Are Bonds Desirable in Tax-Deferred Accounts?

Marcel Marekwica*

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Comments welcome!

Abstract

This paper analyzes the consumption investment problem over the life cycle for investors with tax-deferred investment opportunities. According to the classical result of Black (1980) and Tepper (1981) with unlimited borrowing and short-selling, bonds should be exclusively held in these accounts. We argue that in tax-systems like the US where tax rebates on capital losses are limited, this is not necessarily true. Since a tax loss carry-forward is a less attractive compensation than an immediate tax rebate payment, the different taxable treatment of realized capital gains and losses increases the desirability of holding stocks in tax-deferred accounts. Our results suggest that this feature of the tax-code can help explaining why private investors violate the Tepper-Black result and hold stocks in their tax-deferred accounts.

JEL Classification Codes: G11, H24

Key Words: asset location, asset allocation, limited capital loss deduction, tax-deferred accounts, tax loss carry-forward, asset location puzzle

*University of Regensburg, Department of Business Management and Economics, Phone: +49-941-943-5083, Email: marcel.marekwica@wiwi.uni-regensburg.de. I would like to thank Alexander Schaefer, Michael Stamos and seminar participants at the Universities of Frankfurt and Regensburg and the NFA 2007 Meeting for helpful comments and discussion. Financial support from Friedrich Naumann Foundation is gratefully acknowledged. Opinions and errors are solely those of the author and not of the institutions with whom the author is affiliated.

1 Introduction

The taxable treatment of private investors' profits is a potentially important factor influencing household portfolio structure. While profits in conventional taxable accounts are taxable, profits in tax-deferred accounts are tax-exempt. As tax-law limits contributions to tax-deferred accounts and early withdrawals are subject to a penalty tax, private investors do not only have to decide which types of assets to hold (the so-called asset allocation problem), but also in which type of account to hold them (the so-called asset location problem).

The departing point for research in this field is the work of Black (1980) and Tepper (1981), who showed that with unlimited borrowing and short-selling, only bonds should be held in tax-deferred accounts. However, many private investors hold substantial amounts of stocks in their tax-deferred retirement accounts. This discrepancy between theory and behavior observed in practice is also referred to as the asset location puzzle (Amromin (2003)). Research in this field has therefore consciously focused on introducing frictions to the Tepper-Black setting that can break the strict preference for bonds in tax-deferred accounts, which is also referred to as a pecking-order portfolio rule. These frictions include the tax-inefficiency of actively managed mutual funds (Shoven and Sialm (2003)), stochastic labor income and/or wealth shocks (Amromin (2003), Dammon et al. (2004) and Huang (2007)) as well as short-selling and borrowing constraints (Garlappi and Huang (2006)).

This paper contributes to this line of research and shows that in addition to these factors, the tax code itself, namely the different taxable treatment of realized capital gains and losses, can help explaining the asset location puzzle.

While profits in taxable accounts are subject to taxation, profits in tax-deferred accounts remain untaxed. This non-linearity in stock returns precludes the possibility of perfect replication of stocks in a tax-deferred account with a portfolio of tax-deferred bonds and stocks. In such a setting, the result of Black (1980) and Tepper (1981) only holds if the investor knows in advance whether the stochastic capital gain of stocks is positive or not. In case of capital losses, stocks are preferentially held in the taxable account to obtain a tax-loss carry-forward. In case of capital gains, however, stocks are preferentially held in tax-deferred accounts to earn the capital gains tax-free. In order to diversify the risk of holding stocks in "the wrong account" it can be optimal to hold mixed portfolios in both the taxable and the tax-deferred account.

Another interesting aspect that arises from the different taxable treatment of capital gains and losses is the fact that a tax loss carry-forward that has not been used until the end of the investor's life is forfeited. This causes the value of the tax loss carry-forward to decrease substantially as the investor gets older.

In general, the taxation of profits has several impacts on private investors' investment behavior. Firstly, due to shrinking returns, investors might increase their present consumption and decrease their present investments. Secondly, if different assets are taxed at different tax-rates, taxation has an impact on optimal asset allocation. Historically, dividends and interest payments have been taxed at a higher rate than long-term capital gains (Sialm (2007)). While the impact of different tax-rates on asset allocation can still be analyzed implicitly by adjusting returns, volatilities, and correlations accordingly, the tax-system confronts private investors with additional tax-rules that deserve explicit modeling.

First, in the 1980s tax-deferred accounts like IRAs and 401(k)s with favorable taxable treatment were introduced to stimulate private savings for retirement. In particular, profits in such accounts are not subject to taxation. Furthermore, tax-deferred accounts allow one to defer the taxation of income until withdrawal, which is an advantage to private investors facing lower personal income tax-rates when retired than when employed. However, early withdrawals from tax-deferred accounts are subject to a 10% penalty tax. As a result of the penalty tax and limits on contributions to tax-deferred accounts, taxable accounts are often used as a retirement saving vehicle, as well. Due to the different taxable treatment of assets in taxable and tax-deferred accounts, it is important to make an informed decision as to which assets to locate in each account.

Second, capital losses cannot be offset against other income and thus do not qualify for tax rebates.[1] This tax-rule has an unfavorable impact on the risk-return-profile of volatile assets as it confronts the investor with tax-payments in case of a capital gain, but limits tax rebates for capital losses. Instead, for realized capital losses the investor receives a tax loss carry-forward that can be carried forward indefinitely and can offset against future capital gains to reduce future tax-burden.

Third, capital gains are not only taxed at a lower rate when being realized long-term, they also come with a tax-timing option. That is, capital gains are not taxable the moment they occur, but the moment they are realized. This option provides the investor with an opportunity to defer the realization of capital gains and to exploit the effect of compound returns, resulting in a reduction of the effective capital gains tax-rate (Chay et al. (2006)). A tax-timing option thus increases the equity exposure. However, tax-timing is associated with a risk of ending up with an unbalanced portfolio. An asset that comes with substantial capital gains in some period

[1] Under current US tax-code private investors can offset the lesser of the amount of realized capital losses exceeding realized capital gains and $ 3,000 against other income. We do not elaborate this special feature of the tax-code in more detail here.

tends to increase its fraction in the investor's portfolio. If the investor does not sell some units of that asset, the investor's portfolio may become too heavily invested in it. Hence, for assets with an unrealized capital gain, postponing the taxation of the unrealized gain and diversification can be opposing desires. If an asset comes with an unrealized capital loss, and wash-sale rules do not apply, Marekwica (2007) extends the classical result of Constantinides (1983) to tax-systems where only a tax loss carry-forward is granted for realized capital losses. He shows that it is optimal to realize that loss immediately even though that realization increases the investor's tax-basis. In addition, the investor's compensation potentially comes as a tax loss carry-forward, which is less attractive than a tax rebate, since the tax loss carry-forward does not pay any interest and potentially remains unused. In this paper, we focus on the impact of tax-deferred investing and limitations on tax rebates for capital losses and ignore the tax-timing option.

While optimal asset allocation is intensively discussed in the economic literature, research on optimal asset location is a more recent field of research going back to Black (1980) and Tepper (1981). They analyze investment strategies of companies running defined-benefit pension plans and conclude that in the absence of tax-timing options, limits on capital loss deduction, and short-selling restrictions, only bonds should be held in defined-benefit pension plans. Auerbach and King (1983) point out that this result can also be applied for private investors with a tax-deferred account. Along this line, Shoven (1999) and Shoven and Sialm (1998) introduce the asset location problem to household investment decisions.

Asset location strategies private investors follow in practice are e.g. analyzed in Bodie and Crane (1997), Poterba and Samwick (2001) and Barber and Odean (2003). According to these studies, private investors fail to exploit the opportunities tax-deferred accounts offer them. Particularly for long-term investment horizons, contribution rates have an important impact on total final wealth (Gomes et al. (2006)). This is due to the tax-exemption of profits in tax-deferred accounts, which has a tremendous impact on total final wealth for long-investment horizons due to the compounding on the taxes saved. It would seem that if there were no early withdrawal penalty, all individuals would contribute to tax-deferred accounts to the limit.

Amromin (2002) and Bergstresser and Poterba (2004) point out that many investors hold considerable amounts of equity in their tax-deferred accounts. Furthermore, according to the study of Benartzi and Thaler (2001), private investors' equity exposure in tax-deferred accounts and pension plans increases with the number of equity funds the investors can choose from. In particular, many investors seem to follow simple $\frac{1}{n}$-diversification strategies.

If investors have limited taxable wealth and face high labor income risk or wealth-shocks, some authors, among them Amromin (2003), Dammon et al. (2004), Amromin (2005) and Huang (2007) argue that bonds can be the preferred asset to hold in the tax-deferred account to avoid the penalty tax on early withdrawals and to smooth consumption over the life cycle. However, even with substantial income shocks, Dammon et al. (2004) show that it usually still remains optimal to hold stocks in the taxable account.

While the impact of the opportunity to invest in a tax-deferred account (see e.g. Shoven and Sialm (1998, 2003), Dammon et al. (2004), Poterba et al. (2004), Garlappi and Huang (2006), Gomes et al. (2006), Huang (2007) and Zaman (2007)), the impact of tax-timing on asset allocation (see e.g. Constantinides (1983, 1984) Dammon et al. (1989), Dammon and Spatt (1996), Dammon et al. (2001, 2004), Chay et al. (2006) and Gallmeyer et al. (2006)) have been intensively discussed in the literature, only limited guidance is available to investors faced with tax-systems where realized capital gains and realized capital losses are subject to a different taxable treatment like in the US. Studies dealing with the portfolio problem in such tax-systems are Ehling et al. (2007) and Marekwica (2007, 2008). However, these studies focus on investors that do not have access to tax-deferred investment opportunities. To the best of our knowledge, no guidance is available to investors with access to taxable and tax-deferred accounts in a setting where realized capital gains and realized capital losses are treated differently. This paper seeks to provide such guidance and shows that the different taxable treatment of capital gains and losses can help explaining the asset location puzzle.

The remainder of this paper proceeds as follows. Section 2 derives the effects driving optimal asset location decisions when tax rebates on capital losses are limited. Section 3 introduces the model and presents our numerical results. Section 4 summarizes and gives some hints for further research.

