# Forward-Looking Betas*

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**Abstract**

Few issues are more important for finance practice than the computation of market betas. Existing approaches compute market betas using historical data. While these approaches differ in terms of statistical sophistication and the modeling of the time-variation in the betas, they are all backward-looking. This paper introduces a radically different approach to estimating market betas. We use information embedded in the prices of individual stock options and index options to compute a forward-looking market beta at the daily frequency. This beta can be computed using option data for a single day, and is able to reflect sudden changes in the structure of the underlying company. Based on an empirical investigation of daily cross-sections of option contracts on thirty underlying companies, we conclude that these forward-looking betas contain information relevant for forecasting future betas that is not contained in historical betas.

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# 1 Introduction

Few concepts are more important for finance practice than the simple idea of market beta, which captures the co-variation of the return on an individual security with the return on the market portfolio, as approximated by a broad market index. Accurate measurement of market betas is critical for important issues such as cost of capital estimation, performance measurement and the detection of abnormal returns.

Existing techniques for beta estimation use historical data. These methods use past data to estimate betas and thus assume that the future will be sufficiently similar to the past to justify simple extrapolation of current or lagged betas. There is widespread agreement that betas are time-varying, and historical methods can easily allow for this. One popular approach uses a rolling window of historical returns to capture time-variation, while other approaches model the time-variation in historical betas more explicitly and in a more sophisticated fashion. However, no matter how sophisticated the modeling of the time-variation in the betas, a historical method may not perform well if historical patterns in the data are unstable.

This paper takes a very different approach. We estimate forward-looking betas by extracting information from prices on equity and index options. The appeal of this procedure is that these option prices are inherently forward-looking, and therefore incorporate information on future betas as opposed to lagged betas. We compare the empirical performance of this beta estimate with that of more traditional historical estimates, and we find that the forward-looking betas contain information that is not contained in historical betas and vice versa.

The approach in this paper is partly inspired by recent developments in the literature on volatility forecasting. For the purpose of volatility forecasting, one can compute a historical estimate using a simple historical standard deviation or a more sophisticated approach such as a GARCH model. Regardless of the level of statistical sophistication, such estimates are all historical and backward-looking. Alternatively, one can compute the implied volatility based on option contracts, which is forward-looking and incorporates the market's expectations. Intuitively one would expect a measure that incorporates market expectations to contain information that is different from historical methods, and sometimes outperform historical methods. Many studies confirm this intuition.[1] The method proposed in this paper is inspired by the same intuition: one would expect a forward-looking method for estimating beta to contain information not encompassed by historical beta. The main difference with the literature on volatility forecasting is that at a technical level, forward-looking betas are more difficult to compute than implied volatilities, which may explain why this idea has not yet been explored more in the academic literature and/or adopted into industry practice.

[1] See studies by Blair, Poon and Taylor (2001), Canina and Figlewski (1993), Christensen and Prabhala (1998), Day and Lewis (1992), Ederington and Guan (2002), Figlewski (1997), Fleming (1998), Jorion (1995), Lamoureux and Lastrapes (1993) and Pong, Shackleton, Taylor and Xu (2004). See Poon and Granger (2003) and Granger and Poon (2005) for surveys.

We are not the first to propose extracting market beta from options. French, Groth and Kolari (1983) combine forward-looking volatility with historical correlation to improve the measurement of betas. Siegel (1995) notes the advantage of a beta based on forward-looking option data, and proceeds to propose the creation of a new derivative, called an exchange option, which would allow for the computation of what he refers to as "implicit" betas. Unfortunately the exchange options discussed by Siegel (1995) are not yet traded, and therefore his method cannot be used in practice to compute forward-looking betas.

We show that market betas can be computed without the creation of a new derivative, by using prices on existing equity options and index options. Our proposed beta is computed using forward-looking estimates of variance and skewness, which can be computed using the methods proposed by Bakshi and Madan (2000), Bakshi, Kapadia and Madan (2003), Britten-Jones and Neuberger (2000), Carr and Madan (2001) and Jiang and Tian (2005). See also Jackwerth and Rubinstein (1996).[2] These methods allow us to retrieve the moments of the underlying distributions for index options and stock options from the cross-section of option prices. We then use a traditional one-factor model and express the forward-looking beta as a function of the variance and the skewness of the underlying distributions.

We implement this model using option contracts for the thirty components of the Dow Jones index as well as data on S&P500 options. Daily option data are obtained from the Ivy DB database over the period 1996-2003. We compare the performance of the forward-looking beta with that of the historical beta. We find that the performance of the historical beta critically depends on the estimation window, with longer estimation windows leading to poorer results. The forecasting performance of the forward-looking beta is better than that of the best-performing historical beta in about half the cases. In some cases an equally weighted average of historical and forward-looking beta outperforms either method, even when both methods have significant predictive power.

The empirical findings in this paper are related to some of the results in the growing literature on the pricing of equity options, and more specifically to results on the differential pricing of equity and index options. Dennis and Mayhew (2002) and Duan and Wei (2005) find that firms with high historical market betas have higher negatively skewed risk-neutral distributions. This paper demonstrates that this result is to be expected if historical betas have some predictive power for forward-looking betas, because forward-looking betas are higher if the underlying risk-neutral distribution is more negatively skewed. We also demonstrate that a stock's forward-looking beta is partly determined by the difference between the skew of the stock's risk-neutral distribution and the skew of the index's risk-neutral distribution. Bakshi, Kapadia and Madan (2003) document differences between the skew of stock and index options. Driessen, Maenhout and Vilkov (2005) use equity options to obtain correlations between stocks in a parametric setup.

The remainder of this paper is organized as follows. Section 2 discusses the derivation of forward-

[2] Most studies refer to these moments as model-free moments. We refer to them as forward-looking moments, in order to maintain consistency with the terminology "forward-looking beta".

looking betas using option data. Section 3 describes the data and elaborates on the empirical implementation. Section 4 presents a case study of our method, using the empirical results for three Dow Jones components. Section 5 summarizes the empirical results for the thirty components of the Dow Jones index, and Section 6 presents a number of robustness checks and provides more discussion of the results. Section 7 concludes.

# 2 Retrieving Forward-Looking Betas from Options Data

## 2.1 Historical Betas and Forward-Looking Betas

Factor betas capture the co-variation between the return on an individual security and the return on a factor. Accurate measurement of betas is critically important, because it is a basic requirement for the meaningful implementation of factor models, which have become popular with practitioners as well as academics. Factor models and betas are used by practitioners for a number of reasons. First, they provide a benchmark for performance measurement, because they indicate the return a portfolio manager ought to have made given the risk present in his portfolio.[3] Second, factor models provide a benchmark for the detection of abnormal returns, as is for instance done in event studies.[4] Third, factor models can be used to determine the required cost of capital.[5]

There is an ongoing discussion on the appropriate choice of factor model and the empirical performance of alternative factor models. This debate is relevant for this paper because we derive our results in the context of a particular factor model, the Capital Asset Pricing Model (CAPM). There is evidence in the literature that some multifactor models can improve on the performance of the CAPM, but this issue remains hotly debated.[6] We merely note that the academic literature remains divided about the performance of the CAPM, and that the CAPM is still the factor model most often used in financial practice. In our empirical application, the CAPM performs relatively well.

The computation of betas for the CAPM is an issue that is just as hotly debated as the choice of factor model. In several classic applications of the CAPM, betas were computed by running a regression of stock returns on market returns, using returns for the past sixty months.[7] This technique is still used in many academic and practitioner approaches to beta estimation. The choice of sixty lagged returns reflects the tension implicit in using historical returns to compute

[3] See Jensen (1968, 1969), Sharpe (1966) and Treynor (1966) for early applications to performance measurement. See Christopherson, Ferson and Glassman (1998), Farnsworth, Ferson, Jackson and Todd (2002) and Ferson and Schadt (1996) for more recent contributions. See also Lintner (1965) and Markowitz (1952).

[4] See MacKinlay (1997), chapter 2 in Campbell, Lo, MacKinlay (1997), and the references therein.

[5] See for instance Fama and French (2004) for a review of this literature.

[6] See Campbell, Lo and MacKinlay (1997), Cochrane (2001), Jagannathan and McGratten (1995), Fama (1991), Ferson (1995, 2004), and the references therein for an overview of this extensive debate. See also Ferson and Korajczyk (1995) and MacKinlay (1995).

[7] See for instance Black, Jensen and Scholes (1972), Fama and MacBeth (1973) and Fama and French (1992, 1993, 1996).

betas: on the one hand one would like to use as long a history as possible to obtain more precise estimates, but on the other hand one does not want to use older returns data because it is likely that the beta changes over time.

Over the last two decades, the modeling of the time-variation in the market beta has taken center stage in the CAPM literature. There is by now widespread consensus that beta is time-varying, although it is less clear how much this time-variation helps the model's empirical performance out-of-sample. A number of different econometric techniques are available to model the time-variation in the beta.[8]

While modeling the time-variation in the beta is helpful, it does not address the criticism that this type of measurement is backward-looking, much like the simple regressions that use sixty months of lagged returns. Many applications of factor models, such as the cost of capital, are inherently forward-looking. Ideally we want an estimate of the future beta. The computation of a historical beta, regardless of whether it is time-varying or not, is a second-best solution that we resort to because the computation of genuinely forward-looking betas is thought to be difficult or impossible. This paper takes a different approach: we argue that it is possible to compute forward-looking betas, using the forward-looking information embedded in option prices.

The idea of retrieving forward-looking betas from options data is not entirely new. French, Groth and Kolari (1983) combine forward-looking volatility with historical correlation to improve the measurement of betas. Siegel (1995) proposes the creation of a new derivative, called an exchange option. This option would allow the holder to either purchase the underlying stock at the strike price, or exchange the option for a certain amount of the index. Siegel (1995) then derives a formula for calculating the beta implicit in this contract using at-the-money prices for this exchange option, as well as prices for the corresponding stock and index option contracts. The "implicit" beta for stock $i$ is then given by

$$\beta_i = \frac{\sigma_i^2 + \sigma_m^2 - \sigma_x^2}{2\sigma_m^2}$$

where $\sigma_i$ is the implied volatility of the individual stock option contract, $\sigma_m$ is the implied volatility of the index option contract, and $\sigma_x$ is the implied volatility of the exchange option contract. Unfortunately, since exchange options are not yet traded, the implicit beta cannot be computed in practice. In this paper, we argue that betas similar in spirit to Siegel's (1995) "implicit" betas can be computed without the creation of a new derivative, by using prices on existing equity and index options. We will refer to these betas as "forward-looking" betas. Alternatively they could be labeled "option-implied" betas, "implied" betas, or "implicit" betas.

[8] On modeling time-variation in betas, see Bollerslev, Engle and Wooldridge (1988), Blume (1971, 1975), Bos and Newbold (1984), Cochrane (2001), Ferson (1995, 2004), Ferson and Harvey (1999), Ferson, Kandel and Stambaugh (1987), Ghysels (1998), Harvey (1989) and Jagannathan and Wang (1996). Companies such as BARRA provide investors and risk managers with time varying estimates of market betas.

## 2.2 The Model

Throughout the paper we will assume that the log-return on stock $i$ follows a single factor model of the form

$$R_i = R_f + \beta_i(R_m - R_f) + \varepsilon_i \tag{1}$$

with the market return given by

$$R_m = \mu_m + u \tag{2}$$

The idiosyncratic shock $\varepsilon_i$ has zero mean and is independent of the market return $R_m$.

## 2.3 Computing Forward-Looking Betas

### 2.3.1 Computing Betas from the Moments of Stock and Index Returns

Consider the moments of the return distribution. To simplify notation, we do not provide time subscripts for the moments, but empirically there will be a different estimate for the moments at each point in time. The variance of the return on stock $i$ is given by

$$VAR_i = \int_{-\infty}^{\infty} (R_i - \mu_i)^2 p[\varepsilon_i] d\varepsilon_i$$

Using the one-factor return model (1), we get

$$VAR_i = \beta_i^2 \int_{-\infty}^{\infty} (R_m - \mu_m)^2 p[R_m] dR_m + \int_{-\infty}^{\infty} \varepsilon_i^2 p[\varepsilon_i] d\varepsilon_i \tag{3}$$

which can be rewritten as

$$VAR_i = \beta_i^2 VAR_m + VAR_{\varepsilon,i} \tag{4}$$

Similarly we obtain for the risk-neutral return skewness

$$SKEW_i = \frac{\beta_i^3 \int_{-\infty}^{\infty} (R_m - \mu_m)^3 p[R_m] dR_m + \int_{-\infty}^{\infty} \varepsilon^3 p[\varepsilon_i] d\varepsilon_i}{\left[\beta_i^2 VAR_m + VAR_{\varepsilon,i}\right]^{3/2}} \tag{5}$$

Substituting in the moments from the one factor model and further rearranging yields

$$SKEW_i = \left(1 + \frac{VAR_{\varepsilon,i}}{\beta_i^2 VAR_m}\right)^{-3/2} SKEW_m + \left(1 + \frac{\beta_i^2 VAR_m}{VAR_{\varepsilon,i}}\right)^{-3/2} SKEW_{\varepsilon,i} \tag{6}$$

We now make the identifying assumption that the skewness of the idiosyncratic shock is zero, $SKEW_{\varepsilon,i} = 0$.[9] We then get the following expression for the skew of the return on stock $i$,

$$SKEW_i = \left(1 + \frac{VAR_{\varepsilon,i}}{\beta_i^2 VAR_m}\right)^{-3/2} SKEW_m \tag{7}$$

[9] We further discuss this assumption in Section 6.1.

which implies that the sign of the skewness must be the same in the market as in the individual stock.

Substituting out $VAR_{\varepsilon,i}$ from (7) using (4) we get

$$SKEW_i = \left(1 + \frac{VAR_i - \beta_i^2 VAR_m}{\beta_i^2 VAR_m}\right)^{-3/2} SKEW_m \tag{8}$$

We can now solve for the market beta of stock $i$

$$\beta_i^{MM} = \left(\frac{SKEW_i}{SKEW_m}\right)^{1/3} \left(\frac{VAR_i}{VAR_m}\right)^{1/2} \tag{9}$$

where the $MM$ superscript indicates that the market beta is computed using the moments of the return distribution. Note that expression (9) indicates that the skewness of the market return has to be non-zero for the market beta to be well-defined in this setup.

At this point (9) is simply a method to compute a market beta. The moments can be computed using either historical data or forward-looking option information. In the next subsection we explain how to use option prices to compute forward-looking moments, which will in turn allow us to compute forward-looking betas.

### 2.3.2 Computing the Moments of Stock Returns from Option Prices

Expression (9) allows us to compute betas from the moments of the marginal distributions of the stock and index returns. In this subsection we explain how to compute these moments using option prices. We employ the methods of Carr and Madan (2001).[10] A brief overview of their results is presented in Appendix A. The key result is that any twice differentiable payoff function can be spanned by a position in bonds, stocks and out-of-the-money options.

Let $q$ denote the probability distribution function under the risk-neutral measure. The skewness under the risk neutral measure is defined as

$$SKEW \equiv \frac{E^q\left[(R - E^q[R])^3\right]}{\left(E^q\left[(R - E^q[R])^2\right]\right)^{3/2}}. \tag{10}$$

Using the uncentered moments we can rewrite skewness as

$$SKEW = \frac{E^q\left[R^3\right] - 3E^q[R]\,E^q\left[R^2\right] + 2E^q[R]^3}{\left(E^q\left[R^2\right] - E^q[R]^2\right)^{3/2}} \tag{11}$$

Following Bakshi, Kapadia and Madan (2003), we define the "Quad" and "Cubic" contracts as having a payoff function equal to the squared return and cubed return respectively, for a given horizon $\tau$. The fair values of these contracts are

[10] See also Bakshi and Madan (2000), Bakshi, Kapadia and Madan (2003), Britten-Jones and Neuberger (2000), Derman and Kani (1998) and Rubinstein (1994).

$$Quad = e^{-r\tau} E^q \left[R^2\right]$$
$$Cubic = e^{-r\tau} E^q \left[R^3\right]$$

Substituting these expressions into the skewness formula (11), we get

$$SKEW = \frac{e^{r\tau} Cubic - 3E^q [R] \, e^{r\tau} Quad + 2E^q [R]^3}{\left(e^{r\tau} Quad - E^q [R]^2\right)^{3/2}} \tag{12}$$

Bakshi, Kapadia and Madan (2003) show that under any martingale pricing measure, the "Quad" and "Cubic" contract prices can be recovered from the market prices on portfolios of out-of-the-money European calls $C(\tau, K)$ and puts $P(\tau, K)$, where $K$ denotes the strike price and $\tau$ denotes the time to maturity.

The price of the "Quad" contract is

$$Quad = \int_S^\infty \frac{2\left(1 - \ln\left[\frac{K}{S}\right]\right)}{K^2} C(\tau, K) dK + \int_0^S \frac{2\left(1 + \ln\left[\frac{S}{K}\right]\right)}{K^2} P(\tau, K) dK. \tag{13}$$

The price of this contract can be interpreted as the forward price of volatility.[11] The price of the *Cubic* contract is

$$Cubic = \int_S^\infty \frac{6 \ln\left(\frac{K}{S}\right) - 3 \ln\left(\frac{K}{S}\right)^2}{K^2} C(\tau, K) dK - \int_0^S \frac{6 \ln\left(\frac{S}{K}\right) + 3 \ln\left(\frac{S}{K}\right)^2}{K^2} P(\tau, K) dK. \tag{14}$$

Appendix B demonstrates that the risk-neutral mean is given by

$$E^q [R] = e^{r\tau} - 1 - \frac{e^{r\tau}}{2} Quad - \frac{e^{r\tau}}{6} Cubic. \tag{15}$$

The variance is given from the *Quad* contract by

$$VAR = e^{r\tau} Quad - E^q [R]^2 . \tag{16}$$

Because the *Quad* and *Cubic* contracts are computed using option prices, the resulting $VAR$ in (16) and $SKEW$ in (12) are forward looking moments, $VAR^{FL}$ and $SKEW^{FL}$. We now have all the ingredients needed to compute the forward-looking beta of stock $i$ using (9) as well as option data on individual stocks and the index. We get

[11] Volatilities derived in this fashion have been studied among others by Britten-Jones and Neuberger (2000), Carr and Madan (2001), Carr and Wu (2003) and Jiang and Tian (2005). The terminology in these papers is "model-free" moments. We use "forward-looking" moments, in order to maintain consistency with the terminology "forward-looking beta". Alternatively, one could use at-the-money implied Black-Scholes volatility as an estimate of volatility. For our sample, the correlations between Black-Scholes implied volatility and forward-looking volatility are between 0.866 and 0.957.

$$\beta_i^{FL} = \left(\frac{SKEW_i^{FL}}{SKEW_m^{FL}}\right)^{1/3} \left(\frac{VAR_i^{FL}}{VAR_m^{FL}}\right)^{1/2} \tag{17}$$

Note that we can think of beta as the ratio of the standard deviation of stock $i$ divided by the standard deviation of the market, multiplied by the correlation between stock $i$ and the market. Therefore, in (17), the term $\left(\frac{SKEW_i^{FL}}{SKEW_m^{FL}}\right)^{1/3}$ captures the forward-looking correlation. French, Groth and Kolari (1983) construct betas that combine forward-looking volatilities with historical correlations in an effort to improve beta measurement. Our forward-looking beta can the thought of as a logical extension of their idea.

The moments computed from options data are risk-neutral moments. The question arises how a risk-neutral beta computed from these moments is related to the physical beta. In general these two betas can differ, but in Appendix C we demonstrate that in an important case, the Merton continuous-time CAPM, the two betas are identical. The intuition for this result is that the risk-neutral distribution is given by a mean shift of the physical distribution, leaving the covariance matrix unchanged.

At a more fundamental level, the question also arises whether risk-neutral forecasts are relevant in the real world? This question also arises in the literature on volatility forecasting. We refer the reader to the references above and specifically to Britten-Jones and Neuberger (2000) for further discussion. We note that even though the implied volatility from options is a risk-neutral volatility measure, we continue to use implied volatility for volatility forecasting, because its empirical performance is relatively good. The same is true of the methods proposed in this paper: the (unobserved) differences between the risk-neutral and the physical beta will translate into (measurable) forecasting errors, and the size of these forecasting errors will determine the method's usefulness.

# 3 Data and Implementation Procedure

## 3.1 Data

We obtain option data from OptionMetrics. OptionMetrics' Ivy DB database is a comprehensive source of high-quality historical data for the US equity and index options markets. We extract the security ID, date, expiration date, call or put identifier, strike price, best bid, best offer, and implied volatility from the option price file. For European options, implied volatilities are calculated using mid-quotes and the Black-Scholes formula. For American options, a binomial tree approach that takes into account the early exercise premium is employed. In our empirical analysis, we focus on the quotes of the components of the Dow Jones 30 (using its composition as of April 8, 2004) and the S&P500 index for the period January 1, 1996 to December 31, 2003. Table 1 contains the ticker symbols and company names for the options data. The equity options for the Dow Jones components are traded on the Chicago Board Options Exchange (CBOE) and are American options.

