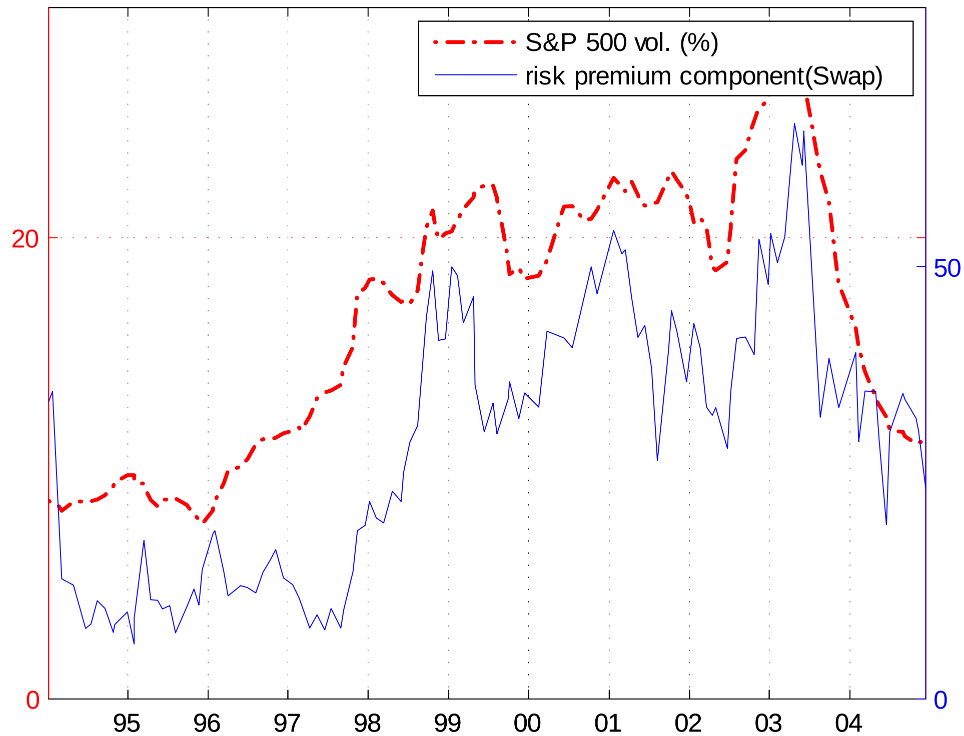

Figure 15. The risk premium in bond spreads vs. S&P 500 volatility as a proxy for systematic risk.

In fact, plotting risk premia together with the market equity volatility and the expected loss component with the average asset volatility, it becomes quite clear that risk premia move closely with systematic volatility whereas the expected loss component is aligned with a measure of total volatility. In summary, we find that the idiosyncratic equity volatility increase in the late nineties documented by Campbell & Taksler (2003) is in fact due to an increase in firm specific asset volatility leading to a higher spread as a result of higher expected losses.

