Liquidity in Asset Pricing
A (partial) survey

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(See Y. Amihud, H. Mendelson and L. Pedersen, *Liquidity and Asset Prices* NOWpublishers.com)

Illiquidity is

1. Market impact cost
   Kyle (1985): $P(y) = \mu + \lambda y$, (y = trade size).
   $\boldsymbol{\lambda}$ = f(information variance$^{(+)}$, noise variance$^{(-)}$)
2. Bid-ask spread
   Stoll (1978) -- risk, Amihud-Mendelson (1980) -- inventory, Bagehot (1977), Glosten-Milgrom (1985) -- information.
3. Search and delay costs, price and execution risk.
4. Commissions, fees, cost of time, etc.

There are **tradeoffs**.

Illiquidity reflects **real costs** of trading. What is its effect?

Cross-section effects of liquidity

<u>The cross-section effect of liquidity</u> – Amihud and Mendelson, JFE 1986:

<u>Securities</u>: transaction costs (%) paid once (at the sale), S_j, dividend stream, d_j.

<u>Investors</u>: expected holding time to liquidation, $1/\boldsymbol{\mu}_k$.

The spread-adjusted return is

$r_{kj} = d_j/V_j - \mu_k \cdot S_j$. (V = value)

For a given vector of V, investors maximize their return.

$r_k^* = Max_j\, r_{kj}$, the net (spread-adjusted) return of investor k.

In bidding for assets, for given d_j and S_j, winners pay the highest V_j ,

$V_j^* = d_j /(r_k^* + \mu_k \cdot S_j)$, which results in the **gross return** on asset *j* being

$$d_j/V_j^* = min_k \{ r_k^* + \mu_k \cdot S_j \}.$$

The equilibrium value V_j^* is given by

$$V_j^* = d_j/r_k^* - \mu_k \cdot S_j \cdot V_j^*/r_k^* .$$

V_j^* = present value of dividend stream – present value of illiquidity costs.

In equilibrium, we prove that …

1. Assets with **higher** S_j are held by investors with **lower** $\boldsymbol{\mu_k}$ (higher $1/\boldsymbol{\mu_k}$ holding period) →**liquidity clientele.**
2. Gross return: an **increasing** and **concave** function of the bid-ask **spread**.
3. Net return: an increasing function of the bid-ask spread.
4. Asset value: **decreasing** and **convex** in the spread (relative to liquid value)

Tests: using 49 (7x7) stock portfolios, by β and S, updating the grouping every year and looking at returns one year ahead. Data: 1961-1980.

Example:

$$R_i = 0.0065 + 0.01 \cdot \beta_i + 0.0021 \cdot ln(S_i)$$

Detailed estimation: piecewise-linear function (i for spread, j for β)

$$R_{py} = a_0 + a_1^* \beta_{py} + \Sigma_i^7 b_i S^{ei}_{py} + \Sigma_i^7 \Sigma_j^7 c_{ij} DP_{ij} + year\ dummies$$

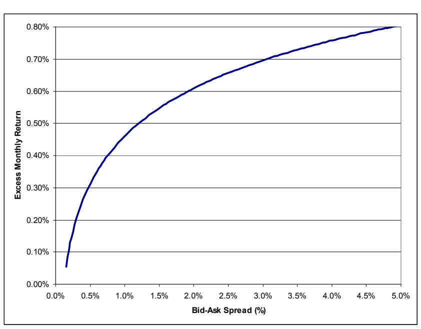
Where S^e is the spread adjusted for the group's mean, DP_{ij} are portfolio dummies, reflecting the effect of the portfolio mean spread.

Result: $b_i > 0$ and **decreases** in *i*. $\Sigma_j^7 c_{ij}$ **increases** in *i*.

→ *R* **is increasing and concave in** *S*. (Also increasing in β.)

The spread-effect subsumes the **size** effect.

***T**he expected excess monthly return (or yield) on a stock as a function of the stock's bid-ask spread.*



The effects of bid-ask spread, β and size in **Nasdaq**, 1976-90. (**Eleswarapu,** JF 1997)

Spread portfolios are updated at the beginning of the year.