2 Effects of Limits on Tax Rebates

We consider a market in which the investor has the opportunity to invest into a risk-free bond paying a pre-tax return of $r > 0$ and a risky stock paying a constant pre-tax dividend rate of $d > 0$, and a risky capital gains rate g_t in period t. The parameters $\tau_g \in (0, 1)$ and $\tau_d \in (0, 1)$ denote the tax-rate on capital gains and losses and on dividends, interest and labor income, respectively. $\alpha_{T,t}$ denotes the fraction of stocks in the taxable account relative to taxable wealth invested in period t. That is, $\alpha_{T,t}$ is the fraction of stocks in the taxable account relative to beginning-of-period-taxable wealth adjusted for income, consumption, and contributions to or withdrawals from the tax-deferred account. $\alpha_{R,t}$ denotes the fraction of stocks relative to tax-deferred retirement wealth invested in period t. In the absence of a limitation on capital loss deduction, the investor's period t gross-return $R_{T,t}$ on a risky asset in a taxable account is given by

$$R_{T,t} = 1 + (1 - \tau_d)\, d + (1 - \tau_g)\, g_t. \tag{1}$$

In our setting however, capital losses only provide the investor with a tax loss carry-forward that can be carried over and offset against capital gains in forthcoming periods.[2]

By G_t we denote the realized capital gain (or loss) of an investor in period t and by L_{t-1} we denote the tax loss carry-forward that has been carried over from period $t-1$ to t, the net capital gain (or loss) T_t in period t that is subject to the capital gains tax is given by

$$T_t \equiv \max\left(G_t + L_{t-1}; 0\right). \tag{2}$$

Realized net capital losses can be indefinitely carried over to forthcoming periods. Thus, the tax loss carry-forward L_t that can be carried over from period t to period $t+1$ is given by

$$L_t \equiv G_t - T_t + L_{t-1}. \tag{3}$$

If $\chi_{g_t > 0}$ denotes the characteristic function which is 1 for $g_t > 0$ and 0 for $g_t \leq 0$, the gross-return on the risky asset in the taxable account $R_{T,t}$ in period t is given by

$$R_{T,t} = 1 + (1 - \tau_d)\, d + \left(1 - \tau_g \chi_{\{g_t > 0\}}\right) g_t = \begin{cases} 1 + (1 - \tau_d)\, d + (1 - \tau_g)\, g_t & \text{if } g_t > 0 \\ 1 + (1 - \tau_d)\, d + g_t & \text{if } g_t \leq 0. \end{cases} \tag{4}$$

The limitation of capital loss deduction in the taxable account is an obvious disadvantage to the investor: There is no longer a tax rebate when facing capital losses, but the investor is still confronted with tax-payments when the capital gain is positive.

The gross-return on the risky asset in the tax-deferred account $R_{R,t}$ at time t is not affected by a limitation on capital loss deduction due to the tax-exemption of profits and is given by

$$R_{R,t} = 1 + d + g_t. \tag{5}$$

Huang (2007) uses a tax-arbitrage argument to show that the investor should only hold bonds

[2]For simplicity, we do not elaborate the impact of the \$ 3,000 amount that can be offset against other income in more detail here. We refer the reader to Marekwica (2008) for this detail. The treatment of capital losses in this manuscript is e.g. in line with the Canadian and many European tax-codes.

in the tax-deferred account when there is no limit on tax rebates for capital loss deduction and the investor has the opportunity to go short in the taxable account. Her tax-arbitrage argument considers the replication costs of one dollar of bonds $C^{(R)}_{B,u}$ and one dollar of stocks $C^{(R)}_{S,u}$ in the tax-deferred account. Assuming that the tax-rate on coupon payments is higher than on capital gains, that is, $\tau_d > \tau_g$, and the interest rate on the risk-free asset is higher than the dividend rate on the stock, i.e. $r > d$, she shows that $C^{(R)}_{B,u}$ and $C^{(R)}_{S,u}$ are given by

$$C^{(R)}_{B,u} = \frac{1+r}{1+(1-\tau_d)\,r} \tag{6}$$

$$C^{(R)}_{S,u} = \frac{1}{1-\tau_g} + \frac{1}{1+(1-\tau_d)\,r}\frac{d\,(\tau_d-\tau_g)-\tau_g}{1-\tau_g} \tag{7}$$

and that the replication cost in the taxable account for one dollar of tax-deferred bonds is higher than for one dollar of tax-deferred stocks, i.e. $C^{(R)}_{B,u} > C^{(R)}_{S,u}$. One can thus replicate one dollar of bonds in tax-deferred wealth by $C^{(R)}_{B,u}$ dollars of taxable wealth in bonds. One dollar of stocks in tax-deferred wealth can be replicated by $\frac{1}{1-\tau_g}$ dollars of stocks and $\frac{1}{1+(1-\tau_d)r}\left(\frac{d(\tau_d-\tau_g)-\tau_g}{1-\tau_g}\right)$ dollars of bonds in the taxable account. Due to the higher replication cost, bonds are the preferred asset to hold in the tax-deferred account in such a setting.

If, however, capital loss deduction is limited, a stock in a tax-deferred account in period t can no longer be perfectly replicated in a taxable account as the replicating portfolio depends on whether the capital gain g_t is positive or not. In this case, stocks in the taxable and the tax-deferred account are only close but no longer perfect substitutes in the sense that the return in the tax-deferred account can no longer be perfectly replicated in the taxable account.

For $g_t \leq 0$, the replicating portfolio for one dollar of stocks in the tax-deferred account is one dollar of stocks and $\frac{\tau_d d}{1+(1-\tau_d)r}$ dollars of bonds in the taxable account. Its replication cost $C^{(R)}_{S,l}$ is given by

$$C^{(R)}_{S,l} = 1 + \frac{\tau_d d}{1+(1-\tau_d)\,r} < C^{(R)}_{B,l} \tag{8}$$

in which $C^{(R)}_{B,l} = \frac{1+r}{1+(1-\tau_d)r}$ is the replication cost for one dollar of bonds in the tax-deferred account. Since the risk-free asset does not come with any capital gains or losses its replication cost is the same as in a tax-system with unlimited capital loss deduction.

For $g_t > 0$ the replicating portfolio thus is the same as in the case with unlimited tax rebates for capital losses and thus consists of $\frac{1}{1-\tau_g}$ stocks and $\frac{1}{1+(1-\tau_d)r}\left(\frac{d(\tau_d-\tau_g)-\tau_g}{1-\tau_g}\right)$ dollars of bonds in the taxable account. As $C^{(R)}_{S,u} < C^{(R)}_{B,u} = C^{(R)}_{B,l}$, the replicating portfolio for both $g_t > 0$ and $g_t \leq 0$ is more expensive for bonds than for stocks in the tax-deferred account.

As one can see, unconditional on the sign of capital gains, bonds are more expensive to replicate in the taxable account than stocks. This implies that given $g_t \leq 0$ or $g_t > 0$ bonds have their preferred location in the tax-deferred account if the replication cost of tax-deferred wealth in the taxable account is state independent at each point in time (Huang (2007)). However, this does not imply that bonds are the preferred asset to hold in the tax-deferred account. This is due to the fact that the replicating portfolios for $g_t \leq 0$ and $g_t > 0$ are different. If the investor chooses the replicating portfolio for $g_t \leq 0$, but g_t turns out to be positive, the return of the replicating portfolio is given by

$$1 + d + g_t (1 - \tau_g) < 1 + d + g_t. \tag{9}$$

In this case, the return of the replicating portfolio is lower than in the tax-deferred account, due to the taxation of capital gains in the taxable account. If, on the other hand, the investor chooses the replicating portfolio for $g_t > 0$, but g_t turns out to be negative, the return of the replicating portfolio is given by

$$1 + d + \frac{g_t}{1 - \tau_g} < 1 + d + g_t. \tag{10}$$

Again, the return of the replicating portfolio is below the return of one unit of stocks in the tax-deferred account. Hence, in addition to the uncertainty about the return of the risky asset, replication itself is another source of risk. In contrast to the setting in Huang (2007) with limits on tax rebates there is no longer a dominating asset location strategy. In the presence of limits on tax rebates for capital losses, it can be optimal to hold stocks in both taxable and tax-deferred accounts for two reasons.

First, whereas so far, the focus of our analysis has been on the impact of asset location decisions for cash flows, the asset location decision also has an impact on a future tax loss carry-forward. An investment strategy that does not invest heavily in stocks in the taxable account in some given period potentially generates a small tax loss carry-forward that has a high probability of being entirely used in future periods In contrast, an investment strategy that does invest heavily in stocks in the taxable account potentially generates a huge tax loss carry-forward that has a lower probability of being entirely used in future periods. It can be optimal to invest in stocks in both the taxable and the tax-deferred account to have the opportunity of generating a small tax loss carry-forward, while avoiding generating a high tax loss carry-forward with a lower probability of being entirely used.

Second, for negative returns, stocks are better held in the taxable account to have the opportunity of being compensated for capital losses with a tax loss carry-forward. For substantial positive returns, however, stocks are better held in the tax-deferred account to benefit from the tax exemption of capital gains in the tax-deferred account. Thus, holding stocks in both taxable and tax-deferred accounts can be part of a diversification strategy. Consequently, limits on tax rebates for capital losses can help explaining the asset location puzzle.

Due to the different taxable treatment in tax-deferred and taxable accounts, shifting assets between these two accounts changes their risk-return profiles. While shifting bonds from a taxable to a tax-deferred account increases their return by $\tau_d r$, shifting stocks from the taxable to the tax-deferred account increases their return by $\tau_d d + \tau_g g_t$ in a tax-system with unlimited capital loss deduction and by $\tau_d d + \tau_g \chi_{\{g_t > 0\}} g_t$ in a tax-system with limits on tax rebates. Thus, stocks are less attractive in the taxable account in the tax-system without tax rebates on capital losses. This is because the taxable treatment for positive returns remains the same, while for negative returns the investor is not compensated for losses in cash, but instead with a tax loss carry-forward that – in contrast to tax rebates – does not pay any interest and bears a risk of never being used. The second source of risk is especially important for aged investors with higher mortality rates.