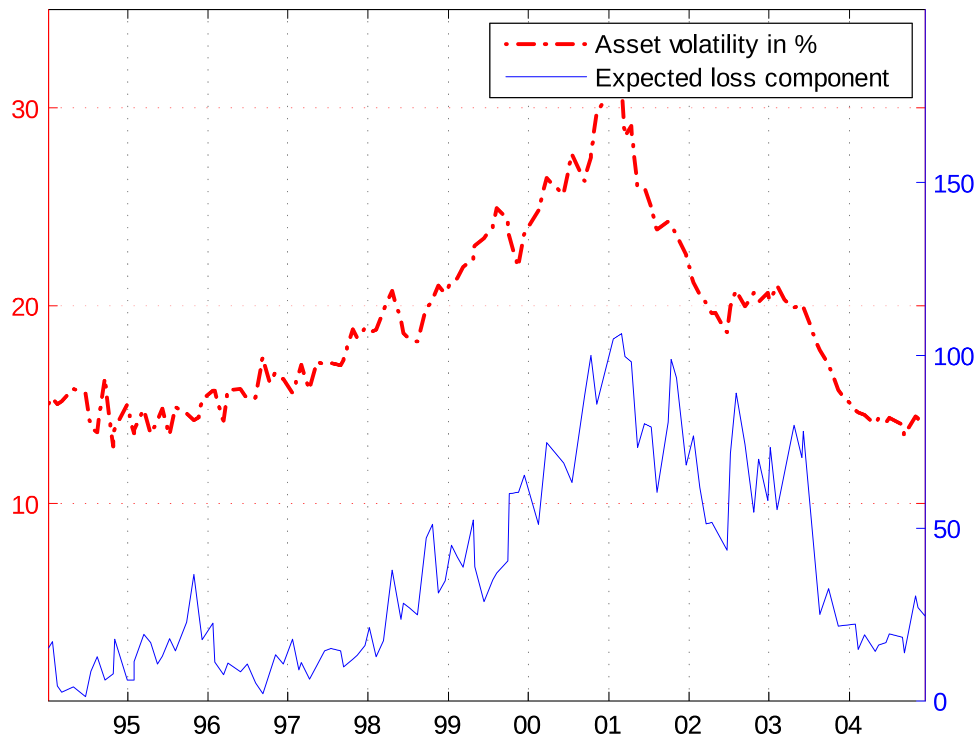

Figure 16: the expected loss component in bond spreads vs. asset volatility which measures total risk.

We now turn to a discussion of the determinants of credit spreads, more precisely we will discuss to what extent our metric for bonds' risk premia can help to explain spread dynamics empirically.

6.3 Explaining credit spreads

We have made extensive use of the contingent claims approach to valuing corporate securities in our analysis. Although the evidence may be mixed on the ability of structural models to correctly predict levels of the risk component of bond spreads, there is ample evidence that they cannot fully explain their time series variation – see for example Collin-Dufresne et al. (2001).[28]

The relative success of our use of structural models in translating equity market risk premia into bond specific risk premia suggests that these models do have merit. However, the typical such model predicts that levels of bond spreads and equity prices depend mainly on two unknowns – asset value and volatility. To date however, the models are almost always silent on risk premia. Studies that have considered risk

[28] Many early studies of corporate bond pricing using structural models resulted in yield spread estimates substantially below their market counterparts - see for example Jones et al. (1984), Jones et al. (1985), Ogden (1987) and Lyden & Saranati (2000). Recent work has produced more mixed evidence (see e.g. Eom et al. (2004) and Ericsson et al. (2006)) and it has been suggested that the reason for the underestimation may be the presence of important non-default components (see e.g. Longstaff et al. (2004) and Ericsson & Renault (2000)).

premia in the context of structural models do not consider their dynamics (see e.g. Huang & Huang (2002) and Leland (2004)). With this in mind, we now consider the possibility that using information about the time series of risk premia can help structural models to explain the variation in credit spreads.

Developing and implementing a structural model with time varying risk premia lies well beyond the scope of this paper.[29] Instead we follow Collin-Dufresne et al. (2001) in working with linear regression analysis with a choice of variables motivated by the theoretical underpinnings of structural models.[30]

We first run the following panel regression

$$\begin{aligned} s_{it} &= \alpha_i + \beta_1 LEV_{i,t} + \beta_2 EVOL_{i,t} + \beta_3 ERET_{i,t} \\ &\quad + \beta_4 SLOPE_t + \beta_5 r_{i,t} + \beta_6 RPI_{it} + \varepsilon_{it} \\ &\quad \text{where } \varepsilon_{it} = \rho \varepsilon_{i,t-1} + \eta_{it} \end{aligned}$$

where LEV denotes a firm's leverage, $EVOL$ and $ERET$ its historical equity volatility and return respectively, $SLOPE$ is the difference between the 10- and 2-year swap rates, r_{it}, the swap rate corresponding to the maturity of the particular bond, and RPI is our equity-implied measure of the spread's risk premium component. We run the regression with and without the risk premium metric in order to gauge the marginal gain in explanatory power by including this variable.

Table 3 reports the results. The first two columns relate to the full sample, whereas the remaining columns provide disaggregated evidence based on firm leverage quartiles. The reported R-squares in the benchmark regressions are comparable to those reported for credit spread level regressions by Campbell & Taksler (2003).[31] The pattern across rating quartiles for the regressions is not dissimilar to what is found by Collin-Dufresne et al. (2001), although they work with credit spread changes rather than levels.