Period	β	Bid-ask spread	Log(Size)
All months	-0.0026 (0.53)	0.0394 (3.05)	0.0004 (0.57)
January	0.0633 (2.94)	0.1749 (2.13)	-0.0048 (1.65)
Non-January	-0.0086 (1.83)	0.0271 (2.34)	0.0008 (1.14)

(*t* – statistic in parentheses).

Results are stronger than for the NYSE, where many transactions take place within the spread; on Nasdaq, transactions are more likely at the quotes.

The effect of turnover -- Datar, Naik and Radcliffe, JFM 1998.

Liquidity is measured by turnover = volume / number of shares.

Turnover proxies for μ in Amihud-Mendelson (1986), 1/ holding period.

Higher μ (lower $1/\mu$) is correlated with greater liquidity (unobserved).

Variables:	All months		Excl. January	
Turnover:	-.04	(8.58)	-.04	(7.91)
Book/Market:	.14	(5.97)	.08	(3.29)
log(Size):	-.05	(4.65)	.02	(1.60)
β:	-.37	(5.76)	-.05	(6.84)

Results persist over subperiods.

Conclusion:

Liquidity---higher turnover---is associated with lower expected returns.

The effect of other illiquidity measures - Brennan-Subrahmanyam, JFE 1996.

Illiquidity is estimated from a Kyle-inspired model (Glosten-Harris 1988):

$$\Delta P_t = \lambda Q_t + \psi(D_t - D_{t-1}) + u_t .$$

Q is the *signed* trade size, D is the ask/bid indicator.

Proportional cost components: $\lambda Q/P$ (variable cost) and ψ/P (fixed cost).

Model: pooled time series & cross section, 25 portfolios (by size and λ):

$$R^e_{it} = \alpha + \Sigma_k \gamma_k {}^* L_{ik} + \beta_i {}^* RM_t + \delta_i {}^* SMB_t + \theta_i HML_t + e_{it} .$$

$L_{ik} = \lambda Q/P$, ψ/P , their squared terms, 1/P, SIZE.

Results:

Return is increasing and concave in $\lambda Q/P$ and increasing and convex in ψ/P .

Effect of illiquidity measure on average return – Amihud, JFM 2002.

Problem with most measures of illiquidity: UNAVAILABILITY.

Need a proxy measure of illiquidity, easily obtained from daily data:

ILLIQ = average of daily (|return| / $volume).

Intuition: proxies market impact. What is the effect on return of a given trading volume. (Signed volume is unavailable.)

Strong positive relation with both $\lambda Q/P$ and ψ/P .

Hasbrouck (2003): Spearman (Pearson) correlation of ILLIQ with λ (modified) is 0.74 (0.47). For portfolios: correlation is 90%.

The effect of ILLIQ on stock expected return (constant omitted), monthly, 1964-1997

Variable	All months	Excl. January	1964-1980	1981-1997
BETA	0.217 (0.64)	0.260 (0.79)	0.297 (0.59)	0.137 (0.30)
ILLIQMA	0.112 (5.39)	0.103 (4.91)	0.135 (3.69)	0.088 (4.56)
LnSIZE	-0.134 (3.50)	-0.073 (2.00)	-0.217 (3.51)	-0.051 (1.14)
StdDev of Return	-0.179 (1.90)	-0.274 (2.89)	-0.136 (0.96)	-0.223 (1.77)
DIVYLD	-0.048 (3.36)	-0.063 (4.28)	-0.075 (2.81)	-0.021 (2.11)
R100	0.888 (3.70)	1.335 (6.19)	0.813 (2.33)	0.962 (2.92)
RYR	0.359 (3.40)	0.439 (4.27)	0.324 (2.04)	0.395 (2.82)

Restricted stock. Price discount of about 1/3. (Silber 1991)

P/E ratio and liquidity (Loderer & Roth, JEF 2003)
Expected returns can be obtained from stock P/E.

The model estimates the effect of illiquidity (relative bid-ask spread) on the P/E ratio, controlling for dividend payout and expected earnings growth (from analysts' reports).

Result: discount of over 20% for a median-spread stock compared to zero-spread stock (1995-2001).