In a tax-system with unlimited tax rebates on capital losses, the tax-deferred account provides an investor with the opportunity of either earning a safe $\tau_d r$ extra dollars per dollar of bonds in the tax-deferred account or $\tau_d d + \tau_g g_t$ extra dollars for each dollar of stocks in the tax-deferred account in period t. In particular, as $\tau_g g_t$ is negative for $g_t < 0$, the tax-exemption of profits in the tax-deferred account can be a disadvantage to the investor since there are no tax rebates on capital losses in tax-deferred accounts. In a tax-system without tax rebates on capital losses, the tax-deferred account provides an investor who has no initial tax loss carry-forward with the opportunity of either earning a safe $\tau_d r$ extra dollars and a tax loss carry-forward of $\tau_g g_t$ dollars, if $g_t < 0$, per dollar of bonds shifted to the tax-deferred account or a safe $\tau_d d$ extra dollars and an extra $\tau_g g_t$ extra dollars if $g_t > 0$, per dollar of stocks shifted to the tax-deferred account.

While in the tax-system with unlimited capital loss deduction, shifting stocks from the taxable to the tax-deferred account increases the order of magnitude of a potential capital loss, this does not hold in a tax-system without capital loss deduction. Thus, shifting stocks from the taxable to the tax-deferred account does not increase the downside risk, but increases the upside potential. However, for negative capital gains, stocks have their preferred location in the taxable account due to the tax loss carry-forward granted for incurred losses and the preferential taxable treatment of bonds in the tax-deferred account.

Table 1 summarizes the effects of shifting assets from the taxable to the tax-deferred account in tax-systems without tax rebates on capital losses and unlimited tax rebates.

Table 1 about here

If, however, the investor is endowed with an initial tax loss carry-forward, potential capital gains in the taxable account not exceeding that tax loss carry-forward can be earned tax-free by making use of the tax loss carry-forward. In that case, shifting stocks to the tax-deferred account only allows the investor to earn $\tau_d d$ extra dollars, whereas shifting bonds to the tax-deferred account allows her to earn $\tau_d r > \tau_d d$ extra dollars per dollar invested in the tax-deferred account. Thus, for investors that are endowed with an initial tax loss carry-forward of substantial size, holding bonds in the tax-deferred account increases their total wealth more than holding stocks in the tax-deferred account.

However, due to penalties on early withdrawal, the opportunity to earn profits tax-free and maximum contribution limits in tax-deferred accounts, taxable and tax-deferred wealth are imperfect substitutes. Besides the impact on total wealth, the distribution on future taxable and tax-deferred wealth can have an impact on the optimal investment decision as well.

3 Numerical Evidence

Having introduced optimal asset location decisions in tax-systems with different taxable treatment of capital gains and losses, we now focus on the optimal asset location and asset allocation over the life cycle in a numerical setting. We consider an economy consisting of investors living for at most T years that can only trade at time $t = 0, 1, \ldots, T$. $F(t)$ denotes the probability that the investor is still alive through period t ($t \leq T$). Investors in that economy derive utility from the consumption of a single good and have CRRA-utility with parameter of risk-aversion $\gamma \in [0, \infty)$. The parameter γ represents the investor's willingness to substitute consumption among different states in time. It also represents the elasticity of consumption, which is given by $\frac{1}{\gamma}$. Given an initial endowment, the investor optimizes the discounted expected utility of lifetime consumption and bequest, subject to the intertemporal budget constraint. Following Dammon et al. (2001), we assume that at the time of death, the investor's remaining wealth is used to purchase an H-period annuity payable to the investor's beneficiary and that the H-period annuity provides the beneficiary with nominal consumption of $A_H W_t (1+i)^{k-t}$ at date k ($t+1 \leq k \leq t+H$), in which $A_H \equiv \frac{r^*(1+r^*)^H}{(1+r^*)^H - 1}$ is the H-period annuity factor, W_t is the sum of the investor's taxable and after-tax tax-deferred wealth, and $r^* \equiv \frac{(1-\tau_d)r-i}{1+i}$ is the after-tax real bond return. For simplicity, the beneficiary is assumed to have the same preferences as the investor. On this assumption, H can be interpreted as a measure for the strength of the investor's bequest motive. High values of H denote a strong bequest motive and low values denote a weak bequest motive. The parameter β represents the investor's utility discount factor and i is a constant annual inflation rate. If C_t denotes the investor's consumption, Z_t the contribution to (or withdrawal from) the tax-deferred account that is limited by some upper bound B_t, i.e. $Z_t \leq B_t$, τ_p is the penalty tax-rate applicable to early withdrawals from tax-deferred accounts, A_t is the investor's age in period t, J is the investor's mandatory retirement age, and N_t is the investor's non-financial income in period t, the investor's optimization problem over the life cycle can be expressed as

$$\max_{C_t,\alpha_{T,t},\alpha_{R,t},Z_t} \mathbb{E}\left[\sum_{t=0}^{T}\beta^t\left[F(t)U\left(\frac{C_t}{(1+i)^t}\right)+[F(t-1)-F(t)]\sum_{k=t+1}^{t+H}\beta^{k-t}U\left(\frac{W_tA_H}{(1+i)^t}\right)\right]\right] \quad (11)$$

subject to

$$W_t = W_{T,t} + (1-\tau_d)\,W_{R,t} \qquad t=1,\ldots,T \quad (12)$$

$$W_{T,t} = N_t\,(1-\tau_d) + Q_{T,t-1}\Big(\alpha_{T,t-1}\,(1+(1-\tau_d)\,d+g_{t-1}) + (1-\alpha_{T,t-1})\,(1+(1-\tau_d)\,r)\Big) - \tau_g T_{t-1} \qquad t=1,\ldots,T \quad (13)$$

$$Q_{T,t} = W_{T,t} - C_t - Z_t\left(1-\tau_d-\tau_p\chi_{\{Z_t<0\}\cap\{A_t<J\}}\right) \qquad t=0,\ldots,T-1 \quad (14)$$

$$W_{R,t} = Q_{R,t-1}\Big(\alpha_{R,t-1}\,(1+d+g_{t-1}) + (1-\alpha_{R,t-1})\,(1+r)\Big) \qquad t=1,\ldots,T \quad (15)$$

$$Q_{R,t} = W_{R,t} + Z_t \qquad t=0,\ldots,T-1 \quad (16)$$

$$Q_{T,t}, Q_{R,t}, C_t \geq 0; \quad \alpha_{T,t}, \alpha_{R,t} \in [0,1]; \quad Z_t \leq B_t \qquad t=0,\ldots,T-1 \quad (17)$$

given the investor's initial taxable wealth $W_{T,0}$, the initial tax-deferred wealth $W_{R,0}$ and the initial tax loss carry-forward L_{-1}. $F(-1)$ is set equal to one to indicate that the investor is alive at the end of period 0. $U(.)$ denotes the utility function of the investor and the beneficiary.

According to equation (11), the investor's current expected utility is a weighted sum of utility from consumption and utility from bequest. Equation (12) defines the investor's beginning-of-period-wealth W_t at time t to be the sum of taxable wealth $W_{T,t}$ and $W_{R,t}$ that part of tax-deferred retirement wealth that does not fall to the treasury at withdrawal.

Equation (13) describes the evolution of wealth in the taxable account. $W_{T,t}$ is the investor's taxable wealth in period t before consumption and investment decisions. $Q_{T,t-1}$ is the investor's taxable wealth invested in period $t-1$ after consumption, contributions or withdrawals, and the tax-payments or tax-rebates resulting from the contribution or withdrawal. Equation (14) is the investor's budget constraint. It shows how the investor's reinvestable wealth in the taxable account depends on consumption as well as the contribution or withdrawal from the tax-deferred account.

Equations (15) and (16) define the evolution of wealth in the tax-deferred account. Equation (15) shows the evolution of wealth $W_{R,t}$ in the tax-deferred account. $Q_{R,t-1}$ is the investor's tax-deferred wealth invested from period $t-1$ to period t. Equation (16) defines $Q_{R,t}$. If the investor dies before retirement age, we assume that the penalty tax on early withdrawal does not apply, which, among other reasons, can be the case if the beneficiary is the investor's spouse.

By letting X_t denote the vector of the investor's state variables, $V_t(.)$ the investor's value function at time t, and $f(t)$ the probability of surviving from period t to $t+1$, and taking into account that

$$\sum_{k=t+1}^{k+H} \beta^{k-t} = \frac{\beta\,(1-\beta)^H}{1-\beta} \tag{18}$$

as shown in Dammon et al. (2001), the optimization problem can also be stated as

$$\begin{aligned} V_t(X_t) = \max_{C_t,\alpha_{T,t},\alpha_{R,t},Z_t} & \Bigg[f(t)U\left(\tfrac{C_t}{(1+i)^t}\right) + f(t)\beta\mathbb{E}_t\left[V_{t+1}\left(X_{t+1}\right)\right] \\ & + (1-f(t))\,\tfrac{\beta\left(1-\beta^H\right)}{1-\beta} U\left(\tfrac{A_H W_t}{(1+i)^t}\right)\Bigg] \end{aligned} \tag{19}$$

for $t = 0, \ldots, T-1$ subject to equations (2), (3) and (12) to (17) with vector of state variables

$$X_t = [W_{T,t}, W_{R,t}, L_{t-1}] \tag{20}$$

and terminal condition $V_T(X_T) = U\left(\frac{A_H W_T}{(1+i)^T}\right)$. Following Dammon et al. (2004), we assume that non-financial income N_t is a constant multiple of W_t, i.e. $n \equiv \frac{N_t}{W_t}$ ($t \in \mathbb{N}_J \equiv \{t \in \mathbb{N} | t \leq J\}$) is a constant during the accumulation phase and a multiple of n that is given by the replacement rate λ during retirement. We further assume that $b_t \equiv \frac{B_t}{W_t}$ is an exogenously given constant during the investor's working age.