For the full sample the R-square increases by 5% to about 36% when including our risk premium variable, a non-trivial increase. The R-square increases for all leverage quartiles, although by varying degrees. The risk premium variable is more important for the lowest leverage firms (an R-square increase by 12%) than for the highest leverage firms (and increase by 2%). This suggests that spreads for firms with lower default risk have higher proportional risk premia. This is consistent with our findings above as well as with the results of BDDFS. As we shall see below, it is also consistent with the predictions of a structural model.

[29] Interesting work in this direction has been done by Chen et al. (2005).

[30] Due to the irregular spacing of our data, we chose for simplicity to work with levels of credit spreads. Clearly a case can be made for working with changes in spreads. However, as we are mainly interested in the marginal importance of an additional variable rather than the absolute level of explanatory power we feel that our choice is adequate. Papers in the field have varied in their approach. See e.g. Campbell & Taksler (2003) and Cremers et al. (2004).

[31] It is important to note that the R-squares measure explained variation over and above the fixed effects. For example, the reported R-square in Table IIa for the fiirst regression is about 31%. In contrast the total R-square for a standard pooled regression is just less than 60%.

7 Structural models and risk premia

Our regression results suggest that risk premia are a relatively more important determinant of high grade debt spreads than for lesser quality bonds. Similarly, we have seen that on average risk premia appear to be more important during periods of low default rates. In high default periods, the expected loss component is more important. As noted above, Berndt et al. (2004) also find that the default premium is higher for high-quality firms. Next, we consider comparative statics of the Leland & Toft (1996) model to determine whether this finding can be explained. The four panels of Figure 16 plot the following quantities against leverage: (i) total spread, expected loss and risk premia, (ii) the ratio of risk premium to total spreads, (iii) ratio of risk neutral to objective 5 year default probabilities and (iv) the difference between risk neutral and objective 5 year default probabilities.