Interest rates are taken from the CRSP Zero Curve file and underlying security prices are taken from the Securities Price file in the OptionMetrics database. As in Bakshi, Cao and Chen (1997), Bakshi, Kapadia and Madan (2003) and Jiang and Tian (2005), we use the average of the bid and ask quotes for each option contract, and we filter out average quotes that are less than \$3/8. We also filter out quotes that do not satisfy standard no-arbitrage conditions. Finally, we eliminate in-the-money options because they are less liquid than out-of-the-money and at-the-money options. We eliminate put options with strike prices of more than 103% of the underlying asset price ($K/S > 1.03$), as well as call options with strike prices of less than 97% of the underlying asset price ($K/S < 0.97$). Table 2 presents descriptive statistics by maturity for the options used in the computations.

### 3.2 Implementing the Estimation of Moments and Betas

Moments are computed by integrating over moneyness. In practice, we do not have a continuum of option prices across moneyness, and we therefore have to make a number of choices regarding implementation. We follow Carr and Wu (2004) and Jiang and Tian (2005) in imposing structure on implied volatilities. Our implementation is specifically designed to improve the quality of the integration procedure. First, as mentioned above, we limit our attention to options on the thirty Dow Jones components to maximize the availability of strike prices and the size of the integration domain. Second, we only estimate the moments for days that have at least two out-of-the money call prices and two out-of-the money put prices available. Third, as in Carr and Wu (2004) and Jiang and Tian (2005), for each maturity we interpolate implied volatilities using a cubic spline across moneyness levels ($K/S$) to obtain a continuum of implied volatilities. The cubic spline is only effective for interpolating between the maximum and minimum available strike price. For moneyness levels below (above) the available moneyness level in the market, we use the implied volatility of the lowest (highest) available strike price.

After implementing this interpolation-extrapolation technique we are able to extract a fine grid of 1000 implied volatilities for moneyness levels between 0.01% and 300%. We then convert these implied volatilities into call and put prices using the following rule: moneyness levels smaller than 100% ($K/S < 1$) are used to generate put prices and moneyness levels larger than 100% ($K/S > 1$) are used to generate call prices. This fine grid of option prices is then used to compute the forward-looking moments by approximating the $Volatility$, *Cubic* and *Quartic* contracts using trapezoidal numerical integration. It is important to note that this procedure does not assume that the Black-Scholes model correctly prices options. It merely provides a translation between option prices and implied volatilities. Also note that our implementation follows Jiang and Tian (2005) and is slightly different from the one in Bakshi, Kapadia and Madan (2003) and Dennis and Mayhew (2002). We document and discuss the benefits of our approach in more detail with the help of a Monte Carlo experiment in Section 6.

We can in principle compute several forward-looking betas for every underlying asset, one for each available option maturity. For each day, we linearly interpolate using the two contracts nearest to the 180-day maturity to get the 180-day VAR and SKEW contracts, always using one contract with maturity longer than 180 days and one contract with maturity less than 180 days. Using these contracts, we obtain estimates of the 180-day forward-looking beta $\beta^{FL}$ using equation (17) for each day $t$.

In implementing historical betas, a number of choices have to be made. For an overview see for instance Damodaran (1999). For our purpose two important choices are the length of the estimation period and the return interval. With respect to the choice of return interval, we are constrained by the fact that we do not have an extensive time series of option data available. To construct a powerful test of the forward-looking beta, we are therefore forced to use daily data in our empirical analysis. Using weekly or monthly returns to compute betas is not a logical choice when one uses a daily data frequency, and thus we estimate historical betas using daily returns. With respect to the length of the estimation period, we implement historical beta in a number of different ways. Because we use 180-day forward-looking beta $\beta^{FL}$, the use of 180 daily returns to compute historical beta is an obvious alternative. On the other hand, many academic studies of the CAPM have followed the approach of Black, Jensen and Scholes (1972) and Fama and MacBeth (1973) and computed historical betas using returns for the past sixty months. Industry providers of betas typically use estimation periods between 2 and 5 years (see Damodaran (1999, p. 88) and Bodie, Kane and Marcus (2007)). In order to investigate the robustness with respect to the estimation window, we therefore investigate historical returns estimated using either five years, one year, or a half year (180 days) of daily returns.[12]

# 4 Three Case Studies

Figure 1 gives a first indication of our empirical results. It is reassuring that our measure of S&P forward-looking volatility in Panel B is very highly correlated with the VIX in Panel A.[13] The average of the forward-looking volatility for the thirty Dow Jones stocks in Panel C is also highly correlated with the VIX. From Panels D and E we can draw some conclusions regarding skewness. First, the S&P500 forward-looking skewness exceeds the average forward-looking skewness of the thirty Dow Jones firms. This finding is consistent with the results in Bakshi, Kapadia and Madan (2003). Note that here and in the rest of the paper, we refer to more negative skewness as higher skewness. Second, the correlation between forward-looking skewness and forward-looking

[12] The use of daily betas has some disadvantages, see for instance Scholes and Williams (1977). However, in the event study literature daily data and short estimation windows have been used extensively. See MacKinlay (1997) for a review. Several recent cross-sectional studies have also started using daily betas, typically constructed using estimation windows shorter than five years. See for example Ang, Chen and Xing (2006) and Barberis, Shleifer and Wurgler (2005).

[13] While the forward-looking beta is expressed in terms of variances, we present volatilities in our empirical results because they are easier to interpret.

volatility for the S&P500 is -0.102, and the correlation between the average forward-looking skewness and average forward-looking volatility for the Dow components is -0.396. Finally, while the forward-looking volatility for the S&P500 is very highly correlated with the average forward-looking volatility for the Dow components (0.778), the correlation between the two skewness time series is only 0.156.

We cannot discuss the results for all thirty Dow Jones components in detail because of space constraints, but we provide a detailed discussion of the empirical results for three of the components: Walt Disney Corporation, Exxon Mobil Corporation and Verizon Communications. In the subsequent section, we discuss results for all thirty stocks and we provide more details about the analysis.

Figure 2 plots the forward-looking skewness and volatility for these three companies. Comparing with Figure 1, it is clear that the volatility for all three companies is positively correlated with S&P500 volatility. In fact, the correlations are 0.71, 0.75 and 0.44 respectively. Skewness patterns are more complex and harder to relate to S&P500 skewness. Verizon skewness substantially increases in the second half of 1998, at a time when S&P500 skewness is also large, but in fact the overall correlation between the two series is -0.21. Disney skewness increases at the end of 1997 when S&P500 skewness increases, but the overall correlation is -0.14. It is difficult to visually identify covariation patterns between Exxon and S&P500 skewness; the overall correlation is 0.07.

Figure 3 presents the time path of betas for Disney. We compare the forward-looking beta with our proxy for the ex post "realized" beta, which is computed as the covariance divided by variance for ex-post daily returns over the 180-day period, depicted in Panel A of Figure 3. We discuss realized beta in more detail below. There is substantial variation in the realized betas, and both the forward-looking beta and the 180-day historical beta do an excellent job at capturing this variation. The 180-day historical beta does slightly better overall. The 5-year historical beta is much too smooth to capture the variation in the 180-day realized beta.

Figure 4 presents a similar analysis for Exxon. Once again the 5-year historical beta does not perform well. The 180-day historical beta substantially outperforms the forward-looking beta. However, closer inspection of the figures indicates that this result is largely due to the patterns in realized beta during 2000 and 2001, when the realized beta becomes very small and even negative. A negative beta is widely regarded as unrealistic, and therefore indicative of a problem with our proxy for the ex post realized beta. In our opinion, our proxy is the best available one despite these shortcomings. For the moment, we note that it is important to look beyond the simple correlation between the realized beta and the forecasts. In the case of Exxon, the 180-day historical beta substantially outperforms the forward-looking beta when judged by correlation, but that is only because the 180-day historical beta is equivalent to a lagged realized beta, and therefore ends up being unrealistically small or negative in the period 2000-2001. The time path for the forward-looking betas is much more plausible.

Figure 5 presents our findings for Verizon Communications. In this case the forward-looking beta substantially outperforms both historical betas. Note how the forward-looking beta nicely captures the increase in the realized beta towards the end of the sample, even though it overshoots in 2002.

The results from the three case studies encourage a full-scale investigation using the 30 Dow Jones components. However, they also illustrate that the realized beta computed over a relatively short period such as 180 days can produce questionable estimates of the true but unknown beta. This caveat is important when interpreting the results below.

# 5 Empirical Results

## 5.1 Estimates of Moments and Betas

We now present empirical results for all thirty Dow Jones components. Figures 6 and 7 present time plots for the forward-looking volatility and skewness. The time averages of the moment estimates are given in Table 3. Earlier, Figure 1 indicated that the skewness of the S&P500 exceeded the average of the skewness of the Dow Jones components. Table 3 indicates that the time average of the S&P500 skew (-1.417) is in fact larger than the time average of the most negatively skewed stock (GE), which has an average skewness of -0.590. Figure 7 also indicates that the individual stocks are positively skewed on some days, although not very often.[14] These results confirm the finding of Bakshi, Kapadia and Madan (2003) and Dennis and Mayhew (2002), who study the 1991-1995 period.

Table 3 also reports the average forward-looking beta (17) for each of the stocks and compares it with the average historical betas. For each day, we compute historical betas using either five years worth of daily data, or alternatively one year or half a year (180 days). Figure 8 presents time plots of the forward-looking and historical betas for each of the thirty stocks and compares this with the time path of the realized betas. We limit ourselves to 180-day historical betas to avoid clutter. Clearly the estimates of forward-looking betas are very reasonable in all cases, and in fact they are similar to the historical betas. Figure 9 graphically compares the average forward-looking beta with the average historical beta. Each "x" marks the mean forward-looking beta and mean historical beta for one of the Dow Jones components. The dashed line is the 45 degree line, and the solid line represents the regression line of historical beta on forward-looking beta. Interestingly, the solid line has a slope larger than one. On average, forward-looking betas are more centered around one than are the historical betas. However, note that raw beta estimates are adjusted in many applications of historical beta. See for instance Damodaran (1999, chapter 4), Bodie, Kane and Marcus (2007, p.284), and the references therein for more on the motivation for these types of adjustments. One popular adjustment is to compute (2/3)estimated beta + (1/3). If we apply this adjustment to the historical betas in Figure 9, the resulting historical and forward-looking betas are even more strongly related, and the regression line is very close to the 45 degree line.

[14] These days are excluded when calculating forward-looking betas.

## 5.2 The Forecasting Performance of the Beta Estimates

To evaluate the relative informational content of the forward-looking and historical betas, we construct a standard out-of-sample forecasting test. A typical forecasting regression regresses the ex-post observed variable to be forecast on the forecasting variables. In our case, the ex-post variable is not directly observed, but this is the case for many applications, such as for example volatility forecasting. We measure the ex-post beta using the realized betas proposed by Andersen et al. (2006). While Andersen et al. use high-frequency data, we compute the average of the ratio of the squared covariance and the squared market return variance, using daily returns data. This implementation is inspired by Schwert's (1987) construction of a measure of realized volatility.

Table 4 contains correlations between the ex-post realized beta and five predictors: the forward-looking beta, the three historical betas, and a mixed beta, which is the simple average of the forward-looking beta and the 180-day historical beta.[15] The right-most column is the average realized beta. We present three different sets of results. In Table 4A we forecast 180-day realized beta. This is a logical choice as a proxy for ex-post beta, because the forward-looking beta is computed using options with a maturity of 180 days. To avoid some of the problems with the 180-day realized beta, as illustrated for instance by the Exxon case in Figure 4, we also investigate realized betas constructed over longer horizons as proxies for ex-post beta in Tables 4B and 4C. Table 4B uses 365 days, and Table 4C uses 730 days.

First consider Table 4A. For each stock, the predictor with the highest correlation is indicated in bold. The forward-looking beta outperforms the other methods ten times, and the same is true for the 180-day historical beta. The mixed beta is the best performer in seven cases, and the 1-year historical beta in three cases. The 5-year historical beta is always outperformed by at least one other method, and it often ranks last. The 180-day historical beta clearly dominates the two other historical methods.

When does the forward-looking beta perform poorly? While we do not have an all-encompassing explanation, it is noteworthy that the forward-looking beta does poorly in a few cases where the average realized beta is either very low or very high. For example, the average realized beta estimates for Johnson and Johnson (JNJ) and Altria (MO), and Procter and Gamble (PG) are 0.676, 0.544 and 0.612 respectively, and the forward-looking betas are negatively correlated with the realized betas in these cases. Table 4A indicates that another case in which the forward-looking method underperforms is Citigroup (C), with an average historical beta of 1.431. Remember from Figure 9 that the forward-looking beta was more centered around one than the historical beta. Perhaps the forward-looking technique has difficulty estimating very large and very small betas. Alternatively, the realized beta estimate may be poor for these stocks.

While there is no obvious pattern among the cases for which the forward-looking betas perform

[15] An investigation of the optimal weights on historical and option-implied betas from a forecasting perspective is beyond the scope of this study. In the volatility forecasting literature, a 50-50 mix between historical and implied volatility often improves on methods with more sophisticated weights (see for example Poon and Granger (2003)).

very well, it is interesting that the method often performs well for technology companies, such as Intel (INTC), IBM, Microsoft (MSFT) and Hewlett-Packard (HPQ). Whether this is due to distinctive patterns in the return distribution and/or whether there is a more structural interpretation, such as more rapid change in these companies, is an interesting topic for further research.

We now turn to the results in Tables 4B and 4C, which are obtained using realized betas constructed with longer horizons. Somewhat unexpectedly, the performance of the forward-looking beta compared to that of the 180-day historical beta is much better in Table 4B than in Table 4A, and this conclusion carries over to Table 4C. The mixed beta also perform well. It is perhaps not surprising that the performance of the 180-day historical beta in Tables 4B and 4C is poorer than in Table 4A, because the horizons used in the construction of realized betas are no longer equal to 180 days. What is harder to explain is why the correlation between realized beta and forward-looking beta is in many cases higher in Tables 4B and 4C than in Table 4A. Finally, it is also interesting that the 1-year and 5-year historical betas do not perform well in Tables 4B and 4C. In summary, in Table 4B either the forward-looking beta or the mixed beta is the best performer in nineteen out of thirty cases. In Table 4C this the case twenty-two times out of thirty.

In Tables 5-8 we investigate univariate as well as multivariate forecasting regressions of realized beta on predicted beta. The univariate forecasting regressions are given by

$$\beta_{t,t+180}^{REAL} = \theta_1 + \theta_2 \beta_t^{HIST} + u_t \tag{18}$$

$$\beta_{t,t+180}^{REAL} = \theta_1 + \theta_2 \beta_t^{FL} + u_t \tag{19}$$

where $\beta_t^{HIST}$ denotes the historical beta. The multivariate forecasting regression is

$$\beta_{t,t+180}^{REAL} = \theta_1 + \theta_2 \beta_t^{FL} + \theta_3 \beta_t^{HIST} + u_t \tag{20}$$

Because of space constraints, the more detailed reporting on the univariate and multivariate forecasting regressions in Tables 5-8 only reports on the best-performing historical beta, which is the 180-day historical beta. Also because of space constraints, we limit ourselves to results for forecasting 180-day realized beta.

Tables 5 and 6 report results for the univariate forecasting regressions. For an ideal forecast the intercept in (18) and (19) is zero, and the slope is one. The R-square is the square of the corresponding correlation reported in Table 4A. While the R-squares indicate respectable explanatory power in many cases, and the slope coefficients are usually significantly estimated, the constant is quite large in most cases.

The right-most columns in Tables 5-6 report the root mean squared errors (RMSEs) for the forward-looking and historical betas, respectively. Note that the forward-looking betas perform relatively better when using the R-squared (and thus correlation) metric than when using the RMSE metric. The explanation for this finding is that while the forward-looking betas contain substantial predictive information, they are relatively more biased than the historical betas. In the forecasting regression, this bias is captured by allowing the constant to be nonzero and the slope to be different from one, but the bias shows up in the RMSE.

It is perhaps even more interesting to consider the multivariate forecasting results in Table 7. The slope coefficients indicate that in most cases, the forward-looking beta carries information in addition to the historical beta and vice versa. This result is consistent with the strong performance of the mixed beta in Table 4. It is also noteworthy that the mixed beta in Table 4 often substantially outperforms the historical and forward-looking betas in cases where the explanatory power of both measures is high. This suggests that when both betas are good forecasts in isolation, they contain different information.

Table 8 provides additional evidence on the incremental information in historical and forward-looking forecasts. We report results from regressing the residuals of the univariate regressions on the other predictor. In most cases, the slope coefficient in these regressions is highly significant, confirming that the forward-looking beta carries information in addition to the historical beta and vice versa.

### 5.3 The Cross-Sectional Performance of the Beta Estimates

The most important implications of factor models are cross-sectional. Our sample is far from ideal to study the cross-section, and we therefore limit the scope of our cross-sectional analysis. We regress average excess returns on average beta in the spirit of Black, Jensen and Scholes (1972). Figure 9 presents the empirical results. For forward-looking beta the $R^2$ is 0.22 and for historical beta the $R^2$ is 0.28.

It is not straightforward to relate our results to the available literature for a number of reasons. First, we use a short sample because of the limited availability of option data. Second, we use individual stock returns rather than portfolios.[16] Third, we use daily returns while most of the literature uses monthly returns. Fourth, we investigate a relatively small cross-sectional sample. Fifth, our sample is limited to large cap stocks and therefore the cross-sectional application has limited power to test a given model. For the purpose of this study, we merely conclude that the forward-looking beta has significant explanatory power, even though the historical beta explains a somewhat larger fraction of the cross-section of asset returns.

## 6 Discussion

In this section, we discuss three important aspects of our forward-looking beta calculations, namely the assumption of a zero idiosyncratic skew, the forecasting power of forward-looking moments on their own, and the accuracy of the forward-looking moment calculations from options.

[16] It is well-known that assessing the performance of the CAPM using individual stocks is subject to testing problems related to measurement error in the betas. See Black, Jensen and Scholes (1972) for an early discussion of this problem. We limit ourselves to thirty stocks because we want to use liquid option contracts.

## 6.1 The Idiosyncratic Skew

Recall that in order to derive beta to be

$$\beta_i^{MM} = \left(\frac{SKEW_i}{SKEW_m}\right)^{1/3}\left(\frac{VAR_i}{VAR_m}\right)^{1/2} \tag{21}$$

we had to assume that the idiosyncratic return component is symmetric. That is, we assumed

$$\begin{aligned} SKEW_i &= \left(1+\frac{VAR_{\varepsilon,i}}{\beta_i^2 VAR_m}\right)^{-3/2} SKEW_m + \left(1+\frac{\beta_i^2 VAR_m}{VAR_{\varepsilon,i}}\right)^{-3/2} SKEW_{\varepsilon,i} &(22)\\ &= \left(1+\frac{VAR_{\varepsilon,i}}{\beta_i^2 VAR_m}\right)^{-3/2} SKEW_m &(23)\end{aligned}$$

We would like to know the bias in $\beta_i^{MM}$ arising from the assumption that $SKEW_{\varepsilon,i} = 0$, if in fact it is not. To this end, consider a a first-order Taylor approximation of the true $\beta_i$ around $\beta_i^{MM}$, where $SKEW_{\varepsilon,i} = 0$ at $\beta_i^{MM}$. We have

$$\beta_i \approx \beta_i^{MM} + \left.\frac{\partial \beta_i}{\partial SKEW_{\varepsilon,i}}\right|_{SKEW_{\varepsilon,i}=0} (SKEW_{\varepsilon,i} - 0)$$

The derivative $\frac{\partial \beta_i}{\partial SKEW_{\varepsilon,i}}$ can be obtained from applying the implicit function theorem to the definition of $SKEW_i$. Define

$$\begin{aligned} F(\beta_i, SKEW_{\varepsilon,i}) &= \left(1+\frac{VAR_{\varepsilon,i}}{\beta_i^2 VAR_m}\right)^{-3/2} SKEW_m + \left(1+\frac{\beta_i^2 VAR_m}{VAR_{\varepsilon,i}}\right)^{-3/2} SKEW_{\varepsilon,i} - SKEW_i \\ &= \left(\frac{VAR_i}{VAR_m}\right)^{-3/2} \beta_i^3 SKEW_m + \left(\frac{VAR_i}{VAR_i - \beta_i^2 VAR_m}\right)^{-3/2} SKEW_{\varepsilon,i} - SKEW_i \end{aligned}$$

where we have used $VAR_{\varepsilon,i} = VAR_i - \beta_i^2 VAR_m$.