When the investor is retired, we do not allow any further contributions and require the investor to withdraw at least a fraction of $\frac{1}{\mathbb{E}[L(A_t)]}$ of the remaining tax-deferred wealth when the investor is aged 70.5 and older where $L(A_t)$ is the remaining life-expectancy of an investor at age A_t. The optimization problem can then be simplified by normalizing both the objective function and the constraints by W_t. We let $w_{R,t} \equiv (1-\tau_d)\frac{W_{R,t}}{W_t}$ denote the fraction of total beginning-of-period-wealth in the tax-deferred account that effectively belongs to the investor. $q_{T,t} \equiv \frac{Q_{T,t}}{W_t}$ is the investor's taxable wealth after transactions relative to beginning-of-period-wealth and $q_{R,t} \equiv (1-\tau_d)\frac{Q_{R,t}}{W_t}$ is the investor's retirement wealth after transactions relative to beginning-of-period-wealth. The investor's initial tax loss carry-forward beginning-of-period-wealth ratio is $l_t \equiv \frac{L_{t-1}}{W_t}$. $z_t \equiv \frac{Z_t}{W_t}$ is the investor's contribution-wealth-ratio, and $c_t \equiv \frac{C_t}{W_t}$ is the investor's consumption-wealth-ratio. Finally,

$$\delta_t \equiv \frac{G_t}{Q_{T,t}} = \alpha_{T,t} g_t \tag{21}$$

is the investor's capital gain per dollar invested in the taxable account from period t to $t+1$. The investor's taxable capital gains wealth ratio $t_t \equiv \frac{T_t}{W_t}$ is then given by

$$t_t = \max\left(\delta_t q_{T,t} + l_{t-1}, 0\right) \tag{22}$$

in which $q_{T,t} = 1 - w_{R,t} - c_t - z_t\left(1 - \tau_d - \tau_p \chi_{\{z_t<0\}\cap\{A_t<J\}}\right)$ defines the ratio between $Q_{T,t}$ and W_t. If

$$\mu_{T,t} = \alpha_{T,t}\Big(1 + (1-\tau_d)\, d + g_t\Big) + (1-\alpha_{T,t})\Big(1 + (1-\tau_d)\, r\Big) \tag{23}$$

and

$$\mu_{R,t} = \alpha_{R,t}\Big(1 + d + g_t\Big) + (1-\alpha_{R,t})\Big(1 + r\Big) \tag{24}$$

denote the investor's return per dollar invested in the taxable account and the tax-deferred account before capital gains tax payments, respectively, the evolution of the investor's total wealth is given by

$$\frac{W_{t+1}}{W_t} = \frac{q_{T,t}\mu_{T,t} - \tau_g t_t + q_{R,t}\mu_{R,t}}{1 - n_{t+1}(1-\tau_d)}. \tag{25}$$

For the evolution of the investor's tax-deferred retirement wealth, it holds that

$$w_{R,t+1} = q_{R,t}\mu_{R,t}\frac{W_t}{W_{t+1}}, \tag{26}$$

and the evolution of the investor's tax loss carry-forward is given by

$$l_t \equiv \frac{L_t}{W_{t+1}} = \frac{L_t}{W_t}\frac{W_t}{W_{t+1}} = \left(\delta_t q_{T,t} - t_t + l_{t-1}\right)\frac{W_t}{W_{t+1}}. \tag{27}$$

By defining $v_t(x_t) \equiv \frac{V_t(X_t)}{\left(\frac{W_t}{(1+i)^t}\right)^{1-\gamma}}$ to be the normalized value function, and $\rho_t \equiv \frac{W_{t+1}}{W_t(1+i)}$ to be the real growth rate of total wealth in period t, the assumption of CRRA-utility assures that the investor's objective function can be rewritten as

$$v_t(x_t) = \max_{c_t,\alpha_{T,t},\alpha_{R,t},z_t} \left[f(t)U(c_t) + f(t)\beta\mathbb{E}\left[v_{t+1}\left(x_{t+1}\right)\rho_t^{1-\gamma}\right] + (1-f(t))\frac{\beta\left(1-\beta^H\right)}{1-\beta}U\left(A_H\right)\right] \tag{28}$$

with vector of state-variables

$$x_t = \left[w_{R,t}, l_{t-1}\right]. \tag{29}$$

The investor's optimization problem can then be rewritten as

$$\max_{\left\{c_t,\alpha_{T,t},\alpha_{R,t},z_t\right\}_{t=0}^{T-1}} v_0\left(x_0\right) \tag{30}$$

subject to $v_T\left(x_T\right) = U\left(A_H\right)$, $q_{T,t}$, $q_{R,t}$, $c_t \geq 0$, $\alpha_{T,t}, \alpha_{R,t} \in [0,1]$, $z_t \leq b_t$ and equations (21) through (27).

For the numerical analysis, it is assumed that annual inflation is $i = 3.5\%$, mandatory retirement age is $J = 66$, such that the investor still works at age 65 and retires when turning 66. It is further assumed that exogenous income n_t is 0.15 during the accumulation phase and 0.105 during retirement, corresponding to a replacement ratio of $\lambda = 70\%$.[3] The risk-free rate is set to $r = 6\%$, the return on equity is lognormally distributed, independent in time, and comes with an expected capital gain of $\mu = 7\%$ ($t \in \mathbb{N}_T$), a standard deviation of $\sigma = 20.7\%$ (which corresponds to a standard deviation of the real return of about 20%), and a constant dividend rate of $d = 2\%$ in each period.

The correct choice of the risk-premium for equity has been subject to numerous theoretical and empirical research (see Siegel (2005) for a survey). While the historical risk-premium has been about 6% in the US since 1872 (Mehra and Prescott (1985)), economists doubt whether this will be true in future periods. We follow the current consensus which is about 3% to 4% (see e.g. Dammon et al. (2001, 2004), Cocco et al. (2005) or Gomes and Michaelides (2005)). The tax-rate on interest, dividends and income is assumed to be $\tau_d = 36\%$. The tax-rate on realized capital gains is assumed to be $\tau_g = 20\%$.[4] This choice of parameters follows Gallmeyer

[3]Ideally, non-financial income should be introduced into our model with its own stochastic process. However, this would require increasing the number of state variables and significantly complicate our numerical analysis. We therefore follow recent literature (see e.g. Dammon et al. (2001, 2004)) and assume the investor's non-financial income to be a constant proportion of her beginning-of-period wealth. However, since present savings do not only increase future wealth, but also future non-financial income, this assumption overstates the importance of present savings.

[4]Since the tax-rate on long-term capital gains is equal to the tax-rate on dividend income, we thereby implicitly

et al. (2006) and is quite similar to that of Dammon et al. (2004).

For this parameter choice, the asset location problem is the problem of whether the investor prefers a safe extra return of $\tau_d r = 0.0216$ and a tax loss carry-forward of $\tau_g g_t$ for $g_t < 0$ per dollar of bonds in the tax-deferred account in period t, or a safe extra return of $\tau_d d = 0.0072$ and an extra return of $\tau_g g_t$ where $g_t \geq 0$ for each dollar of stocks held in the tax-deferred account in period t.

We consider an investor who makes decisions annually starting at age 20 ($t = 0$). The maximum age the investor can attain is set to 100 years ($T = 80$). It is assumed that the relative risk-aversion of the investor is $\gamma = 3$ and the annual subjective utility discount factor is $\beta = 0.96$. The data for the survival probabilities of our investor were set equal to the survival probabilities for female investors according to the 2001 Commissioners Standard Ordinary Mortality Table.

During the accumulation phase until retirement age J, the maximum contribution is set to $b_t = 5\%$ ($t \in \mathbb{N}_{J-1}$). When the investor is retired, we do not allow any further contributions and require an investor at age A_t with remaining life-expectancy of $L(A_t)$ to withdraw at least a fraction of $\frac{1}{\mathbb{E}[L(A_t)]}$ of the remaining tax-deferred wealth when the investor is aged 70.5 and older. Initially, H is set to $H = 10$ in the bequest function, indicating that the investor wished to provide the beneficiary with an income stream for the next 10 years. This set of parameter values is referred to as the base case scenario. Table 2 summarizes these base-case parameters.

Table 2 about here

Our optimization problem can be solved numerically using backward induction. The state-space for the vector of endogenous state-variables $x_t = [w_{R,t}, l_{t-1}]$ is spanned over a 51×51 grid with equally distributed grid points on $[0, 1]$ and $[-0.5, 0]$, respectively, for each period t. For values of $[w_{R,t}, l_{t-1}]$ within the grid, cubic spline interpolation is performed. The integral of the expectation in equation (30) is computed using Gaussian quadrature.

In the following, we present our numerical results for the optimal consumption-investment problem in the presence of taxes, limits on tax rebates for capital losses and short-sale restrictions. The model is first solved for the base-case parameters. To demonstrate the effect of limits on tax rebates for capital losses and tax-deferred investment opportunities, the model is also solved for a setting with unlimited tax rebates for capital losses and a setting without a tax-deferred account. These two settings serve as benchmarks for our forthcoming analysis.

assume that the capital gains qualify for long-term treatment. The case with $\tau_g = \tau_d = 0.36$ is analyzed in section 3.4.

The optimal consumption policy is considered in section 3.1, and the optimal investment policy is analyzed in section 3.2. Section 3.3 provides a simulation analysis on the optimal path and shows the evolution of the investor's state and decision variables over the life cycle. Section 3.4 examines the sensitivity of the optimal investment and liquidation policies to various model parameters.

3.1 Optimal Consumption Policy

We begin the discussion of our numerical results by presenting the investor's optimal consumption policy over the life cycle depending on age, the fraction of wealth in the tax-deferred account and an initial tax loss carry-forward the investor is potentially endowed with. As benchmark cases, we consider an investor in a tax-system with unlimited capital loss deduction and an investor who does not have the opportunity of locating funds in a tax-deferred account.

The relation between the optimal consumption-wealth ratio and age depends heavily on the investor's bequest motive. As the investor ages, her mortality rate increases and the impact of the bequest motive on her consumption decision increases. Investors with strong bequest motives will decrease their consumption-wealth ratio; while in contrast, investors with low levels of bequest motive will increase their consumption-wealth ratio. The bequest motive used in the base-case is not so strong that the investor decreases her consumption throughout her entire life nor is it so weak that she increases her consumption throughout her entire life. Due to tax-effects and a replacement ratio below one, the investor decreases her consumption until retirement age and increases it again afterwards.