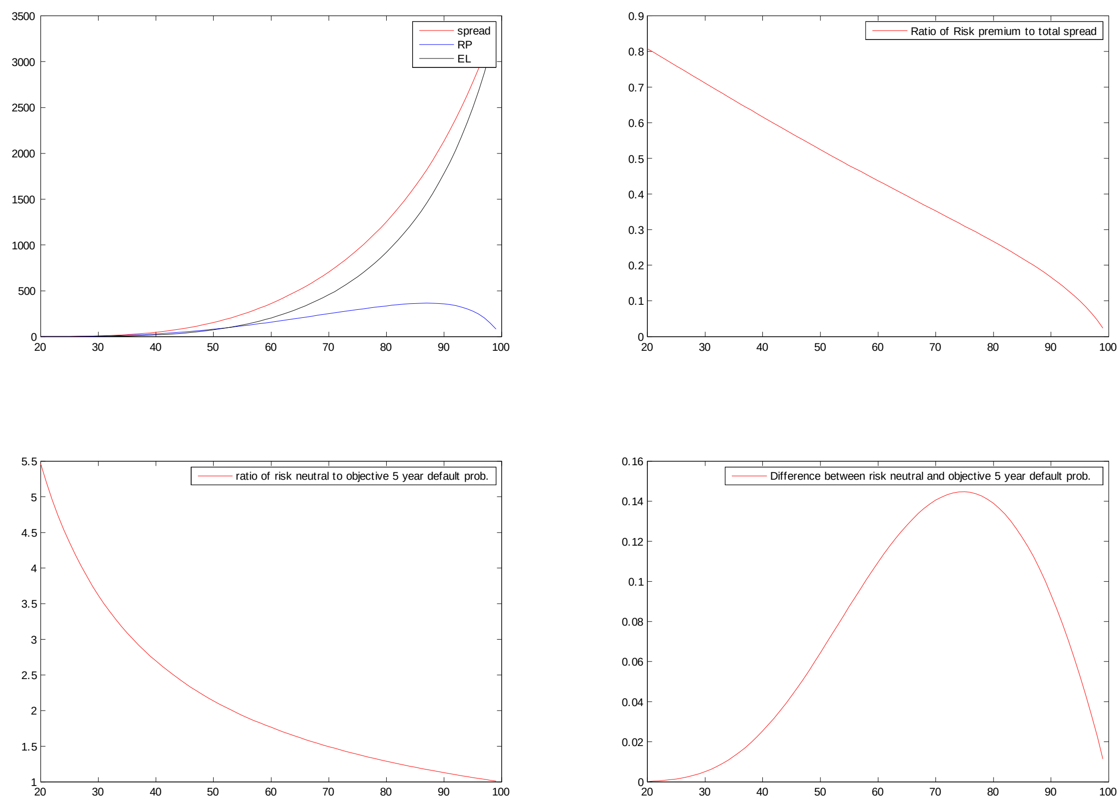

Figure 16

The first two are relevant to our results on bond spread components whereas the last two are intended for comparison with the results of the BDDFS study. The first panel shows, not surprisingly, that the expected loss spread and total spread increase monotonically in leverage. The risk premium component on the other hand does not. For low levels of leverage it increases in tandem with the expected loss spread. But for some (high) level of leverage, it decreases and eventually disappears as the distress becomes more and more certain. This is intuitive, as when default is certain, debt needs to compensate for the imminent losses, but there is in fact little risk. The second panel shows that the fraction of the risk premium in the total spread decreases monotonically as leverage increases. This is consistent with our regression coefficients on the risk premium being highest in the lowest leverage quartile and monotonically decreasing as leverage increases.

The third panel illustrates the same effect using a proxy for the risk premium similar to what BDDFS use in their empirical study. The ratio of risk neutral to objective default probabilities behaves much like the ratio of the risk premium spread component to the total yield spread - it decreases monotonically with leverage. This suggests that the empirical finding of BDDFS, also found in our study is consistent with the prediction of a structural model.[32] The final panel plots the absolute difference between risk neutral and objective default probabilities - the resulting pattern echoes what can be seen in the first panel: for extremely high and low default probabilities, the difference between the risk neutral and objective probabilities disappears.[33] Again - this is intuitive. Both probabilities have to converge to zero or one at the extremes.

8 Concluding discussion

Investors in credit markets need a framework to assess whether a given defaultable security is fairly priced. The spread itself may not be an adequate metric to respond to this question. The investor needs to know if the spread contains (i) acceptable compensation for expected default losses and (ii) a sufficient risk premium to induce participation.

With a methodology capable of disentangling risk premia and expected losses, we measure risk premia in a large panel of US corporate bond data spanning a ten year period. We find, like previous work, that the risk premium is highly time varying. We also document similar time series patterns as previous work. We find that the expected loss and default components behave differently over time. The risk premium is at its most important for high grade debt, whereas the expected loss component increases monotonically with the default probability. We show that the time series variation of the risk premium is closely related to the overall market volatility whereas the expected loss component appears more closely related to the average total volatility across firms.

Perhaps our two most important findings are that (i) the time series variation observed in the risk

[32] For robustness, we have also established that this result can be generated within the much simpler Merton (1974) model. It would thus seem that it is not very sensitive to the choice of a particular model for the computation of the risk premia.

[33] Berndt et al. (2006) work with the difference in intensities rather than ratios.

premium in bond markets can be replicated using equity market measured risk premia translated to corporate bond risk premia and (ii) that including our risk premium metric in a linear regression of bond spreads on theoretical determinants of corporate bond risk premia increases explanatory power, suggesting that time varying risk premia is a desirable feature of future structural credit risk models.

The risk premium we have measured is a translation of risk premia measured in equity markets. As such it does not capture risk premia that may be specific to fixed income markets. We have already discussed the sensitivity of our results to a swap market specific factor. Another example of market specific risks that influence prices has been documented by Newman & Rierson (2004), who show that issuance activity may play a role in the pricing for seasoned securities in particular market segments. Recent work by Berndt et al. (2006) suggests the presence of a credit market specific risk factor. In addition, recent work documents the commonality of illiquidity risk within and across markets. The unexplained part of our market risk premia may well contain information about some or all of these additional risk premia.

