Testing Portfolio Efficiency with Conditioning Information

Wayne E. Ferson
Boston College
Andrew F. Siegel*
University of Washington

First draft July 28, 2000, this revision: February 19, 2006

We develop asset pricing models' implications for portfolio efficiency when there is conditioning information in the form of a set of lagged instruments. A model of expected returns identifies a portfolio that should be minimum variance efficient with respect to the conditioning information. Our tests refine previous tests of portfolio efficiency, using the conditioning information optimally. We reject the efficiency of all static or time-varying combinations of the three Fama-French (1996) factors with respect to the conditioning information and also the *conditional* efficiency of time-varying combinations of the factors, given standard lagged instruments.

*Wayne E. Ferson is the Collins Chair in Finance, Boston College, 140 Commonwealth Ave., Chestnut Hill, MA. 02467 and a Research Associate, National Bureau of Economic Research [phone: (617)552-6431, fax: 552-0431, www2.bc.edu/~fersonwa, email wayne.ferson@bc.edu]. Andrew F. Siegel is the Grant I. Butterbaugh Professor of Finance and Management Science and Adjunct Professor of Statistics, University of Washington, Box 353200, Seattle, WA. 98195-3200 [phone: (206)543-4476, fax: 685-9392, e-mail asiegel@u.washington.edu]. This paper has benefited from comments of an anonymous referee and from Tom George, Jay Shanken and from workshops at the 2000 Pacific Northwest Finance Conference, the 2001 Western Finance Association, the Thirteenth Annual Conference on Financial Economics and Accounting/Fifth Maryland Finance Symposium (2002), the 2003 American Finance Association and at the following Universities: Arizona, CUNY Baruch, Iowa, McGill, New York University, Penn State, Toronto, Washington, and Wisconsin.

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Testing the mean variance efficiency of a given portfolio has long been an important topic in empirical asset pricing, since most asset pricing models say that particular portfolios are efficient.[1] Classical efficiency tests, as studied by Gibbons (1982), Jobson and Korkie (1982), Stambaugh (1982), MacKinlay (1987), Gibbons, Ross and Shanken (1989) and others, ask if a tested portfolio lies "significantly" inside a sample mean variance boundary. These studies form the boundary from fixed-weight combinations of the tested asset returns. However, many studies in asset pricing now condition on predetermined variables to model conditional expected returns, correlations and volatility, and portfolio weights may be functions of the predetermined variables. This paper studies tests of portfolio efficiency in the presence of such conditioning information.

We develop a new framework for testing asset pricing theories in the presence of conditioning information. The framework is based on "unconditional" efficiency as defined by Hansen and Richard (1987). An unconditionally efficient portfolio, as defined by Hansen and Richard, uses the conditioning information to minimize the unconditional variance of return for its unconditional mean. We refer to this efficiency concept, using more descriptive language, as efficiency *with respect to the information*, Z. We show that when the model of economic interest implies $E(mR|Z) = 1$, where Z is a set of observable lagged instruments, then particular specifications for m imply that certain portfolios should be efficient with

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[1] The Capital Asset Pricing Model (CAPM, Sharpe, 1964) implies that a market portfolio should be mean variance efficient. Multiple-beta asset pricing models such as Merton (1973) imply that a combination of the factor portfolios is minimum variance efficient (Chamberlain, 1983; Grinblatt and Titman, 1987). The consumption CAPM implies that a maximum correlation portfolio for consumption is efficient (Breeden, 1979). More generally, any stochastic discount factor model implies that a maximum correlation portfolio for the stochastic discount factor is minimum variance efficient (e.g., Hansen and Richard, 1987).

respect to Z. Since different expected return models imply different specifications for m, we can test the economic models by testing the efficiency hypothesis. We develop the new framework by analogy with the classical tests of portfolio efficiency when conditioning information is ignored. Along the way, we present generalizations for a number of earlier results.

A central empirical issue in this paper is whether optimal "dynamic strategies" can improve on the unconditional Sharpe ratio. For example, some hedge funds claim that by moving in and out of assets based on conditioning information, they can achieve unconditional Sharpe ratios far in excess of static investments. Recent academic studies use conditioning information to expand the set of available returns with particular dynamic strategies. For example, the "factors" or assets' returns may be multiplied by lagged instruments, as in Shanken (1990), Hansen and Jagannathan (1991), Cochrane (1996), Jagannathan and Wang (1996) or Ferson and Schadt (1996). This "multiplicative" approach corresponds to dynamic strategies whose portfolio weights are linear functions of the lagged instruments. In this paper the weights may be any well-behaved functions of the given conditioning information. This expands the set of portfolio returns to the maximum possible extent, thereby using the conditioning information optimally.

The primary empirical motivation for our refinement of the way conditioning variables are employed is to use the information optimally. Recent evidence calls into question the usefulness of standard lagged instruments, once bias and sampling errors are accounted for (e.g. Ghysels (1997), Carlson and Chapman (2000), Goyal and Welch (2003, 2004), Simin

(2005), Ferson, Sarkissian and Simin, 2003). We find that when similar variables are used optimally they do have information that enhances returns and efficiency tests.

A second motivation for our approach is tractability. In the standard multiplicative approach, with N asset returns and L lagged instruments, a $NL \times NL$ covariance matrix must be inverted. With our approach the matrices are $N \times N$. A third motivation is robustness. As discussed below, our approach should be robust to certain misspecifications.

We find that the multiplicative approach, using standard instruments and adjusting for sampling errors, typically has no more ability to reject models than tests that ignore the conditioning information altogether. Our tests that use the same information efficiently perform better. We find that the new tests can reject efficiency in settings where traditional tests do not.

The rest of the paper is organized as follows. Section 1 further motivates tests with respect to conditioning information and presents the main ideas. Section 2 develops the specific tests. The data are described in Section 3 and section 4 presents the main empirical results. The robustness of the results is addressed in Section 5. Section 6 concludes the paper.

1. Asset Pricing, Portfolio Efficiency and Conditioning Information

Most empirical tests of asset pricing models follow from the fundamental valuation equation:

$$E\{m_{t+1}R_{t+1}|Z_t\} = \underline{1}, \tag{1}$$

where R_{t+1} is an N-vector of test asset gross returns, Z_t is the *conditioning information*, a vector of observable variables at time t, m_{t+1} is a *stochastic discount factor*, and $\underline{1}$ is an N-vector of ones. An asset pricing model implies a specification for the stochastic discount factor, and a common approach to testing an asset pricing model is to examine necessary conditions that follow from (1). For example, multiplying both sides of (1) by the elements of Z_t and then taking the unconditional expectations leads to a *multiplicative approach*:

$$E\{m_{t+1}(R_{t+1} \otimes Z_t)\} = E\{\underline{1} \otimes Z_t\}. \tag{2}$$

The moment conditions in Equation (2) ask the stochastic discount factor to "price" the dynamic strategy payoffs, $R_{t+1} \otimes Z_t$, on average (or "unconditionally"), where $E\{\underline{1} \otimes Z_t\}$ are the average prices. However, the multiplicative approach in Equation (2) captures only a portion of the information in Equation (1). By using "the right" functions of Z_t we can do better.

Equation (1) is equivalent to Equation (3), holding for *all* bounded integrable functions *f*(.):

$$E\left\{m_{t+1}\left[R_{t+1}f\left(Z_t\right)\right]\right\} = E\{\underline{1}f(Z_t)\}. \tag{3}$$

Clearly, Equation (2) is a special case of (3), which may be seen by stacking (3) while taking $f\left(Z_t\right)$ to be each of the instruments in turn. Thus, Equation (2) asks the stochastic discount factor to price only a subset of the strategies allowed by Equation (3).

In this paper we develop tests of asset pricing models based on the following version of Equation (3):

$$E\left\{m_{t+1}x'(Z_t)R_{t+1}\right\} = \underline{1} \quad \forall x(Z_t) : x'(Z_t)\underline{1} = 1. \tag{4}$$

Equation (4) uses all portfolio weight functions $x(Z)$ in place of the general functions in Equation (3), subject only to the restrictions that the weights are bounded integrable functions that sum to 1.0.[2]

While studies of conditional asset pricing typically use a version of Equation (2), our objective is to move to Equation (4). There are several strong motivations. The first is to use all of the information in Z_t for forming portfolio returns. The intuition is that if we ask the model to price a larger set of portfolio returns, as through Equation (4), then a smaller set of m_{t+1}'s can do the job. The tests using Equation (4) will be able to reject more models than tests using Equation (2).

The second motivation for using Equation (4) is tractability. While it may seem difficult to work in the infinite-dimensional space of all possible $x(Z)$, the closed-form solutions in Ferson and Siegel (2001) provide tractable expressions from which we construct the tests. The key is the optimal weight function that minimizes the unconditional variance of $x'(Z_t)R_{t+1}$ for its unconditional mean, over the functions $x(Z)$. Ferson and Siegel (2001) posit a parametric model for the conditional mean vector, $\mu(Z_t)$, and the conditional covariance matrix $\Sigma(Z_t)$, for the returns R_{t+1}. They find the optimal $x(Z_t)$ as a function of $\mu(Z_t)$ and

[2] Equation (**4**) follows by multiplying (**1**) by the elements of the portfolio weight vector $x(Z)$ and summing, using the fact that the weights sum to 1.0, then taking the unconditional expectation. Because of the portfolio weight restriction, Equation (4) is an implication of, not equivalent to (3). Equation (4) retains the dynamic asset allocation decisions allowed by (3)—moving funds from one asset to another based on conditioning information—but leaves out the opportunity to save more or less, altering the overall scale of the investment based on conditioning information. In Equation (4), the portfolio weights almost always sum to 1.0 at each realization of Z. In equation (3), since both sides of the equation may be arbitrarily scaled by a constant, the unconditional expectation of the portfolio weights sum to 1.0 (see Abhyankar, Basu and Stremme, 2002). Restricting to weights that almost always sum to 1.0 in Equation (4) allows us to work with portfolio returns and portfolio efficiency concepts, as opposed to asset prices and payoffs. Working with prices and payoffs, it would be necessary in any event, to normalize the prices to achieve stationarity for empirical work.

$\Sigma(Z_t)$. We use these solutions, reproduced in the Appendix, to implement our tests. Implementing the solutions with N assets and L lagged instruments requires $N \times N$ covariance matrices, whereas the multiplicative approach requires the inversion of matrices with the dimension $NL \times NL$.

The third motivation for our approach is potential robustness. Ferson and Siegel (2001) show that the expressions we use in our tests are likely to be robust to extreme observations, because the optimal portfolio weights are conservative in the face of extreme realizations of Z_t. Ferson and Siegel (2003) apply these expressions to the Hansen-Jagannathan (1991) bounds and find evidence of robustness in that setting. Bekaert and Liu (2004) argue that equation (4) is inherently robust to misspecifying the conditional moments of returns. The intuition is that with the wrong moments $\mu(Z_t)$ and $\Sigma(Z_t)$, the "optimal" $x(Z)$ is suboptimal. However, it remains a valid, if now ad-hoc, dynamic strategy. Thus the tests may sacrifice power, but remain valid. The key to obtaining these advantages is the relation of Equation (4) to the concept of minimum variance efficient portfolios.

1.1 Asset Pricing Models and Portfolio Efficiency

Minimum variance efficient portfolios are those which have minimum variance among portfolios with the same mean return. Asset pricing models are related to portfolio efficiency because an asset pricing model implies a stochastic discount factor and a specification for the stochastic discount factor indicates a portfolio that should be minimum-variance efficient. If we reject the portfolio's efficiency, we reject the asset pricing model.