Now

$$\begin{aligned} \left.\frac{\partial \beta_i}{\partial SKEW_{\varepsilon,i}}\right|_{SKEW_{\varepsilon,i}=0} &= -\left.\frac{F_2'(\beta_i, SKEW_{\varepsilon,i})}{F_1'(\beta_i, SKEW_{\varepsilon,i})}\right|_{SKEW_{\varepsilon,i}=0} \\ &= \frac{-\left(\frac{VAR_i}{VAR_i - \beta_i^2 VAR_m}\right)^{-3/2}}{3\left(\frac{VAR_i}{VAR_m}\right)^{-3/2} \beta_i^2 SKEW_m} \\ &= -\frac{1}{3\beta_i^2}\left(\frac{VAR_{\varepsilon,i}}{VAR_m}\right)^{3/2}\left(\frac{1}{SKEW_m}\right) \end{aligned}$$

So that when using the Taylor approximation, the bias in $\beta_i^{MM}$ will be

$$\beta_i^{MM} - \beta_i \approx \frac{1}{3\beta_i^2}\left(\frac{VAR_{\varepsilon,i}}{VAR_m}\right)^{3/2}\left(\frac{SKEW_{\varepsilon,i}}{SKEW_m}\right)$$

Notice that the first two terms are always positive. Assuming that $SKEW_m$ is negative, which is the case empirically, then $\beta_i^{MM}$ will be biased downward if $SKEW_{\varepsilon,i}$ is positive and vice versa.

Notice also that the bias will be the largest when $\beta_i$ is close to zero, and when $SKEW_m$ is close to zero.

It is difficult to verify the appropriateness of this assumption using options data. We note that if there is a systematic asymmetry pattern across the idiosyncratic distribution of firms, we would expect the estimated forward-looking betas to be systematically higher or lower than the historical betas. This is not the case. However, we did observe that forward-looking betas were more centered around one than historical betas. This finding may be due to the assumption of a symmetric idiosyncratic return component, if the idiosyncratic skew is negative for low beta stocks and positive for high beta stocks.

### 6.2 The Forecasting Performance of Forward-Looking Moments

Our forecasting analysis has thus far been focused on the performance of the forward-looking beta (17). A natural question that arises is to what extent the functional form of the forward-looking beta impacts on this forecasting performance. In other words, if forward-looking moments ought to be important for forecasting beta, to what extent can we forecast betas better by using forward-looking moments in a more flexible way? Table 9 addresses this question by reporting forecasting regressions where the forward-looking moments are used as regressors. We report on forecasting regressions that use various permutations of the forward-looking moments. In all cases coefficients of multiple correlation are reported.[17] In order to investigate the additional explanatory power of the historical beta, we also report on a forecasting regression that includes all regressors used in the table as well as the 180-day historical beta.

The results in Tables 9 are of substantial interest and allow for several conclusions. First, when using all four forward-looking moments in forecasting (in the next to last column in Table 9), the resulting coefficient of multiple correlation is often much higher than the correlation coefficient associated with forward-looking beta (in the first column of Table 9).[18] Second, the relative importance of the forward-looking moments, as judged by the correlation coefficients for univariate regressions, strongly differs between companies. Third, in the large majority of cases, the forecasting performance of $VAR_i$ and $SKEW_i$ combined (in column 7) is better than that of $VAR_m$ and $SKEW_m$ combined (in column 6). Fourth, even in cases where the coefficient of multiple correlation using all four moments is very high (in column 8), adding the historical beta as a regressor improves forecasting performance. Finally, we also investigated alternative forecasting horizons (not reported). Consistent with the results in Tables 4B and 4C, using 365-day and 730-day realized betas leads to substantially higher coefficients of multiple correlation.

[17] The conclusions do not change when using the square roots of the adjusted R-squares.

[18] Bakshi, Kapadia and Madan (2003, p.114) derive an expression for co-skewness as a function of the stock's market beta as well as moments of the distribution of the stock return and the market return. One possible interpretation of these results is therefore that co-skewness is relevant for forecasting realized beta.

### 6.3 The Accuracy of the Moment Computations

As we do not have a continuum of option prices available to compute the moments, it is inevitable that certain biases will be induced in the estimation. We conducted a number of Monte Carlo experiments to determine the importance of these biases on the estimation of volatility and skewness, and we used the results of these Monte-Carlo experiments to guide our empirical implementation. There are four types of biases that are of particular concern in the estimation of volatility and skewness: discretization of strike prices, truncation of the integration domain, asymmetry of the integration domain and the position of the underlying stock price in the strike price interval. Jiang and Tian (2005) examine the effects of discretization and truncation on the computation of forward-looking volatility. We closely follow their setup and extend their analysis to the computation of forward-looking skewness. We also include an analysis of asymmetry and the position of the underlying stock in the strike price interval, which are biases that mainly affect the calculation of skewness.

To examine the size of the approximation errors induced by these biases, we generate option prices using Heston's (1993) stochastic volatility model (HSV) with standard parameterization based on the empirical results of Bakshi, Cao and Chen (1997): $\theta = 0.04$, $\kappa = 2$, $\sigma_v = 0.225$ and $\rho = -0.5$. We set the initial instantaneous variance ($V$) to be equal to the long-run variance $V = \theta = 0.04$, the initial stock price equal to $S_0 = 100$, and the risk-free rate equal to $r_f = 0.05$. Because we are interested in a 180-day beta, we set the time to maturity equal to 180 days, $T = 180/365$. To measure the variance and the skewness of the distribution implied by the HSV parameterization we use the following approach. To ensure that we do not encounter negative stock prices or negative variances we use Ito's Lemma to convert the risk-neutral stochastic volatility dynamics in HSV to $d\ln S$ and $d\ln V$. Once we discretize the equations we get the following risk neutral dynamics

$$\ln(S_{t+\Delta t}) = \ln(S_t) + \left(r - \frac{1}{2}V_t\right)\Delta t + \sqrt{V_t}\sqrt{\Delta t}\varepsilon^S_{t+\Delta t} \tag{24}$$

$$\ln(V_{t+\Delta t}) = \ln(V_t) + \frac{1}{V_t}\left((\kappa+\lambda)\left(\frac{\kappa\theta}{\kappa+\lambda} - V\right) - \frac{1}{2}\sigma_V^2\right)\Delta t + \frac{1}{\sqrt{V_t}}\sigma\sqrt{\Delta t}\varepsilon^V_{t+\Delta t}$$

$$\varepsilon^V_{t+\Delta t} = \rho\varepsilon^S_{t+\Delta t} + \sqrt{1-\rho^2}\varepsilon_{t+\Delta t}$$

where $\Delta t = 1/252$ (assuming that there are 124 trading days in 180 calendar days). By iterating through these equations 124 times, we can generate a 180-day stock price path. We repeat this exercise $250,000$ times, calculate the log returns $\ln\left(\frac{S_T}{S_0}\right)$ of each path and compute volatility, which is equal to 0.2037, and skewness, which is equal to $-0.4610$. To verify the accuracy of this simulation-based approach, we compare the call price with strike $K = 100$ obtained using simulation, $C_{SIM}$, with the closed form solution $C_{HEST}$. We find $C_{HEST} = 6.5905$ and $C_{SIM} = 6.5780$.

We now have benchmarks to evaluate the accuracy of our estimation procedure. The first bias we investigate results from the discreteness of the strike prices. To compute the moments arbitrarily precisely, we need a continuum of option prices from 0 to plus infinity, while in reality we only have prices at fixed strike price levels. In the first part of this experiment we generate HSV call and put prices with different discrete strike price intervals $d$ ranging from $d = 0.1$ to $d = 5$ using the integration domain $[Su^{-1}, Su]$ with $u = 2$. The percentage approximation errors for the volatility and skewness estimates are plotted in Figure 11 for both simple trapezoidal integration using only observed prices (dotted line) and for the interpolation-extrapolation technique of Jiang and Tian (2005) described in Section 3 (solid line). We conclude that when using the interpolation-extrapolation technique of Jiang and Tian, the bias is negligible for realistic discreteness.

We next investigate the bias resulting from the truncation of the integration domain. We keep the strike price interval constant at $d = 0.1$ and we vary the width of the integration domain $[Su^{-1}, Su]$ by changing $u$ between 1.1 and 2. Figure 12 plots the percentage approximation errors in the volatility and skewness estimation for simple trapezoidal integration using only observed prices (dotted line) and for the interpolation-extrapolation technique (solid line). We see that it is difficult to estimate skewness accurately when the width of the integration domain is small. Therefore, as mentioned above, we choose a sample of stocks with liquid option data.

The other biases that we investigate are induced by the asymmetry of the integration domain and the place of the underlying stock price in the strike price interval. These biases have not been investigated in previous studies, perhaps because they have little effect on the measurement of volatility, but they are important for the measurement of skewness because skewness in effect compares the two sides of the distribution.

If the integration domain is asymmetric, there is more information about one side of the log return distribution than about the other. To illustrate the bias resulting from this, we generate HSV call and put prices with a constant strike price interval $d = 0.1$ and we vary the integration domain from $[Su_L^{-1}, Su_H]$ where $u = 1.6$, $u_L = u - du$, $u_H = u + du$ and we vary $du$ between $-0.3$ and 0.3. Figure 13 plots the percentage approximation errors in the volatility and skewness estimation for simple trapezoidal integration using only observed prices (dotted line) and for the interpolation-extrapolation technique (solid line). For the interpolation-extrapolation technique used the errors are very small for the relevant strike prices.

The bias resulting from the place of the underlying stock price in the strike price interval is best explained through an example. If the current index value is 892 and the strike price interval is \$5 ($d = 5$) then the first OTM put will have a strike of 890 and the first OTM call will have a strike of 895. Simply due to the fact that the largest mass of the function is in the middle of the integration domain, the estimate will be negatively biased in this case. For this last experiment, we generate HSV call and put prices with a constant discrete strike price interval $d = 2.5$ and integration domain $u = 2$, but we generate these prices while varying the initial stock price according to $S_0 + dS$ where $dS$ varies between $-2.5$ and 2.5. In this experiment the strike prices do not change, only the initial stock price $S_0$ changes. Figure 14 plots the percentage approximation errors in volatility and skewness for simple trapezoidal integration using only observed prices (dotted line) and for the interpolation-extrapolation technique (solid line). As seen in the figure, employing the interpolation-extrapolation technique yields errors that are very small.

We conclude from these four experiments that the integration technique introduced by Jiang and Tian (2005), described in Section 3, does a good job of mitigating potential biases. We therefore use this approach in our empirical implementation.

# 7 Concluding Remarks

Market betas are one of the most important concepts in the practice and theory of finance, and for many interesting applications of market betas, such as computing the cost of capital, out-of-sample performance is key. Currently, market betas are obtained by using regression techniques on historical data. Many historical implementations still use a simple rolling regression approach, while other approaches allow more explicitly for time-varying betas. However, no matter how sophisticated the approach, historical betas implicitly assume that the past offers a good guide to the future.

This paper presents a radically different approach that extracts betas from option data. Because option data contains information about the future, this approach is inherently forward-looking. The approach is inspired by the literature on volatility forecasting, where a number of authors have compared the forecasting performance of implied volatility with that of more traditional historical methods. The strength of our approach is that betas can be computed using a single cross-section of option data, which may be an important advantage when a company experiences (potential) changes in its operating environment or capital structure.

We test our method using a very conservative approach, by investigating how the method compares with historical methods on average, including stable periods. We find that the forward-looking estimates perform relatively well. Forward-looking betas that were extracted for equities with liquid options often outperform the historical market beta in predicting the realized beta in the following period. In some cases, combining historical betas with forward-looking betas further improves results. The forward-looking betas also explain a sizeable amount of the cross-sectional variation in expected returns.

Much remains to be done. The computation of the forward-looking beta uses moments extracted from options data. While we have made full use of recent innovations in the implementation of these procedures, and added some innovations ourselves, it may be that the out-of-sample performance of the forward-looking betas can be improved through more efficient estimation of moments. Also, we have not provided an explanation for the relatively poor performance of forward-looking betas for stocks with very small realized betas. Another remaining question is if forward-looking beta performs even better in situations and time periods that can a priori be labeled as inherently unstable. It may also prove interesting to investigate the optimal combination of forward-looking and historical betas, which is of course related to the previous question. Finally, we note that it may be possible to compute forward-looking betas that are estimated for a particular purpose using the relevant statistical loss function, for instance in a portfolio model (Granger (1969)).

Our technique can be used for a variety of applications, such as the detection of abnormal returns in event studies, and the uncovering of abnormal returns for portfolio management. The very definition of the word "event" indicates that some aspects of the firm or the firm's environment change, and therefore it will prove interesting to contrast the results obtained using forward-looking betas with those obtained using historical betas. One particularly promising application is the computation of the cost of capital for newly merged companies.

The main focus in this paper has been on forecasting 180-day realized betas, which are relevant for certain applications such as abnormal returns. For other applications, such as cost of capital calculations, longer-horizon betas may be needed. We plan to investigate the performance of forward-looking betas in this context by using LEAPS as well as option contracts with longer maturities traded on non-U.S markets. Finally, in this paper we compute forward-looking moments and forward-looking betas using option prices for a given day. While this is the most obvious initial approach to investigate the method's merits, the performance of the forward-looking betas may be improved by adjusting these betas using a pre-determined rule, or by smoothing betas and/or moments using information extracted from option prices on other days. The optimal use and optimal smoothing of information contained in option prices is left for future work.

# 8 Appendix A: Forward-Looking Moments

Carr and Madan (2001) show that any twice continuously differentiable payoff function $f(S)$ can be replicated with bonds, the underlying stock and the cross section of out-of-the-money options. For convenience, we replicate their argument here. The fundamental theorem of calculus implies that for any fixed $F$

$$
\begin{aligned}
f(S) &= f(F) + 1_{S>F} \int_F^S f'(u)du - 1_{S<F} \int_S^F f'(u)du \\
&= f(F) + 1_{S>F} \int_F^S \left[ f'(F) + \int_F^u f''(v)dv \right] du - 1_{S<F} \int_S^F \left[ f'(F) + \int_u^F f''(v)dv \right] du
\end{aligned}
$$

Because $f'(F)$ is not a function of $u$ we are able to apply Fubini's theorem

$$
f(S) = f(F) + f'(F)(S-F) + 1_{S>F} \int_F^S \int_v^S f''(v)dudv + 1_{S<F} \int_S^F \int_S^v f''(v)dudv
$$

Now integrate over $u$

$$\begin{aligned} f(S) &= f(F) + f'(F)(S-F) + 1_{S>F} \int_F^S f''(v)(S-v)dv + 1_{S<F} \int_S^F f''(v)(v-S)dv \\ &= f(F) + f'(F)(S-F) + \int_F^\infty f''(v)(S-v)^+dv + \int_0^F f''(v)(v-S)^+dv \end{aligned}$$

If we set $F$ equal to the initial stock price, $F = S_0$, and integrate over $K$ instead of $v$, where $K$ is interpreted as the strike, we are left with the spanning equation

$$f(S) = [f(S_0) - f'(S_0)S_0] + f'(S_0)S + \int_{S_0}^\infty f''(K)(S-K)^+dK + \int_0^{S_0} f''(K)(K-S)^+dK$$

From this equation we see that the payoff $f(S)$ is spanned by a $[f(S_0) - f'(S_0)S_0]$ position in bonds, $f'(S_0)$ position in shares of the stock and a $f''(K)dK$ position in out-of-the-money options.

# 9 Appendix B: The Risk-Neutral Return Mean

Under the risk-neutral distribution we know that the return on the stock must earn the risk free rate, therefore

$$\exp(r\tau) = E_t^q\left[\frac{S(t+\tau)}{S(t)}\right] = E_t^q\left[\exp(R(t,\tau))\right]$$

Bakshi, Kapadia and Madan (2003) show that when applying $\exp[R] = 1+R+R^2/2+R^3/6+o(R^3)$, we get

$$\exp(r\tau) = 1 + E_t^q\left[R(t,\tau)\right] + \frac{1}{2}E_t^q\left[R(t,\tau)^2\right] + \frac{1}{6}E_t^q\left[R(t,\tau)^3\right]$$

Reorganizing and using the *Quad* and *Cubic* definitions yields the mean return

$$E_t^q\left[R(t,\tau)\right] = e^{r\tau} - 1 - \frac{e^{r\tau}}{2}Quad - \frac{e^{r\tau}}{6}Cubic$$

# 10 Appendix C: Risk Neutralization in Merton's CAPM

**Stock and Market Processes Under P**

The processes for the individual firm and for the market are given by:

$$dS_{it} = \alpha_i S_{it}dt + \sigma_i S_{it}dw_{1it}$$

$$dS_{Mt} = \alpha_M S_{Mt}dt + \sigma_M S_{Mt}dw_{2t}$$

The correlation between the Brownian motions is given by

$$w_{1it} = \rho_i w_{2t} + \sqrt{1-\rho_i^2}w_{3it}$$

where $w_{2t}$ and $w_{3it}$ are independent

$$dS_{it} = \alpha_i S_{it}dt + \sigma_i S_{it}\rho_i dw_{2t} + \sigma_i S_{it}\sqrt{1-\rho_i^2}dw_{3it}$$

Now we apply Ito's Lemma to obtain the processes for the discounted values of the individual stock and the market index

$$d(e^{-rt}S_{it}) = (\alpha_i - r)\alpha_i S_{it}dt + \sigma_i S_{it}\rho_i dw_{2t} + \sigma_i S_{it}\sqrt{1-\rho_i^2}dw_{3it} \tag{25}$$

$$d(e^{-rt}S_{Mt}) = (\alpha_M - r)\alpha_M S_{Mt}dt + \sigma_M S_{Mt}dw_{2t} \tag{26}$$

Define the risk-reward ratios $\{\gamma_{2t}; t \geq 0\}$ and $\{\gamma_{3it}; t \geq 0\}$ which make (25) and (26) Q-martingales. For the market we have

$$d(e^{-rt}S_{Mt}) = (\alpha_M - r - \gamma_2\sigma_M)\alpha_M S_{Mt}dt + \sigma_M S_{Mt}d(w_{2t} + \gamma_2), \text{ where}$$

$$\gamma_2 = \frac{\alpha_M - r}{\sigma_M}$$

Since we know that $\gamma_2$ satisfies the Novikov condition, we can apply the Girsanov theorem, implying that $\overline{w_{2t}} = w_{2t} + \gamma_2$ is a Brownian motion under Q. Now applying the same reasoning to the process for the firm, we get

$$\begin{aligned} d(e^{-rt}S_{it}) &= (\alpha_i - r - \rho_i\sigma_i\gamma_2 - \sqrt{1-\rho_i^2}\sigma_i\gamma_{3i})\alpha_i S_{it}dt \\ &\quad + \sigma_i S_{it}\rho_i d(w_{2t} + \gamma_2) + \sigma_i S_{it}\sqrt{1-\rho_i^2}d(w_{3it} + \gamma_{3i}), \text{ where} \end{aligned}$$

$$\gamma_{3i} = \frac{\alpha_i - r - \rho_i\sigma_i\left(\frac{\alpha_M - r}{\sigma_M}\right)}{\sqrt{1-\rho_i^2}\sigma_i} \tag{27}$$

Note that if the correlation $\rho_i = 0$ we get the conventional risk reward ratio $\gamma_{3i} = \frac{\alpha_i - r}{\sigma_i}$. Rearranging (27) we get the following expression

$$(\alpha_i - r) = \frac{\rho_i\sigma_i}{\sigma_M}(\sigma_M - r) + \sqrt{1-\rho_i^2}\sigma_i\gamma_{3i}$$

If the risk associated with $w_{3it}$ is diversifiable we get

$$(\alpha_i - r) = \beta_i(\alpha_M - r)$$

where $\beta_i = \frac{\rho_i\sigma_i}{\sigma_M}$ follows the definition of the continuous-time CAPM.

**Stock and Market Processes Under Q**

Market

$$\begin{aligned} dS_{Mt} &= (\alpha_M - r - \gamma_2\sigma_M)\alpha_M S_{Mt}dt + \sigma_M S_{Mt}d(w_{2t} + \gamma_2) \\ &= (\alpha_M - r - \left(\frac{\alpha_M - r}{\sigma_M}\right)\sigma_M)\alpha_M S_{Mt}dt + \sigma_M S_{Mt}d(w_{2t} + \gamma_2) \\ &= rS_{Mt}dt + \sigma_M S_{Mt}d\overline{w_{2t}} \end{aligned}$$

Individual firm

$$
\begin{aligned}
dS_{it} &= (\alpha_i - r - \rho_i\sigma_i\gamma_2 - \sqrt{1-\rho_i^2}\sigma_i\gamma_{3i})\alpha_i S_{it}dt \\
&\quad +\sigma_i S_{it}\rho_i d(w_{2t}+\gamma_2) + \sigma_i S_{it}\sqrt{1-\rho_i^2}d(w_{3it}+\gamma_{3i}) \\
&= \left(\alpha_i - r - \rho_i\sigma_i\left(\frac{\alpha_M - r}{\sigma_M}\right) - \sqrt{1-\rho_i^2}\sigma_i\left(\frac{\alpha_i - r - \rho_i\sigma_i\left(\frac{\alpha_M-r}{\sigma_M}\right)}{\sqrt{1-\rho_i^2}\sigma_i}\right)\right)\alpha_i S_{it}dt \\
&\quad +\sigma_i S_{it}\rho_i d(w_{2t}+\gamma_2) + \sigma_i S_{it}\sqrt{1-\rho_i^2}d(w_{3it}+\gamma_{3i}) \\
&= rS_{it}dt + \sigma_i S_{it}\rho_i d\overline{w_{2t}} + \sigma_i S_{it}\sqrt{1-\rho_i^2}d\overline{w_{3it}}
\end{aligned}
$$

Notice that $\rho_i$, $\sigma_i$ and $\sigma_M$ are the same under $P$ (the physical measure) and under $Q$ (the risk-neutral measure). Extracting $\beta_i = \frac{\rho_i\sigma_i}{\sigma_M}$ under either measure will yield the same results. In this case, risk-neutralization is equivalent to mean-shifting the distribution without affecting the covariance terms.