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10 Appendices

10.1 Default risk premia in reduced form models

The default intensity λ^P of a firm is the instantaneous mean arrival rate of the first event of a non explosive counting process N. Conditional on survival to time t and all additional available information, the probability of default in a short time between t and $t + \Delta$ is approximately $\lambda^P \Delta$. In a Cox process framework, the probability of survival of an obligor for some time h, conditional on survival up to time t is

$$P_t(\tau > t + h) = E_t^P \left[\exp\left\{ -\int_t^{t+h} \lambda_s^p ds \right\} \right], \tag{7}$$

with E_t^P denoting the expectation operator conditional on the information available up to time t.

In the absence of arbitrage and market frictions, there exists a risk neutral probability measure.[34] In reduced form models, it is the change from historical to risk-neutral default intensities that defines the risk premium. Information about risk-neutral default intensities can be extracted from market prices of corporate bonds or credit derivatives. The historical default probabilities need to be inferred from a different source such as historical default frequencies conditional on rating categories or alternative measures such as Moody's KMV expected default frequencies (EDFs).[35]

In order to describe the concept of risk premia in reduced form credit risk models, we will borrow from a discussion in Lando (2003), who provides an illustrative setting where under the historical probability measure P, we can write the dynamics of the default intensity as follows (assuming deterministic interest rates):

$$d\lambda_t = \gamma(\bar{\lambda} - \lambda_t)dt + \sigma_\lambda \sqrt{\lambda_t} dW_t, \tag{8}$$

where W is a Brownian motion under P, γ is the speed of mean reversion, $\bar{\lambda}$ the long run mean and σ_λ is the volatility of the intensity process. Under an equivalent measure Q :

$$d\lambda_t = (\gamma\bar{\lambda} - (\kappa - \psi_\lambda)\lambda_t)dt + \sigma_\lambda \sqrt{\lambda_t} d\tilde{W}_t, \tag{9}$$

where $\tilde{W}$ is a Brownian motion under Q.

The parameter ψ_λ is the price of risk for a unit change in the intensity. Note that the stopping time of default that has an intensity process λ with a process defined in (8) under the probability measure P is no longer a stopping time under Q with intensity dynamics defined in (9). It is however a stopping time with intensity process $\lambda_t^Q = \mu_t \lambda_t$, where μ_t is assumed constant for simplicity.[36]

[34] See Harrison & Kreps (1979) and Delbaen & Schachermayer (1994) for technical conditions.
[35] See e.g. Berndt et al. (2004) and Driessen (2005). .
[36] See Lando (2003) for details.

Now, consider a h maturity zero coupon bond with zero recovery. Its price at time t is

$$B_{t,h} = E_t^Q \left[\exp\left\{ -\int_t^{t+h} (r_s + \lambda_s^Q) ds \right\} \right], \tag{10}$$

where r_s is the short term risk free rate. An application of Ito's lemma permits us to write the instantaneous expected excess return for the bond as

$$R_B(t) - r_t = F_\lambda \sigma_\lambda \psi_\lambda \lambda_t + (\mu - 1)\lambda_t 1_{\{N_t=0\}}, \tag{11}$$

where $R_B(t)$ is the drift rate of the corporate bond under P and F_λ is the loading for intensity risk. The excess return on the bond thus consists of two distinct components. First, there is a positive contribution from the risk of a jump to default itself – if $\mu \neq 1$. Second, the bond is subject to price volatility due to fluctuations in the default intensity, contributing to the risk premium whenever ψ_λ is nonzero. The first type of risk is commonly referred to as default event risk, whereas the second can intuitively be thought of as spread risk.

Jarrow et al. (2005) show that under certain conditions, asymptotically, $\mu \to 1$, implying that default event risk should not be priced. Driessen (2005) shows empirically that μ exceeds 1. This is supported by Berndt et al. (2004) who estimate the risk premium as the ratio between historical and risk neutral default intensities, which corresponds to μ in equation (11). They obtain λ^p by calibrating Moody's KMV Expected default frequencies to $\{P_t(\tau < s); s \in (t, \infty)\}$ in equation (7), and use market prices for default swaps to recover λ^Q from equation (10). Berndt et al. (2006) investigate the source for common variation in the portion of returns on default swaps that is not explained by changes in risk-free rates or expected default losses. Their estimate for risk premia corresponds to $(\mu - 1)\lambda_t 1_{\{N_t=0\}}$ in equation (11).