Consider first the special case where there is no conditioning information, and the asset pricing equation is $E(mR) = \underline{1}$. The following results are well known. Given a portfolio return R_p, there exists a stochastic discount factor of the form $m = a + bR_p$, if and only if R_p is minimum variance efficient. An example is the classical CAPM of Sharpe (1964), as discussed by Dybvig and Ingersoll (1982). There exists a stochastic discount factor that is linear in a k-vector of benchmark returns or "factors," $R_B : m = A + B'R_B$, if and only if some combination of the factor returns is minimum variance efficient. This is the case of an exact k-factor beta pricing model, as discussed by Chamberlain (1983), Grinblatt and Titman (1987), Shanken (1987), and Ferson and Jagannathan (1996). Finally, if the stochastic discount factor is a fixed function of observable data and parameters: $m = m(X, \theta)$, a portfolio that maximizes the squared correlation with $m(X, \theta)$ must be minimum variance efficient. Examples include the consumption-based model of Lucas (1978) and Breeden (1979), and its more recent generalizations. See Ferson (1995) for a review of these results.

This paper extends these examples to the context of efficiency with respect to conditioning information. We show that an asset pricing model implies a specification of the stochastic discount factor for Equation (1), which implies that particular portfolios are minimum variance efficient with respect to the information Z, as formally defined below. Using Equation (4), we then develop empirical tests of the hypothesis that a portfolio is efficient in this sense.

1.2 Portfolio Efficiency with Respect to Conditioning Information

We first define efficiency with respect to the information, Z_t. Consider the set of all portfolios of the N test assets in R_{t+1}, where the weights that determine the portfolio at time t are functions of the given information Z_t at time t. The gross return on such a portfolio with weight $x(Z_t)$, is $x'(Z_t)R_{t+1}$. The restrictions on the portfolio weight function are that the weights must sum to 1.0 (almost surely in Z_t), and that the expected value and second moments of the portfolio return are well defined. This set of portfolio returns determines a mean-standard deviation frontier, as shown by Hansen and Richard (1987). This frontier depicts the *unconditional* means versus the *unconditional* standard deviations of the portfolio returns. A portfolio is defined to be efficient with respect to the information Z_t, if and only if it is on this mean standard deviation frontier.

Proposition 1. *(Hansen and Richard, 1987, Corollary 3.1) Given* N *test asset gross returns,* R_{t+1}, *a given portfolio with gross return* $R_{p,t+1}$ *is* ***minimum-variance efficient with respect to the information*** $\mathbf{Z_t}$ *if and only if Equation (5) (equivalently, Equation 6) is satisfied for all* $x(Z_t)$: $x'(Z_t)\underline{1}=1$ *almost surely, where the relevant unconditional expectations exist and are finite:*

$$Var\left(R_{p,t+1}\right) \le Var\left[x'\left(Z_t\right)R_{t+1}\right] \quad \text{if} \quad E\left(R_{p,t+1}\right) = E\left[x'\left(Z_t\right)R_{t+1}\right] \tag{5}$$

$$E\left[x'\left(Z_t\right)R_{t+1}\right] = \gamma_0 + \gamma_1 Cov\left[x'\left(Z_t\right)R_{t+1}; R_{p,t+1}\right]. \tag{6}$$

Equation (5) is the *definition* of minimum variance efficiency with respect to Z. It states that $R_{p,t+1}$ is on the minimum variance boundary formed by all possible portfolios that use the test assets and the conditioning information. Equation (6) states that the familiar expected return - covariance relation from Fama (1973) and Roll (1977) must hold using efficient–with-respect-to-Z portfolios. The expected returns on all dynamic portfolio strategies are linear functions of their covariances with $R_{p,t+1}$. In Equation (6), the coefficients γ_0 and γ_1 are fixed scalars that do not depend on the functions *x*(.) or the realizations of Z_t.

1.3 Asset Pricing Models and Efficiency with Respect to Information

Most asset pricing models specify a stochastic discount factor. In particular, linear factor models say that *m* is linear in one or more factors. Proposition 2 shows that when there is conditioning information, testing linear factor models in Equation (4) amounts to testing for the efficiency of a portfolio of the factors with respect to the information.

Proposition 2. *Given* $\{R_{t+1}, Z_t\}$ *and a stochastic discount factor* m_{t+1} *such that Equation (4) holds, then if* $m_{t+1} = A + B'R_{B,t+1}$, *where* $R_{B,t+1}$ *is a k-vector of benchmark factor returns, and* A *and* B *are a constant and a fixed* k-*vector, there exists a portfolio,* $R_{p,t+1} = w'R_{B,t+1}$, $w'\underline{1} = 1$, *where* w *is a constant* N-*vector, and* $R_{p,t+1}$ *is efficient with respect to the information* Z_t.

Proof: See the Appendix for all proofs.

The intuition of Proposition 2 is the same as the case with no conditioning information, except that the set of returns is expanded to all $x'(Z)R$. Thus, linear factor models can easily be tested in the set of all dynamic strategy returns implied by Equation (4), as we illustrate below.

We now consider the case of a general $m(X,\theta)$ and allow for time-varying weights in the efficient portfolio. This requires the definition of portfolios that are *maximum correlation with respect to Z*.

Definition. *A portfolio* R_P *is* ***maximum correlation for a random variable,*** **m*****, with respect to conditioning information Z****, if:*

$$\rho^2(R_P, m) \geq \rho^2[x'(Z)R, m] \quad \forall x(Z): x'(Z)\underline{1} = 1, \tag{7}$$

where $\rho^2(.,.)$ *is the squared unconditional correlation coefficient*.

Proposition 3. *If a given* m *satisfies Equation (4), then a portfolio* R_P *that is maximum correlation for* m *with respect to* Z *must be minimum variance efficient with respect to* Z.

Proposition 2 is clearly a special case of Proposition 3, because if m_{t+1} is linear in $R_{B,t+1}$, a linear regression maximizes the squared correlation. We use Proposition 3 in our tests as follows. Given a stochastic discount factor, *m*, we test the model by constructing a portfolio (using the methods described below) that is maximum correlation for this *m* with respect to *Z*, and then test the implication that the portfolio is efficient with respect to *Z*.

With the preceding results we can consider a case where the model implies a stochastic discount factor that is linear in *k* factor-portfolios, allowing for time-varying weights.

Corollary. *Given* $\{R_{t+1}, Z_t\}$ *and a stochastic discount factor* m_{t+1} *such that Equation (4) holds, then if a maximum correlation portfolio for* m_{t+1} *with respect to* Z_t *has nonzero weights only on the* k*-vector of benchmark factor returns* $R_{B,t+1}$*, an efficient-with-respect-to-*Z *portfolio of the factor returns* $R_{B,t+1}$ *is efficient with respect to* Z *in the full set of test asset returns.*

With conditioning information, efficient portfolios generally have time-varying weights. The situation described in the Corollary is a "dynamic" version of mean variance intersection, as developed by Huberman, Kandel and Stambaugh (1987). The Corollary follows because the factor portfolio in question satisfies the condition of Proposition 3 and so is efficient with respect to *Z* in the full set of assets. If it loads only on the subset of factor returns it must also be efficient in those returns. Thus, the full set and subset minimum variance boundaries must touch at the point defined by the maximum correlation portfolio.

The Corollary does not say that *all* efficient combinations of the factor returns are efficient in the full set of returns. The intersection point corresponds to a particular zero beta rate. Other points on the subset boundary may be inside the full set boundary. If the risk-free rate is fixed, however, the efficient set becomes a ray and the efficient portfolio for a given set of risky assets is unique. In this case, any efficient combination of the factor portfolios must be efficient in the full set of assets. For example, assuming a fixed risk-free rate, one hypothesis that we consider below is that some combination (that depends on *Z*) of the three Fama and French (1996) factors is maximum correlation for some stochastic discount factor. The test is to find the efficient-with-respect-to-*Z* portfolio of the Fama-French factors and see if this time-varying combination is efficient with respect to *Z* in the sample of test assets.

1.4 Discussion

The presence of conditioning information impacts tests of asset pricing models in three general ways. First, conditioning information relates to the set of payoffs we ask the model to price. Second, conditioning information relates to the specification of the functional form of the SDF. Third, the asset pricing statement, Equation (1), would ideally apply to conditional moments given a public information set Ω, but an empiricist can only measure *Z*, a proper subset of Ω.

The first issue with respect to conditioning information is the set of payoffs that we ask the model to price. By using the given conditioning information *Z* in different ways we generate different payoffs from the test assets, *R*. As explained above, our approach asks the model to price all portfolios $x(Z)'R$, where $x(Z)'\underline{1} = 1$. Expanding the set of payoffs in this way, we restrict the set of *m*'s that can price those payoffs. Our tests should therefore reject models that previous approaches would not reject.

The second related issue is the functional form of the SDF. Different asset pricing models imply different functional forms. Our approach can handle general functions of measurable data, $m(X,\theta)$. If we reject the efficiency with respect to *Z*, of a portfolio that has maximum correlation with $m(X,\theta)$ with respect to *Z*, we reject the hypothesis that $E\left[m\left(X,\theta\right)R \mid Z\right]=\underline{1}$. By iterated expectations, we therefore reject the model that says $E\left[m(X,\theta)R \mid \Omega\right]=\underline{1}$.

The third issue arises because the asset pricing theory says $E(mR|\Omega)=\underline{1}$, but the full information set Ω cannot be measured. There are two cases. In the first case, the SDF is a known function of measurable data and parameters, as described above, and we can test $E(mR|Z) =\underline{1}$. The inability to measure all of Ω results only in a potential loss of power in this case.

A more difficult case arises when the SDF, $m(\Omega)$, is a function of unobservable parts of Ω. In this case it is not known how to test a model that says $E(m(\Omega)R|\Omega) = \underline{1}$. While it remains true that $E(m(\Omega)R|Z)= \underline{1}$, that is no help if $m(\Omega)$ can not be measured.[3] Hansen and Richard (1987) describe a version of this problem in terms of portfolio efficiency. Consider a conditional version of the CAPM in which $m(\Omega) = a(\Omega) + b(\Omega)R_m$ and the market portfolio R_m is *conditionally efficient* given Ω (meaning minimum conditional variance given Ω subject to the conditional mean return given Ω). Hansen and Richard show that the conditional efficiency of R_m given Ω does not imply conditional efficiency given Z. If we can only observe Z we can test the efficiency of R_m using Z, but this does not allow us to reject the conditional CAPM. Cochrane (2001) calls this result the "Hansen-Richard critique." By analogy with the Roll (1977) critique that the CAPM can't be tested because we can't measure the entire market portfolio, the Hansen-Richard critique implies that the *conditional* CAPM can't be tested (even if we could measure the market portfolio) because we can't measure all the information, Ω. This problem is by no means unique to our paper. In the spirit of virtually all empirical studies, we therefore focus on cases where the SDF is assumed to depend on measurable data only.

1.5 Testing Conditional Efficiency

[3] Hansen and Jagannathan (1991) develop an SDF given by $m^* = E(m|R)$ and they show how to form the projection m^*. However, m^* can not be used to test the original model because it prices the returns by construction.

Our approach is to test the (unconditional) efficiency of a portfolio R_P with respect to Z. An alternative approach is to test the *conditional* efficiency given Z. The Hansen-Richard critique says that such tests do not allow inferences about efficiency given Ω, but tests of conditional efficiency given observable instruments Z have nevertheless been of historical interest in the literature. Hansen and Hodrick (1983) and Gibbons and Ferson (1985) test conditional efficiency given Z, assuming constant conditional betas. Campbell (1987) and Harvey (1989) restrict the form of a market price of risk. Shanken (1990) restricts the form of time-varying conditional betas. Tests of conditional efficiency given Z may be handled in our framework as a special case, choosing the functional form for $m(X,\theta)$.