However, the optimal consumption-wealth ratio does not only depend on age, but also on the investor's initial tax loss carry-forward and the distribution of her wealth to the taxable and the tax-deferred account. The higher the tax loss carry-forward is, the more advantageous the risk-return profile of equity in the taxable account, and thus the higher the increase of potential future consumption for each dollar that is not consumed today. The higher the investor's fraction of wealth in the tax-deferred account, the higher the investor's expected return on her total wealth. Thus, to generate the same expected wealth in the forthcoming period, an investor who has a high fraction of tax-deferred wealth can consume more than an investor who has a small fraction of tax-deferred wealth.

Figure 1 about here

Figure 1 depicts the optimal consumption-wealth ratios of an investor in our base-case setting who is endowed with either no initial tax loss carry-forward ($l = 0$, left graph), or an initial tax loss carry-forward of 10% of her beginning-of-period wealth ($l = -0.1$, right graph).

The investor's optimal consumption depends on both age and the fraction of tax-deferred wealth. The optimal consumption is strictly increasing in tax-deferred wealth except for very high levels of tax-deferred wealth where the investor has to withdraw from her tax-deferred account to finance consumption. This is due to the fact that tax-deferred wealth provides the investor with higher returns than taxable wealth due to the tax-exemption of profits. Due to the tax-exemption of profits in tax-deferred accounts, one dollar of tax-deferred wealth is worth more than one dollar of taxable wealth if the investor can avoid the penalty-tax on early withdrawal (Dammon et al. (2004), Poterba (2004)). Hence, with a higher level of tax-deferred wealth, the investor can attain the same level of next-period-wealth by investing a lower amount of wealth today. Furthermore, the investor can both increase her present consumption by consuming some part of the difference between the amount of wealth needed in the absence of a tax-deferred investment opportunity and the amount needed in the presence of tax-deferred investment opportunities, and investing the remainder. Young investors, however, face penalties on early withdrawals from tax-deferred accounts.

Prior to the retirement age of 66, the investor's consumption gradually decreases. This reflects the fact that at retirement age, the investor's non-financial income sharply decreases. To keep up a certain level of consumption, the investor builds up capital she can use to compensate for the decrease in non-financial income when she reaches retirement age.

Over the life cycle, the investor faces two structural changes in the taxable treatment of her wealth. First, having attained retirement age, the investor is not allowed to make any further contributions to the tax-deferred account, but is allowed to withdraw from it. For an investor with very high levels of tax-deferred wealth, the opportunity to withdraw funds from the tax-deferred account is an advantage since it allows her to consume without being confronted with the tax-penalty on early withdrawal. Hence, having attained retirement age consumption increases significantly.[5] Investors with lower levels of tax-deferred wealth, however, does not consider the opportunity of withdrawing funds from the tax-deferred account a tremendous advantage, since she is endowed with sufficient amounts of taxable wealth. The upper bound on contributions of zero prevents such investors from shifting taxable wealth to tax-deferred wealth for earning higher returns. The investor thus can no longer increase returns by shifting the location of assets, which reduces the intertemporal rate of consumption, and

[5]Due to the assumption of non-financial income increasing taxable wealth, the base-case investor can never end up in such a state. In section 3.4 we consider an investor who does not have non-financial income.

makes current consumption more attractive due to the decreasing premium on consumption deferral. Consequently, the consumption level increases having attained retirement age. Second, at age 70.5 the investor is subject to the minimum withdrawal rules which force her to withdraw at least a fraction, equal to one divided by her remaining life expectancy, from her tax-deferred account. Due to this forced withdrawal, the investor's return is decreasing since assets in the tax-deferred account are not subject to taxation of profits while assets in the taxable account are. Hence, the appeal of current consumption increases as the premium for a deferral of consumption decreases. This is why starting at age 70.5, the slope of the investor's consumption increases with time even stronger than before.

Compared to the benchmark case with no tax-deferred account (not shown here), the investor's optimal consumption is significantly higher, since the tax-deferred investment opportunity allows the investor to generate substantially higher returns than investments in the taxable account. Compared to the benchmark case with unlimited capital loss deduction, the investor's optimal consumption does not change significantly.

Figure 1 further shows that the level of an initial tax loss carry-forward does not have a significant impact on optimal consumption. However, an initial tax loss carry-forward does have a tremendous impact on the optimal investment policy. In particular, it has a strong impact on optimal asset location.

3.2 Optimal Investment Policy

Having discussed the optimal consumption policy in the previous section, we now turn to the optimal investment policy in the presence of tax-deferred investment opportunities and limitations on capital loss deductions.

In the absence of tax-deferred investment opportunities (not shown here), there is no longer an asset location problem. In that case, differences in the investor's equity exposure only depend on the remaining investment horizon and the level of the initial tax loss carry-forward. Not very surprisingly, the investor's equity exposure tends to increase with an increasing initial tax loss carry-forward. This is due to the fact that in the presence of a tax loss carry-forward, the risk-return-profile of the asset becomes more advantageous to the investor. Furthermore, for low levels of initial tax loss carry-forward, the investor's equity exposure decreases when the investor ages. This is due to the fact that because by investing into equity, the investor faces a high probability of generating a tax loss carry-forward that potentially cannot be used anymore.

Without limits on capital loss deduction, the tax-arbitrage argument of Huang (2007) suggests that the investor should hold her entire tax-deferred wealth in bonds. Our numerical analysis confirms this finding.

Figure 2 about here

Figure 2 depicts the relation between the investor's optimal equity exposure in both the taxable and the tax-deferred account and the fraction of beginning-of-period-wealth in the tax-deferred account and her age for an investor in our base-case setting who is either endowed with no initial tax loss carry-forward ($l = 0$, upper graphs), endowed with an initial tax loss carry-forward of 4% of beginning-of-period wealth ($l = -0.04$, middle graphs), or endowed with an initial tax loss carry-forward of 10% of her beginning-of-period wealth ($l = -0.1$, lower graphs).

The upper left graph shows the optimal equity proportion in the taxable account for an investor who is not endowed with an initial tax loss carry-forward. The upper right graph shows the optimal fraction of stocks in the tax-deferred account. When the investor is young, more stocks are held in the taxable account as the certain tax-gift in the tax-deferred account of $\tau_d r$ extra dollars and the tax loss carry-forward of $\tau_g g_t$ in period t is very attractive when she is young and can expect to make use of the tax loss carry-forward at a significant probability. In contrast, if the investor is old, stocks are preferentially held in the tax-deferred account. This is due to the fact that for an old investor, the probability of using the entire tax loss carry-forward in future periods is much lower than for a young investor. That is, a safe extra return of $\tau_d d$ plus an extra return of $\tau_g g_t$ if $g_t > 0$ in period t tends to be more attractive for a young investor than a safe $\tau_d r$ extra dollars and a tax loss carry-forward of $\tau_g g_t$ dollars if $g_t < 0$. However, to hold a well diversified portfolio, it can be rational to hold stocks in both the taxable and the tax-deferred account.[6]

During the accumulation phase, the investor takes advantage of that diversification strategy and holds stocks both in her tax-deferred and her taxable account. Having attained retirement age, she preferentially locates bonds in the tax-deferred account. This preference is caused by two tax-effects.

First, having passed age 70.5, the investor is subject to the minimum withdrawal rules such that an increase in the return on tax-deferred wealth does not only increase tax-deferred wealth,

[6]The optimal equity proportion of one in the taxable account for high levels of tax-deferred wealth is a side-effect of the modeling. Whereas tax-deferred wealth is measured as beginning-of-period tax-deferred wealth, the optimal equity proportion in the TA is given as a fraction of the investor's taxable wealth after consumption, contributions to and withdrawals from the taxable account. This modeling allows us to reduce the number of decision variables. Since the investor's optimal consumption is roughly 10%, she has no taxable wealth left that can be reinvested and thus any equity proportion in the taxable account is optimal.

but also increases the minimum withdrawal in forthcoming periods, which in turn decreases the appeal of tax-deferred investing.

Second, having passed retirement age, the investor is no longer allowed to contribute to the tax-deferred account. Since the investor's equity exposure in the taxable account is above the equity exposure in the tax-deferred account, and taxable wealth can no longer be decreased by contributing to the tax-deferred account, the optimal equity exposure in the tax-deferred account is decreasing. Accordingly, the minimum distribution rules increase the investor's taxable wealth, which in turn causes her to decrease her taxable equity exposure when she passes age 70.5.

When investors are very old, these effects are outweighed by the high probability of generating a tax loss carry-forward that cannot be used anymore due to increasing mortality rates. Consequently, the tax loss carry-forward has a low value for the investor, which causes her to increase her stock holdings in the tax-deferred account and decrease her equity exposure in the taxable account when approaching her maximum age of 100.

Our results show that the different taxable treatment of capital gains and losses can help explaining why it can be optimal to hold mixed portfolios in both the taxable and the tax-deferred account. In contrast to Amromin (2003) and Dammon et al. (2004) who require quite strong labor income or wealth shocks (a 0.5% probability of loosing 99% of labor income or a requirement to consume 50% of total wealth) our assumptions that are sufficient to cause the investor to hold mixed portfolios seem quite mild. This result is due to the reason that in Amromin (2003) and Dammon et al. (2004) shocks only come with a low probability are only temporary while the different taxable treatment of capital gains and losses in our setting has a permanent impact.

The middle left graph of Figure 2 shows the optimal equity exposure in the taxable account of an investor who is endowed with an initial tax loss carry-forward of 4% of her initial total wealth. Compared to the case without a tax loss carry-forward, there are two important differences.

First, the investor's equity exposure in the tax-deferred account at high age decreases while her equity exposure in the taxable account increases in comparison to the case with no initial tax loss carry-forward. If the investor is endowed with an initial tax loss carry-forward, her equity exposure in the taxable account is significantly above her equity exposure in the absence of an in initial tax loss carry-forward. This is due to the fact that the tax loss carry-forward changes the risk-return-profile of equity in the taxable account in an attractive manner. The increase in the investor's equity exposure is such that her tax loss carry-forward suffices to cover potential capital gains at a satisfactory probability. Consequently, the investor decreases her equity exposure in the tax-deferred account to avoid getting too heavily invested into equity.

Second, in the presence of a small tax loss carry-forward, a retired investor holds stocks in the tax-deferred account for lower levels of tax-deferred wealth than in the absence of a small tax loss carry-forward. This is because when endowed with a substantial tax loss carry-forward, the investor holds 100% of stocks in the taxable account for a lower level of initial tax-deferred wealth and thus can only increase her equity exposure by holding additional stocks. To avoid ending up with a very small equity exposure, the investor holds some tax-deferred equity as an imperfect substitute for taxable equity as well. Again, bonds are the preferred asset in the tax-deferred account during retirement.