Other models of the effect of liquidity:
Constantinides, JPE 1986
The investor is maximizing expected CRRA utility of consumption.
A riskless and a risky asset. The risky asset is costly to trade.
The riskless asset pays continuous dividend.
The value of the risky asset varies continuously.
The investor wants to maintain a constant ratio of the assets.
This requires continuous trading. Costly.
Solution: a no-trade zone around the optimal ratio.
Greater volatility, higher trading costs → wider no-trade zone.
Cost of illiquidity: the monetary equivalence of the loss in expected utility of deviating from the optimal ratio. Second order.

Huang, JET 2003 -- Risk due to illiquidity.
Explains the high liquidity premium as reflecting an additional risk.
The net return is risky even if the asset's dividend is riskless, due to time to uncertain liquidation.

Model:
* Risk-averse investors maximize expected CRRA utility of final consumption.
* Two riskless assets – one liquid, one illiquid (proportional transaction costs), paying continuous dividend.
* Initial endowment. Possibly a steady income.
* Short sale is prohibited. Borrowing constraint.
* Random liquidity shocks (Poisson arrival) that force sale.

Investors need to accumulate wealth to accommodate liquidity shocks.
Early negative liquidity shocks with no borrowing – strong negative shock to consumption and to utility.
→ The net-of-transaction-costs return of the illiquid asset is risky.
Optimal policy: invest in a combination of the two assets for some range of liquidity premium.
Wider range for higher risk aversion, trading costs and prob. of shock arrival.

No borrowing constraint → liquidity premium equals the PV of trading costs, as in the case of fixed horizon (equating net returns).
Borrowing constraint → higher liquidity premium, reflecting the risk.
Older investors are more likely to hold the illiquid asset.
Liquidity premium is higher for smaller relative supply of liquid asset.

Liquidity risk

<u>Exposure to market liquidity risk</u> – Pastor and Stambaugh, JPE 2003.

Greater exposure to liquidity risk → higher expected return.

Liquidity measure $\gamma_{i,t}$: obtained from estimating for each month *t* the daily model

$$R^e_{i,d} = \theta_i + \Phi_i * R_{i,d-1} + \gamma_i * sign(R^e_{i,d-1}) * V_{i,d-1} + e_{i,d} .$$

$R^e_{i,d}$ = excess return over $R_{m,d}$. V = volume in $.

$\gamma_i < 0$, the liquidity "cost" = return reversal after trading $1 million of stock *i*.

larger return reversal (more negative γ) → larger cost, lower liquidity.

γ_i is more negative for less-liquid stocks.

The model is estimated each month for each stock (daily data).

Market liquidity – γ_t – is the average across stocks in each month.

$$\Delta\gamma_t = (m_t/m_1)\ \Sigma_i\ (\gamma_{i,t} - \gamma_{i,t-1})\ /N_t\ ,$$ (average across stocks)

m_t = total $ value at the end of month *t-1* of stocks included in month *t*.

$$\Delta\gamma_t = a + b*\Delta\gamma_{t-1} + c*(m_t/m_1)*\gamma_{t-1} + L_t\ .$$ ◄◄

A stock's exposure to (co-movement with) market liquidity is measured by β^L_i :

$$R_{i,t} = \beta^0_i + \beta^L_i * L_t + \beta^M_i * RM_t + \beta^S_i * SMB_t + \beta^H_i * HML_t + e_{i,t}\ .$$

The analysis uses predicted β^L_i to reduce EIV (using 7 variables).

10 β^L portfolios. Average β^L ranges from -5.8 to +2.5 (1966-1999).

β^L_i is positively correlated with size and liquidity.

Stocks are sorted into 10 portfolios by their (predicted) β^L_i .

<u>Results</u>: Liquidity risk is priced:

α from market or FF models is strongly increasing in β^L_i .

<u>Pricing liquidity risk</u> (liquidity βs) -- Acharya and Pedersen, JFE 2004.

			<u>sign of cov</u>
$Cov(R_{it} - C_{it}, RM_t - CM_t)$ =		$Cov(R_{it}, RM_t)$	Ordinary
	┌	+ $Cov(C_{it}, CM_t)$	(+) Stock liquidity risk
3 liquidity betas	│	- $Cov(R_{it}, CM_t)$	(–) Exposure to mkt liquidi
	└	- $Cov(C_{it}, RM_t)$	(–) Stock liquidity response to RM

Dividing by $Var(RM_t - CM_t)$ yields 4 βs.