The conditional efficiency of a portfolio R_P given Z is equivalent to the existence of an SDF, $m=a(Z) + b(Z)R_P$. The conditional efficiency of a combination of K factor-returns, R_B, is equivalent to the existence of an SDF, $m = A(Z) + B(Z)'R_B$. The restrictions on the coefficients as functions of Z follow from the requirement that the model correctly prices the factor returns R_B. The coefficients are: $A(Z) = [1 + \Sigma_j \lambda_j E(R_{Bj}|Z)/var(R_{Bj}|Z)]/E(R_o|Z)$ *and* $[B_j(Z) = -\lambda_j /[E(R_o|Z)var(R_{Bj}|Z)]$, where $\lambda_j = E(R_{Bj} - R_0|Z)$ and R_0 is the conditional zero-beta return for R_B (that is, $Cov(R_0,R_p|Z) = 0$). When $K=1$ and $R_{Bj} = R_p$ we have the single-factor model coefficients. (See Ferson and Jagannathan, 1996.)

With our approach we can test conditional efficiency by constructing the maximum correlation portfolio for the indicated m with respect to Z. This portfolio, call it R_p^*, should be efficient with respect to Z. We test conditional efficiency by testing the efficiency of R_p^* with respect to Z. Note that R_p^* will be different from R_P when the coefficients $A(Z)$ or $B(Z)$

are time varying functions of Z. Thus, for example, the conditional CAPM does not imply that the market portfolio is efficient with respect to Z. However, the model does identify a portfolio R_p^* that should be efficient with respect to *Z*, and this can be tested using our approach.

2. Testing Efficiency

2.1 No Conditioning Information

Classical tests for the efficiency of a given portfolio involve restrictions on the intercepts of a system of time-series regressions. If r_t is the vector of *N excess* returns at time *t*, measured in excess of a risk-free or zero-beta return, and $r_{p,t}$ is the excess return on the tested portfolio, the regression system is

$$r_t = \alpha + \beta r_{p,t} + u_t; \quad t = 1,...,T, \tag{8}$$

where *T* is the number of time-series observations, β is the *N*-vector of betas and α is the *N*-vector of alphas. The portfolio r_p is minimum-variance efficient and has the given zero-beta return only if α=0.

It is well known that the classical test statistics for the hypothesis that α=0 in Equation (8) can be written in terms of squared Sharpe ratios (e.g., Jobson and Korkie, 1982). Consider the simplest case of the Wald Statistic:

$$W = T\hat{\alpha}'[Cov(\hat{\alpha})]^{-1}\hat{\alpha} = T\left(\frac{\hat{S}^2(R) - \hat{S}^2(R_p)}{1 + \hat{S}^2(R_p)}\right) \dot{\sim} \chi^2(N) \tag{9}$$

where $\hat{\alpha}$ is the OLS or ML estimator of α and $Cov(\hat{\alpha})$ is its asymptotic covariance matrix. The term $\hat{S}^2(R_p)$ is the sample value of the squared Sharpe ratio of R_p: $S^2(R_p) = \left[E(r_p)/\sigma(r_p)\right]^2$. The term $\hat{S}^2(R)$ is the sample value of the maximum squared Sharpe ratio that can be obtained by portfolios of the assets in R (including R_p):

$$S^2(R) = \max_x \left\{ \frac{[E(x'r)]^2}{Var(x'r)} \right\}. \tag{10}$$

The Wald statistic has an asymptotic chi-squared distribution with N degrees of freedom under the null hypothesis[4].

Since the Sharpe ratio is the slope of a line in the mean-standard deviation space, Equation (9) suggests a graphical representation. The Wald statistic measures the distance between the sample frontier and the location of the tested portfolio, inside the frontier. Kandel (1984), Roll (1985), Gibbons, Ross and Shanken (1989) and Kandel and Stambaugh (1987, 1989) develop this interpretation.

2.2 Tests with Conditioning Information

In this paper we use statistics similar to the classical statistic of Equation (9). With conditioning information, its asymptotic distribution, under the null hypothesis, is not known

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[4] When the Wald statistic in (9) is multiplied by $\left(\frac{N+1}{T(T-2)}\right)\left[\frac{1+E(r_p)^2}{\sigma(r_p)^2}\right]^{-1}$, the result has an exact F distribution in finite samples under the assumption that the (r_t, r_{pt}) in (8) are normally distributed. Gibbons, Ross and Shanken (1989) use this result to study finite sample biases in transformations of the Wald statistic and we use it below in some of our experiments.

to be chi-squared in general. There may also be alternative statistics with better sampling properties. Thus, by moving to Equation (4) and conditioning information we raise some new statistical questions for future research. Our examples focus on the classical-looking statistic as a natural extension of the literature.

Classical tests that ignore conditioning information restrict the maximization in Equation (10) to *fixed-weight* portfolios, where x is a constant vector. In contrast, efficient portfolios with respect to the information Z maximize the squared Sharpe ratio over *all portfolio weight functions*, $x(Z)$. Maximizing over a larger set of weights we get a larger maximum Sharpe ratio.

Jobson and Korkie (1982) show that the test statistic in Equation (9) may be interpreted as the relative performance of the portfolio of the test assets that is the "most-mispriced" by R_P. This portfolio is also called the "active" portfolio by Gibbons, Ross and Shanken (1989) and the "optimal orthogonal portfolio" by MacKinlay (1995). We use a version of this portfolio in some of our empirical examples. The portfolio has weights proportional to $\left[Cov(\hat{\alpha})\right]^{-1}\hat{\alpha}$ in the classical case with no conditioning information. With conditioning information the portfolio's weight function is time-varying. We derive the most mispriced portfolio, assuming an arbitrary fixed "zero-beta" rate, γ_0.

Consider any portfolio formed using weights x_p with return $R_{p,t+1}=x_p'R_{t+1}$, where x_p may depend on Z_t. The portfolio has unconditional expected return $E(x_p'R_{t+1})=\mu_p$ and variance $var(x_p'R_{t+1}) = \sigma_p^2$. Define the ***most mispriced portfolio***, R_c, with respect to R_p as the dynamic strategy that maximizes α_c^2/σ_c^2, where σ_c^2 is the unconditional variance of R_c, $\mu_c = E(R_c)$

and $\alpha_c = \mu_c - [\gamma_o + (\mu_p - \gamma_o)\sigma_{cp} / \sigma_p{}^2]$ is the unconditional alpha of R_c with respect to R_p, where $\sigma_{cp} = Cov(R_c, R_p)$. This generalizes the most mispriced, active or optimal orthogonal portfolio to the set of all dynamic trading strategies, but the "mispricing" is still defined with respect to the unconditional moments. We show how to construct this portfolio as a dynamic strategy[5].

Let R_s be the portfolio return that maximizes the squared unconditional Sharpe ratio in (10) over all portfolio weight functions $x(Z)$, when the excess returns are $r \equiv R - \gamma_0$. The portfolio R_s has unconditional mean return μ_s and variance $\sigma_s{}^2$.

Proposition 4. *The most mispriced portfolio* R_c *with respect to a given portfolio* R_p *and zero-beta rate,* γ_0*, may be found as a fixed linear combination of* R_p *and the efficient-with-respect to* Z *portfolio,* R_s*:*

$$R_c = \frac{\left(\frac{\mu_s - \gamma_0}{\sigma_s^2}\right) R_s - \left(\frac{\mu_p - \gamma_0}{\sigma_p^2}\right) R_p}{\left(\frac{\mu_s - \gamma_0}{\sigma_s^2}\right) - \left(\frac{\mu_p - \gamma_0}{\sigma_p^2}\right)}, \tag{11}$$

or

$$R_c = \left(R_s - \frac{\sigma_{sp}}{\sigma_p^2} R_p\right) \Big/ \left(1 - \frac{\sigma_{sp}}{\sigma_p^2}\right). \tag{12}$$

The most mispriced portfolio R_c has weights that depend on Z; these are presented with the proof in the Appendix. Note that the portfolio R_c is uncorrelated with R_p, according to Equation (12). The portfolio R_c is essentially the regression error of R_s projected on R_p,

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[5] There are alternative, equivalent definitions for the most mispriced portfolio. For example, the portfolio can maximize the ratio of squared alpha to residual variance, given R_p. It can also maximize the squared unconditional Sharpe ratio subject to orthogonality with R_p.

normalized so that the weights sum to 1.0. The portfolio R_c may be found by starting with R_s and then removing its component that is correlated with R_p. A combination of R_p and its most-mispriced portfolio R_c is an efficient portfolio with respect to Z.

2.3 Empirical Strategy

Our empirical examples compare the classical approach using no conditioning information, the multiplicative approach to conditioning information, and the efficient use of the conditioning information. The specifics depend on the example.

When we test the efficiency of a given portfolio, R_p, then $\hat{S}^2\left(R_p\right)$ is formed using the normal maximum likelihood estimators of the mean and variance of the portfolio's excess return. We use the average one-month US Treasury bill return as the fixed risk-free or zero-beta rate.[6] The squared Sharpe ratio of the boundary, $\hat{S}^2\left(R\right)$, differs according to the way conditioning information is used. When there is no conditioning information we use the fixed-weight solution to (10) evaluated at the maximum likelihood estimates.

When the information is used multiplicatively we define an expanded set of returns $\hat{R}_t = R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$, where R_{ft} is the one-month Treasury bill return for month t. We then proceed as in the fixed-weight case, using the returns $\hat{R}_t$ in place of R_t.

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[6] In our sample, stock returns in excess of the average Treasury rate are virtually empirically indistinguishable from returns in excess of the monthly bill rate. We conducted some experiments using either measure in the test statistics and obtain very similar results.

When the information is used efficiently, $\hat{S}^2(R)$ is formed using the sample mean and variance of $\hat{x}'(Z)R$ where $\hat{x}(Z)$ is the sample version of the efficient-with-respect-to-Z portfolio from Ferson and Siegel (2001), described in the Appendix as Equation (18). This dynamic portfolio strategy defines the mean-variance frontier that contains all dynamic strategy returns. The average value of the one-month Treasury bill determines the risk-free rate in these solutions.

The solution for $\hat{x}(Z)$ from Ferson and Siegel is a function of the assumed parametric models for $\mu(Z_t) = E(R_{t+1}|Z_t)$ and $\Sigma(Z_t) = var(R_{t+1}|Z_t)$. Most of our examples adopt a simple regression specification, where $\mu(Z_t)$ is the linear regression function and $\Sigma(Z_t)$ is the covariance matrix of the regression residuals, which is held fixed over time. We also consider models with time-varying $\Sigma(Z_t)$ as a check on the robustness of our results. [7]

We evaluate the tests using simulations. To generate data consistent with the null hypothesis that a given portfolio R_p is efficient, we replace its return with a portfolio that is efficient, based on the specification of the asset-return moments in the simulation. With this substitution, we then construct the null distribution of the test statistic using the artificial data in the same way that we use the actual data to get the sample value of the statistic. The details of the implementation are discussed in the Appendix.

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[7] Future research should further examine the effects of alternative specifications for the conditional moments of returns. Previous approaches to conditional asset pricing directly specify ad-hoc functional forms for the dynamic strategies $x(Z)$. We use the optimal strategies, which are given functions of $\mu(Z)$ and $\Sigma(Z)$, but we must specify these functional forms. Pushing the selection of the functional forms closer to the data is an improvement because the functional forms of asset return moments can be evaluated independently of portfolio performance.