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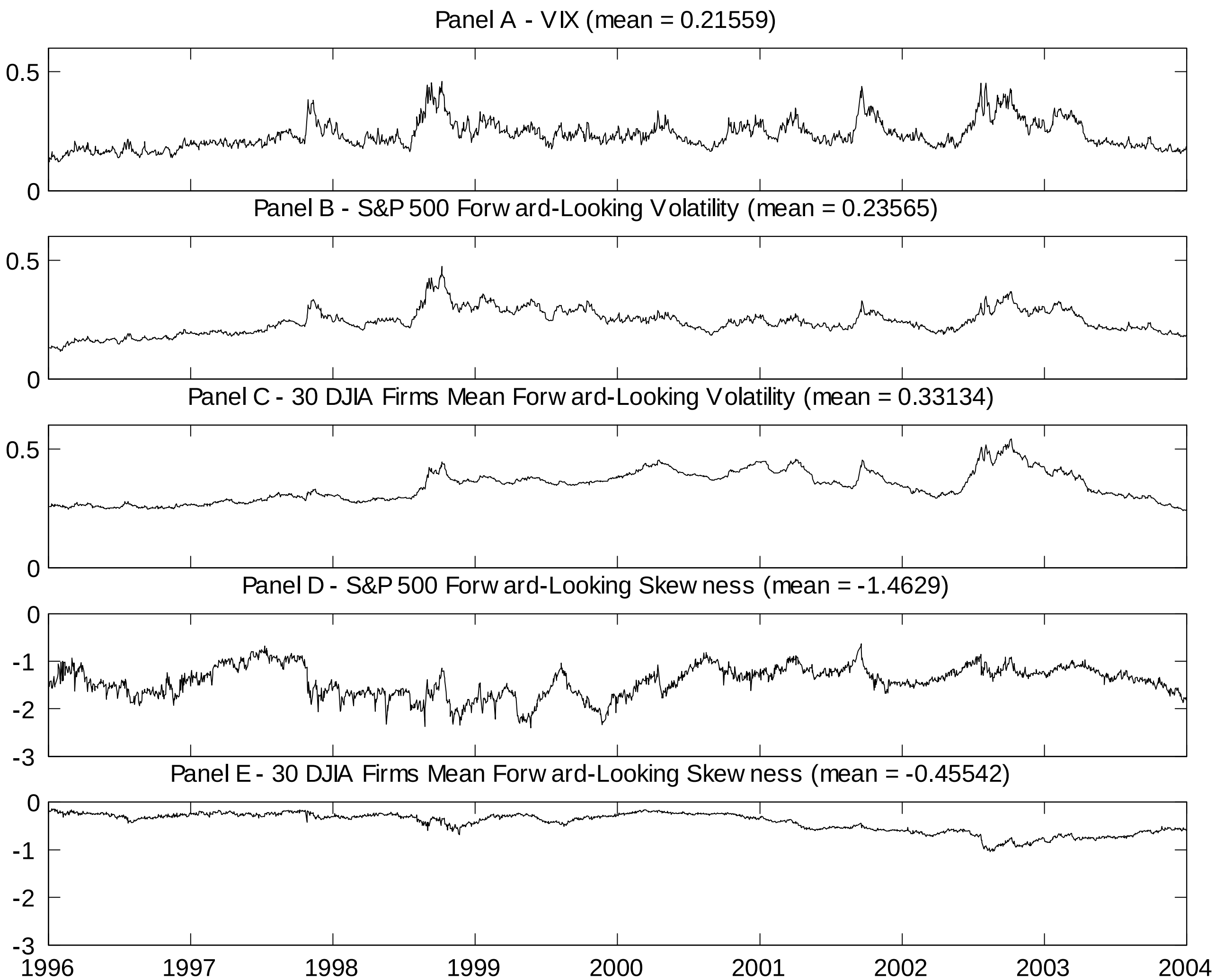
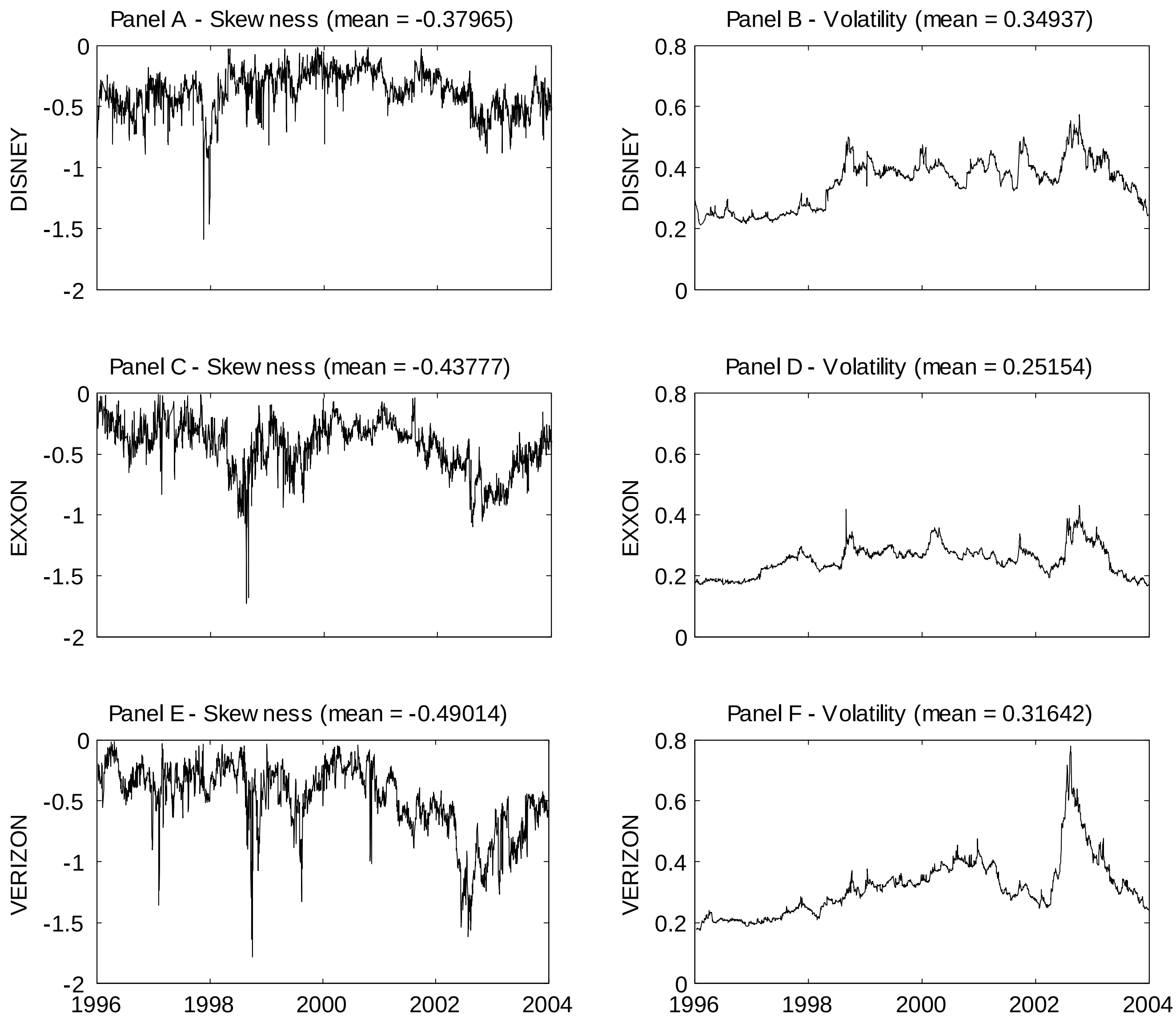
**Figure 1**
**Forward-Looking Volatility and Skewness**




Notes to Figure: Panel A shows the value of the VIX index during 1996-2003. Panel B plots the S&P 500 forward-looking volatility for the same period. Panel C plots the average of the forward-looking volatility for the Dow Jones 30 components. Panel D plots S&P 500 forward-looking skewness and Panel E plots the average of the forward-looking skewness for the Dow Jones 30 components.

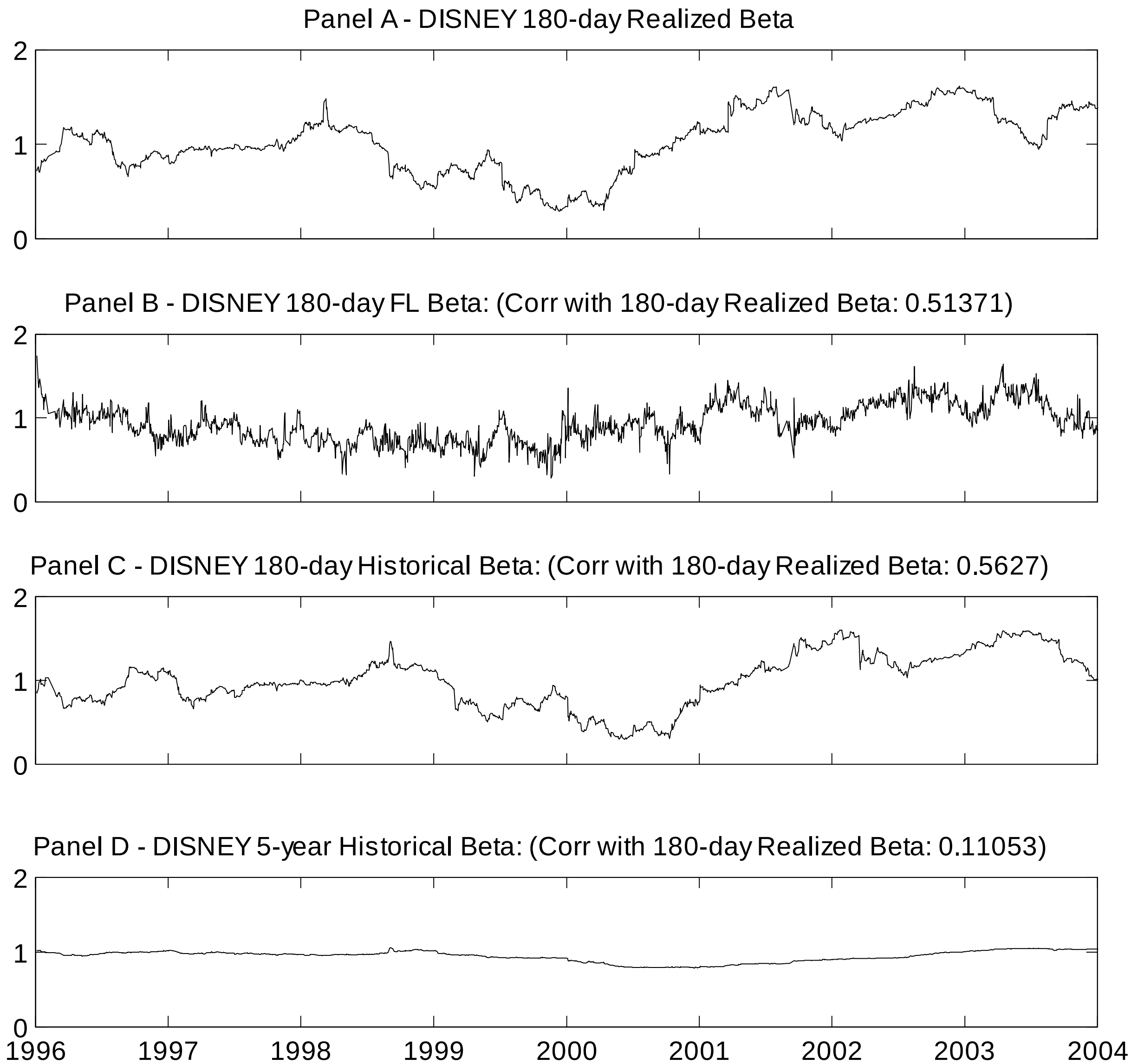
**Figure 2**
**Forward-Looking Moments for Disney, Exxon and Verizon**




Notes to Figure: We plot the time series of the forward-looking moments needed for computing the forward-looking beta for Walt Disney Corporation (DIS), Exxon Mobil Corporation (XOM) and Verizon Communications (VZ). The other moments needed are the forward-looking volatility and skewness for the S&P 500 in Figure 1. We use moments for 180-day returns in all cases. The moments are calculated at a daily frequency for the period January 1, 1996 to December 31, 2003.

**Figure 3**

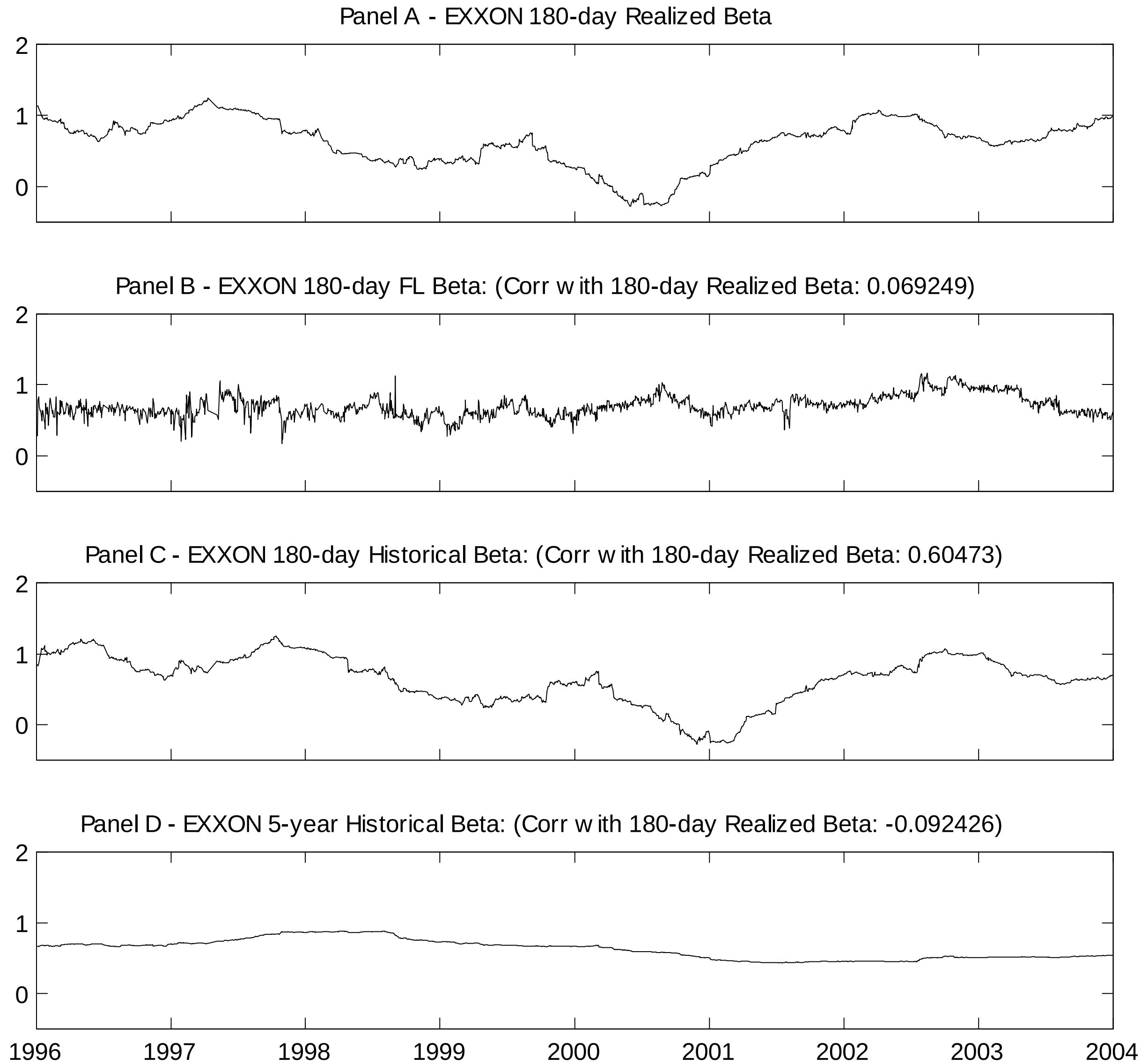
**Realized, Forward-Looking and Historical Betas for Disney**




Notes to Figure: We plot the time-series of different types of betas for Walt Disney Corporation (DIS) for the period January 1, 1996 to December 31, 2003. Panel A plots the time series of ex-post 180-day realized betas ($\beta^{REAL}$). On each day a 180-day realized beta is computed using the returns for the following 180 days. Panel B plots the time series of forward-looking betas. On each day, a 180-day forward-looking beta ($\beta^{FL}$) is computed. Panel C plots the time series of 180-day historical betas, computed using the previous 180 days of returns. Panel D plots the time series of 5-year historical betas, computed using the previous 1800 days of returns.

**Figure 4**

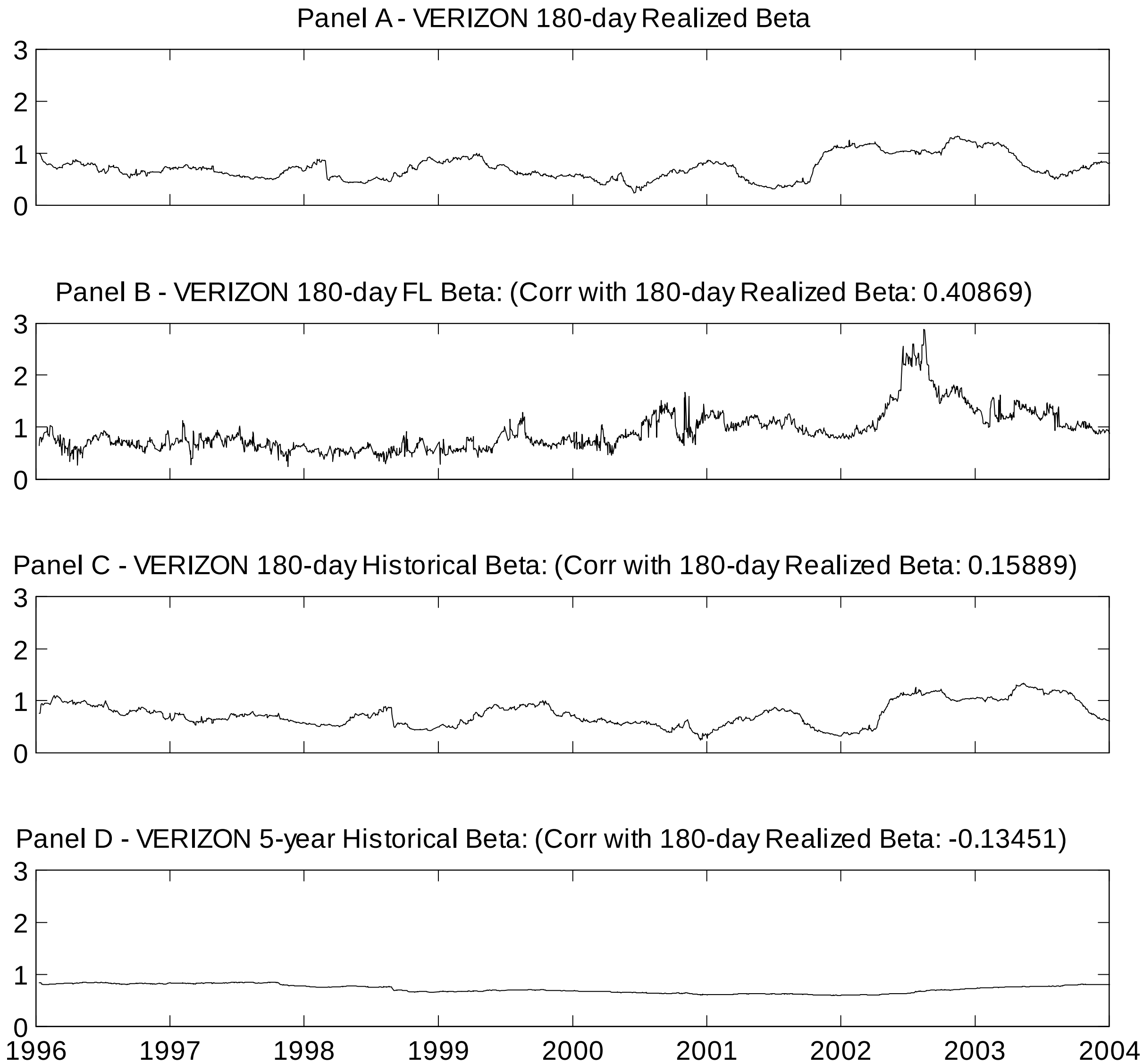
**Realized, Forward-Looking and Historical Betas for Exxon**




Notes to Figure: We plot the time-series of different types of betas for Exxon Mobil Corporation (XOM) for the period January 1, 1996 to December 31, 2003. Panel A plots the time series of ex-post 180-day realized betas ($\beta^{REAL}$). On each day a 180-day realized beta is computed using the returns for the following 180 days. Panel B plots the time series of forward-looking betas. On each day, a 180-day forward-looking beta ($\beta^{FL}$) is computed. Panel C plots the time series of 180-day historical betas, computed using the previous 180 days of returns. Panel D plots the time series of 5-year historical betas, computed using the previous 1800 days of returns.