Appendix: The Leland & Toft Model

The value of the firm is the same as in Leland (1994). The value of debt is given by

$$\mathcal{D}(v_t) = \frac{C}{r} + \left(N - \frac{C}{r}\right)\left(\frac{1 - e^{-rM}}{rM} - I(v_t)\right) + \left((1-\alpha)L - \frac{C}{r}\right)J(v_t)$$

The bankruptcy barrier

$$L = \frac{\frac{C}{r}\left(\frac{A}{rM} - B\right) - \frac{AP}{rM} - \frac{\tau C x}{r}}{1 + \alpha x - (1-\alpha)B}$$

where

$$\begin{aligned} A = {} & 2ye^{-rM}\phi\left[y\sigma\sqrt{M}\right] - 2z\phi\left[z\sigma\sqrt{M}\right] \\ & - \tfrac{2}{\sigma\sqrt{M}} n\left[z\sigma\sqrt{M}\right] + \tfrac{2e^{-r\Upsilon}}{\sigma\sqrt{M}} n\left[y\sigma\sqrt{M}\right] + (z-y) \end{aligned}$$

$$B = -\left(2z + \frac{2}{z\sigma^2 M}\right)\phi\left[z\sigma\sqrt{M}\right] - \frac{2}{\sigma\sqrt{M}} n\left[z\sigma\sqrt{M}\right] + (z-y) + \frac{1}{z\sigma^2 M}$$

and $n[\cdot]$ denotes the standard normal density function.

The components of the debt formulae are

$$I(v) = \frac{1}{r\Upsilon}\left(i\left(v\right) - e^{-r\Upsilon}j(v)\right)$$

$$i(v) = \phi\left[h_1\right] + \left(\frac{v}{L}\right)^{-2a}\phi\left[h_2\right]$$

$$j(v) = \left(\frac{v}{L}\right)^{-y+z}\phi\left[q_1\right] + \left(\frac{v}{L}\right)^{-y-z}\phi\left[q_2\right]$$

and

$$J(v) = \frac{1}{z\sigma\sqrt{M}}\begin{pmatrix} -\left(\frac{v}{L}\right)^{-a+z}\phi\left[q_1\right]q_1 \\ \\ +\left(\frac{v}{L}\right)^{-a-z}\phi\left[q_2\right]q_2 \end{pmatrix}$$

Finally

$$q_1 = \frac{-b - z\sigma^2 M}{\sigma\sqrt{M}}$$

$$q_2 = \frac{-b + z\sigma^2 M}{\sigma\sqrt{M}}$$

$$h_1 = \frac{-b - y\sigma^2 M}{\sigma\sqrt{M}}$$

$$h_2 = \frac{-b + y\sigma^2 M}{\sigma\sqrt{M}}$$

and

$$y = \frac{r - \beta - 0.5\sigma^2}{\sigma^2}$$

$$z = \frac{\sqrt{y^2\sigma^4 + 2r\sigma^2}}{\sigma^2}$$

$$x = y + z$$

$$b = \ln\left(\frac{v}{L}\right)$$

10.2 Bond pricing

Next we need a pricing formula for the corporate bond obligation . To this end, we apply a bond pricing model that takes discrete coupons, nominal repayment and default recovery into account.[37] To express the value of the bond we make use of two building blocks, a binary option $H\left(v_t, t; S\right)$ and a dollar-in-default claim $G\left(v_t, t; S\right)$. The former pays off \$1 at maturity S if the firm has not defaulted before that,

[37] This bond pricing model was used in Ericsson & Reneby (2004) and was shown to compare well to reduced form bond pricing models.

the latter pays off \$1 upon default should this occur before S; the value of both depend upon the firms asset value v_t and current time t. The formulae for the binary option and the dollar-in-default claim are, for a given default barrier L.

Proposition 1 ***A straight coupon bond.*** *The value of a coupon bond with M coupons c paid out at times $\{t_i : i = 1..M\}$ is*

$$\begin{aligned}\mathcal{B}(v_t, t) &= \sum_{i=1}^{M-1} c \cdot H(v_t, t; t_i) \\ &\quad + (c+p) \cdot H(v_t, t; T) \\ &\quad + R \cdot p \cdot G(v_t, t; T)\end{aligned}$$

The formulae for H and G are given in the appendix.