Our approach to generating the null distributions involves replacing the tested portfolio with one that is efficient given the data generating process. We conduct some experiments to assess the accuracy of the empirical *p*-values produced by this approach. We take a context with no lagged instruments and normality, where the exact finite sample *p*-values are known (see Gibbons, Ross and Shanken, GRS, 1989). We first generate a sample of normal returns from a population with mean and covariance matrix equal to our sample estimates. We estimate the test statistic on the normal data. The tested portfolio, the SP500, is not efficient in this sample and the exact *p*-value from the *F* distribution is taken to be the "correct" *p*-value. We run simulations to see if our approach generates reliable empirical *p*-values. The sample means and covariances from the normal sample define "population" parameters for a simulation. To generate the null distribution, we replace the SP500 with the portfolio that is efficient at these parameter values and we resample the unexpected returns to generate 1,000 artificial samples. We compute the test statistic in Equation (9) on each artificial sample. The empirical *p*-value is the fraction of the 1,000 trials in which the test statistic exceeds the value computed on the normal return sample that calibrates the simulations. Averaging the *p*-values across 100 normal samples, we find that the average empirical *p*-values and the GRS *p*-values are similar, which increases the confidence in our approach.[8]

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[8] The *p*-values averaged over 100 normal samples are summarized below:

	industries 1963-94	industries 1963-72	industries 1973-82	industries 1983-92	Size/BM 1963-94
Avg. GRS	0.073	0.035	0.066	0.061	0.000
Avg. empirical	0.043	0.036	0.069	0.059	0.000

3. The Data

To model the conditioning information, we use a number of lagged variables that have long been prominent in the conditional asset pricing literature. These include: (1) the lagged value of a one-month Treasury bill yield (see Fama and Schwert (1977), Ferson (1989), Breen et al. (1989) or Shanken, 1990); (2) the dividend yield of the market index (see Fama and French, 1988); (3) the spread between Moody's Baa and Aaa corporate bond yields (see Keim and Stambaugh, (1986) or Fama, 1990); (4) the spread between ten-year and one-year constant maturity Treasury bond yields (see Fama and French, 1989) and (5); the difference between the one-month lagged returns of a three-month and a one-month Treasury bill (see Campbell, 1987).

We provide results using two alternative methods of grouping common stocks into portfolios. The first sample comprises twenty five industry portfolios (from Harvey and Kirby, 1996) measured for the period February, 1963 to December, 1994.[9] The portfolios are created by grouping common stocks according to their SIC codes and forming value-weighted averages (based on beginning-of-month values) of the total returns within each group of firms.

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[9] We are grateful to Campbell Harvey for providing these data.

Table 1 shows the industry classifications for the 25 portfolios, and summary statistics of the returns.

The second grouping follows Fama and French (1996). Individual common stocks are placed into five groups according to their prior equity market capitalization, and independently into five groups on the basis of their ratios of book value to market value of equity per share. This 5 by 5 classification scheme results in a sample of 25 portfolio returns. These are the same portfolios used by Ferson and Harvey (1999), who provide details and summary statistics.

This project has matured over a length of time, providing the opportunity to investigate the results over a "hold-out" sample. The hold-out sample period is January, 1995 through December, 2002. We use 25 size x book-to-market and Industry portfolios from Kenneth French and update the other series with fresh data.[10] The hold-out sample results are interesting in view of recent evidence, cited above, that some of the lagged instruments may have lost their predictive power for stock returns in recent data. Table 1 illustrates this, reporting the adjusted *R*-squares from regressing the industry returns on the lagged instruments over the 1963-1994 period and the 1995-2002 sample. The *R*-squares are substantially lower in the more recent period. Running the regressions on the 25 size and book-to-market portfolios produces a similar result. The average adjusted *R*-squared over 1963-1994 is 10.5%, while over 1995-2002 it is only 1.4%.

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[10] We use a subset of the 48 value-weighted industry portfolios provided by French to match the definitions in Table 1. We confirm that the matched industry returns produce similar summary statistics and regression *R*-squares on the lagged instruments as our original data, over the 1963-1994 period.

4. Empirical Results

4.1 Inefficiency of the SP500 Relative to Industry Portfolios

Table 2 summarizes results for the 25 industry portfolios for the 1963-94 period, three ten-year subperiods and the holdout sample, 1995-2002. The tested portfolio, R_p, is the SP500. In Panel A there is no conditioning information. Substituting the maximum likelihood estimates of $\hat{S}^2(R_p)$ and $\hat{S}^2(R)$ into (9) gives the sample value of the test statistic. Referring to the asymptotic distribution, the right-tail *p*-value is 0.48 for the full sample and $0.14 - 0.39$ in the ten-year subperiods. The test produces little evidence to reject the null hypothesis. During the holdout sample the Sharpe ratios are substantially higher, and so is the value of the test statistic. The asymptotic *p*-value is 0.001 for this period.

Panel A of Table 2 also reports 5% critical values and empirical *p*-values based on Monte Carlo simulation assuming normality, and based on a resampling approach that does not assume normality. In addition, we report *p*-values from the exact *F* statistic, under the assumption of normality. Consistent with Gibbons, Ross and Shanken (1989) the Wald Test rejects too often when the asymptotic distribution is used. The asymptotic *p*-values are smaller than both the empirical *p*-values and the *p*-values from the *F*-test. The smallest empirical *p*-value in the panel is 0.43. Thus, when we correct for finite sample bias there is no evidence against the efficiency of the market index in the industry portfolios, given that no conditioning information is used in the tests. The *p*-values from the *F* distribution are also close to the empirical *p*-values from the Monte Carol simulations in most of the subsamples. If the test asset returns were normally distributed we should expect to get the same results.

Panel B of Table 2 summarizes tests using the "multiplicative" returns, $\hat{R}_t = R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$. With 25 industry portfolios, the market return and five instruments plus a constant (L=6), there are 156 "returns." One disadvantage of the multiplicative approach is that the size of the system quickly becomes unwieldy. It is not possible to construct the Wald Test for the ten year subperiods, as the sample covariance matrix is singular.

Over the full sample period the value of the Wald Test statistic using the multiplicative returns is 348.6. The asymptotic p-value is close to zero. However, we expect a finite-sample bias and the simulations confirm the bias. Based on the empirical p-values the tests reject the efficiency of the SP500 at either the 2% (Monte Carlo) or 44% (resampling) levels. The Gibbons-Ross-Shanken p-value assuming normality is 3%. Thus, the finite sample results are sensitive to the data generating process. This makes sense, because even if R_t is approximately normal, the products of returns and the elements of Z_{t-1} are not normal, and the Monte Carlo simulation assumes normality. We therefore place more trust in the resampling results. Correcting for finite sample bias with the resampling scheme, we find no evidence to reject the efficiency of the market index using the multiplicative design.

Panel C uses the conditioning information Z optimally. With this approach the size of the covariance matrices to be inverted does not increase with the use of conditioning information, so results for the subperiods can be obtained. This illustrates the tractability of our approach compared to the multiplicative approach. The value of the statistic given by Equation (9) is 161.84 in the full sample, $164.98 - 203.29$ in the ten-year subperiods and 148.2 in the holdout sample. The empirical p-values are 0.5% or less in the full sample and each ten-year subperiod, and 4.4% or less in the holdout sample. Thus, we find strong evidence to reject the hypothesis that the SP500 index is efficient in the design that uses the conditioning information optimally.

The results based on the efficient-with-respect-to-*Z* frontier are fairly robust to the method of simulation (Monte Carlo or resampling). This makes sense in view of the "robust" behavior of the portfolio weights that define the efficient-with-respect-to-*Z* boundary. As described by Ferson and Siegel (2001), these portfolios put small weights on the extreme observations of *Z* so that the portfolios are dominated by the center of the distribution. Whereas, the weights of the multiplicative approach are unbounded linear functions of *Z*.

In summary, we can reject the hypothesis that the market index is mean variance efficient when the conditioning information is used optimally. The tests that use the conditioning information optimally can reject the model when the multiplicative approach cannot. We even find marginal rejections during the holdout sample period, where Table 1 illustrated that the predictive power of the lagged instruments is relatively low.

Figure 1 illustrates the test, showing the sample frontier of fixed-weight portfolios that ignore the conditioning information and the efficient frontier with respect to *Z*. The test statistics are related to the differences between the squared slopes of the lines drawn through the SP500 versus the lines tangent to the frontiers. The figure shows how the optimal use of conditioning information produces a larger test statistic.

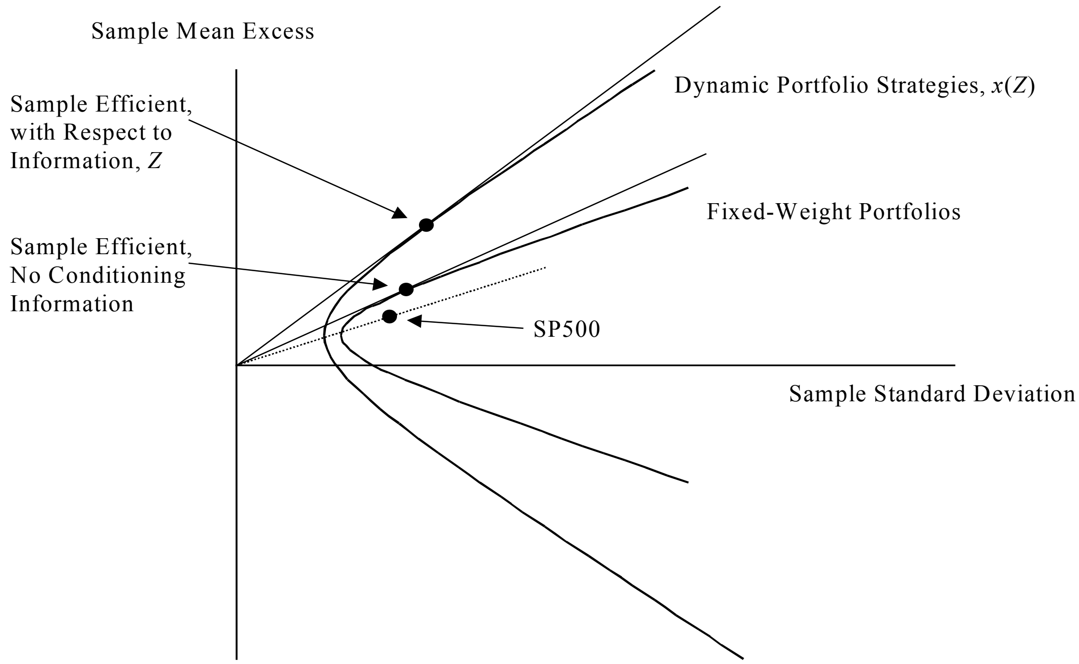

Figure 1. The test statistic for the efficiency of the SP500 compares the squared slope of the line through the tested portfolio with the line through the sample efficient portfolio. As the slopes diverge, the test statistic is larger. Testing for efficiency with respect to the information, *Z*, the test statistic is larger than when the information is ignored.

4.2 Alternative Test Assets

Recent studies use portfolios grouped on firm size and book-to-market ratios, and find that a market index is not efficient in these returns (e.g. Fama and French, 1992). Table 3 presents results where the portfolios are grouped on size and book-to-market. The full sample and holdout results for industries from Table 2 are repeated in the right hand column for comparison purposes.

In panel A of Table 3 there is no conditioning information. Consistent with previous studies, the efficiency of the SP500 is rejected in the size × book-to-market portfolio design for the 1963-1994 period. However, in the 1995-2002 period, the efficiency of the market index is not rejected when the finite sample bias in the statistics is corrected. This is consistent with a weakening of the size and book-to-market effects during 1995-2002.

In panel B of Table 3, the test assets are the multiplicative returns. The asymptotic *p*-values suggest rejections of the efficiency hypothesis, but the resampling results indicate a strong, finite-sample bias. The empirical *p*-value based on resampling is 4.4% with the 25 size × book-to-market portfolios over the 1963-1994 sample.

In panel C the test assets are all portfolios of the form $x'(Z_{t-1})R_t$. The resampling *p*-values are 0.3% or less in the size × book-to-market design, including the 1995-2002 subsample. As in Table 2, the Monte Carlo and resampling *p*-values are similar. Thus, once again we find that efficiency can be rejected with our approach, in settings where the classical approach does not reject efficiency. The results show that expanding the set of dynamic strategies using our results makes a substantial difference, even in the size × book-to-market portfolio design.