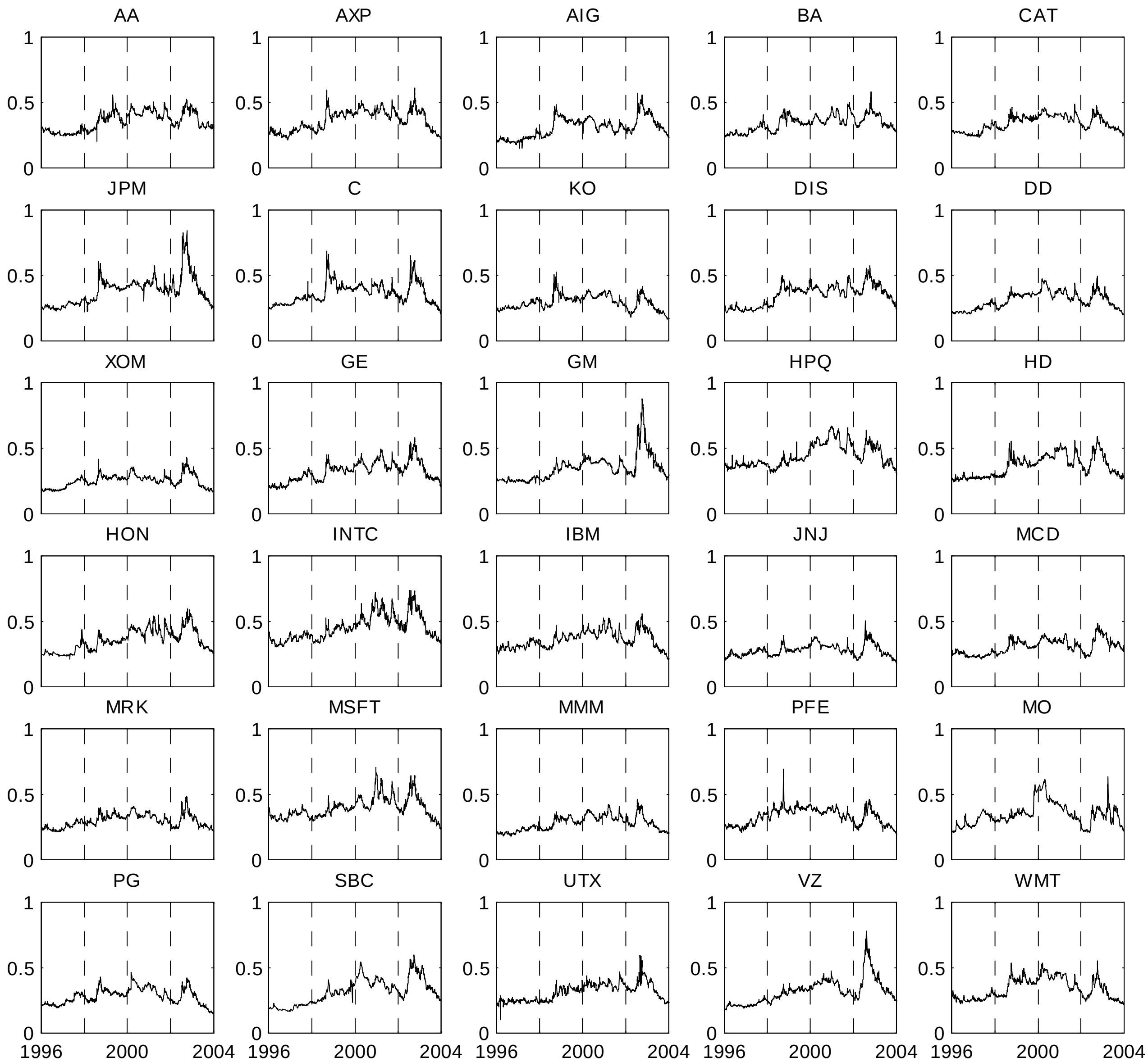
**Figure 5**

**Realized, Forward-Looking and Historical Betas for Verizon**



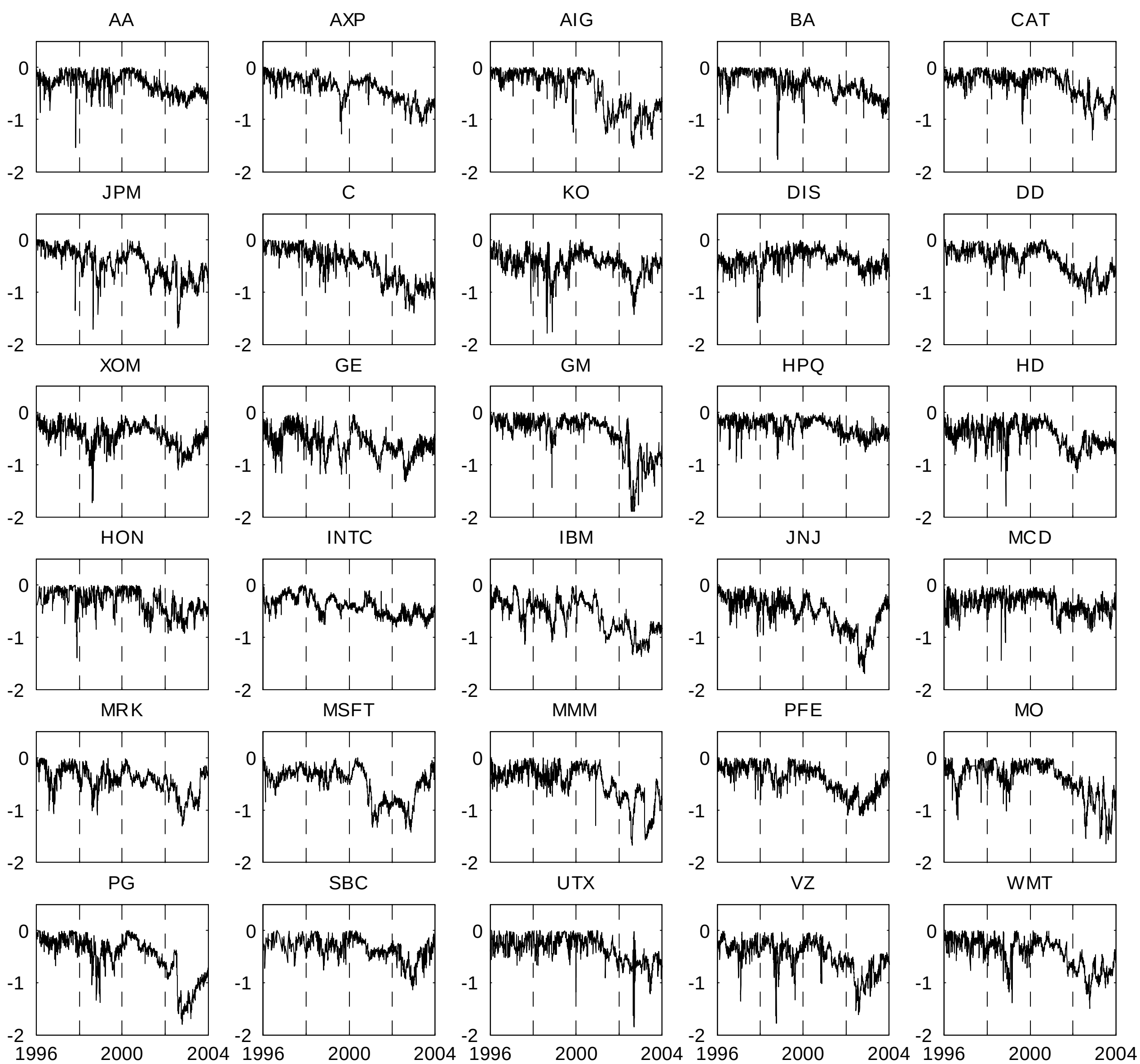

Notes to Figure: We plot the time-series of different types of betas for Verizon Communications (VZ) for the period January 1, 1996 to December 31, 2003. Panel A plots the time series of ex-post realized betas ($\beta^{REAL}$). On each day a 180-day realized beta is computed using the returns for the following 180 days. Panel B plots the time series of forward-looking betas. On each day, a 180-day forward-looking beta ($\beta^{FL}$) is computed. Panel C plots the time series of 180-day historical betas, computed using the previous 180 days of returns. Panel D plots the time series of 5-year historical betas, computed using the previous 1800 days of returns.

**Figure 6**
**Forward-Looking Volatility for the Dow Jones 30 Components**



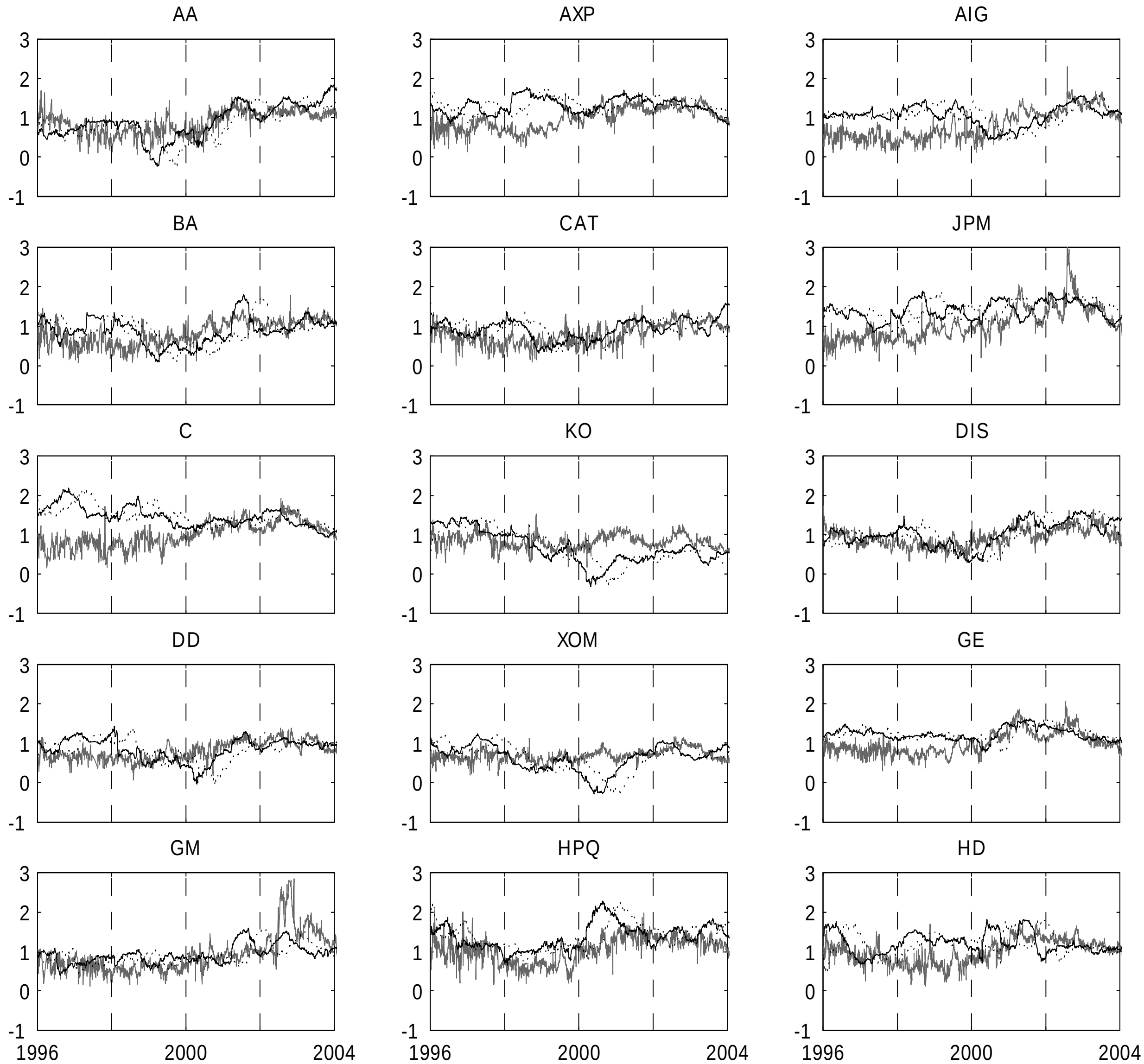

Notes to Figure: We plot the forward-looking volatility for each of the Dow Jones 30 components at a daily frequency for the period January 1, 1996 to December 31, 2003.

**Figure 7**
**Forward-Looking Skewness for the Dow Jones 30 Components**




Notes to Figure: We plot the forward-looking skewness for each of the Dow Jones 30 components at a daily frequency for the period January 1, 1996 to December 31, 2003.

**Figure 8**
**Beta Estimates for the Dow Jones 30 Components**




Notes to Figure: We plot three different betas for each of the Dow Jones 30 components for the period January 1, 1996 to December 31, 2003. 180-day realized beta is plotted in solid black, 180-day forward-looking beta is plotted in solid grey and 180-day historical beta is plotted in dotted black. For realized beta, on each day the beta for the following 180 days is plotted.

**Figure 8 continued**
**Beta Estimates for the Dow Jones 30 Components**

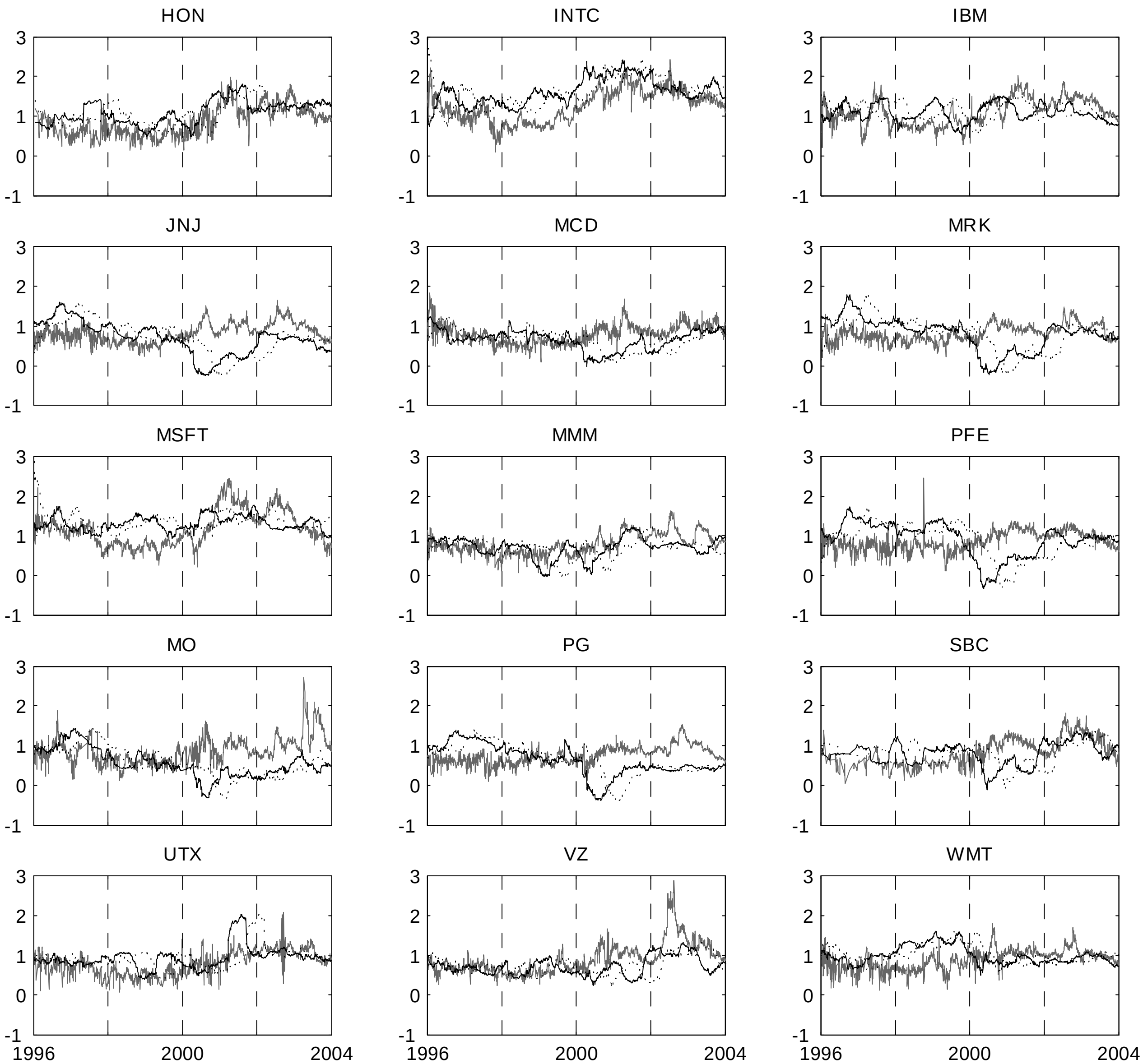
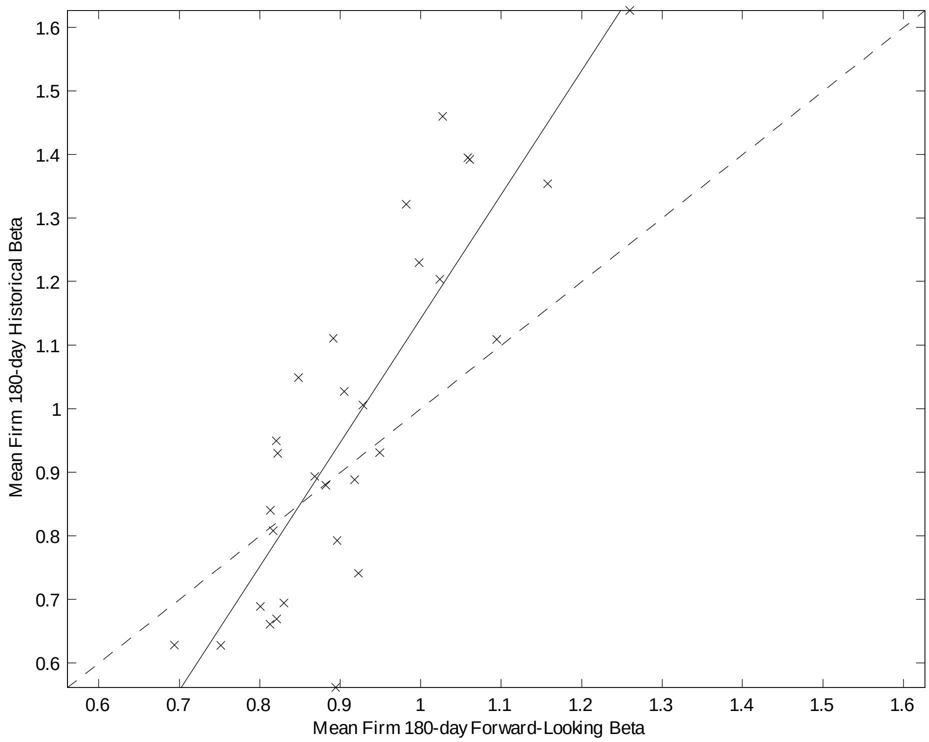


**Figure 9**

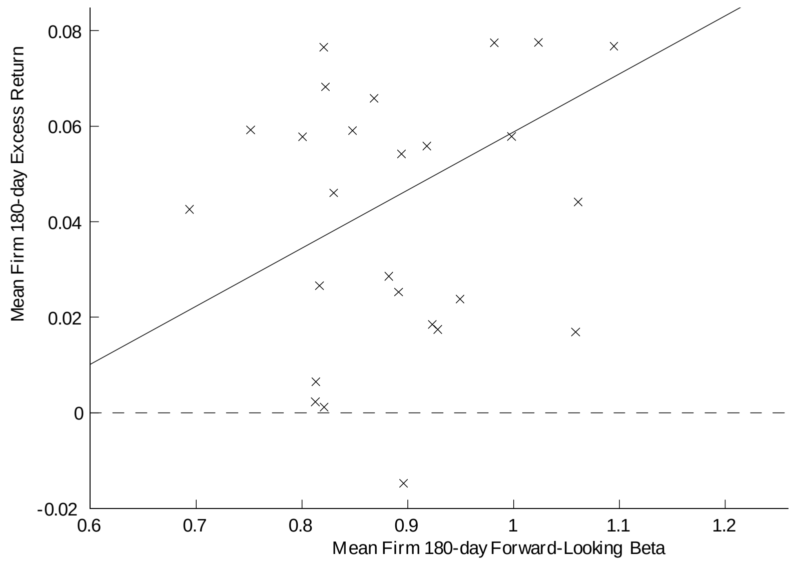
**Mean Historical Beta versus Mean Forward-Looking Beta**



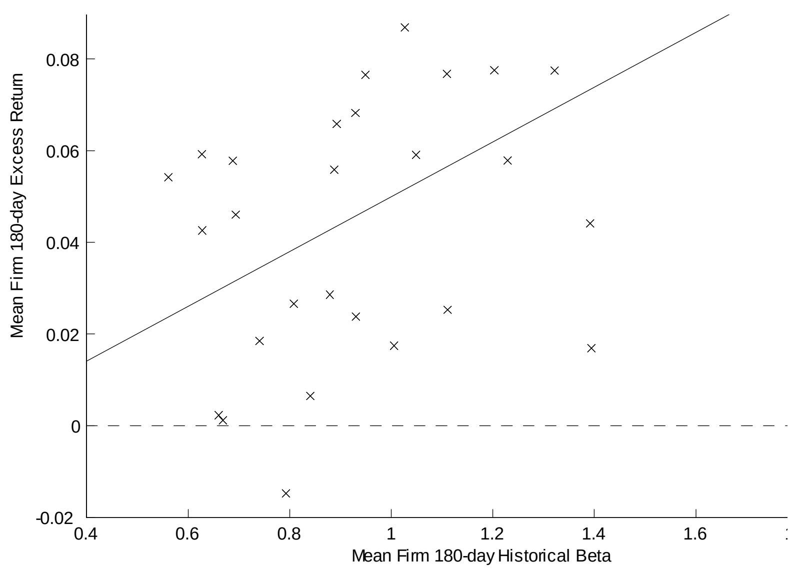

Notes to Figure: We plot the average historical beta for the thirty Dow Jones components versus the average forward-looking beta. The averages are computed for the period January 1, 1996 to December 31, 2003. The solid line is the regression line of mean historical beta on mean forward-looking beta.