<u>Pricing model</u>:

$$E_{t-1}(R_{it} - Rf_t) = E_{t-1}(C_i) + \lambda_{t-1}(\beta_1 + \beta_2 - \beta_3 - \beta_4)$$

where

$$\lambda_{t-1} = E_{t-1}(RM_t - CM_t - Rf).$$

Test results are consistent with the model's predictions. (Tests use cost =*ILLIQ*.)

Stock returns are increasing significantly in the sum of βs: $\lambda_{t-1} > 0$.

The cross-section coefficients on the liquidity βs all have the predicted signs.

Martinez, Nieto, Rubio and Tapia (*IREF* 2005)

Pricing kernel: $E_{t-1}[M_t(1+R_{jt})] = 1$

Assume $M_t = d_0 + d_1 RM_t + d_2 L_t$, L = replicating liquidity-based portfolio

The *conditional* version is

$M_t = d_{0,t-1} + d_{1,t-1} RM_t + d_{2,t-1} L_t$

with $d_{i,t-1} = d_{k,0} + d_{k,1} \log(B/M_{t-1})$. B/M = aggregate B/M ratio

Estimation model, cross-section:

$E(R_j) = c_0 + c_1 \beta_{j,m} + c_2 \beta_{j,bm} + c_3 \beta_{j,mbm} + c_4 \beta_{j,L} + c_5 \beta_{j,Lbm}$.

$\boldsymbol{\beta}_{j,mbm} = cov(R_{jt}, bm_{t-1}RM_t)/var(bm_{t-1}RM_t)$. $\boldsymbol{\beta}_{j,Lbm} = cov(R_{jt}, bm_{t-1}L_t)/var(bm_{t-1}L_t)$.

The others are the return beta with the respective return index.

L_t are: high-minus-low spread, Pastor-Stambaugh's L, Amihud's *ILLIQ*.

Estimation model, cross-section (Fama-Macbeth method):

$E(R_j) = c_0 + c_1 \beta_{j,m} + c_2 \beta_{j,bm} + c_3 \beta_{j,mbm} + c_{54} \beta_{j,L} + c_5 \beta_{j,Lbm}$.

c_4 and c_5 estimate the effect of the unconditional and conditional liquidity βs.

The model is estimated on Spanish stock data, 1993-2000.

Results:

The coefficients of both unconditional and conditional liquidity βs are negative and significant when using L = ILLIQ.

(The liquidity βs are negative on overage).

In some specifications, the conditional liquidity β is more important than the unconditional β.

Effects of liquidity *over time*

Does a change in the liquidity of a security affect its value?

(the security's "fundamentals" remain unchanged)

Amihud, Mendelson and Wood, JPM 1990:

The Oct. 19, 1987 Crash.

There was a re-evaluation of the market liquidity in the week before the crash.

Sharp declines when program trading kicked in.

Liquidity lower than previously thought → lower stock value.

Test and evidence:

1) Stocks whose liquidity declined more suffered greater price declines.

2) Stocks whose liquidity recovered more by October-end had greater price increase.

Do changes in individual stocks' liquidity affect their values?

(the security's "fundamentals" remain unchanged)

Evidence:

Listing on an exchange (Kadlec & McConnell, JF 1994)

But: (i) self-selection; (ii) not a pure liquidity event.

Implications:

If the answer is positive, liquidity-increasing policies – both by regulators and issuers – are value-increasing.

Effects of changes in liquidity --Amihud, Mendelson & Lauterbach, JFE 1997:

Study the effects of a change in trading method on the Tel Aviv Stock Exchange.

Stocks were moved in batches from a once-a-day call auction to a semi-continuous "variable price" trading.

Process: executive committee selected the stocks.

Criterion: "high marketability" (volume, size), but experimenting.

Transfer is announced, then done a few days later.

Important: The company is not consulted, i.e.,

The decision is exogenous to the company.

→ No self-selection effect on the results.

This is a pure microstructure event.

Data and Methodology

17 transfer events of 120 stocks, 1988-1994.

CAR is calculated for Announcement-5 to Transfer+30.

Estimation period is T+31 to T+160.