4.3 Expanding the Mean Variance Boundary

The evidence so far shows that the market index return lies "significantly" inside the mean-variance boundaries when the conditioning information is used optimally. However, these results only indirectly address the question of inferences about the mean variance boundaries themselves. If the Sharpe ratio of a given portfolio is estimated with greater precision than the maximum Sharpe ratio in a set of returns, as seems likely, then we may be able to draw inferences about efficiency for a given portfolio and yet be unable to draw reliable inferences about the efficient frontiers themselves. Inferences about the frontiers themselves are interesting from the perspective of questions about which strategies may expand the mean variance investment opportunity set.

In this section we ask if the use of conditioning information expands the mean variance boundary at a point tangent to the average risk-free rate. Table 4 presents the tests. Here we replace the market index with a portfolio of the test assets whose weights are proportional to $\Sigma^{-1}\mu$, where Σ is the unconditional covariance matrix and μ is the unconditional mean of the excess returns. Using the same parameters as in the data generating process of the simulations, this is a portfolio on the "population" mean-variance boundary with no conditioning information. We then test for the efficiency of this portfolio instead of the SP500, as in the previous tables. In panel A, we examine the mean variance boundary constructed using the multiplicative approach. The resampled *p*-values are 0.464 and 0.686, thus providing no evidence that the multiplicative approach expands the boundary. These results are consistent with studies such as Carlson and Chapman (2000) that question the usefulness of the standard lagged instruments in the multiplicative design.[11]

In Panel B of Table 4 the test assets are all portfolios of the form $x'(Z_{t-1})R_t$. In the 1963-94 period the resampled *p*-values are 0.1% and 2.5% for the two portfolio grouping methods, showing that when the conditioning information is used optimally the mean variance boundary is expanded. However, in the holdout sample we do not reject the null hypothesis. This is consistent with the low explanatory power of the lagged variables during the holdout

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[11] Using a fixed risk-free rate these tests may sacrifice power, as there may be other regions, corresponding to other values of the zero-beta rate, where the two boundaries are reliably distinct.

sample, as indicated in Table 1. While the efficiency of the market index can be rejected during this period, the maximum Sharpe ratio on the fixed-weight frontier is closer to the efficient-with-respect-to-Z boundary than is the market index.

The tests of Table 4 have an interesting interpretation when they are applied to the size × book-to-market portfolios and the market index. Fama and French (1996) construct three factors designed to capture the average returns of portfolios grouped by size and book-to-market, the Fama-French "3 factor model." If betas on these factors describe the cross-section of expected returns, a combination of the factors is efficient. The Fama-French factors can approximately be obtained through combinations of the size and book-to-market portfolios. Therefore, the tests in Table 4 reject a fortiori a static (fixed-weight) version of the Fama-French 3-factor model over 1963-94, but not for 1995-2002. However, given that the statistical noise involved in estimating the maximum Sharpe ratio for 26 test assets will differ from that involving three factors, it is interesting to examine the multifactor models explicitly.

4.4 Testing Static Combinations of the Fama-French Factors

This section presents tests for the efficiency of fixed-weight combinations of the three Fama and French factors. The hypothesis may be stated as $m = a + b_1 R_m + b_2 R_{HML} + b_3 R_{SMB}$, where the coefficients are fixed over time. R_m is the gross return of the market index. R_{HML} is the one-month Treasury bill gross return plus the excess return of high book-to-market over low book-to-market stocks, and R_{SMB} is similarly constructed using small and large market-capitalization stocks. In testing this model we replace the first and last portfolios in the industry or size × book-to-market design with the returns R_{HML} and R_{SMB}, to insure that the factor portfolios are a subset of the tested portfolio returns.

Table 5 presents the tests. In Panel A there is no conditioning information. Based on the asymptotic *p*-values we would reject the efficiency of the Fama-French factors at the 5% level, except in the size × book-to-market portfolio design over 1963-1994. However, adjusting for finite sample bias the only rejection occurs for the industry portfolios. Fama and French (1997) also find that their factors don't explain industry portfolio returns very well.

In Panel B the multiplicative approach to conditioning information is used. The resampled *p*-values strongly reject the model for 1963-94. This is consistent with studies such as Ferson and Harvey (1999) who find that the Fama-French factors do not explain time-varying expected returns over a similar sample period. Once again, we cannot examine the multiplicative approach over the holdout sample because the covariance matrices are too large to invert.

Panel C of Table 5 presents the tests relative to the efficient-with-respect-to-*Z* frontier. The tests confirm the value of using the conditioning information optimally. We observe strong rejections of the static version of the Fama-French model, both over 1963-1994 and in the 1995-2002 sample, and for both portfolio designs. The test results are consistent with the intuition that Sharpe ratios can be estimated with greater precision on a smaller number of assets (the Fama French factors in Table 5) than they can on a larger number of assets (the 26 portfolios in Table 4). Thus, the tests using the conditioning information optimally can reject the Fama French factors even when they could not reject the hypothesis that the mean variance boundary fails to expand, as is the case for the 1995-2002 sample.

4.5 Testing Dynamic Multifactor Models

The empirical results so far show that the optimal use of conditioning information expands the mean variance boundary of monthly portfolio returns for the sample before 1995, even when a multiplicative approach does not, and that the stock market index and fixed combinations of the Fama-French factors lie inside the expanded boundary, even during the 1995-2002 holdout sample. This section illustrates tests of multifactor benchmarks with time-varying weights.

Let R_B denote the vector of benchmark factor returns (eg., a market index and the Fama-French factors). We develop two examples. In the first example, $m(Z) = a + b\, w'(Z)R_B$, where a and b are constants and $w'(Z)\underline{1} = 1$. In the language of Huberman, Kandel and Stambaugh (1987), this says there is mean-variance "intersection" of the efficient-with-respect-to-Z boundary formed from R_B and the boundary of all the test assets, including R_B. Equivalently, a dynamic portfolio $w'(Z)R_B$ is efficient with respect to Z, and there is a single-beta pricing model for the unconditional mean returns of all portfolios, based on the portfolio $w'(Z)R_B$. We refer to this as the hypothesis of "dynamic intersection."

The second example tests conditional efficiency. This implies that the stochastic discount factor is $m(Z) = A(Z) + B(Z)'R_B$, where $A(Z)$ and $B(Z)$ are specified in Section 1.5. A time-varying combination of the K factor portfolios is conditionally minimum variance efficient given Z, and there is a k-beta pricing relation for the *conditional* mean returns of the test assets. According to this model, a maximum correlation portfolio with respect to Z for $A(Z)$ + $B(Z)'R_B$, will be efficient with respect to Z. A special case is a conditional CAPM, when k=1 and R_B is the market return.

The hypotheses of conditional efficiency and dynamic intersection are related as follows. Both hypotheses specify that a particular time-varying combination of the assets should be efficient with respect to *Z*. Conditional efficiency specifies that the combination involves all of the test assets through the maximum correlation portfolio. Dynamic intersection restricts the time-varying combination to the factor portfolios.[12]

Table 6 summarizes the tests for dynamic intersection. The tests ask if the efficient-with-respect-to-*Z* frontier formed from all portfolios of the three Fama-French factor returns touches the efficient-with-respect-to-*Z* frontier of the test assets at a point tangent to the risk-free rate. The sample values of the test statistics are smaller, in every case, than the values in Table 5. This is because a time-varying combination of the Fama-French factors has a larger maximum Sharpe ratio in the sample than a fixed-weight combination. The simulations reveal that the 5% critical values of the test statistics are fairly close to those in Table 5, and the *p*-values of the test statistics in Table 6 are not as small as the values in Table 5. Still, the hypothesis of dynamic intersection is strongly rejected for the 1963-94 sample, with *p*-values of 0.1% or less.

During 1995-2002 the tests in Table 6 marginally reject intersection, with *p*-values of 3.9% in the industry portfolio design and 5.5% in the size and book/market design. In Panel

[12] Dynamic intersection is stronger in general than conditional efficiency. Dynamic intersection says that the efficient-with-respect-to-*Z* boundaries of the test assets and of the factor returns share a common point. Conditional efficiency says the *conditional* boundaries have a common point, for each realization of *Z*. Efficient-with-respect-to-*Z* portfolios must also be conditionally efficient, as shown by Hansen and Richard (1987). However, in conditional efficiency, the tangency to the common point is a zero-beta rate that may vary with *Z* over time. It follows that a rejection of conditional efficiency with a given risk-free rate, as in our empirical examples, does not imply a rejection of dynamic intersection.

A of Table 5, when no conditioning information was used over the same sample period, the *p*-values for the tests of intersection were 7.7% and 15.7%. Thus, the evidence against the hypothesis that a combination of the three Fama-French factors touches the boundary of the test assets is stronger, even during the 1995-2002 period, when the conditioning information is used optimally.

Table 7 presents the tests of conditional efficiency. The tests work by estimating the conditional means and covariances of the returns using the maximum likelihood estimators and then forming the discount factor $A(Z) + B(Z)'R_B$, using the expressions for $A(Z)$ and $B(Z)$ described earlier. We then construct the weights of the maximum correlation portfolio to this SDF using Equation (21). The tests then ask if this maximum correlation portfolio is on the efficient-with-respect-to-*Z* boundary at a point tangent to the risk-free rate. We test the conditional efficiency of the market index (Panel A) and the conditional efficiency of a time-varying combination of the three Fama-French factors (Panel B). We reject both models over 1963-1994 in both the industry and the size x book/market portfolio designs. The bootstrapped *p*-values are 1.2% or less. We also reject conditional efficiency in the 1995-2002 sample period, with *p*-values of 1.6% or less. Thus, when the conditioning information is used optimally our tests can reject conditional versions of both the CAPM and the Fama-French three-factor model.

5. Robustness

The tests were illustrated under the assumptions that the conditional mean returns are linear functions of the instruments and the conditional covariance matrix is fixed. While this is a common set of assumptions, there are many ways to model conditional moments and it is interesting to consider our framework under alternative specifications. We present some experiments with time-varying covariances. First, it is important to understand how the rejections of efficiency in our examples are inherently robust to misspecified conditional moments.

We test efficiency in the returns constructed from the test assets using the conditioning information. If we use the correct specification the solutions for $x(Z)$ are optimal and the mean variance boundaries they define include the returns of all portfolio functions. If we incorrectly specify the conditional moments the portfolio weights $x(Z)$ are not optimal, but they still generate valid dynamic strategy returns. With the wrong conditional moments we essentially test efficiency in a smaller set of constructed returns, but if we reject the efficiency of a given portfolio on the subset, it implies rejection on the larger set of returns. Therefore, if we reject the efficiency of a given portfolio with incorrectly specified conditional moments, it implies a rejection when the conditional moments are correct.

The robustness to misspecified moments does not necessarily apply when the specification of the tested portfolio depends on the data generating process. For example, the conditional efficiency tests use a maximum correlation portfolio as the tested portfolio. If we get the moments wrong the portfolio is not maximum correlation, and it is no longer implied to be minimum variance efficient. Thus, our rejections of the conditional models in Table 7 could reflect a misspecified data generating process. However, Ferson, Siegel and Xu (2005) show that the weights of the portfolio that maximizes the correlation with respect to Z are "robust" to extreme observations, similar to the efficient-with-respect-to-Z solutions. Thus, the approach is a priori likely to be robust to alternative data generating processes, and the examples in the next subsection illustrate some robustness.

While the rejections for a given portfolio are theoretically robust to incorrectly parameterizing the conditional moments, the results of the simulations may be sensitive to the data generating process. The simulated null distribution of the test statistic may depend on the specification of the conditional moments. We conduct some experiments to evaluate the sensitivity of the tests to alternative specifications for the conditional moments. We focus on the second moments in these experiments.

5.1 Conditional Heteroskedasticity

The assumption of a fixed covariance matrix is a strong one, so it is important to assess the sensitivity of our results to this assumption. We modify the simulations so that the data generating process incorporates conditional heteroskedasticity as a function of the lagged instruments. However, the "artificial analyst" in the simulations treats the data the same way that we did in our original experiments, estimating the test statistics as if the data were homoskedastic. The idea of these experiments is to see how our inferences, based on statistics that ignore heteroskedasticity, might be affected by heteroskedasticity in the data.