The value of the bond is equal to the value of the coupons (c), the value of the nominal repayment (P) plus the value of the recovery in a default (Rp). Each payment is weighted with a claim capturing the value of receiving \$1 at the respective date.

Note that the above formula for the reference bond is not directly related to the debt structure of the firm. Specifically, coupon payments to the bond are unaffected by the debt redemption schedule elaborated in the Leland & Toft model. The choice of model affects the bond formula solely via the default barrier L.

10.3 Building blocks for bond valuation

First, define default as the time (τ) the asset value hits the default boundary from above, $\ln \frac{v_\tau}{V_{B,\tau}} \equiv 0$. Then define $G(v, t)$ as the value of a claim paying off \$1 in default:

$$G(v_t, t) \equiv E^B \left[e^{-r(\tau - t)} \cdot 1 \right]$$

We let E^B denote expectations under the standard pricing measure. The value of G is given by

$$G(v_t, t) = \left(\frac{v_t}{V_{B,t}} \right)^{-\theta}$$

with the constant given by

$$\theta = \frac{\sqrt{(h^B)^2 + 2r} + h^B}{\sigma}$$

and

$$h^B = \frac{r - \beta - 0.5\sigma^2}{\sigma}$$

Define the dollar-in-default with maturity $G(v_t, t; T)$ as the value of a claim paying off \$1 in default *if* it occurs before T

$$G(v_t, t; T) \equiv E^B \left[e^{-r(\tau - t)} \cdot 1 \cdot \left(1 - I_{\tau \nless T}\right) \right]$$

and define the binary option $H(v_t, t; T)$ as the value of a claim paying off \$1 at T if default has *not* occurred before that date

$$H(v_t, t; T) \equiv E^B \left[e^{-r(T - t)} \cdot 1 \cdot I_{\tau \nless T} \right]$$

$I_{\tau \nless T}$ is the indicator function for the *survival event,* i.e. the event that the asset value (v_T) has not hit the barrier prior to maturity $(\tau \nleq T)$. The price formulae for the last two building blocks are given below. They contain a term that expresses the probabilities (under different measures) of the survival event – or, the *survival probability.* To clarify this common structure, we first state those probabilities in the following lemma.[38]

Lemma 2 *The probabilities of the event* $(\tau \nleq T)$ *(the "survival event") at* t *under the probability measures* $Q^m : m = \{B, G, obj\}$ *are*

$$Q^m (\tau \nleq T) = \phi \left(k^m \left(\frac{v_t}{V_{B,t}} \right) \right) - \left(\frac{v_t}{V_{B,t}} \right)^{-\frac{2}{\sigma} h^m} \phi \left(k^m \left(\frac{V_{B,t}}{v_t} \right) \right)$$

where

$$\begin{aligned} k^m (x) &= \frac{\ln x}{\sigma \sqrt{T - t}} + h^m \sqrt{T - t} \\ h^G &= h^B - \theta \cdot \sigma = -\sqrt{(h^B)^2 + 2r} \\ h^{obj} &= \end{aligned}$$

$\phi(k)$ *denotes the cumulative standard normal distribution function with* $\in$*ration limit* k*.*

The probability measure Q^G is the measure having $G(v_t, t)$ as numeraire (the Girsanov kernel for going to this measure from the standard pricing measure is $\theta \cdot \sigma$). Using this lemma we obtain the pricing formulae for the building blocks in a convenient form. The price of a down-and-out binary option is

$$H(v_t, t; T) = e^{-r(T - t)} \cdot Q^B (\tau \nleq T)$$

[38] The probabilities are previously known, as is the formula the down-and-out binary option in Lemma 10.3 (see for example Björk (1998)).