**Figure 10**

**Panel A: Mean Excess Return versus Mean Forward-Looking Beta**



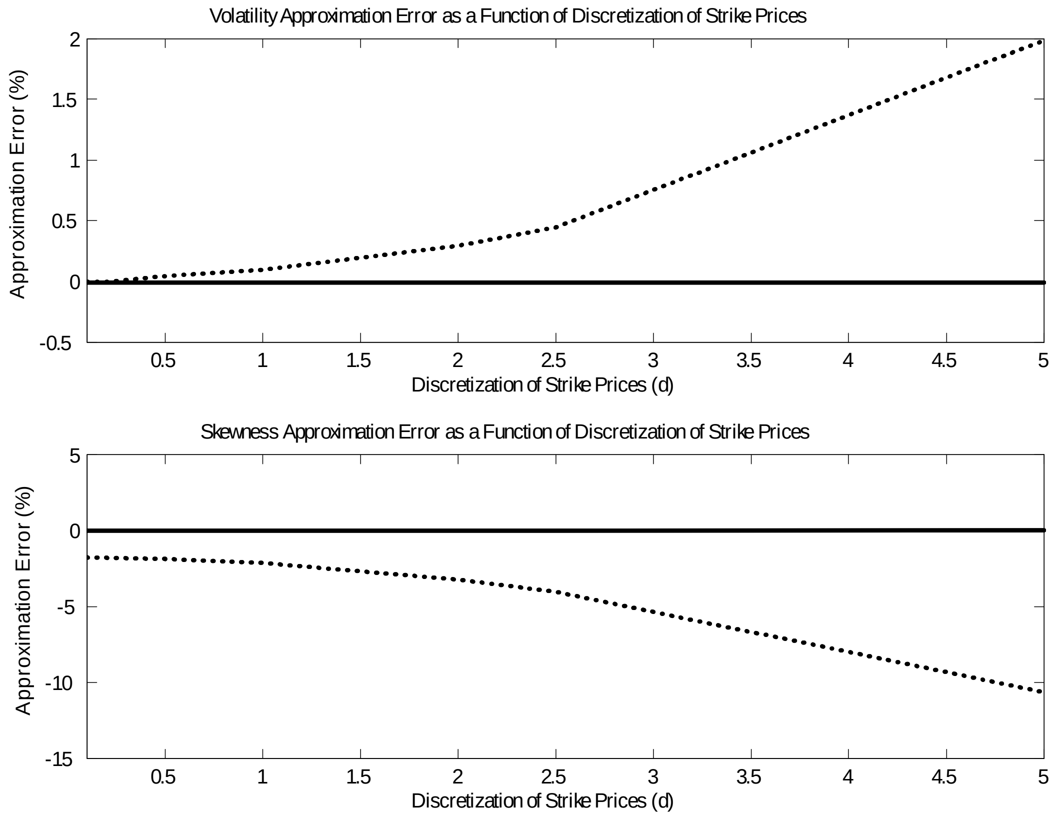

**Panel B: Mean Excess Return versus Mean Historical Beta**




Notes to Figure: We plot the average excess return for each of the thirty Dow Jones components against the average forward-looking beta (Panel A) and the average historical 180-day beta (Panel B). The averages are computed for the period January 1, 1996 to December 31, 2003. We perform a cross-sectional regression of average excess returns on average forward-looking beta in Panel A and average historical 180-day beta in Panel B. The solid lines are the resulting regression lines. The $R^2$ of the regressions are 0.22 and 0.28 respectively.

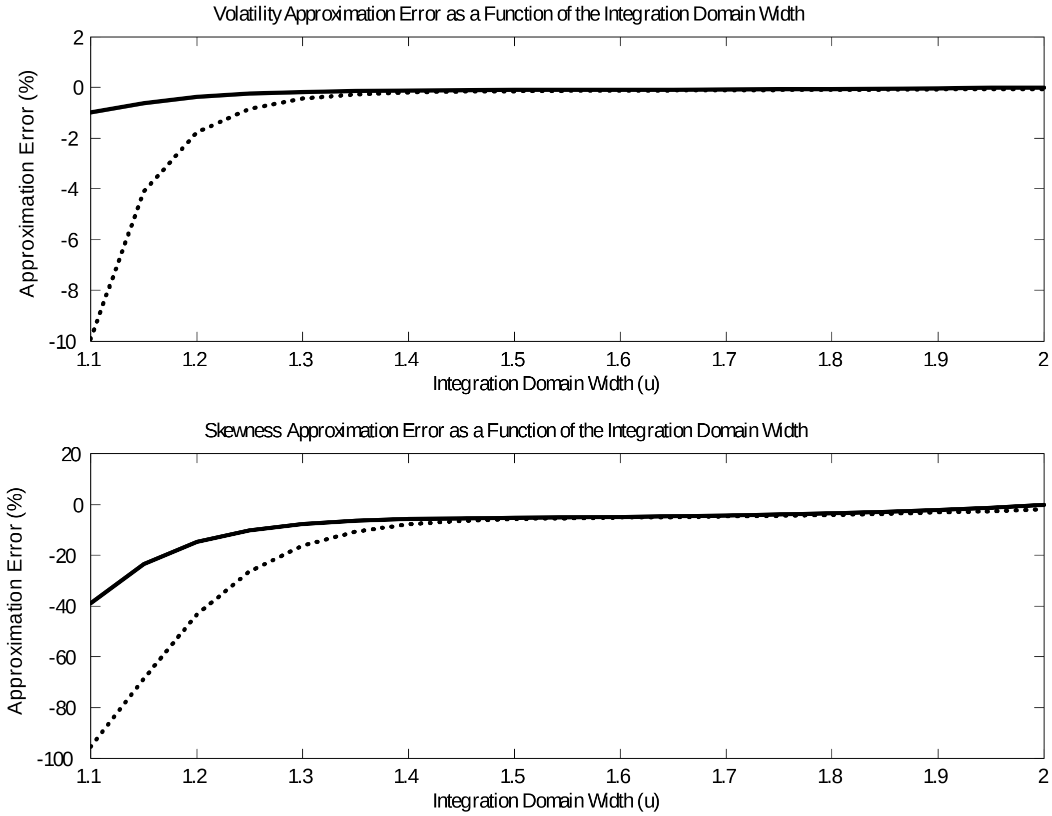
**Figure 11**

**Volatility and Skewness versus Strike Price Interval**




Notes to Figure: We plot the percentage approximation errors induced by the discretization of strike prices ($d = [0.1, 5]$), for the simple integration approach (dotted) and the interpolation-extrapolation (solid) technique of Jiang and Tian (2005). Heston stochastic volatility call and put prices are generated with parameters ($\theta = 0.04$, $\kappa = 2$, $\sigma_v = 0.225$, $\rho = -0.5$ and $V = 0.04$) for the strike price range of $K = [Su^{-1}, Su]$ where $u = 2$. The approximation error is calculated as the percent difference between the estimated moments and the actual moments. The actual moments are obtained by generating $250,000$ Heston stochastic volatility 180-day log returns and computing the sample standard deviation (0.2037) and skewness ($-0.4610$).

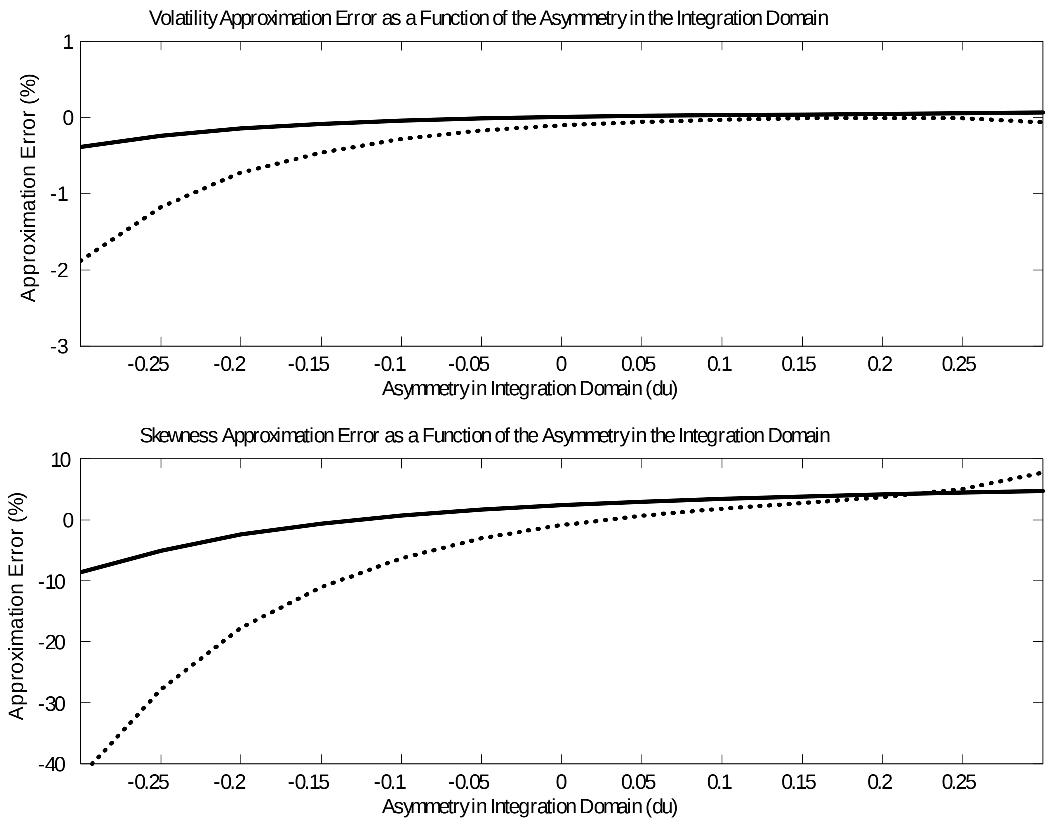
**Figure 12**
**Volatility and Skewness versus Width of Integration Domain**




Notes to Figure: We plot the percentage approximation errors induced by the width of the integration domain ($u = [1.1, 2]$), for the simple integration approach (dotted) and the interpolation-extrapolation (solid) technique of Jiang and Tian (2005). Heston stochastic volatility call and put prices are generated with parameters ($\theta = 0.04$, $\kappa = 2$, $\sigma_v = 0.225$, $\rho = -0.5$ and $V = 0.04$) for the strike price range of $K = [Su^{-1}, Su]$ with discrete strike price interval $d = 0.1$. The approximation error is calculated as the percent difference between the estimated moments and the actual moments. The actual moments are obtained by generating $250,000$ Heston stochastic volatility 180-day log returns and computing the sample standard deviation (0.2037) and skewness ($-0.4610$).

**Figure 13**

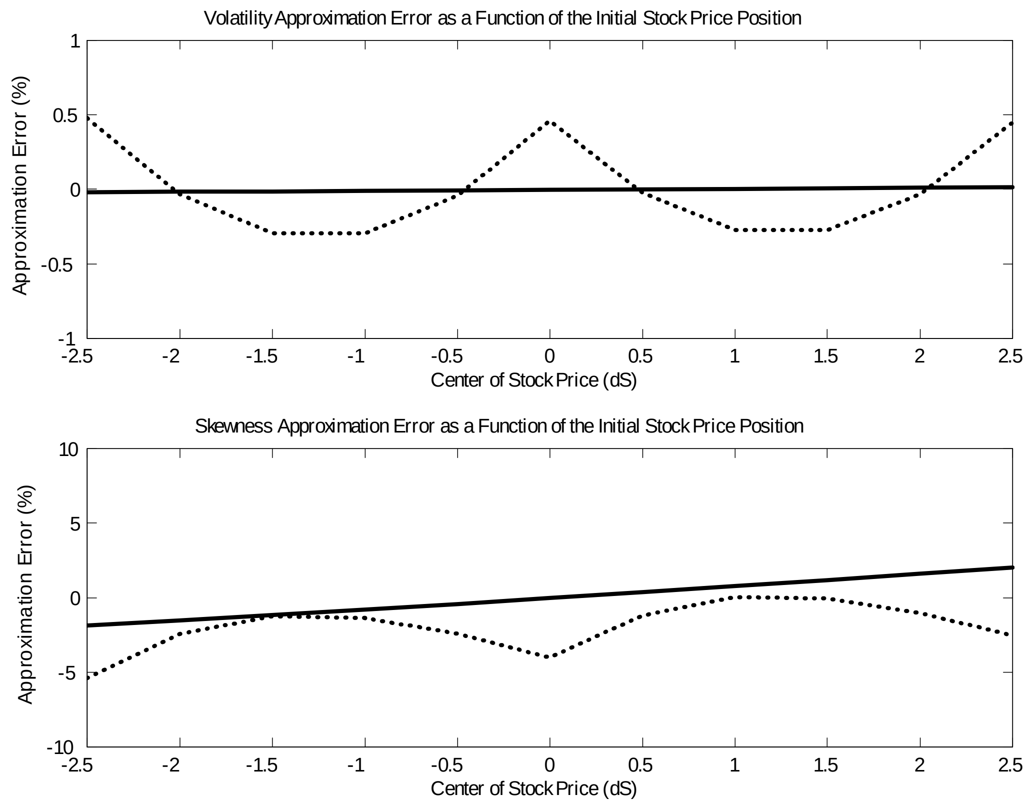
**Volatility and Skewness versus Asymmetry of Integration Domain**




Notes to Figure: We plot the percentage approximation errors induced by the asymmetry of the integration domain $du = [-0.3, 0.3]$ for the simple integration approach (dotted) and the interpolation-extrapolation (solid) technique of Jiang and Tian (2005). Heston stochastic volatility call and put prices are generated with parameters ($\theta = 0.04$, $\kappa = 2$, $\sigma_v = 0.225$, $\rho = -0.5$ and $V = 0.04$) for the strike price range of $K = [Su_L^{-1}, Su_H]$ where $u_L = u - du$, $u_H = u + du$, $u = 1.6$ and discrete strike price interval $d = 0.1$. The approximation error is calculated as the percent difference between the estimated moments and the actual moments. The actual moments are obtained by generating $250,000$ Heston stochastic volatility 180-day log returns and computing the sample standard deviation (0.2037) and skewness ($-0.4610$).

**Figure 14**

**Volatility and Skewness versus Stock Price Position in Strike Price Interval**




Notes to Figure: We plot the percentage approximation errors induced by the location of the center of the stock price $S + dS$, where $dS = [-2.5, 2.5]$ for the simple integration approach (dotted) and the interpolation-extrapolation (solid) technique of Jiang and Tian (2005). Heston stochastic volatility call and put prices are generated with parameters ($\theta = 0.04$, $\kappa = 2$, $\sigma_v = 0.225$, $\rho = -0.5$ and $V = 0.04$) for the strike price range of $K = [Su^{-1}, Su]$ where $u = 2$ and discrete strike price interval $d = 2.5$. The approximation error is calculated as the percent difference between the estimated moments and the actual moments. The actual moments are obtained by generating $250,000$ Heston stochastic volatility 180-day log returns and computing the sample standard deviation (0.2037) and skewness ($-0.4610$).

**Table 1**
**Dow Jones 30 Components**

| Secids | Tickers | Company Name |
|---|---|---|
| 101204 | AA | ALCOA INC |
| 101375 | AXP | AMERICAN EXPRESS CO |
| 101397 | AIG | AMERICAN INTL GROUP INC |
| 102265 | BA | BOEING CO |
| 102796 | CAT | CATERPILLAR INC DEL |
| 102936 | JPM | J P MORGAN CHASE CO |
| 103049 | C | CITIGROUP INC |
| 103125 | KO | COCA COLA CO |
| 103879 | DIS | DISNEY WALT CO |
| 103969 | DD | DU PONT E I DE NEMOURS & CO |
| 104533 | XOM | EXXON MOBIL CORP |
| 105169 | GE | GENERAL ELEC CO |
| 105175 | GM | GENERAL MTRS CORP |
| 105700 | HPQ | HEWLETT PACKARD CO |
| 105759 | HD | HOME DEPOT INC |
| 105785 | HON | HONEYWELL INTL INC |
| 106203 | INTC | INTEL CORP |
| 106276 | IBM | INTERNATIONAL BUSINESS MACHS |
| 106566 | JNJ | JOHNSON & JOHNSON |
| 107318 | MCD | MCDONALDS CORP |
| 107430 | MRK | MERCK & CO INC |
| 107525 | MSFT | MICROSOFT CORP |
| 107616 | MMM | 3M CO |
| 108948 | PFE | PFIZER INC |
| 108965 | MO | ALTRIA GROUP INC |
| 109224 | PG | PROCTER & GAMBLE COMPANY |
| 109775 | SBC | SBC COMMUNICATIONS INC |
| 111459 | UTX | UNITED TECHNOLOGIES CORP |
| 111668 | VZ | VERIZON COMMUNICATIONS |
| 111860 | WMT | WAL MART STORES INC |

Notes to Table: For each of the components of the Dow Jones 30, we give the ticker symbol and the optionmetrics security ID. We use the components of the Dow Jones 30 as of April, 8, 2004.

**Table 2**
**Descriptive Statistics for Option Data**

| | Calls | | | | | | Puts | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 8 < DTM < 181 | | | 180 < DTM < 366 | | | 8 < DTM < 181 | | | 180 < DTM < 366 | | |
| Ticker | # of Quotes | Mean Price | Mean Imp. Vol. | # of Quotes | Mean Price | Mean Imp. Vol. | # of Quotes | Mean Price | Mean Imp. Vol. | # of Quotes | Mean Price | Mean Imp. Vol. |
| AA | 15824 | 2.01 | 0.35 | 5679 | 2.91 | 0.33 | 16986 | 1.71 | 0.38 | 5505 | 2.48 | 0.36 |
| AXP | 21727 | 3.33 | 0.36 | 9240 | 5.74 | 0.34 | 26317 | 2.62 | 0.39 | 10633 | 4.03 | 0.37 |
| AIG | 20611 | 3.28 | 0.29 | 9472 | 5.78 | 0.28 | 25538 | 2.47 | 0.33 | 11092 | 4.01 | 0.32 |
| BA | 18550 | 1.97 | 0.33 | 11065 | 3.21 | 0.31 | 20384 | 1.65 | 0.34 | 10789 | 2.46 | 0.33 |
| CAT | 17400 | 2.08 | 0.32 | 7866 | 3.51 | 0.31 | 19721 | 1.86 | 0.35 | 9193 | 2.89 | 0.34 |
| JPM | 20309 | 2.58 | 0.36 | 9086 | 4.33 | 0.34 | 24524 | 2.12 | 0.39 | 11604 | 3.34 | 0.38 |
| C | 20065 | 2.01 | 0.35 | 9671 | 3.46 | 0.32 | 26117 | 1.61 | 0.39 | 11773 | 2.51 | 0.36 |
| KO | 16232 | 1.87 | 0.28 | 8635 | 3.04 | 0.27 | 17784 | 1.59 | 0.31 | 8867 | 2.37 | 0.29 |
| DIS | 15347 | 1.73 | 0.34 | 7159 | 2.89 | 0.33 | 14694 | 1.49 | 0.36 | 6838 | 2.12 | 0.36 |
| DD | 15002 | 1.96 | 0.29 | 7260 | 3.13 | 0.29 | 16431 | 1.74 | 0.31 | 7734 | 2.59 | 0.31 |
| XOM | 14809 | 1.93 | 0.24 | 7035 | 3.34 | 0.24 | 16838 | 1.67 | 0.27 | 8036 | 2.51 | 0.26 |
| GE | 20680 | 2.78 | 0.31 | 9942 | 4.50 | 0.30 | 27130 | 2.10 | 0.35 | 13062 | 2.99 | 0.33 |
| GM | 19183 | 2.09 | 0.34 | 9311 | 3.82 | 0.32 | 22007 | 1.93 | 0.38 | 10463 | 3.16 | 0.35 |
| HPQ | 23633 | 3.44 | 0.46 | 10202 | 5.91 | 0.43 | 23020 | 2.85 | 0.48 | 10052 | 4.39 | 0.44 |
| HD | 17992 | 1.96 | 0.36 | 9800 | 3.28 | 0.34 | 20122 | 1.58 | 0.39 | 10035 | 2.35 | 0.36 |

Notes to Table: The sample contains call and put prices for the S&P 500 and the Dow Jones 30 components during the period January 1, 1996 to December 31, 2003. We apply the same moneyness and maturity filters used by Jiang and Tian (2005). Prices and implied volatilities are extracted from the price file in the Ivy Options Database.