Since it may not be possible to agree on the right model for conditional heteroskedasticity, we use two alternative approaches. In the first approach the heteroskedasticity in returns is driven by a factor model for unexpected returns, where the conditional betas on the common factor (the CRSP value-weighted stock index return) are linear functions of the lagged instruments. The linear conditional betas, $\beta(Z)$, are estimated by regressing the unexpected returns on the index return, and the products of the index return and the lagged instruments, similar to Shanken (1990), Cochrane (1996), Ferson and Schadt (1996) and others. The time-varying beta is the regression coefficient on the index plus the coefficients on the product terms multiplied by the lagged instruments. The conditional covariance matrix of the returns is modeled as $\Sigma(Z)=\beta(Z)\beta'(Z)\sigma_f^2+\Sigma_\mu$, where Σ_μ is the fixed covariance matrix of the factor model residuals and σ_f^2 is the fixed conditional variance of the common factor (estimated from the residuals of the linear regression on the lagged *Z*).

The second approach to modeling heteroskedasticity follows Davidian and Carroll (1987) and Ferson and Foerster (1994), among others. The conditional standard deviations of the returns are assumed to be linear functions of the lagged instruments. To estimate this model the absolute residuals from the linear expected return models are regressed on the instruments. The fitted value, multiplied by $\sqrt{\pi/2}$, is the conditional standard deviation. The conditional covariances are then modeled as the products of the standard deviations and the fixed conditional correlations, where the correlations are estimated from the residuals of the mean equations.

Table 8 presents the results of using the linear beta model for conditional heteroskedasticity. The models tested in tables 6 and 7 are evaluated again under heteroskedastic data. In panels A and B we test conditional efficiency. The null hypothesis is imposed in this case by restricting the conditional means of the returns according to the conditional multibeta model, which now features linear conditional betas. Panel C tests dynamic intersection with the Fama-French factors. Here the null hypothesis is imposed by substituting portfolios under the null, that are efficient with respect to Z, but now given the heteroskedastic data generating process. Table 9 summarizes the results of using the linear standard deviation model for heteroskedasticity.

The results in tables 8 and 9 are similar. The sample values of the test statistics are identical to the previous tables when testing dynamic intersection, but slightly different when testing conditional efficiency. This is because when testing conditional efficiency the specification of the stochastic discount factor changes under heteroskedasticity.[13] The differences are small, however. We experiment by computing the sample values of the various test statistics, either using the heteroskedastic structure in the calculations or ignoring it, and the sample values are not very sensitive to the choice. This lack of sensitivity foreshadows the main results of tables 8 and 9.

The experiments in tables 8 and 9 show that the size of the tests may be affected by misspecified conditional second moments. The empirical 5% critical values of the tests are different from those in the previous tables. The differences are small using the 1995-2002 sample, but larger using the 1963-1994 sample. However, these differences do not change the inferences. The empirical p-values are close to those in tables 6 and 7. We conclude that our original inferences are robust to conditional heteroskedasticity, even though the test statistics were computed ignoring the heteroskedasticity.

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[13] Under conditional efficiency the stochastic discount factor is $A(Z) + B(Z)'R_B$, and the coefficients $A(Z)$ and $B(Z)$ change when the data generating process changes.

6. Conclusions

We develop a new framework for testing asset pricing models in the presence of lagged conditioning information. The approach requires the model to correctly price all the dynamic portfolio returns that may be constructed from a set of test assets, where the portfolio weights may be functions of the conditioning information. By requiring the model to price a large set of payoffs, the tests can reject models that previous approaches would not reject.

Our tests examine the (unconditional) mean variance efficiency of a portfolio with respect to the conditioning information, a version of efficiency studied previously by Hansen and Richard (1987) and Ferson and Siegel (2001). This paper shows how different specifications for a model's stochastic discount factor identify portfolios that should be efficient with respect to the conditioning information. If we reject the efficiency of the portfolio, we reject the asset pricing model. We illustrate the approach with versions of the Capital Asset Pricing model, the Fama-French (1996) factors, and a dynamic version of mean-variance intersection (Huberman, Kandel and Stambaugh, 1987).

Using a standard set of lagged instruments and test portfolios, the efficiency of the SP500 index and all time-varying combinations of the Fama-French factor returns are rejected. In the same setting, the commonly-used "multiplicative" approach to conditioning information does not significantly expand the mean variance boundary, when compared to ignoring the conditioning information altogether, nor can it reject all the models. A holdout sample illustrates that the predictive power of the lagged variables declines after 1995, but even during this period the optimal use of the conditioning information enhances the results.

Our paper suggests opportunities for future research. In our empirical examples we use the average Treasury bill return as the fixed risk-free rate. We provide analytical results for a general zero-beta rate. The empirical results may be sensitive to the choice of the zero beta rate. Therefore, it should be interesting in future research to apply our framework in a setting where the zero beta rate is a parameter to be estimated, perhaps by extending results in Kandel (1984). Some of our results use a maximum correlation, mimicking portfolio. It should be possible to study models in which the correlation is less than the maximum, as would be implied by missing assets for example, perhaps by extending results in Kandel and Stambaugh (1989). Future applications of our framework should also consider alternative test assets, asset pricing models and data generating processes. International asset pricing and portfolio performance evaluation may be especially interesting applications.

Appendix

Efficient Portfolio Solutions. The portfolio weights for efficient portfolios in the presence of conditioning information are derived by Ferson and Siegel (2001). Consider N risky assets with returns R and a riskless asset returning R_f. In N×1 column-vector notation, we have

$$R = \mu(Z) + \varepsilon \tag{13}$$

There can be any number of conditioning variables, Z, in the expected return vector, μ(Z). The noise term ε is assumed to have conditional mean zero given Z and nonsingular conditional covariance matrix $\boldsymbol{\Sigma}_\varepsilon(Z)$.

Define portfolio P by letting the 1×N row vector $x'(Z) = (x_1(Z), ..., x_N(Z))$ denote the portfolio share invested in each of the N risky assets, investing (or borrowing) at the riskless rate the amount $1 - x'(Z)\underline{1}$, where $\underline{1} = (1,...,1)'$ denotes the column vector of ones. The observed return on this portfolio will be $R_f + x'(Z)(R - R_f\underline{1})$, with unconditional expectation and variance as follows:

$$\mu_p = R_f + E[x'(Z)(\mu(Z) - R_f\underline{1})] \tag{14}$$

$$\sigma_P^2 = E\left\{x'(Z)\left[\left(\mu(Z) - R_f\underline{1}\right)\left(\mu(Z) - R_f\underline{1}\right)' + \boldsymbol{\Sigma}_\varepsilon(Z)\right]x(Z)\right\} - \left(\mu_p - R_f\right)^2 \tag{15}$$

$$= E\left[x'(Z)Q^{-1}x(Z)\right] - \left(\mu_p - R_f\right)^2$$

where we have defined the N×N matrix

$$Q = Q(Z) = \left\{E\left[(R - R_f\underline{1})(R - R_f\underline{1})'|Z\right]\right\}^{-1} = \left[(\mu(Z) - R_f\underline{1})(\mu(Z) - R_f\underline{1})' + \boldsymbol{\Sigma}_\varepsilon(Z)\right]^{-1} \tag{16}$$

Define the constant ζ as follows:

$$\zeta = E\{[\mu(Z) - R_f\underline{1}]'Q[\mu(Z) - R_f\underline{1}]\} \tag{17}$$

Theorem 1. (Ferson and Siegel, 2001) Given the unconditional expected return μ_P, N risky assets, and a riskless asset, the unique portfolio having minimum unconditional variance is determined by the weights:

$$x'(Z) = \frac{\mu_P - R_f}{\zeta}\left[\mu(Z) - R_f \underline{1}\right]' Q \,. \tag{18}$$

Proof. See Ferson and Siegel (2001).

Proof of Proposition 2. By the definition of covariance, $E\left[m_{t+1}x'(Z_t)R_{t+1}\right] = 1$ implies

$$E\left[x'(Z_t)R_{t+1}\right] = \left\{1 - Cov\left[m_{t+1}, x'(Z_t)R_{t+1}\right]\right\} / E\left(m_{t+1}\right). \tag{19}$$

Now, using $m_{t+1} = A + B'R_{B,t+1}$, we find that Equation (6) is satisfied, with $R_{p,t+1} = w'R_{B,t+1}$, $w \equiv B/(\underline{1}'B), \gamma_0 = [A + B'E(R_{B,t+1})]^{-1}$, and $\gamma_1 = -\gamma_0(\underline{1}'B)$. ■

Proof of Proposition 3. Regress m on R_P using a simple regression: $m = a + bR_P + u$, where without loss of generality a and b are constants and $E(u) = E\left(uR_P\right) = 0$. If R_P is maximum correlation with respect to Z, then the error also satisfies: $E\left[ux'(Z)R\right] = 0 \;\; \forall x(Z): x'(Z)\underline{1} = 1$. If this were not true for some $x(Z)$, then $x'(Z)R$ enters an expanded regression with R_P and $x'(Z)R$ on the right-hand side. Since the regression maximizes the squared correlation, this contradicts the assumption that R_P is maximum correlation. Now, substitute the regression into (4) to obtain $E\left[\left(a + bR_P + u\right)x'(Z)R\right] = 1 = E\left[\left(a + bR_P\right)x'(Z)R\right] \;\; \forall x(Z): x'(Z)\underline{1} = 1$. Proposition 2 now establishes that R_P is efficient with respect to Z. ■

*Theorem 2***.** (Ferson, Siegel and Xu, 2005). The solution, $x_m(Z)$ to the maximization:

$$\underset{x(Z)}{Max}\,\rho^2\left[x'(Z)R, F\right] \text{ s.t. } x'(Z)\underline{1} = 1, \tag{20}$$

where F is any random variable, is given by:

$$x_m(Z) = \frac{\Lambda\underline{1}}{\underline{1}'\Lambda\underline{1}} + \left(\Lambda - \frac{\Lambda\underline{1}\underline{1}'\Lambda}{\underline{1}'\Lambda\underline{1}}\right)\{-\lambda_1\mu(Z) - \lambda_2 E(RF|Z)\}, \text{ where} \tag{21}$$

$\Lambda=\Lambda(Z)=\{E(RR' \mid Z)\}^{-1} = \{\mu(Z)\mu'(Z)+\Sigma_\varepsilon(Z)\}^{-1}$, $\gamma_1(Z) = 1/(\underline{1}'\Lambda\underline{1})$, $\gamma_\mu(Z) = \underline{1}'\Lambda\mu(Z)/(\underline{1}'\Lambda\underline{1})$,

$$\gamma_F(Z) = \underline{1}'\Lambda E(RF'|Z)/(\underline{1}'\Lambda\underline{1}),$$

$$\varphi(Z) = [\Lambda - \Lambda\underline{1}\,\underline{1}'\Lambda/(\underline{1}'\Lambda\underline{1})], \quad \gamma_{\mu\mu}(Z) = \mu(Z)'\varphi(Z)\mu(Z),$$

and $\gamma_{\mu F}(Z) = \mu(Z)'\varphi(Z)E(RF'|Z)$, where:

$$\lambda_1 = \frac{-\gamma_1[E(F) - \gamma_{\mu F}] + \gamma_\mu\gamma_F}{\gamma_\mu[E(F) - \gamma_{\mu F}] + \gamma_F[\gamma_{\mu\mu} - 1]},$$

$$\lambda_2 = \frac{-\gamma_1[\gamma_{\mu\mu} - 1] - \gamma_\mu^{\ 2}}{\gamma_\mu[E(F) - \gamma_{\mu F}] + \gamma_F[\gamma_{\mu\mu} - 1]},$$

and:

$$\gamma_1 = E[\gamma_1(Z)], \quad \gamma_\mu = E\left[\gamma_\mu(Z)\right], \quad \gamma_F = E[\gamma_F(Z)]$$

$$\gamma_{\mu\mu} = E\left[\gamma_{\mu\mu}(Z)\right], \text{ and } \gamma_{\mu F} = E\left[\gamma_{\mu F}(Z)\right],$$

Proof of Proposition 4. Observe that $\alpha_c \equiv \mu_c - [\gamma_0 + (\mu_p - \gamma_0)\sigma_{cp}/\sigma_p^2]$ depends on the portfolio R_c only through its mean and covariance with R_p. It follows that R_c must have minimal variance among all portfolios with its mean and covariance with R_p. From Ferson, Siegel and Xu (2005, Eq. 6) the optimal weights $x_c(Z)$ corresponding to R_c are given by:

$$x_c(Z) = \frac{\Lambda \underline{1}}{\underline{1}'\Lambda \underline{1}} + \left(\Lambda - \frac{\Lambda \underline{1}\underline{1}'\Lambda}{\underline{1}'\Lambda \underline{1}} \right) [a\mu(Z) + bE(RR_p|Z)] , \tag{22}$$

where a and b are constants. Substituting $R_p = R' x_p(Z)$ we obtain:

$$x_c = (1-b)\frac{\Lambda 1}{1'\Lambda 1} + a\left(\Lambda - \frac{\Lambda 1 1'\Lambda}{1'\Lambda 1} \right)\mu(Z) + bx_p(Z) . \tag{23}$$

Comparing equation (23) with Equation (19) in Ferson and Siegel (2001) we conclude that the most mispriced portfolio must be formed by combining an efficient-with-respect-to-*Z* portfolio with R_p.