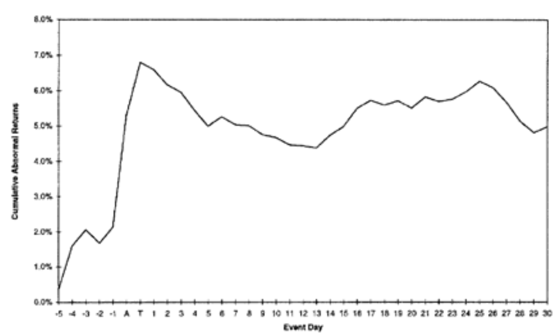
The post-event estimation avoids selection-induced bias.

By the liquidity hypothesis: increased liquidity → higher value.

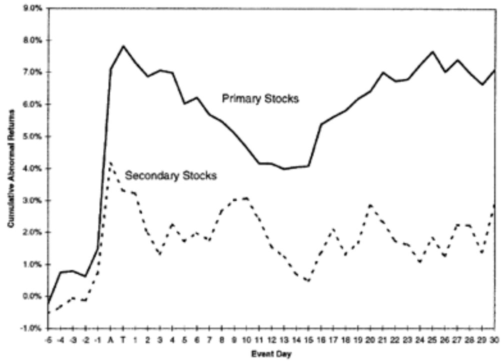
Also, higher liquidity

(i) improves efficiency,

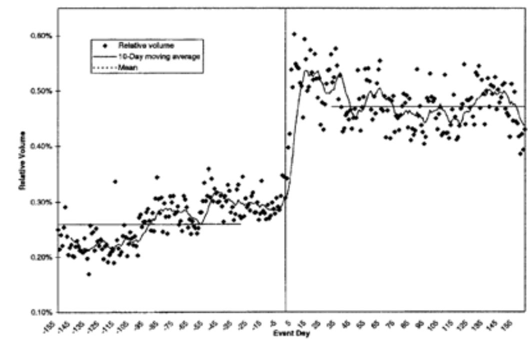
(ii) reduces 'noise' volatility.



Cumulative abnormal returns on stocks transferred from the auction trading method to variable-price trading method.



Spillover (externality) effect:
CAR on stocks moving to the variable-price trading method and on stocks of the same companies not moving to the new method.



Effect on liquidity:
Relative volume on stocks moving from the call method to the v-price method.

<u>Efficiency improvement</u>:

higher liquidity →

1) Faster adjustment of prices to information.

$$R_{jt} = \alpha_j + \beta_j \cdot RM_t + \beta^{\ell}_j \cdot RM_{t-1} + \varepsilon_{jt}.$$

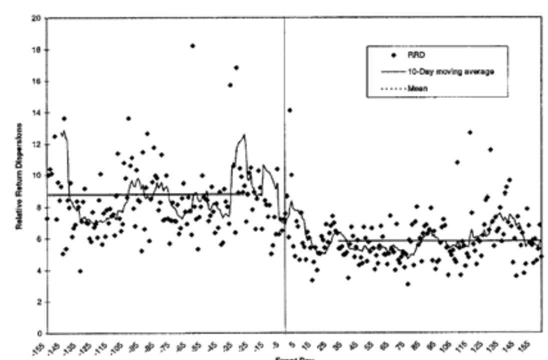
Lagged price adjustment to the market under the C-Method.

The lag disappears under the V-Method.

Sum of *βs* remains the same.

2) Reduced residual variance (partly noise).

The declines in (abs) β^{ℓ}_j and in residual volatility are significantly greater for stocks with greater increase in liquidity (volume).



Effect on volatility:
Residual return dispersion on stocks moving to the variable-price method.

<u>The effect of liquidity on returns over time</u> - Amihud, JFM 2002:

Excess return = *RM-Rf* is considered “risk premium.”

<u>Equity premium puzzle</u>: the mean (*RM-Rf*) is higher than what is expected by reasonable risk aversion parameters.

Proposition: (*RM-Rf*) reflects both risk and illiquidity premia.

→ Expected excess return should rise in expected illiquidity.

Evidence: Illiquidity is highly autoregressive.

→ Stock price should decline in unexpected illiquidity.