The price of a dollar-in-default claim with maturity T is

$$G(v_t, t; T) = G(v_t, t) \cdot \left(1 - Q^G(\tau \nleq T)\right)$$

To understand this second formula, note that the value of receiving a dollar if default occurs prior to T must be equal to receiving a dollar-in-default claim with infinite maturity, less a claim where you receive a dollar in default conditional on it *not* occurring prior to T:

$$G(v_t, t; T) = G(v_t, t) - e^{-r(T-t)} E^B \left[G(v_T, T) \cdot I_{\tau \nleq T}\right]$$

Using a change of probability measure, we can separate the variables within the expectation brackets (see e.g. Geman et al. (1995)).

$$\begin{aligned} G(v_t, t; T) &= G(v_t, t) - e^{-r(T-t)} E^B \left[G(v_T, T)\right] \cdot E^G \left[I_{\tau \nleq T}\right] \\ &= G(v_t, t) \cdot \left(1 - Q^G(\tau \nleq T)\right) \end{aligned}$$

10.4 Objective default probabilities

$$P_t(\tau > T_i) = N(d^P_{T_i}(\frac{v_t}{L})) - (\frac{v_t}{L})^{-2\frac{\mu_v - 0.5\sigma^2}{\sigma^2}} N(d^P_{T_i}(\frac{L}{v_t}))$$

$$\begin{aligned} \text{with } d^P_{T_i}(\frac{v_t}{L}) &= \frac{\ln(\frac{v_t}{L}) + \left(\mu_v - 0.5\sigma^2\right)(T_i - t)}{\sigma\sqrt{T_i - t}} \\ \text{and } d^B_{T_i}(\frac{L}{v_t}) &= \frac{\ln(\frac{L}{v_t}) + \left(\mu_v - 0.5\sigma^2\right)(T_i - t)}{\sigma\sqrt{T_i - t}} \\ \mu_v &= \text{the realized mean of the time series of } v(t) \end{aligned}$$

Table 3: Importance of risk premium for credit spreads

The results are based on the following panel regression during the period between January 1994 and December 2004.

$$s_{it} = \alpha_i + \beta_1 LEV_{i,t} + \beta_2 EVOL_{i,t} + \beta_3 ERET_{i,t} + \beta_4 SLOPE_t + \beta_5 r_{i,t} + \beta_6 RPI_{it} + \varepsilon_{it}$$
$$\text{where } \varepsilon_{it} = \rho\varepsilon_{i,t-1} + \eta_{it}$$

where LEV denotes a firm's leverage, $EVOL$ and $ERET$ its historical equity volatility and return respectively, $SLOPE$ is the difference between the 10- and 2-year Swap yields, r_{it}, the Swap rate corresponding to the maturity of the particular bond, and RPI is our equity implied measure of the bond specific risk premium.

	All		1st quartile		2nd quartile		3rd quartile		4th quartile	
Leverage	4.901	4.431	1.839	1.296	3.402	2.850	4.261	3.690	7.166	7.053
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Historical equity vol.	0.778	0.715	1.349	1.112	0 .995	.822	0 .674	0.638	1.004	0.996
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Equity return	0.001	0.001	0.001	0.001	0.001	.001	0.003	0.003	0.001	0.001
	(0.000)	(0.000)	(0.010)	(0.032)	(0.03)	(0.09)	(0.000)	(0.000)	(0.107)	(0.108)
Slope	7.114	9.593	6.923	11.38	7.351	11.637	13.520	16.548	-1.848	-1.930
	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)	(0.000)	(0.715)	(0.703)
Swap-yield	-24.292	-22.616	-7.718	-7.235	-14.618	-13.440	-23.68	-21.230	-53.463	-52.742
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Risk premium		0.364		0.637		.567		0.368		0.076
		(0.000)		(0.000)		(0.000)		(0.000)		(0.000)
Constant	-76.96		-6.36	-3.29	-25.01	-20.42	-127.17	-111.14	-300.2	-300.6
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2- within	13.00%	13.54%	10.05%	12.19%	11.87%	13.32%	14.56%	14.94%	18.44%	18.50%
R^2- between	27.23%	33.72%	0.07%	10.67%	12.14%	35.49%	17.98%	29.46%	14.70%	16.74%
R^2- overall	30.9%	36.08%	8.76%	20.35%	15.60%	31.07%	20.62%	27.74%	22.40%	23.83%
N	33.626*	33.626*	8828	8828	6,605	6,605	9078	9078	6763	6763
Number of groups	**988**	**988**	**359**	**359**	**579**	**579**	**479**	**479**	**361**	**361**