Without loss of generality we represent the efficient-with-respect-to-*Z* frontier using the following two portfolios. Let R_0 denote the efficient portfolio with (unconditional) mean $\mu_0 = \gamma_0$ and unconditional variance σ_0^2. Note that the covariance between R_0 and R_s is $\sigma_{os} = 0$ as R_0 is a zero-beta asset for R_s.

Consider the system of three assets (R_o, R_s, R_p), which has mean vector (γ_0, μ_s, μ_p) and alpha vector with respect to R_p equal to

$\alpha' = (\alpha_0, \alpha_s, \alpha_p) = \left[-(\mu_p - \gamma_o)\sigma_{op} / \sigma_p^2, \mu_s - \gamma_0 - (\mu_p - \gamma_0)\sigma_{sp} / \sigma_p^2, 0 \right]$. The covariance matrix for these assets is

$$V = \begin{bmatrix} \sigma_o^2 & 0 & \sigma_{op} \\ 0 & \sigma_s^2 & \sigma_{sp} \\ \sigma_{op} & \sigma_{sp} & \sigma_p^2 \end{bmatrix} \tag{24}$$

Note that the efficient-with-respect to *Z* frontier and portfolio R_p are accessible as fixed-weight portfolios within this system.

We maximize the mispricing by maximizing the squared Sharpe ratio within an isomorphic system of assets defined as $(R_0^+, R_s^+, R_p^+) = (R_0 - \gamma_0 + \alpha_0, R_s - \mu_s + \alpha_s, R_p - \mu_p)$, constructed so that the alphas of the original system are equal to the means in the isomorphic system: $E(R_0^+, R_s^+, R_p^+) = (\alpha_0, \alpha_s, 0)$ and we define the zero beta rate in the isomorphic system to be zero. The variances for any fixed portfolio weight function will be the same in the original and the isomorphic system. Thus, the mispricing to be maximized, α_c^2 / σ_c^2 in the original system is equal to the squared Sharpe ratio μ_c^2 / σ_c^2 in the isomorphic system. It follows that the most mispriced portfolio weights x_c are proportional to $V^{-1}\alpha$.

We show that the portfolio R_o has zero weight in the most mispriced portfolio, which establishes that R_c is a combination of R_s and R_p.. To see this, note that when we multiply the first row of V^{-1} by α, the result is proportional

to

$$\left(\begin{vmatrix} \sigma_s^2 & \sigma_{sp} \\ \sigma_{sp} & \sigma_p^2 \end{vmatrix}, -\begin{vmatrix} 0 & \sigma_{sp} \\ \sigma_{op} & \sigma_p^2 \end{vmatrix}, \begin{vmatrix} 0 & \sigma_s^2 \\ \sigma_{op} & \sigma_{sp} \end{vmatrix} \right) \left(\alpha_0, \alpha_s, 0\right)' = \alpha_0 \left(\sigma_s^2 \sigma_p^2 - \sigma_{sp}^2\right) + \alpha_s \sigma_{op} \sigma_{sp}$$

$$= \left[-\left(\mu_p - \gamma_0\right) \sigma_{op} / \sigma_p^2 \right] \left(\sigma_s^2 \sigma_p^2 - \sigma_{sp}^2\right) + \left[\mu_s - \gamma_0 - \left(\mu_p - \gamma_0\right) \sigma_{sp} / \sigma_p^2 \right] \sigma_{op} \sigma_{sp}$$

$$= \left[-\left(\mu_p - \gamma_0\right) \sigma_{op} \sigma_s^2 \right] + \left(\mu_s - \gamma_0\right) \sigma_{op} \sigma_{sp} = -\sigma_{op} \sigma_s^2 \left[\left(\mu_p - \gamma_0\right) - \left(\mu_s - \gamma_0\right) \sigma_{sp} / \sigma_s^2 \right] = 0$$

where the last equality follows from the fact that R_s is efficient with a zero-beta rate of γ_0 and thus $\mu_p = \gamma_0 + \left(\mu_s - \gamma_0\right) \sigma_{sp} / \sigma_s^2$.

Since R_o does not appear in the most mispriced portfolio, we may maximize α_c^2 / σ_c^2 over the restricted isomorphic system (R_s^+, R_p^+). The optimal weights $(w_s, w_p)'$ will be proportional to

$$\begin{pmatrix}\sigma_s^2 & \sigma_{sp}\\ \sigma_{sp} & \sigma_p^2\end{pmatrix}^{-1}\begin{pmatrix}\alpha_s\\ \alpha_p\end{pmatrix}=\begin{vmatrix}\sigma_s^2 & \sigma_{sp}\\ \sigma_{sp} & \sigma_p^2\end{vmatrix}^{-1}\begin{pmatrix}\sigma_p^2 & -\sigma_{sp}\\ -\sigma_{sp} & \sigma_s^2\end{pmatrix}\begin{pmatrix}\mu_s-\gamma_0-\left(\mu_p-\gamma_0\right)\sigma_{sp}/\sigma_p^2\\ 0\end{pmatrix}$$

$$=\begin{vmatrix}\sigma_s^2 & \sigma_{sp}\\ \sigma_{sp} & \sigma_p^2\end{vmatrix}^{-1}\left[\mu_s-\gamma_0-\left(\mu_p-\gamma_0\right)\sigma_{sp}/\sigma_p^2\right]\sigma_p^2\begin{pmatrix}1\\ -\sigma_{sp}/\sigma_p^2\end{pmatrix}$$

hence (w_s,w_p) is proportional to $(1,-\sigma_{sp}/\sigma_p^2)$, which establishes Equation (12) after normalization. To establish Equation (11), substitute for σ_{sp} using $\mu_p=\gamma_0+(\mu_s-\gamma_0)\sigma_{sp}/\sigma_s^2$.

■

Evaluating the Tests by Simulation. We conduct simulation experiments to evaluate the test statistics. Consider first a case with no conditioning information. In Monte Carlo experiments we draw random samples from a normal distribution with mean vector and covariance matrix set equal to the maximum likelihood (ML) estimates from our data. Under one null hypothesis the fixed-weight portfolio R_p should be minimum variance efficient. We replace R_p in the simulations by a fixed-weight portfolio whose weights maximize the Sharpe ratio at the ML estimates. Thus, each artificial sample is drawn from a population in which the tested portfolio R_p is efficient. Each simulation experiment produces 1,000 artificial samples, and we estimate the relevant test statistic on each sample. The empirical 5% critical value is the value above which 5% of the 1,000 statistics lie. The empirical *p*-value is the fraction of the 1,000 statistics that are larger than the value obtained in the original sample. The logic is that if this *p*-value is small, it is unlikely that the sample statistic comes from a population in which the null hypothesis is true.

The Monte Carlo results may be sensitive to the assumption of normally distributed data. We therefore resample from the original data instead of a normal distribution, using a parametric bootstrap approach. A regression of the returns on the lagged conditioning information defines the conditional mean function and the data matrix of sample residuals. We choose randomly selected rows, with replacement, from the matrix of the sample residuals; the number of draws matches the length of the time series in the relevant subperiod. We use the conditional mean functions, evaluated at the simulated Z, and add the independently resampled residuals (unexpected returns) to obtain the simulated returns.

When conditioning information is involved the distribution of Z is taken from the empirical distribution of the 5 lagged instruments. In order to capture the strong serial dependence of these instruments we model Z_t as a vector AR(1) process. The sample AR(1) coefficient matrix is a parameter of the simulations. We resample from the matrix of residuals of the AR(1) model and build the time series of the Z_t's recursively in each simulation trial.

Under the null hypothesis the artificial samples are drawn from a population in which the tested portfolio R_P is efficient with respect to Z. The precise manner in which this is accomplished depends on the situation. When the null hypothesis places a given portfolio on the efficient-with-respect-to-Z frontier, we simply replace the tested portfolio return with the time-varying combination of test assets that is ex ante efficient given the data generating process (Tables 2 through 4). When the null hypothesis specifies that a fixed weight combination of factors is efficient, we replace the first factor with the ex ante efficient portfolio (Table 5).

The Corrollary to Proposition 3 describes the case of dynamic intersection. In this case we exploit the most mispriced portfolio of Proposition 4 in order to generate data that satisfy the null hypothesis. We first form a portfolio that is efficient with respect to Z within the set of $k-1$ of the factor portfolios for the given data generating process. This portfolio, call it R_{k-1}, will be inefficient in the full set of assets. We then use Proposition 4 to compute the most mispriced portfolio by R_{k-1}. A combination of R_{k-1} and its most-mispriced portfolio is efficient in the full sample of test assets given the data generating process. We replace the *k*-th factor with the most mispriced portfolio. With this replacement, the *k* factor-portfolios satisfy the null hypothesis that they are efficient in the full set of test assets (Table 6). When the null hypothesis specifies the conditional efficiency of a combination of the benchmark returns, R_B, we satisfy the null hypothesis by replacing the conditional mean functions of the test assets with the expressions implied by the equivalent conditional beta pricing restriction: $\mu(Z) = \gamma_o + \Sigma_{j=1}^{k} \beta_j(Z) E[R_{Bj} - \gamma_0 | Z]$, where $\beta_j(Z)$ is the vector of conditional betas on the *j*-th benchmark return (Table 7).

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Table 1. Monthly Industry Returns

Monthly returns on 25 portfolios of common stocks are from Harvey and Kirby (1996). The portfolios are value-weighted within each industry group, based on the SIC codes as shown. *Mean* is the sample mean of the gross (one plus rate of) return, σ is the sample standard deviation and ρ_1 is the first order autocorrelation of the monthly return. R^2 is the adjusted coefficient of determination in percent from the regression of the returns on the lagged instruments. The sample period is February of 1963 through December of 1994 (383 observations). $R^2_{HOLDOUT}$ is for the 1995-2002 holdout sample (96 observations). Negative adjusted *R*-squares are reported as 0.0.