With increasing length of the investment horizon, i.e. for younger investors, a higher tax loss carry-forward becomes more attractive, as the investor may expect to live longer and thus the probability of making use of the tax loss carry-forward increases. That is, with decreasing age, the investor increases her equity exposure in the taxable account and decreases her equity exposure in the tax-deferred account.

The lower left graph of Figure 2 shows the optimal equity exposure in the taxable account of an investor who is endowed with a substantial initial tax loss carry-forward of 10% of her total wealth ($l = -0.1$). The lower right graph shows her optimal equity exposure in the tax-deferred account. While for a small tax loss carry-forward, the investor holds equity in both the taxable and the tax-deferred account at high age, with substantial initial tax loss carry-forward, equity is strictly preferred in the taxable account over the entire life cycle.

A welfare analysis indicates that for our base case parameter setting, welfare gains from following an optimal asset location strategy compared to the "pecking order" asset location strategy of preferring bonds in tax-deferred accounts are quite small and do not exceed 0.1%. This finding is mostly driven by the fact that a substantial fraction of the investor's utility is derived from present consumption and the investor is non-financial income, such that the investor's future level of wealth is only partly driven by her current investment decision.

3.3 Unconditional Strategies

Having analyzed the investor's optimal investment policy given specific values of the state variables, we next turn to the investor's optimal unconditional investment policy over the life cycle. While the graphs in sections 3.1 and 3.2 provide a good intuition about the impact of the state variables on the investor's optimal equity exposure and the effects that drive these results, they do not reveal how likely the investor ends up in which state.

To analyze the investor's optimal investment strategy over the life cycle we run 50,000 simulations on our optimal grid. We consider an investor who enters the market at age 20, who neither faces unrealized capital gains or losses, who is not endowed with an initial tax loss carry-forward and whose initial wealth is \$ 10,000.

Table 3 summarizes the simulation results on the evolution of the state and decision variables in our base case parameter setting.

Table 3 about here

Table 3 shows the investor's wealth-level W, her consumption-wealth ratio c, her taxable and her retirement equity exposure α_{TA} and α_{RA}, respectively. It further shows the investor's optimal contribution-wealth ratio, the evolution of her fraction of wealth in the retirement account and the level of her initial tax loss carry-forward over the life cycle.

While the investor's consumption-wealth ratio c remains quite stable over her life cycle with a little drop past retirement, the investor's optimal shift of wealth z between the different types of accounts varies substantially with age. While at age 30 with low initial retirement wealth, the investor contributes to the limit to exploit the tax-exemption of profits in the tax-deferred retirement account, contributions are substantially lower at age 50, when the investor holds about 90% of her total wealth in the tax-deferred account already. To avoid the penalty tax on early withdrawals, the investor decreases her contribution substantially to around 0.7%. Having passed retirement age, the investor at age 70 starts withdrawing money from her retirement account to keep her consumption at a substantial level despite the fact that at retirement age her non-financial income drops by 30%.

At all ages considered in table 3, it can be optimal to hold mixed portfolios in the taxable and the tax-deferred account. At age 30, the investor's fraction of wealth in the tax-deferred retirement account is still quite small and the remaining investment horizon is still long such that a potential tax loss carry-forward is valuable to the investor. Consequently, at young age the investor's fraction of wealth in the tax-deferred retirement account is small. As the investor gets older, her optimal equity exposure in the retirement account increases with age. However, at age 90, the investor tends to decrease her optimal equity in the tax-deferred retirement account again since the minimum withdrawal rules decrease the attractiveness of holding assets with high returns in tax-deferred environments.

The evolution of the investor's tax loss carry-forward over the life cycle is heavily related to the evolution of her fraction of retirement wealth. In states where her retirement wealth is substantial, the investor can only end up with a quite small tax loss carry-forward since her tax-deferred wealth does not provide her with a tax loss carry-forward. Consequently at young age where her fraction of wealth in the retirement account is still small and at high age when (due to the minimum withdrawal rules) her fraction of retirement is again small, the level of her tax loss carry-forward is bigger.

3.4 Comparative Static Analysis

In this section, we provide some comparative static results by varying the maximum percentage of beginning-of-period-wealth that qualifies for tax rebates, ignoring non-financial income, increasing the tax-rate on capital gains τ_g to the tax-rate on dividends and interest τ_d, increasing the volatility of equity, considering the mortality table of a male investor, and ignoring minimum withdrawal rules.

If the investor does not receive any non-financial income and has to finance her entire consumption from her investments, her consumption is much lower when she is young and strictly increases to the level of an investor endowed with exogenous income.[7] Her optimal investment strategy remains similar to that of an investor who is endowed with non-financial income.

Increasing the volatility of stocks from $\sigma = 20.7\%$ to $\sigma = 31.05\%$, corresponding to a volatility of real stock returns of about 30%, causes the investor to sharply decrease her equity exposure. Furthermore, with $\sigma = 31.05\%$, the investor prefers stocks in her tax-deferred account when not being endowed with an initial tax loss carry-forward. Again, this is due to the fact that the higher volatility provides the investor with a higher increase in wealth when holding stocks in the tax-deferred account and a higher tax loss carry-forward when holding stocks in the taxable account compared to the base-case.

In the base-case scenario, we have considered the mortality table of a female investor. If, instead, one considers the mortality table of a male investor, our results do not change significantly (not shown here). While the optimal consumption policy remains about the same as for a female investor, the optimal asset location at the end of the life cycle slightly differs. Since men are subject to higher mortality rates, male investors start shifting stocks from their taxable to their tax-deferred accounts at a lower age than female investors. While female investors start this shifting around age 88, male investors already start shifting stocks at around

[7] In fact, her optimization problem is the same as for an investor with non-financial income in the last period.

age 81.

The two most important changes in the taxable treatment of the investor's wealth occur, firstly, when attaining retirement age, when the investor is no longer allowed to contribute to her tax-deferred account but the penalty-tax on early withdrawal no longer applies, and secondly at age 70.5, when minimum distribution rules apply. In a tax-system where early withdrawals from the tax-deferred account are not subject to a penalty-tax, i.e. $\tau_p = 0$, the investor with high level of tax-deferred wealth decreases her consumption less than in the base-case with a penalty-tax on early withdrawal of $\tau_p = 10\%$. However, she still decreases her consumption. This is due to the fact that to finance her consumption, she has to withdraw from her tax-deferred account, which pays a tax-free return, whereas for low levels of tax-deferred wealth, she can consume from her taxable wealth, which only pays an after-tax return.

If minimum distribution rules do not apply and the investor is never forced to withdraw funds from her tax-deferred account for legal reasons, the optimal fraction of stocks in the taxable account is significantly below the level of an investor in a tax-system where minimum distribution rules apply for an investor endowed with some tax-deferred wealth.[8] This is due to the fact that the investor only withdraws funds from her tax-deferred account for consumption, since investments in the tax-deferred account result in higher returns than investments in the taxable account. Consequently, the investor's taxable wealth is primarily used for consumption and thus a low volatility in it is a desirable feature. This is why the equity exposure in the taxable account is quite low compared to the case with a tax-system where minimum distribution rules apply. For not getting too underinvested in equity, the investor increases her equity exposure in the tax-deferred account in such a setting.

In our analysis, we have so far assumed that capital gains are taxable when they occur. Many tax-systems around the world do not tax capital gains until they are realized, which provides the investor with a tax-timing opportunity. As tax-timing can only be performed in the taxable account, a tax-timing option increases the attractiveness of holding stocks in the taxable account. Zhou (2007) argues that optimal asset location should depend on the frequency of capital gains realizations since this frequency affects the tax burden on stocks. The study of Chay et al. (2006) shows that due to the tax-timing option, the effective capital gains rate $\tau_{g,e}$ that the investor faces is lower than the capital gains tax-rate τ_g. According to their study, the effective capital gains tax-rate of an investor with capital gains tax-rate

[8] As in the base-case setting, the high fraction of stocks in the taxable account for high levels of tax-deferred wealth is a side-effect.

$\tau_g = 20\%$ is $\tau_{g,e} = 14\%$.

Figure 3 about here

Figure 3 shows the optimal asset location and asset allocation of an investor who is confronted with a capital gains tax-rate of $\tau_g = 14\%$, the effective capital gains tax-rate of Chay et al. (2006). An investor who faces such a low capital gains tax-rate strictly prefers bonds in her retirement account, due to the high tax-burden on these assets. These findings suggest that in the presence of a tax-timing option, the asset location puzzle cannot entirely be explained by asymmetric treatment of capital gains and losses in taxable and tax-deferred accounts and other factors like the desire for liquidity might haven an important impact as well. However, to describe the advantages of the tax-timing option in one single value, Chay et al. (2006) require some simplifying assumptions. In particular, they assume that the optimal liquidation price is independent of the purchase price, which ignores the fact that with unrealized capital gains, investors tend to hold a higher fraction of that asset than without to avoid the capital gains tax payment. This assumption thus casts doubt on whether the tax-timing opportunity causes investors to prefer bonds in their tax-deferred accounts.

In the base-case, it has been assumed that the tax-rate on capital gains τ_g is $\tau_g = 20\%$, which is below the dividends tax-rate that we set as $\tau_d = 36\%$. The tax-rate on capital gains is only lower than the tax-rate on dividends if capital gains are realized long-term. In case capital gains are realized short-term, the tax-rate on capital gains is equal to that of dividends and $\tau_g = \tau_d$. Considering all capital gains short-term gains, τ_g increases to $\tau_g = 36\%$. This increase in the tax-rate on capital gains has a tremendous impact on optimal asset location for an investor who is not endowed with an initial tax loss carry-forward, as shown in Figure 4.