**Table 2 continued**
**Descriptive Statistics for Option Data**

| | Calls | | | | | | Puts | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 8 < DTM < 181 | | | 180 < DTM < 366 | | | 8 < DTM < 181 | | | 180 < DTM < 366 | | |
| Ticker | # of Quotes | Mean Price | Mean Imp. Vol. | # of Quotes | Mean Price | Mean Imp. Vol. | # of Quotes | Mean Price | Mean Imp. Vol. | # of Quotes | Mean Price | Mean Imp. Vol. |
| HON | 18010 | 1.82 | 0.36 | 7574 | 3.18 | 0.33 | 19422 | 1.61 | 0.39 | 8837 | 2.39 | 0.35 |
| INTC | 28421 | 3.36 | 0.44 | 13988 | 5.76 | 0.42 | 30386 | 2.60 | 0.47 | 14034 | 3.96 | 0.45 |
| IBM | 33352 | 3.91 | 0.35 | 14386 | 7.24 | 0.32 | 40633 | 3.03 | 0.40 | 16812 | 4.94 | 0.37 |
| JNJ | 15846 | 2.39 | 0.27 | 8358 | 4.16 | 0.26 | 19085 | 1.90 | 0.30 | 10534 | 2.77 | 0.29 |
| MCD | 13983 | 1.58 | 0.30 | 7150 | 2.39 | 0.28 | 14367 | 1.31 | 0.32 | 7022 | 1.76 | 0.31 |
| MRK | 18864 | 2.69 | 0.29 | 9094 | 4.72 | 0.28 | 22407 | 2.21 | 0.32 | 10556 | 3.51 | 0.31 |
| MSFT | 28376 | 3.52 | 0.38 | 14846 | 5.69 | 0.37 | 34780 | 2.55 | 0.43 | 16780 | 3.80 | 0.40 |
| MMM | 21520 | 3.00 | 0.27 | 9087 | 5.59 | 0.26 | 25925 | 2.59 | 0.31 | 10944 | 4.23 | 0.29 |
| PFE | 21992 | 2.49 | 0.33 | 10216 | 4.22 | 0.31 | 24174 | 2.01 | 0.35 | 11725 | 2.96 | 0.34 |
| MO | 22847 | 1.79 | 0.34 | 11339 | 2.58 | 0.33 | 26429 | 1.71 | 0.35 | 12999 | 2.47 | 0.34 |
| PG | 18252 | 2.72 | 0.26 | 8736 | 4.56 | 0.26 | 22010 | 2.24 | 0.29 | 10822 | 3.40 | 0.28 |
| SBC | 10046 | 1.56 | 0.32 | 4972 | 2.43 | 0.32 | 11235 | 1.39 | 0.35 | 5492 | 2.04 | 0.34 |
| UTX | 19912 | 2.77 | 0.29 | 8101 | 4.87 | 0.28 | 24156 | 2.35 | 0.33 | 10104 | 3.71 | 0.31 |
| VZ | 13665 | 1.82 | 0.29 | 6823 | 2.92 | 0.28 | 15963 | 1.66 | 0.34 | 7346 | 2.63 | 0.32 |
| WMT | 17465 | 2.08 | 0.34 | 9509 | 3.60 | 0.32 | 22889 | 1.66 | 0.38 | 12038 | 2.53 | 0.36 |
| SPX | 124285 | 20.79 | 0.20 | 59812 | 42.68 | 0.19 | 175826 | 15.15 | 0.27 | 71561 | 31.85 | 0.26 |

**Table 3**
**Standard Deviation, Skewness, Forward-Looking and Historical Betas**

| Tickers | STDEV | SKEW | FLB | HISTB (180-day) | HISTB (1-year) | HISTB (5-year) |
|---|---|---|---|---|---|---|
| AA | 0.361 | -0.345 | 0.913 | 0.863 | 0.874 | 0.803 |
| AXP | 0.366 | -0.421 | 0.987 | 1.330 | 1.340 | 1.289 |
| AIG | 0.311 | -0.482 | 0.845 | 1.047 | 1.049 | 1.053 |
| BA | 0.344 | -0.351 | 0.876 | 0.873 | 0.874 | 0.843 |
| CAT | 0.340 | -0.332 | 0.821 | 0.922 | 0.934 | 0.937 |
| JPM | 0.386 | -0.480 | 1.075 | 1.426 | 1.426 | 1.390 |
| C | 0.368 | -0.514 | 1.034 | 1.477 | 1.501 | 1.557 |
| KO | 0.294 | -0.467 | 0.829 | 0.678 | 0.703 | 0.833 |
| DIS | 0.352 | -0.383 | 0.935 | 0.998 | 0.985 | 0.947 |
| DD | 0.314 | -0.411 | 0.817 | 0.836 | 0.836 | 0.854 |
| XOM | 0.254 | -0.438 | 0.698 | 0.621 | 0.633 | 0.626 |
| GE | 0.326 | -0.590 | 1.008 | 1.235 | 1.244 | 1.194 |
| GM | 0.353 | -0.427 | 0.940 | 0.929 | 0.933 | 0.963 |
| HPQ | 0.445 | -0.272 | 1.065 | 1.383 | 1.387 | 1.328 |
| HD | 0.370 | -0.443 | 1.027 | 1.209 | 1.212 | 1.250 |
| HON | 0.365 | -0.324 | 0.889 | 1.102 | 1.099 | 1.032 |
| INTC | 0.454 | -0.411 | 1.266 | 1.619 | 1.618 | 1.533 |
| IBM | 0.362 | -0.578 | 1.102 | 1.118 | 1.124 | 1.085 |
| JNJ | 0.281 | -0.543 | 0.838 | 0.702 | 0.707 | 0.831 |
| MCD | 0.308 | -0.359 | 0.813 | 0.653 | 0.656 | 0.756 |
| MRK | 0.302 | -0.436 | 0.821 | 0.811 | 0.814 | 0.913 |
| MSFT | 0.399 | -0.494 | 1.174 | 1.356 | 1.358 | 1.352 |
| MMM | 0.283 | -0.487 | 0.800 | 0.685 | 0.706 | 0.701 |
| PFE | 0.333 | -0.415 | 0.875 | 0.891 | 0.888 | 0.991 |
| MO | 0.340 | -0.425 | 0.893 | 0.567 | 0.572 | 0.711 |
| PG | 0.280 | -0.515 | 0.757 | 0.636 | 0.652 | 0.821 |
| SBC | 0.346 | -0.380 | 0.899 | 0.795 | 0.779 | 0.772 |
| UTX | 0.314 | -0.368 | 0.823 | 0.949 | 0.943 | 0.895 |
| VZ | 0.319 | -0.487 | 0.924 | 0.746 | 0.742 | 0.715 |
| WMT | 0.344 | -0.450 | 0.909 | 1.032 | 1.032 | 1.108 |
| SPX | 0.246 | -1.417 | | | | |

Notes to Table: For each of the firms in the Dow Jones 30 and the S&P 500 index, we present descriptive statistics for moments and computed market betas. We report the mean forward-looking standard deviation (STDEV), the mean forward-looking skew (SKEW), the mean 180-day forward-looking beta (FLB) and the mean historical beta (HISTB) for three different estimation windows.

**Table 4A**
**Correlation Between 180-day Realized Beta and Predictors**

| | Correlation with REALB (180-day) | | | | | Mean REALB |
|---|---|---|---|---|---|---|
| **Tickers** | **FLB** | **HISTB (180-day)** | **MIXB** | **HISTB (1-year)** | **HISTB (5-year)** | |
| AA | 0.605 | 0.631 | **0.716** | 0.592 | 0.125 | 0.937 |
| AXP | -0.025 | **0.185** | 0.074 | -0.021 | 0.029 | 1.308 |
| AIG | 0.131 | **0.547** | 0.427 | 0.403 | -0.165 | 1.048 |
| BA | 0.372 | 0.323 | **0.461** | 0.250 | -0.491 | 0.867 |
| CAT | 0.369 | 0.291 | **0.416** | 0.251 | -0.267 | 0.922 |
| JPM | **0.279** | 0.011 | 0.206 | -0.074 | -0.141 | 1.410 |
| C | -0.418 | **0.472** | -0.059 | 0.347 | 0.379 | 1.431 |
| KO | 0.087 | 0.746 | 0.731 | **0.747** | 0.567 | 0.640 |
| DIS | 0.545 | 0.568 | **0.662** | 0.536 | 0.080 | 1.025 |
| DD | 0.275 | 0.502 | **0.559** | 0.474 | -0.023 | 0.841 |
| XOM | 0.089 | **0.600** | 0.553 | 0.443 | -0.080 | 0.609 |
| GE | 0.464 | 0.395 | **0.531** | 0.233 | -0.242 | 1.230 |
| GM | **0.524** | 0.175 | 0.457 | 0.322 | -0.329 | 0.934 |
| HPQ | 0.514 | 0.416 | **0.515** | 0.335 | -0.069 | 1.381 |
| HD | **0.016** | -0.209 | -0.124 | -0.182 | -0.206 | 1.215 |
| HON | **0.621** | 0.417 | 0.593 | 0.370 | 0.157 | 1.110 |
| INTC | **0.590** | 0.342 | 0.518 | 0.459 | 0.081 | 1.627 |
| IBM | **0.211** | -0.281 | 0.018 | -0.503 | -0.347 | 1.095 |
| JNJ | -0.336 | **0.682** | 0.491 | 0.617 | 0.529 | 0.676 |
| MCD | 0.003 | **0.556** | 0.459 | 0.486 | 0.264 | 0.641 |
| MRK | -0.343 | **0.590** | 0.446 | 0.533 | 0.352 | 0.798 |
| MSFT | **0.210** | -0.040 | 0.144 | -0.022 | -0.304 | 1.315 |
| MMM | **0.352** | 0.104 | 0.291 | 0.106 | -0.097 | 0.695 |
| PFE | -0.475 | **0.601** | 0.447 | 0.495 | 0.137 | 0.894 |
| MO | -0.158 | 0.657 | 0.418 | **0.662** | 0.553 | 0.544 |
| PG | -0.445 | **0.686** | 0.553 | 0.611 | 0.581 | 0.612 |
| SBC | 0.097 | 0.195 | 0.196 | **0.203** | -0.118 | 0.788 |
| UTX | **0.377** | 0.046 | 0.258 | -0.041 | -0.197 | 0.945 |
| VZ | **0.409** | 0.157 | 0.361 | 0.246 | -0.128 | 0.726 |
| WMT | -0.382 | **0.494** | 0.072 | 0.378 | -0.004 | 1.009 |

Notes to Table: For each of the firms in the Dow Jones 30, we present the correlation between on the one hand the realized 180-day beta (REALB), and on the other hand either 180-day forward-looking (FLB) beta, 180-day historical beta (HISTB 180-day), mixed equally weighted forward-looking and historical beta (MIXB), 1-year historical beta (HISTB 1-year) or 5-year historical beta (HISTB 5-year). We also report the average realized beta for each firm over this period. The sample period is January 1, 1996 to December 31, 2003.

**Table 4B**
**Correlation Between 1-year Realized Beta and Predictors**

| | Correlation with REALB (1-year) | | | | | Mean REALB |
|---|---|---|---|---|---|---|
| **Tickers** | **FLB** | **HISTB (180-day)** | **MIXB** | **HISTB (1-year)** | **HISTB (5-year)** | |
| AA | 0.654 | 0.638 | **0.744** | 0.606 | 0.074 | 0.976 |
| AXP | -0.122 | **-0.045** | -0.115 | -0.234 | -0.206 | 1.308 |
| AIG | 0.222 | 0.391 | **0.411** | 0.170 | -0.401 | 1.063 |
| BA | 0.413 | 0.317 | **0.481** | 0.161 | -0.676 | 0.878 |
| CAT | **0.538** | 0.290 | 0.524 | 0.237 | -0.442 | 0.927 |
| JPM | **0.330** | -0.087 | 0.204 | -0.227 | -0.365 | 1.414 |
| C | -0.441 | 0.350 | -0.170 | 0.279 | **0.381** | 1.425 |
| KO | 0.117 | **0.750** | 0.747 | 0.728 | 0.508 | 0.633 |
| DIS | 0.655 | 0.556 | **0.708** | 0.441 | -0.074 | 1.033 |
| DD | 0.417 | 0.429 | **0.592** | 0.373 | -0.205 | 0.837 |
| XOM | 0.131 | **0.447** | 0.435 | 0.334 | -0.274 | 0.616 |
| GE | **0.501** | 0.214 | 0.482 | -0.003 | -0.394 | 1.229 |
| GM | 0.620 | 0.418 | **0.622** | 0.487 | -0.450 | 0.953 |
| HPQ | **0.519** | 0.343 | 0.476 | 0.208 | -0.277 | 1.368 |
| HD | **-0.142** | -0.183 | -0.218 | -0.305 | -0.615 | 1.198 |
| HON | **0.639** | 0.360 | 0.575 | 0.257 | 0.014 | 1.132 |
| INTC | 0.601 | 0.496 | **0.607** | 0.480 | -0.014 | 1.622 |
| IBM | **0.185** | -0.288 | -0.005 | -0.385 | -0.439 | 1.088 |
| JNJ | -0.233 | **0.594** | 0.461 | 0.471 | 0.473 | 0.657 |
| MCD | 0.014 | **0.484** | 0.408 | 0.408 | 0.197 | 0.630 |
| MRK | -0.299 | **0.506** | 0.380 | 0.372 | 0.257 | 0.787 |
| MSFT | **0.256** | -0.053 | 0.173 | -0.055 | -0.415 | 1.289 |
| MMM | **0.513** | 0.053 | 0.363 | 0.169 | -0.253 | 0.705 |
| PFE | -0.338 | **0.479** | 0.383 | 0.313 | 0.017 | 0.897 |
| MO | -0.026 | 0.652 | 0.509 | **0.654** | 0.505 | 0.551 |
| PG | -0.424 | **0.652** | 0.524 | 0.608 | 0.565 | 0.608 |
| SBC | 0.232 | 0.244 | **0.321** | 0.032 | -0.298 | 0.797 |
| UTX | **0.441** | 0.023 | 0.284 | 0.013 | -0.246 | 0.945 |
| VZ | **0.582** | 0.212 | 0.510 | 0.199 | -0.334 | 0.726 |
| WMT | -0.471 | **0.344** | -0.127 | 0.242 | -0.063 | 0.995 |

Notes to Table: For each of the firms in the Dow Jones 30, we present the correlation between on the one hand the realized 1-year beta (REALB), and on the other hand either 180-day forward-looking (FLB) beta, 180-day historical beta (HISTB 180-day), mixed equally weighted forward-looking and historical beta (MIXB), 1-year historical beta (HISTB 1-year) or 5-year historical beta (HISTB 5-year). We also report the average realized beta for each firm over this period. The sample period is January 1, 1996 to December 31, 2003.

**Table 4C**
**Correlation Between 2-year Realized Beta and Predictors**

| | Correlation with REALB (2-year) | | | | | Mean REALB |
|---|---|---|---|---|---|---|
| Tickers | FLB | HISTB (180-day) | MIXB | HISTB (1-year) | HISTB (5-year) | |
| AA | **0.731** | 0.544 | 0.716 | 0.512 | -0.066 | 1.015 |
| AXP | -0.213 | **-0.191** | -0.258 | -0.325 | -0.444 | 1.302 |
| AIG | **0.421** | -0.108 | 0.285 | -0.339 | -0.818 | 1.081 |
| BA | **0.520** | 0.179 | 0.443 | 0.010 | -0.831 | 0.890 |
| CAT | **0.650** | 0.245 | 0.570 | 0.197 | -0.540 | 0.954 |
| JPM | **0.269** | -0.158 | 0.133 | -0.252 | -0.517 | 1.419 |
| C | -0.379 | 0.409 | -0.068 | 0.287 | **0.492** | 1.388 |
| KO | 0.213 | 0.608 | **0.653** | 0.538 | 0.296 | 0.596 |
| DIS | **0.662** | 0.330 | 0.554 | 0.208 | -0.366 | 1.048 |
| DD | **0.592** | 0.207 | 0.531 | 0.105 | -0.505 | 0.842 |
| XOM | 0.245 | 0.236 | **0.294** | 0.148 | -0.580 | 0.631 |
| GE | **0.388** | 0.025 | 0.313 | -0.179 | -0.421 | 1.214 |
| GM | **0.678** | 0.434 | 0.670 | 0.533 | -0.466 | 0.998 |
| HPQ | **0.225** | 0.052 | 0.152 | -0.115 | -0.742 | 1.344 |
| HD | -0.538 | -0.137 | -0.466 | **-0.106** | -0.802 | 1.203 |
| HON | **0.597** | 0.227 | 0.482 | 0.160 | -0.149 | 1.142 |
| INTC | 0.299 | 0.301 | **0.331** | 0.227 | -0.361 | 1.622 |
| IBM | **0.070** | -0.110 | -0.003 | -0.304 | -0.551 | 1.086 |
| JNJ | -0.007 | **0.334** | 0.331 | 0.263 | 0.322 | 0.619 |
| MCD | 0.259 | 0.309 | **0.443** | 0.190 | 0.039 | 0.627 |
| MRK | -0.054 | **0.197** | 0.181 | 0.091 | 0.061 | 0.755 |
| MSFT | **-0.048** | -0.150 | -0.100 | -0.173 | -0.373 | 1.262 |
| MMM | **0.603** | 0.018 | 0.398 | -0.001 | -0.547 | 0.707 |
| PFE | -0.064 | **0.159** | 0.158 | 0.023 | -0.201 | 0.877 |
| MO | 0.208 | 0.554 | **0.598** | 0.525 | 0.247 | 0.521 |
| PG | -0.327 | **0.576** | 0.496 | 0.514 | 0.400 | 0.571 |
| SBC | **0.523** | -0.184 | 0.237 | -0.333 | -0.581 | 0.816 |
| UTX | **0.529** | 0.119 | 0.396 | 0.035 | -0.252 | 0.953 |
| VZ | **0.659** | 0.034 | 0.488 | -0.045 | -0.603 | 0.744 |
| WMT | -0.602 | **0.107** | -0.434 | 0.032 | 0.058 | 0.977 |

Notes to Table: For each of the firms in the Dow Jones 30, we present the correlation between on the one hand the realized 2-year beta (REALB), and on the other hand either 180-day forward-looking (FLB) beta, 180-day historical beta (HISTB 180-day), mixed equally weighted forward-looking and historical beta (MIXB), 1-year historical beta (HISTB 1-year) or 5-year historical beta (HISTB 5-year). We also report the average realized beta for each firm over this period. The sample period is January 1, 1996 to December 31, 2003.