ILLIQ is autoregressive:

$$lnAILLIQ_t = c_0 + c_1 \cdot lnAILLIQ_{t-1} + v_t .$$

Expected excess return rises in expected illiquidity:

$$E(RM – Rf)_t = f_0 + f_1 \cdot lnAILLIQ_t^E = g_0 + g_1 \cdot lnAILLIQ_{t-1} .$$

*Un*expected return should decline in *un*expected illiquidity:

$$(RM – Rf)_t = g_0 + g_1 \cdot lnAILLIQ_{t-1} + g_2 \cdot lnAILLIQ_t^U + w_t .$$

Hypotheses: $g_1 > 0$, $g_2 < 0$.

Expected *ILLIQ* <u>raises</u> expected return. *Un*expected *ILLIQ* <u>lowers</u> current prices.

<u>Two effects</u>:

1) Liquidity effect – the same for all size stocks.

2) Substitution effect (flight to liquidity) – Substitution from small to large stocks.

The effect of expected and unexpected illiquidity on excess annual stock returns, 1964-1997.

	RM-Rf	Excess returns on size-based portfolios				
		RSZ_2 -Rf	RSZ_4 -Rf	RSZ_6 -Rf	RSZ_8 -Rf	RSZ_{10}-Rf
Constant	14.740 (4.37)	19.532 (5.12)	17.268 (5.04)	14.521 (4.32)	12.028 (3.55)	4.686 (1.58)
$LnAILLIQ_{y-1}$	10.226 (2.74)	15.230 (3.92)	11.609 (3.31)	9.631 (2.74)	7.014 (1.84)	-0.447 (0.14)
$LnAILLIQ_y^U$	-23.567 (4.52)	-28.021 (3.91)	-24.397 (3.63)	-20.780 (3.41)	-18.549 (3.50)	-14.416 (3.39)
R-squared	0.512	0.523	0.450	0.435	0.413	0.249
D-W	2.55	2.42	2.64	2.47	2.39	2.28

(Jones (WP 2003) examines the effect of lagged bid-ask spread on the Dow Jones Index, 1900-1998 (not including the unexpected effect). Similar results.)

The effect of expected and unexpected illiquidity on excess monthly stock returns, 1964-1997.

	RM-Rf	Excess returns on size-based portfolios				
		RSZ_2 -Rf	RSZ_4 -Rf	RSZ_6 -Rf	RSZ_8 -Rf	RSZ_{10} –Rf
Constant	-3.876 (1.97)	-4.864 (2.03)	-4.335 (2.12)	-4.060 (2.13)	-3.660 (2.05)	-1.553 (0.99)
$LnMILLIQ_{m-1}$	0.712 (2.120)	0.863 (2.110)	0.808 (2.33)	0.761 (2.36)	0.701 (2.30)	0.319 (1.18)
$LnMILLIQ_m^U$	-5.520 (4.42)	-6.513 (4.53)	-5.705 (4.34)	-5.238 (4.12)	-4.426 (4.04)	-3.104 (3.38)
$JANDUM_m$	5.280 (4.20)	8.067 (5.03)	5.446 (4.08)	4.232 (3.45)	3.000 (2.64)	1.425 (1.47)
R-squared	0.144	0.188	0.140	0.119	0.089	0.049
D-W	1.98	1.99	1.96	1.99	2.03	2.14

Liquidity and bond yields

Liquidity and Treasury securities yields - Amihud and Mendelson, JF 1991:

Treasury bills and notes with less than 6 months to maturity have **identical** cash flows, but bills are **more liquid** (they are "**on the run**").

489 note, each paired with 2 bills whose maturities straddle the notes' maturities. (37 days during April-Nov 1987).

Average maturity: 97.4 days. (Excluded: maturity < 30.)

Results:	Spread (%)	yield (%)	Brokerage fees
Notes	0.03	6.52	\$78.125/\$1mil
Bills	0.0078	6.09	\$12.5/\$1mil

If there are in addition higher fixed-costs in notes trading, the yield differential should be greater for short maturities.

Results: ΔYield = const. + 12.03·(1/maturity) - 0.014·Coupon

The flight to liquidity - Amihud, WP 2003

The **note-bill yield spread** is affected by liquidity needs.