Figure 4 about here

Figure 4 shows that when capital gains are realized short-term, an investor who is not endowed with an initial tax loss carry-forward prefers holding stocks in the tax-deferred account to holding stocks in the taxable account. This result is due to changes in the investor's risk-return profile caused by the increase of the capital gains rate. By increasing τ_g from 20% to 36%, the asset location decision depends on whether the investor prefers a safe extra return of $\tau_d r = 0.0216$ and a tax loss carry-forward of $\tau_g g$ for $g_t \leq 0$ per dollar of bonds in the tax-deferred account in period t, or a safe extra return of $\tau_d d = 0.0072$ and an extra return of $\tau_g g_t$ in case that $g_t > 0$ for each dollar of stocks held in the tax-deferred account. The increase of τ_g thus increases the potential increase in wealth in the tax-deferred account for positive capital gains, while in the taxable account it only provides the investor with a higher tax loss carry-forward in case of negative capital gains. Since the opportunity to earn extra money by locating stocks in the tax-deferred account is more attractive than the opportunity to increase the tax loss carry-forward by holding stocks in the taxable account, the investor prefers stocks in the tax-deferred account when her capital gains are realized short-term.

When the investor attains retirement age and is not endowed with significant tax-deferred wealth, she increases her equity exposure in the taxable account significantly. Having passed that significant increase in equity, however, her equity exposure decreases with age. The strong increase in her equity exposure is due to the fact that having passed retirement age; she no longer has the opportunity to contribute to a tax-deferred account. On the one hand, this causes her to increase her consumption (not shown here), as the premium for consumption deferral is decreasing. On the other hand, she can no longer increase her return by shifting funds from the taxable to the tax-deferred account. To avoid decreasing her return too sharply, she increases her equity exposure. Having increased her fraction of stocks in the taxable account at retirement age, she then decreases this fraction again in time. This decrease is due to the fact that her mortality increases, which in turn increases her probability of potentially generating a tax loss carry-forward that she cannot use anymore.

4 Conclusion

This paper analyzes optimal asset location and asset allocation decisions in the presence of tax-deferred investment opportunities and an asymmetric treatment of capital gains and losses. While capital gains are subject to taxation, capital losses only qualify for a tax loss carry-forward that can be offset against realized capital gains in future periods. This paper shows that given such an asymmetric treatment of capital gains and losses, it can be optimal to break the "pecking order" portfolio rule and hold mixed portfolios in both types of accounts and thereby helps explaining the asset location puzzle. Even though utility gains from following an optimal asset location strategy are quite small, it is usually not optimal to hold as many bonds as possible in the tax-deferred account. There are two important reasons for this result.

First, limitations on tax rebates for capital losses worsen stocks' risk-return profile in the taxable account by compensating the investor with a tax loss carry-forward instead of a tax rebate. Especially for aged investors facing high mortality rates, a tax loss carry-forward is potentially worthless if the investor does not survive long enough to offset it with positive capital gains or receive tax rebates on it. Furthermore, in contrast to compensation by tax refunds, a tax loss carry-forward does not pay any interest.

Second, while for positive capital gains holding stocks in the tax-deferred account can result in a higher increase in total wealth, for negative capital gains holding stocks in the taxable account results in the same return on the stock, but leaves the investor with the tax-free return of bonds and a tax loss carry-forward that provides her with a tax-advantage in forthcoming periods. Thus, investors might want to hold stocks in both taxable and tax-deferred accounts to diversify this risk. Hence, limits on tax rebates for capital losses can help explaining why private investors hold substantial amounts of their tax-deferred wealth in stocks.

Inevitably this paper ignores several important factors. In particular, it makes a simplifying assumption on the taxation of capital gains, namely that capital gains are taxable when they occur. Many tax-systems around the world do not tax capital gains until they are realized which provides the investor with a tax-timing opportunity. Computations with effective capital gains tax-rates according to Chay et al. (2006) suggest that the optimal asset location decision might change when the investor has a tax-timing opportunity. However, their effective tax-rates rely on some simplifying assumptions that are not met in this paper.

An interesting avenue for further research is to extend our model with an endogenous tax-timing opportunity. Due to the forgiveness of capital gains when being bequeathed, aged investors facing higher mortality rates might consider pre-tax capital gains more important than after-tax capital gains and might thus prefer stocks to bonds in their taxable accounts. It would be interesting to know how our results on optimal asset location change in a tax-system that allows for tax-timing. We leave this problem for further research.

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Table 1: **Effects of Shifting Asset from Taxable to Tax-Deferred Account**

Asset	Effect of shift on	Unlimited tax rebates	No tax rebates
Bonds	Return	$\tau_d r$	$\tau_d r$
	Extra tax loss carry-forward	-	-
Stocks	Return	$\tau_d d + \tau_g g_t$	$\tau_d d + \tau_g g_t \chi_{\{g_t \geq 0\}}$
	Extra tax loss carry-forward	-	$\tau_g g_t \chi_{\{g_t < 0\}}$

Table 2: **Base-case Parameters**

Description	Parameter	Value
Risk-aversion	γ	3
Length of investment horizon	T	80
Number of years annuity beneficiary	H	10
Mandatory retirement age	J	66
Utility discount factor	β	0.96
Dividend rate	d	2%
Expected return stock	μ	7%
Standard deviation stock	σ	20.7%
Interest payment of bond	r	6%
Inflation rate	i	3.5%
Non-financial income	n	15%
Tax-rate on dividends, interest and income	τ_d	36%
Tax-rate on capital gains	τ_g	20%

This table reports the parameter values used in the base-case.

Evolution of State and Decision Variables over the Life Cycle

Panel A - Age 30

Percentile	W	c	α_T	α_R	z	w_R	l
1%	11,336	9.1%	39.8%	0%	5%	26.9%	-25.7%
10%	13,379	9.2%	43.5%	0%	5%	29.9%	-11.2%
50%	16,299	9.2%	44.4%	1.5%	5%	33.6%	-1%
90%	19,801	9.4%	57%	6.6%	5%	38.2%	0%
99%	23,177	9.4%	69.4%	10.7%	5%	43.1%	0%
Mean	16,512	9.3%	50.8%	3.8%	5%	34.3%	-7.6%
Std	2,545	0.1%	12.3%	4.7%	0%	6.5%	11.2%

Panel B - Age 50

Percentile	W	c	α_T	α_R	z	w_R	l
1%	29,863	9.1%	9%	22.6%	0.4%	89.4%	-11.6%
10%	38,142	9.1%	9.4%	22.6%	0.6%	89.6%	-4.3%
50%	51,229	9.1%	18.4%	23.3%	0.7%	89.9%	-0.1%
90%	69,927	9.1%	100%	23.4%	0.8%	90%	0%
99%	90,313	9.1%	100%	23.4%	1.1%	90%	0%
Mean	52,994	9.1%	47.4%	23%	0.7%	89.8%	-3.2%
Std	12,851	0%	48.2%	0.4%	0.3%	0.3%	5%

Panel C - Age 70

Percentile	W	c	α_T	α_R	z	w_R	l
1%	95,912	8.1%	100%	23.3%	-1.9%	92.7%	-3.2%
10%	130,434	8.1%	100%	23.3%	-1.9%	92.9%	-1.1%
50%	194,122	8.1%	100%	23.3%	-1.8%	93.1%	0%
90%	289,889	8.1%	100%	23.3%	-1.5%	93.1%	0%
99%	407,812	8.1%	100%	23.3%	-1.4%	93.1%	0%
Mean	204,374	8.1%	100%	23.3%	-1.7%	93%	-0.9%
Std	65,895	0%	0%	0%	0.2%	0.2%	1.4%

Panel D - Age 90

Percentile	W	c	α_T	α_R	z	w_R	l
1%	194,590	9.1%	20.2%	0%	-9.2%	21.7%	-28.5%
10%	285,794	9.1%	21.1%	0%	-7.6%	25.2%	-12.1%
50%	457,864	9.1%	33.4%	6.9%	-6.3%	30%	-1.4%
90%	736,840	9.2%	46.4%	38.2%	-5.3%	36.3%	0%
99%	1,092,906	9.3%	60.3%	44.3%	-4.5%	44.3%	0%
Mean	491,275	9.2%	36.3%	17.9%	-6.6%	31.5%	-8.4%
Std	189,058	0.1%	17.1%	21.6%	1.9%	9%	12.3%

Table 3: This table shows the evolution of the investor's state and decision variables over the life cycle. W denotes the investor's wealth-level, c her consumption-wealth ratio, α_T and α_R her equity exposure in the taxable and tax-deferred retirement account, z her contribution-wealth ratio. w_R is the investor's beginning-of-period-wealth in the tax-deferred account and l her initial beginning-of-period- tax loss carry-forward to wealth ratio.

Optimal consumption over the life cycle in the base-case

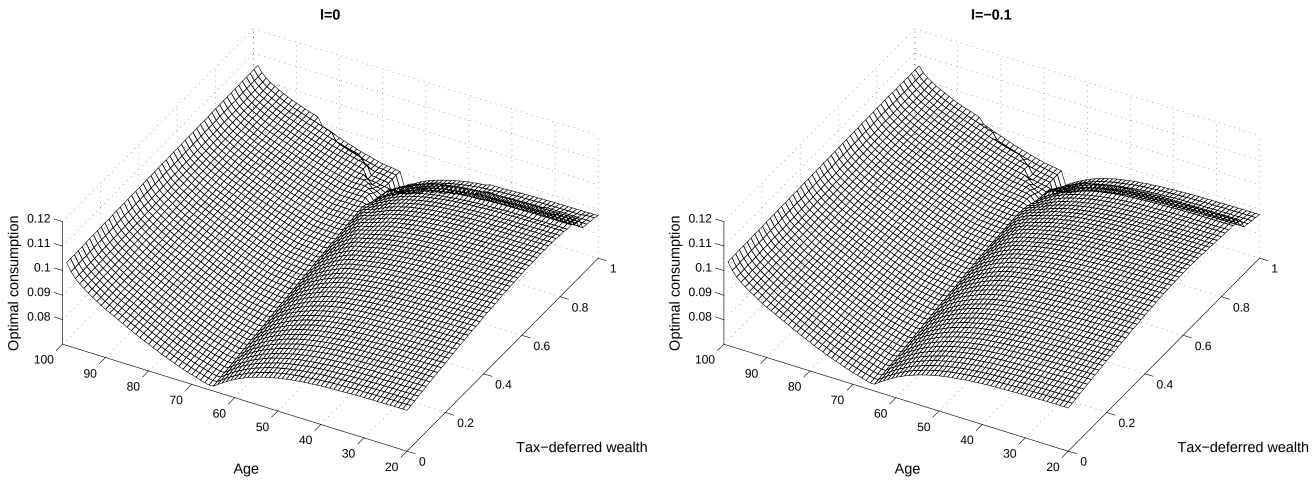

Figure 1: Optimal consumption-wealth-ratio (optimal consumption) for a female investor in the base-case scenario who has an initial tax loss carry-forward of zero ($l = 0$) or 10% of her initial wealth ($l = -0.1$) as a function of her age and her initial fraction of tax-deferred wealth.