	Industry	SIC codes	Mean	σ	ρ_1	R^2	$R^2_{HOLDOUT}$
1	Aerospace	372, 376	1.0107	0.0644	0.13	9.9	1.1
2	Transportation	40, 45	1.0094	0.0648	0.08	9.1	0.0
3	Banking	60	1.0086	0.0631	0.10	4.3	2.4
4	Building materials	24, 32	1.0097	0.0608	0.09	10.4	0.0
5	Chemicals/Plastics	281, 282, 286-289, 308	1.0094	0.0525	-0.01	8.0	2.5
6	Construction	15-17	1.0109	0.0760	0.16	10.2	0.0
7	Entertainment	365, 483, 484, 78	1.0135	0.0662	0.14	5.7	0.0
8	Food/Beverages	20	1.0117	0.0449	0.05	6.6	0.2
9	Healthcare	283, 384, 385, 80	1.0113	0.0524	0.01	2.4	0.0
10	Industrial Mach.	351-356	1.0089	0.0587	0.05	8.2	0.0
11	Insurance/Real Estate	63-65	1.0095	0.0581	0.15	6.4	2.3
12	Investments	62, 67	1.0097	0.0453	0.05	8.7	4.1
13	Metals	33	1.0075	0.0610	-0.02	4.5	0.2
14	Mining	10, 12, 14	1.0108	0.0535	0.01	7.2	0.3
15	Motor Vehicles	371, 551, 552	1.0095	0.0584	0.11	10.6	0.0
16	Paper	26	1.0095	0.0536	-0.02	6.9	2.4
17	Petroleum	13, 29	1.0102	0.0518	-0.02	4.4	0.0
18	Printing/Publishing	27	1.0114	0.0586	0.21	11.3	0.0
19	Professional Services	73, 87	1.0111	0.0693	0.13	8.4	2.8
20	Retailing	53, 56, 57, 59	1.0106	0.0597	0.15	8.7	3.7
21	Semiconductors	357, 367	1.0080	0.0559	0.08	9.0	0.0
22	Telecommunications	366, 381, 481, 482, 489	1.0085	0.0412	-0.05	5.4	8.8
23	Textiles/Apparel	22, 23	1.0100	0.0613	0.21	11.0	0.0
24	Utilities	49	1.0078	0.0392	0.02	6.8	4.3
25	Wholesaling	50, 51	1.0109	0.0614	0.13	10.7	0.0

Table 2

Tests of the mean variance efficiency of the Standard and Poors 500 stock index in a sample of industry portfolio returns. The monthly returns on 25 industry-sorted portfolios of common stocks are measured, for the sample period February 1963 through December 1994 (T=383 observations), and ten-year subperiods. A holdout sample from January, 1995 through December, 2002 (96 observations) is also shown. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. NA denotes not applicable, when the number of assets is larger than the number of time series observations and the covariance matrix of the returns is singular. Asymptotic *p*-values are from the chi-squared distribution. GRS *p*-values are from the *F* distribution, after the test statistic is rescaled to have an exact *F* distribution assuming normality as in Gibbons, Ross, and Shanken (1989).

Subperiod	63-72	73-82	83-92	63-94	95-02
Panel A: Test assets $R_{t,}$ no conditioning information:					
Wald Statistic	32.8	26.3	29.8	24.8	51.3
asymptotic *p*-value	0.14	0.39	0.23	0.48	0.001
GRS *p*-value	0.51	0.75	0.65	0.65	0.13
Monte Carlo 5% Critical Value	52.8	52.3	50.8	51.9	211.4
empirical *p*-value	0.43	0.71	0.58	0.59	0.70
Resampling 5% Critical Value	60.1	63.9	62.3	40.0	231.4
empirical *p*-value	0.52	0.81	0.65	0.58	0.72
Panel B: Test assets are $R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$:					
Wald Statistic	NA	NA	NA	348.6	NA
asymptotic *p*-value				0.00	
GRS *p*-value				0.03	
Monte Carlo 5% Critical Value				328.0	
empirical *p*-value				0.02	
Resampling 5% Critical Value				476.0	
empirical *p*-value				0.44	
Panel C: Test assets are all Portfolios $x'\left(Z_{t-1}\right)R_t$:					
Test Statistic	203.3	188.6	165.0	161.8	148.2
Monte Carlo 5% Critical Value	125.7	121.6	121.6	133.3	139.9
empirical *p*-value	0.000	0.000	0.001	0.002	0.029
Resampling 5% Critical Value	117.3	130.6	121.6	118.8	144.9
empirical *p*-value	0.003	0.005	0.003	0.001	0.044

Table 3

Tests of the mean variance efficiency of the Standard and Poors 500 stock index return. The industry data are monthly returns on 25 industry-sorted portfolios of common stocks, for the sample period February 1963 through December 1994 (T=383 observations). The size/BM returns are 25 portfolios of stocks sorted on market capitalization and book-to-market ratio, for the sample period July 1963 through December 1994 (T=378 observations). A holdout sample covers January 1995 through December, 2002 (96 observations). The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. Asymptotic p-values are from the chi-squared distribution. NA indicates that the sample size does not allow the statistic to be calculated.

Sample	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Test assets $R_{t,}$ no conditioning information:				
Sample Statistic	83.0	74.1	24.8	51.3
asymptotic p-value	0.000	0.000	0.48	0.001
Monte Carlo 5% Critical Value	41.0	114.6	51.9	211.4
empirical p-value	0.000	0.192	0.59	0.70
Resampling 5% Critical Value	45.1	131.5	40.0	231.4
empirical p-value	0.000	0.277	0.58	0.72
Panel B: Test assets are $R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$:				
Sample Statistic	517.1	NA	348.6	NA
asymptotic p-value	0.000		0.000	
Monte Carlo 5% Critical Value	342.0		328.0	
empirical p-value	0.000		0.02	
Resampling 5% Critical Value	508.8		476.0	
empirical p-value	0.040		0.44	
Panel C: Test assets are all Portfolios $x'(Z_{t-1})R_t$:				
Sample Statistic	272.7	210.4	161.8	148.2
Monte Carlo 5% Critical Value	120.8	131.9	133.3	139.9
empirical p-value	0.000	0.000	0.002	0.029
Resampling 5% Critical Value	107.6	135.1	118.8	144.9
empirical p-value	0.000	0.003	0.001	0.044

Table 4

Tests of the null hypothesis that conditioning information does not expand the mean variance boundary. The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index return. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. Asymptotic *p*-values are from the chi-squared distribution. NA indicates that the sample size does not allow the test statistic to be calculated.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Test assets are $R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$:				
Sample Statistic	356.3	NA	304.3	NA
asymptotic *p*-value	0.000		0.000	
Monte Carlo 5% Critical Value	338.2		326.7	
empirical *p*-value	0.033		0.141	
Resampling 5% Critical Value	520.4		486.0	
empirical *p*-value	0.464		0.686	
Panel B: Test assets are all Portfolios $x'(Z_{t-1}) R_t$:				
Sample Statistic	155.8	77.7	128.8	63.8
Monte Carlo 5% Critical Value	122.3	127.7	133.4	134.0
empirical *p*-value	0.000	0.458	0.067	0.806
Resampling 5% Critical Value	108.7	138.3	118.8	148.1
empirical *p*-value	0.001	0.539	0.025	0.779

Table 5

Tests of the null hypothesis that a fixed-weight combination of the three Fama-French factors is efficient. The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a value-weighted index. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratio and a value-weighted return. In each design the first and 25th portfolio returns are replaced with the returns of the HML and SMB factors, respectively. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds. Asymptotic *p*-values are from the chi-squared distribution. NA indicates that the sample size does not allow the test statistic to be calculated.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Test assets are R_t:				
Sample Statistic	35.0	49.5	43.0	55.5
asymptotic *p*-value	0.089	0.002	0.014	0.004
Resampling 5% Critical Value	41.6	64.0	39.2	61.2
empirical *p*-value	0.117	0.157	0.021	0.077
Panel B: Test assets are all Portfolios $R_{ft} + (R_t - R_{ft}) \otimes Z_{t-1}$:				
Sample Statistic	521.9	NA	415.0	NA
asymptotic *p*-value	0.000		0.000	
Resampling 5% Critical Value	319.3	NA	313.7	NA
empirical *p*-value	0.000		0.000	
Panel C: Test assets are $x'\left(Z_{t-1}\right) R_t$:				
Sample Statistic	340.6	181.6	180.1	174.6
Resampling 5% Critical Value	70.5	128.0	75.6	118.4
empirical *p*-value	0.000	0.003	0.000	0.001

Table 6

Tests of dynamic intersection. The null hypothesis is that an efficient-with-respect-to-Z combination of the three Fama French factors touches the efficient-with-respect-to-Z frontier of the test assets at a tangency from the risk-free rate. Industry refers to monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index. The first and 25th portfolio returns are replaced with the returns of the HML and SMB factors. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Sample Statistic	268.1	124.3	125.5	118.9
Resampling 5% Critical Value	73.3	127.1	79.6	114.1
empirical *p*-value	0.000	0.055	0.001	0.039

Table 7

Tests of Conditional Efficiency. The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index. In each design the first and 25th portfolio returns are replaced with the returns of the HML and SMB factors. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Conditional Efficiency of the Market Index				
Sample Statistic	339.2	131.7	189.7	143.0
Resampling 5% Critical Value	101.1	88.2	91.4	83.2
empirical *p*-value	0.000	0.008	0.002	0.006
Panel B: Conditional Efficiency of the Fama-French Factors				
Sample Statistic	363.0	132.5	144.4	137.8
Resampling 5% Critical Value	98.1	97.7	117.8	94.2
empirical *p*-value	0.000	0.016	0.012	0.015

Table 8

The Impact of Conditional Heteroskedasticity. The simulated data incorporate conditional heteroskedasticity through a factor model with conditional betas that are linear functions of the lagged instruments. The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index. In each design the first and 25th portfolio returns are replaced with the returns of the HML and SMB factors. The conditioning information consists of a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Conditional Efficiency of the Market Index				
Sample Statistic	344.5	131.4	184.9	144.3
Resampling 5% Critical Value	56.1	84.3	75.5	82.3
empirical *p*-value	0.000	0.008	0.004	0.005
Panel B: Conditional Efficiency of the Fama-French Factors				
Sample Statistic	366.6	140.9	143.4	145.1
Resampling 5% Critical Value	67.1	97.1	78.0	91.7
empirical *p*-value	0.000	0.012	0.010	0.007
Panel C: Dynamic Intersection with Fama-French Factors				
Sample Statistic	268.1	124.3	125.5	118.9
Resampling 5% Critical Value	90.8	123.6	95.3	122.6
empirical *p*-value	0.000	0.046	0.003	0.054

Table 9

The Impact of Conditional Heteroskedasticity. The simulated data incorporate conditional heteroskedasticity through a model in which the conditional standard deviations are linear functions of the conditioning variables and the conditional correlations are constant over time. The industry data are monthly returns on 25 industry-sorted portfolios of common stocks and a market index return. The size/BM returns are for 25 portfolios of stocks sorted on market capitalization and book-to-market ratios and a market index. In each design the first and 25th portfolio returns are replaced with the returns of the HML and SMB factors. The conditioning variables are a lagged Treasury bill yield, dividend yield, excess bill return, and yield spreads of long over short-term Government bonds and low-grade over high-grade corporate bonds.

	size/BM		industry	
	63-94	95-02	63-94	95-02
Panel A: Conditional Efficiency of the Market Index				
Sample Statistic	371.3	152.5	211.3	158.4
Resampling 5% Critical Value	82.9	87.2	74.1	88.9
empirical *p*-value	0.000	0.001	0.000	0.001
Panel B: Conditional Efficiency of the Fama-French Factors				
Sample Statistic	354.4	163.3	142.1	163.2
Resampling 5% Critical Value	93.2	113.4	85.8	103.1
empirical *p*-value	0.000	0.004	0.003	0.001
Panel C: Dynamic Intersection Using Fama-French Factors				
Sample Statistic	268.1	124.3	125.5	118.9
Resampling 5% Critical Value	76.5	131.8	88.4	106.3
empirical *p*-value	0.000	0.066	0.000	0.024