(This effect varies and declines over time.)

Model: $SPRD_t^U = \alpha + \beta RM_t + \gamma |RM_{t-1}| + e_t$

$SPRD_t$: The yield spread (matching bill with 2 two-yr notes).

$SPRD_t^U$: Residuals from applying an ARMA(2,2) model to $SPRD_t$. R^2 = 0.64.

RM_t: Market return (equally-weighted), from Friday of previous week to the day $SPRD_t$ is estimated (usually Friday). Period: 7/6/1984 - 11/15/1996.

Hypothesis: $\beta < 0$, $\gamma > 0$.

		α	RM_t	$\lvert RM_{t-1}\rvert$	R^2
All	(644)	-0.004	-1.264 (3.47)	0.893 (3.04)	0.055
$RM > 0$	(452)	0.002	-1.070 (2.17)	0.650 (1.79)	0.017
$RM < 0$	(192)	-0.025	-2.602 (5.45)	0.640 (1.01)	0.147
Exclude tails (RM_{t-1} +/-2%, 596)		-0.003	-0.926 (2.42)	0.532 (1.31)	0.013
Two subperiods					
Subperiod I	(321)	-0.013	-1.333 (3.57)	1.388 (3.03)	0.090
Subperiod II	(323)	0.003	-0.549 (1.64)	0.585 (1.52)	0.014

[Std.Dev.($SPRD_t^U$) = 0.115 and 0.065 in subperiod I and II.]

Corporate bonds:

- The yield spread is considered a risk and default premium.
- Some studies show: corporate bonds have high Sharp ratios.
- Corporate bonds are usually less liquid than Treasury bonds, especially the high-yield ones.

→ The yield spread may reflect both risk\default premium and illiquidity premium.

Problem: liquidity of corporate bonds is hard to measure.

Liquidity costs and corporate bond yields - Chen, Lesmond & Wei, WP 2003:
Use Lesmond, Ogden & Trzcinka method (RFS 1999).
"True" return behaves as

$$R^*_t = \beta 1 * Duration_t * \Delta Rf_t + \beta 1 * Duration_t * \Delta S\&P\ Index_t + u_t$$

We observe actual return R_t

Let $-\alpha_1$ and α_2 be the effective sell and buy side costs (approx. half-spread).

If the bond price should change but it does not, it is because transaction costs exceed the would-be price change.
→ the width of the no-trade band, $\alpha_2 - \alpha_1$, provides information on trading costs.

Method:
Use information on no-trade days.
Estimate α_1 and α_2 by the limited dependent variable method.
Result: $\alpha_2 - \alpha_1$ increases with maturity and with the decline in the bond rating.

$\alpha_2 - \alpha_1$ in b.p. by rating, short-term:	AAA – 8,	BBB – 31,	CCC – 606.
$\alpha_2 - \alpha_1$ in b.p. by maturity for AAA bonds: Short – 8,		Medium – 25,	Long – 59.

Liquidity effect:
Estimate a cross-section model of bond yield spreads as a function of $\alpha_2 - \alpha_1$ (and other variables: maturity, age, amount outstanding, rating, volatility).
Result: $\alpha_2 - \alpha_1$ has a positive and highly significant effect on yield spread.
Liquidity alone explains 13% (20%) of the cross-sectional variation of investment (speculative) grade bonds.

Bond yields and liquidity betas – deJong & Driessen, 2005.

Use two illiquidity indices:
(1) Illiquidity of stocks
(2) Illiquidity of Treasury bonds.

Result:
Higher liquidity beta (= greater exposure to these illiquidity factors, especially to (2))
→ higher bond yields.

Other examples:

1. Similar **options**, one liquid and one illiquid.
   The illiquid options have lower implied volatility (cheaper).

2. Difference in price between ADRs and foreign stocks.
   The ADR premium is an increasing function of the ADR liquidity in the US (relative to that of the foreign stock).

3. Closed-end country funds discount is smaller if the fund liquidity is greater (relative to the foreign market's liquidity).

4. Hedge funds: the return on a portfolio of funds with lock-up period is higher than on funds without lockup.

Conclusion:

Liquidity is an important determinant of securities prices.

The higher the liquidity – the higher the price (and the lower the required or expected return).