Optimal fraction of stocks in taxable and tax-deferred wealth in the base-case

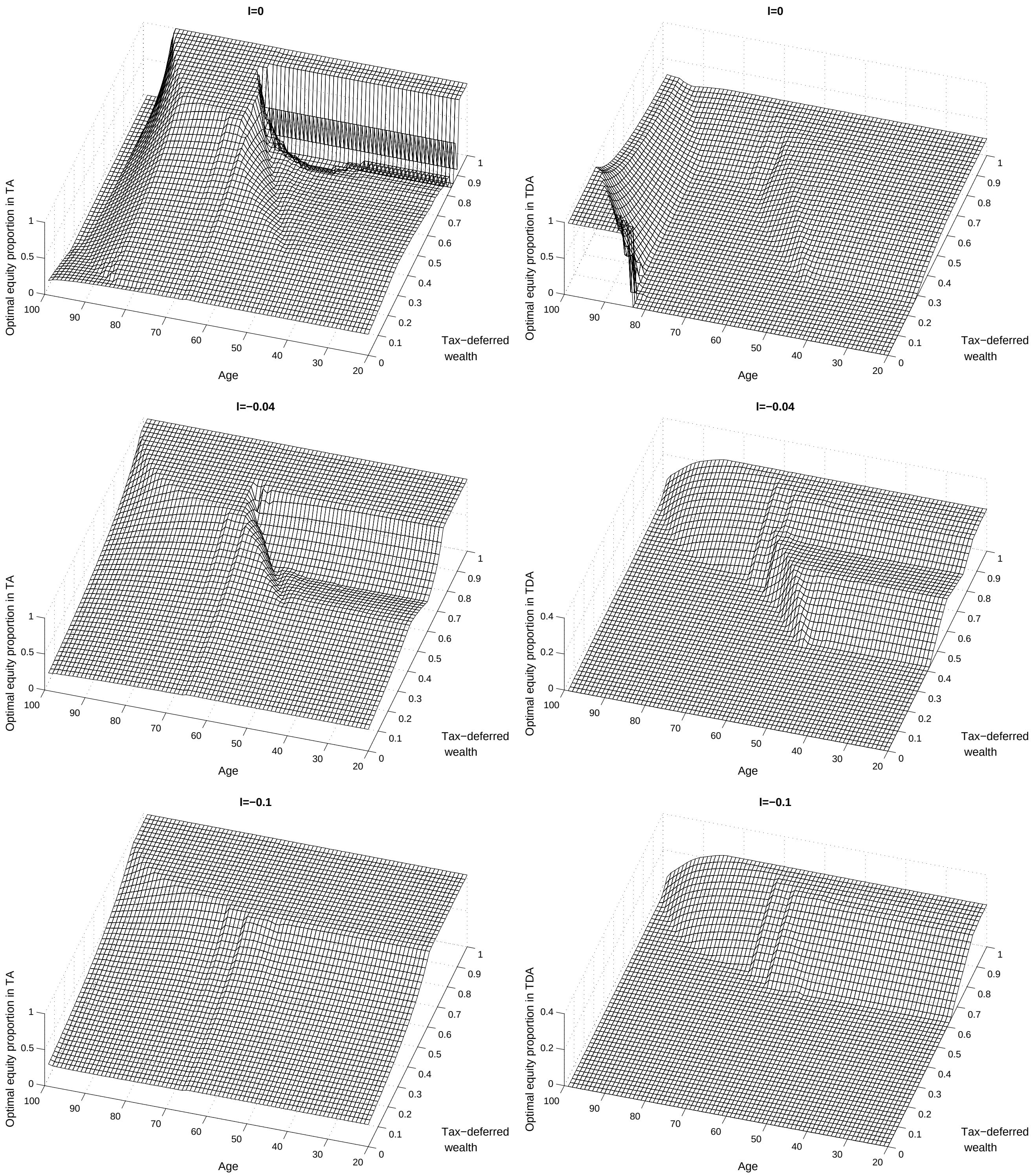

Figure 2: Optimal stock holding relative to taxable and tax-deferred wealth after consumption, contributions, and withdrawals for a female investor in the base-case scenario in taxable account (TA, left graphs), and tax-deferred account (TDA, right graphs) who is endowed with an initial tax loss carry-forward of zero ($l = 0$, upper graphs), 4% of her initial wealth ($l = -0.04$, middle graphs) or 10% of her initial wealth ($l = -0.1$, lower graphs) as a function of age and initial fraction of tax-deferred wealth.

Optimal fraction of stocks in taxable and tax-deferred wealth for $\tau_g = 14\%$

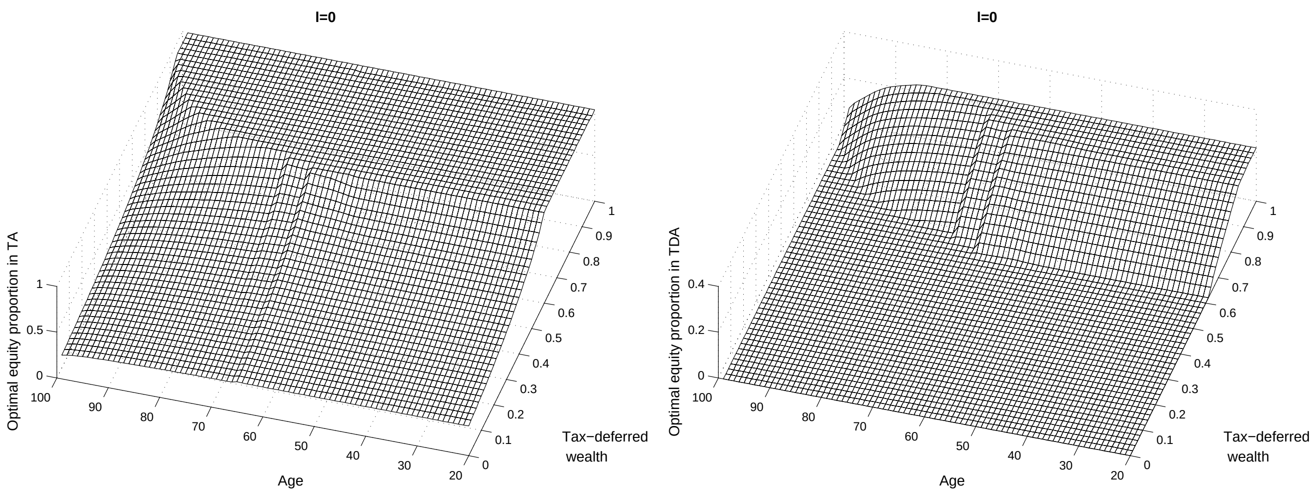

Figure 3: Optimal stock holding relative to taxable and tax-deferred wealth after consumption, contributions, and withdrawals for a female investor in taxable account (TA, left graph) and tax-deferred account (TDA, right graph) who is endowed with an initial tax loss carry-forward of zero ($l = 0$), when the tax-rate on capital gains corresponds to the effective capital gains rate of Chay et al. (2006) of $\tau_g = 14\%$.

Optimal fraction of stocks in taxable and tax-deferred wealth for $\tau_g = 36\%$

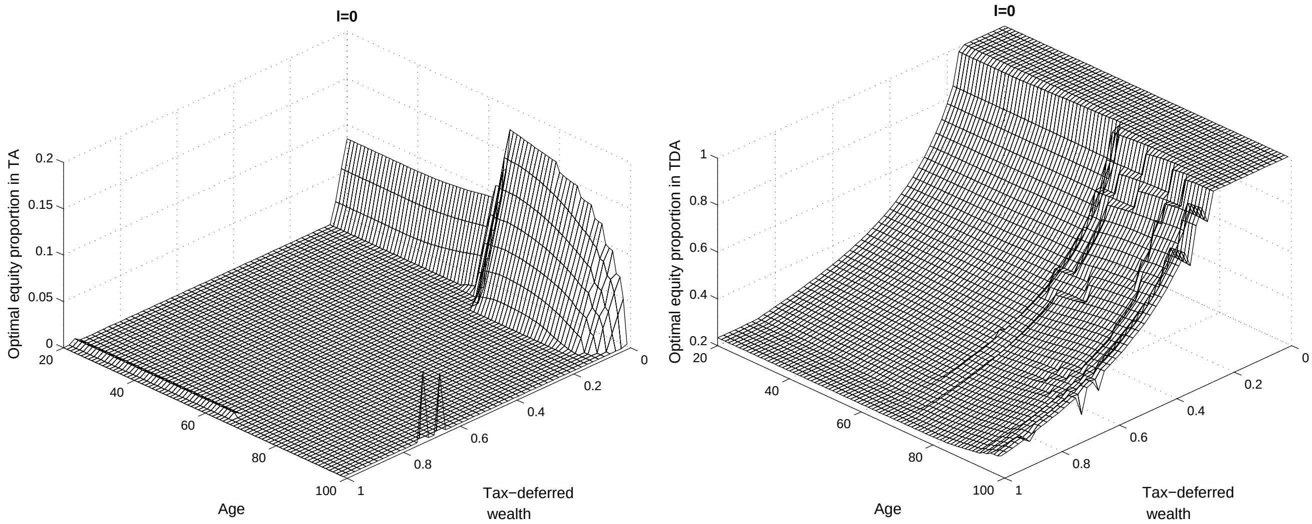

Figure 4: Optimal stock holding relative to taxable and tax-deferred wealth after consumption, contributions, and withdrawals for a female investor in taxable account (TA, left graph) and tax-deferred account (TDA, right graph) who is endowed with an initial tax loss carry-forward of zero ($l = 0$), when capital gains are considered short-term gains and $\tau_g = 36\%$.