**Table 5**
**Univariate Forecasting Regression of 180-day Realized Beta on 180-day Forward-Looking Beta**

| Tickers | Constant | | FLB | | adj-Rsq | RMSE |
|---|---|---|---|---|---|---|
| AA | 0.101 | (0.913) | 0.916 | (8.155) | 0.366 | 0.339 |
| AXP | 1.326 | (15.927) | -0.018 | (-0.242) | 0.000 | 0.473 |
| AIG | 0.971 | (16.645) | 0.092 | (1.324) | 0.017 | 0.468 |
| BA | 0.477 | (4.123) | 0.445 | (3.693) | 0.138 | 0.355 |
| CAT | 0.620 | (8.172) | 0.368 | (4.631) | 0.136 | 0.302 |
| JPM | 1.243 | (20.282) | 0.154 | (3.132) | 0.078 | 0.540 |
| C | 1.774 | (23.285) | -0.331 | (-5.160) | 0.174 | 0.610 |
| KO | 0.479 | (3.127) | 0.195 | (1.036) | 0.007 | 0.467 |
| DIS | 0.305 | (2.979) | 0.770 | (7.572) | 0.296 | 0.288 |
| DD | 0.564 | (6.115) | 0.339 | (3.630) | 0.075 | 0.310 |
| XOM | 0.469 | (3.712) | 0.201 | (1.078) | 0.008 | 0.363 |
| GE | 0.969 | (24.361) | 0.260 | (6.382) | 0.215 | 0.338 |
| GM | 0.684 | (19.138) | 0.266 | (9.145) | 0.274 | 0.403 |
| HPQ | 0.822 | (11.917) | 0.525 | (8.158) | 0.264 | 0.448 |
| HD | 1.200 | (14.085) | 0.014 | (0.165) | 0.000 | 0.438 |
| HON | 0.657 | (11.695) | 0.509 | (9.142) | 0.386 | 0.364 |
| INTC | 0.992 | (12.097) | 0.501 | (6.956) | 0.347 | 0.491 |
| IBM | 0.936 | (12.218) | 0.145 | (2.284) | 0.044 | 0.352 |
| JNJ | 1.171 | (10.179) | -0.590 | (-4.105) | 0.113 | 0.554 |
| MCD | 0.638 | (7.753) | 0.004 | (0.032) | -0.001 | 0.387 |
| MRK | 1.361 | (10.743) | -0.685 | (-3.931) | 0.117 | 0.513 |
| MSFT | 1.210 | (25.664) | 0.090 | (2.517) | 0.044 | 0.463 |
| MMM | 0.414 | (4.724) | 0.351 | (3.541) | 0.124 | 0.308 |
| PFE | 1.701 | (15.949) | -0.923 | (-7.893) | 0.225 | 0.570 |
| MO | 0.699 | (8.326) | -0.174 | (-2.001) | 0.024 | 0.621 |
| PG | 1.207 | (10.461) | -0.787 | (-5.860) | 0.198 | 0.533 |
| SBC | 0.705 | (8.263) | 0.093 | (0.874) | 0.009 | 0.450 |
| UTX | 0.640 | (9.781) | 0.370 | (4.133) | 0.141 | 0.348 |
| VZ | 0.502 | (12.260) | 0.243 | (5.467) | 0.166 | 0.417 |
| WMT | 1.332 | (17.486) | -0.356 | (-5.128) | 0.145 | 0.398 |

Notes to Table: We report the results of the univariate regression of realized 180-day beta (REALB) on forward-looking 180-day beta (FLB). For each regression, we present the regression coefficients and the adjusted R-squared. The root mean squared error (RMSE) is also reported. The regressions are performed on daily data for the period January 1, 1996 to December 31, 2003. T-statistics are presented in parentheses.

**Table 6**
**Univariate Forecasting Regression of 180-day Realized Beta on 180-day Historical Beta**

| Tickers | Constant | | HISTB (180-day) | | adj-Rsq | RMSE |
|---|---|---|---|---|---|---|
| AA | 0.364 | (5.398) | 0.664 | (9.473) | 0.398 | 0.363 |
| AXP | 1.046 | (7.673) | 0.197 | (2.055) | 0.034 | 0.247 |
| AIG | 0.478 | (5.503) | 0.545 | (6.810) | 0.299 | 0.247 |
| BA | 0.593 | (7.204) | 0.314 | (4.242) | 0.104 | 0.404 |
| CAT | 0.634 | (7.256) | 0.312 | (3.573) | 0.084 | 0.292 |
| JPM | 1.394 | (9.736) | 0.011 | (0.111) | 0.000 | 0.326 |
| C | 0.697 | (5.426) | 0.497 | (5.682) | 0.223 | 0.248 |
| KO | 0.150 | (3.019) | 0.724 | (12.254) | 0.556 | 0.295 |
| DIS | 0.457 | (5.653) | 0.569 | (7.440) | 0.322 | 0.299 |
| DD | 0.417 | (4.925) | 0.507 | (5.824) | 0.252 | 0.279 |
| XOM | 0.252 | (4.032) | 0.574 | (7.624) | 0.360 | 0.305 |
| GE | 0.735 | (4.183) | 0.401 | (2.927) | 0.156 | 0.175 |
| GM | 0.773 | (9.566) | 0.173 | (2.272) | 0.030 | 0.310 |
| HPQ | 0.817 | (7.033) | 0.408 | (5.464) | 0.173 | 0.355 |
| HD | 1.459 | (11.682) | -0.202 | (-2.170) | 0.043 | 0.419 |
| HON | 0.653 | (5.646) | 0.415 | (4.192) | 0.174 | 0.318 |
| INTC | 1.121 | (6.329) | 0.312 | (2.705) | 0.116 | 0.403 |
| IBM | 1.418 | (11.945) | -0.289 | (-2.826) | 0.079 | 0.351 |
| JNJ | 0.193 | (3.314) | 0.688 | (10.850) | 0.465 | 0.323 |
| MCD | 0.300 | (6.413) | 0.523 | (8.015) | 0.309 | 0.245 |
| MRK | 0.327 | (4.353) | 0.581 | (7.992) | 0.348 | 0.371 |
| MSFT | 1.358 | (16.767) | -0.032 | (-0.562) | 0.001 | 0.314 |
| MMM | 0.623 | (11.778) | 0.105 | (1.512) | 0.010 | 0.338 |
| PFE | 0.360 | (4.749) | 0.599 | (9.700) | 0.360 | 0.385 |
| MO | 0.177 | (3.547) | 0.646 | (8.625) | 0.432 | 0.296 |
| PG | 0.178 | (3.432) | 0.682 | (10.485) | 0.471 | 0.302 |
| SBC | 0.635 | (7.521) | 0.193 | (2.032) | 0.037 | 0.404 |
| UTX | 0.901 | (11.760) | 0.046 | (0.798) | 0.002 | 0.399 |
| VZ | 0.611 | (6.906) | 0.154 | (1.353) | 0.024 | 0.307 |
| WMT | 0.505 | (4.757) | 0.489 | (4.329) | 0.244 | 0.227 |

Notes to Table: We report the results of the univariate regression of realized 180-day beta (REALB) on 180-day historical beta (HISTB). For each regression, we present the regression coefficients and the adjusted R-squared. The root mean squared error (RMSE) is also reported. The regressions are performed on daily data for the period January 1, 1996 to December 31, 2003. T-statistics are presented in parentheses.

**Table 7**
**Bivariate Forecasting Regression of 180-day Realized Beta on**
**180-day Forward-Looking Beta and 180-day Historical Beta**

| Tickers | Constant | | FLB | | HISTB (180-day) | | adj-Rsq |
|---|---|---|---|---|---|---|---|
| AA | -0.004 | (-0.036) | **0.592** | **(4.833)** | **0.464** | **(5.723)** | 0.514 |
| AXP | **1.072** | **(7.107)** | -0.050 | (-0.706) | **0.214** | **(2.450)** | 0.038 |
| AIG | **0.388** | **(3.447)** | 0.102 | (1.700) | **0.549** | **(6.612)** | 0.319 |
| BA | **0.268** | **(2.380)** | **0.407** | **(3.404)** | **0.277** | **(3.593)** | 0.218 |
| CAT | **0.461** | **(4.692)** | **0.313** | **(3.569)** | **0.221** | **(2.278)** | 0.175 |
| JPM | **1.508** | **(11.425)** | **0.232** | **(4.352)** | **-0.244** | **(-2.514)** | 0.114 |
| C | **1.088** | **(7.009)** | **-0.236** | **(-3.886)** | **0.398** | **(4.735)** | 0.302 |
| KO | -0.075 | (-0.657) | **0.267** | **(2.229)** | **0.728** | **(12.997)** | 0.570 |
| DIS | 0.111 | (0.951) | **0.533** | **(4.871)** | **0.417** | **(5.392)** | 0.441 |
| DD | 0.149 | (1.428) | **0.331** | **(3.703)** | **0.504** | **(6.078)** | 0.323 |
| XOM | **0.282** | **(2.994)** | -0.045 | (-0.294) | **0.578** | **(7.183)** | 0.360 |
| GE | **0.670** | **(4.872)** | **0.211** | **(4.317)** | **0.281** | **(2.227)** | 0.284 |
| GM | **0.777** | **(9.641)** | **0.303** | **(8.139)** | -0.137 | (-1.379) | 0.288 |
| HPQ | **0.716** | **(7.401)** | **0.426** | **(4.357)** | 0.153 | (1.490) | 0.278 |
| HD | **1.430** | **(10.236)** | 0.032 | (0.388) | **-0.205** | **(-2.184)** | 0.044 |
| HON | **0.605** | **(6.488)** | **0.471** | **(7.473)** | 0.078 | (0.822) | 0.390 |
| INTC | **1.043** | **(6.855)** | **0.537** | **(7.220)** | -0.060 | (-0.506) | 0.349 |
| IBM | **1.277** | **(10.542)** | **0.204** | **(3.510)** | **-0.363** | **(-3.767)** | 0.161 |
| JNJ | **0.444** | **(4.018)** | **-0.263** | **(-2.160)** | **0.644** | **(10.408)** | 0.486 |
| MCD | **0.199** | **(2.341)** | 0.110 | (1.394) | **0.540** | **(8.030)** | 0.320 |
| MRK | **0.619** | **(4.493)** | **-0.303** | **(-2.099)** | **0.527** | **(7.267)** | 0.368 |
| MSFT | **1.330** | **(17.095)** | **0.112** | **(2.807)** | -0.107 | (-1.789) | 0.060 |
| MMM | **0.403** | **(4.432)** | **0.346** | **(3.331)** | 0.022 | (0.273) | 0.124 |
| PFE | **0.748** | **(4.013)** | **-0.337** | **(-2.569)** | **0.494** | **(6.420)** | 0.379 |
| MO | **0.241** | **(2.931)** | -0.066 | (-0.815) | **0.637** | **(8.609)** | 0.435 |
| PG | 0.239 | (1.777) | -0.062 | (-0.578) | **0.661** | **(8.655)** | 0.471 |
| SBC | **0.574** | **(4.756)** | 0.075 | (0.763) | **0.185** | **(1.997)** | 0.043 |
| UTX | **0.708** | **(11.566)** | **0.406** | **(3.643)** | -0.103 | (-1.151) | 0.150 |
| VZ | **0.525** | **(6.898)** | **0.257** | **(5.039)** | -0.048 | (-0.433) | 0.168 |
| WMT | **0.802** | **(5.139)** | **-0.243** | **(-3.450)** | **0.414** | **(3.562)** | 0.305 |

Notes to Table: We report the results of the multivariate regression of realized 180-day beta (REALB) on forward-looking beta (FLB) and 180-day historical beta (HISTB). For each firm, we present the regression coefficients (bolded when significant at the 5% level) and the adjusted R-squared. The regressions are performed on daily data for the period January 1, 1996 to December 31, 2003. T-statistics are presented in parentheses.

**Table 8**
**Regression of Univariate Residuals on Betas**

| | Residuals from HISTB regressed on FLB | | | | | Residuals from FLB regressed on HISTB | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Tickers** | **Constant** | | **Coefficient** | | **adj-Rsq** | **Constant** | | **Coefficient** | | **adj-Rsq** |
| AA | -0.413 | (-3.758) | 0.452 | (4.215) | 0.148 | -0.306 | (-4.276) | 0.355 | (4.945) | 0.179 |
| AXP | 0.047 | (0.563) | -0.047 | (-0.649) | 0.004 | -0.271 | (-2.009) | 0.203 | (2.145) | 0.036 |
| AIG | -0.086 | (-1.589) | 0.102 | (1.710) | 0.029 | -0.574 | (-6.452) | 0.549 | (6.679) | 0.308 |
| BA | -0.352 | (-3.337) | 0.402 | (3.434) | 0.126 | -0.239 | (-3.132) | 0.274 | (3.637) | 0.092 |
| CAT | -0.238 | (-3.135) | 0.290 | (3.495) | 0.092 | -0.189 | (-2.084) | 0.205 | (2.241) | 0.042 |
| JPM | -0.162 | (-2.648) | 0.151 | (3.061) | 0.074 | 0.227 | (1.670) | -0.159 | (-1.742) | 0.025 |
| C | 0.220 | (2.972) | -0.212 | (-3.474) | 0.092 | -0.528 | (-4.278) | 0.358 | (4.343) | 0.139 |
| KO | -0.221 | (-2.044) | 0.266 | (2.225) | 0.031 | -0.493 | (-10.261) | 0.727 | (12.822) | 0.566 |
| DIS | -0.417 | (-3.771) | 0.446 | (3.919) | 0.146 | -0.348 | (-4.249) | 0.349 | (4.390) | 0.172 |
| DD | -0.270 | (-3.241) | 0.331 | (3.710) | 0.095 | -0.421 | (-5.297) | 0.504 | (6.093) | 0.268 |
| XOM | 0.030 | (0.314) | -0.044 | (-0.314) | 0.000 | -0.347 | (-5.399) | 0.559 | (7.216) | 0.344 |
| GE | -0.192 | (-4.990) | 0.191 | (4.686) | 0.137 | -0.314 | (-2.279) | 0.254 | (2.366) | 0.079 |
| GM | -0.206 | (-5.255) | 0.219 | (6.306) | 0.192 | 0.092 | (1.164) | -0.100 | (-1.321) | 0.013 |
| HPQ | -0.277 | (-3.874) | 0.260 | (3.843) | 0.078 | -0.129 | (-1.288) | 0.093 | (1.340) | 0.012 |
| HD | -0.033 | (-0.383) | 0.032 | (0.388) | 0.001 | 0.246 | (1.972) | -0.203 | (-2.186) | 0.044 |
| HON | -0.274 | (-4.583) | 0.308 | (5.296) | 0.171 | -0.056 | (-0.610) | 0.051 | (0.634) | 0.004 |
| INTC | -0.398 | (-4.495) | 0.314 | (4.162) | 0.154 | 0.056 | (0.367) | -0.035 | (-0.341) | 0.002 |
| IBM | -0.211 | (-2.934) | 0.192 | (3.308) | 0.085 | 0.382 | (3.404) | -0.341 | (-3.549) | 0.115 |
| JNJ | 0.201 | (2.035) | -0.240 | (-1.951) | 0.035 | -0.414 | (-7.285) | 0.589 | (9.012) | 0.384 |
| MCD | -0.087 | (-1.364) | 0.107 | (1.374) | 0.014 | -0.342 | (-7.313) | 0.523 | (8.025) | 0.310 |
| MRK | 0.217 | (2.042) | -0.264 | (-1.880) | 0.026 | -0.373 | (-5.395) | 0.460 | (6.589) | 0.248 |
| MSFT | -0.113 | (-2.396) | 0.096 | (2.704) | 0.050 | 0.125 | (1.622) | -0.092 | (-1.763) | 0.014 |
| MMM | -0.261 | (-2.930) | 0.326 | (3.250) | 0.108 | -0.014 | (-0.245) | 0.021 | (0.272) | 0.000 |
| PFE | 0.185 | (1.743) | -0.211 | (-1.901) | 0.018 | -0.276 | (-3.446) | 0.310 | (4.682) | 0.124 |
| MO | 0.057 | (0.752) | -0.064 | (-0.779) | 0.005 | -0.353 | (-7.022) | 0.623 | (8.125) | 0.411 |
| PG | 0.029 | (0.333) | -0.039 | (-0.408) | 0.000 | -0.260 | (-5.185) | 0.409 | (6.453) | 0.211 |
| SBC | -0.067 | (-0.777) | 0.074 | (0.746) | 0.006 | -0.146 | (-1.672) | 0.183 | (1.944) | 0.034 |
| UTX | -0.291 | (-4.402) | 0.354 | (3.908) | 0.130 | 0.085 | (1.125) | -0.089 | (-1.486) | 0.009 |
| VZ | -0.183 | (-4.425) | 0.198 | (4.557) | 0.114 | 0.028 | (0.349) | -0.037 | (-0.379) | 0.001 |
| WMT | 0.202 | (3.085) | -0.222 | (-3.657) | 0.075 | -0.391 | (-3.942) | 0.379 | (3.592) | 0.172 |

Notes to Table: We report the results of regressing the residuals from the univariate regressions on the other predictor. For each firm, we present the regression coefficients (bolded when significant at the 5% level) and the adjusted R-squared. The regressions are performed on daily data for the period January 1, 1996 to December 31, 2003. T-statistics are presented in parentheses.

**Table 9**
**Coefficients of Multiple Correlation for Regressing 180-day Realized Beta on Model-Free Moments**

| Tickers | FLB | $Var_m$ | $Skew_m$ | $Var_i$ | $Skew_i$ | $Var_m$ + $Skew_m$ | $Var_i$ + $Skew_i$ | $Var_m$ + $Skew_m$ + $Var_i$ + $Skew_i$ | $Var_m$ + $Skew_m$ + $Var_i$ + $Skew_i$ + HIST |
|---|---|---|---|---|---|---|---|---|---|
| AA | 0.605 | 0.137 | 0.479 | 0.112 | 0.525 | 0.481 | 0.526 | 0.693 | 0.751 |
| AXP | 0.025 | 0.393 | 0.049 | 0.364 | 0.160 | 0.396 | 0.427 | 0.476 | 0.479 |
| AIG | 0.131 | 0.310 | 0.200 | 0.219 | 0.217 | 0.337 | 0.262 | 0.429 | 0.678 |
| BA | 0.372 | 0.337 | 0.514 | 0.133 | 0.237 | 0.566 | 0.317 | 0.632 | 0.692 |
| CAT | 0.369 | 0.264 | 0.335 | 0.234 | 0.305 | 0.392 | 0.377 | 0.487 | 0.500 |
| JPM | 0.279 | 0.274 | 0.015 | 0.384 | 0.185 | 0.285 | 0.385 | 0.390 | 0.434 |
| C | 0.418 | 0.072 | 0.131 | 0.104 | 0.476 | 0.169 | 0.476 | 0.477 | 0.584 |
| KO | 0.087 | 0.254 | 0.076 | 0.383 | 0.037 | 0.289 | 0.399 | 0.410 | 0.785 |
| DIS | 0.545 | 0.126 | 0.430 | 0.128 | 0.356 | 0.431 | 0.398 | 0.579 | 0.712 |
| DD | 0.275 | 0.226 | 0.345 | 0.533 | 0.382 | 0.378 | 0.650 | 0.743 | 0.746 |
| XOM | 0.089 | 0.291 | 0.223 | 0.393 | 0.116 | 0.334 | 0.459 | 0.545 | 0.736 |
| GE | 0.464 | 0.176 | 0.249 | 0.129 | 0.270 | 0.278 | 0.271 | 0.440 | 0.524 |
| GM | 0.524 | 0.162 | 0.338 | 0.419 | 0.512 | 0.419 | 0.526 | 0.581 | 0.587 |
| HPQ | 0.514 | 0.281 | 0.425 | 0.516 | 0.019 | 0.467 | 0.520 | 0.686 | 0.688 |
| HD | 0.016 | 0.005 | 0.067 | 0.117 | 0.174 | 0.068 | 0.216 | 0.262 | 0.433 |
| HON | 0.621 | 0.215 | 0.594 | 0.344 | 0.407 | 0.600 | 0.454 | 0.692 | 0.699 |
| INTC | 0.590 | 0.031 | 0.228 | 0.565 | 0.441 | 0.243 | 0.629 | 0.673 | 0.683 |
| IBM | 0.211 | 0.038 | 0.284 | 0.271 | 0.043 | 0.302 | 0.286 | 0.380 | 0.514 |
| JNJ | 0.336 | 0.154 | 0.230 | 0.389 | 0.218 | 0.308 | 0.399 | 0.422 | 0.762 |
| MCD | 0.003 | 0.018 | 0.213 | 0.280 | 0.128 | 0.227 | 0.331 | 0.432 | 0.628 |
| MRK | 0.343 | 0.087 | 0.266 | 0.473 | 0.002 | 0.304 | 0.486 | 0.589 | 0.749 |
| MSFT | 0.210 | 0.012 | 0.075 | 0.105 | 0.229 | 0.075 | 0.231 | 0.276 | 0.294 |
| MMM | 0.352 | 0.358 | 0.251 | 0.237 | 0.199 | 0.398 | 0.323 | 0.452 | 0.454 |
| PFE | 0.475 | 0.007 | 0.303 | 0.329 | 0.116 | 0.309 | 0.346 | 0.601 | 0.746 |
| MO | 0.158 | 0.172 | 0.125 | 0.431 | 0.069 | 0.238 | 0.451 | 0.465 | 0.706 |
| PG | 0.445 | 0.128 | 0.259 | 0.489 | 0.235 | 0.320 | 0.543 | 0.651 | 0.775 |
| SBC | 0.097 | 0.325 | 0.215 | 0.106 | 0.438 | 0.354 | 0.446 | 0.587 | 0.612 |
| UTX | 0.377 | 0.062 | 0.234 | 0.193 | 0.267 | 0.234 | 0.288 | 0.345 | 0.349 |
| VZ | 0.409 | 0.174 | 0.049 | 0.361 | 0.484 | 0.194 | 0.495 | 0.504 | 0.512 |
| WMT | 0.382 | 0.443 | 0.504 | 0.042 | 0.078 | 0.607 | 0.085 | 0.674 | 0.703 |

Notes to Table: For each of the firms in the Dow Jones 30, we present the coefficient of multiple correlation for the regression of the realized 180-day beta (REALB) on various combinations of the forward looking moments: the market variance $Var_m$, the market skewness $Skew_m$, the firm variance $Var_i$, and the firm skewness $Skew_i$. We also present results for a regression on the four moments and the 180-day historical beta, as well as for a regression on forward-looking beta. The sample period is January 1, 1996, to December 31, 2003.