

## Issues in valuation of privately held firms

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# **ISSUES IN VALUATION OF PRIVATELY HELD FIRMS**

## **ABSTRACT**

We examine how Danish investors and corporate financial advisers apply present value approaches in valuing privately held companies. We interview 39 corporate advisers and private equity investors who apply present value approaches as a valuation tool for privately held firms (market coverage of approximately 90%). Our findings indicate that many of the participants comply with the recommendations outlined in the valuation literature. However, the findings also indicate that some practitioners apply the present value approaches incorrectly. For example, some participants do not adjust for differences in financial leverage when applying peers' beta estimates.

Our research also confirms that valuation of privately held firms involves a great deal of judgement. Especially if the literature does not provide clear cut answers. For example, estimating the marketability discount and control premium often involve guesstimate among the participants. Further, several participants argue that the literature in some cases is inadequate. For example, some argue that beta estimates from peers are imprecise and contain severe measurement problems. In the extreme cases they replace beta estimates from peers with a (subjective) evaluation of fundamental risk drivers.

## ISSUES IN VALUATION OF PRIVATELY HELD FIRMS

This study is a comprehensive field study that presents evidence on how financially sophisticated corporate financial advisers and private equity funds apply present value approaches in valuing privately held firms. Previous studies on valuation have primarily been concerned with listed companies and to our knowledge no prior studies have empirically explored how practitioners address problems related to the use of present value approaches on privately held firms. Our Danish field study includes 15 private equity funds and 27 corporate financial advisers, with an acceptance rate of 96%. 14 private equity funds and 25 corporate financial advisers adopt specifically a present value approach for valuation of privately held firms which corresponds to market coverage of 90%. The high acceptance rate and the wide use of present value approaches increase the chances of obtaining valuable insights into valuation of privately held firms. Further, the external validity is also considered high which is unusual in field studies.

Our field study is valuable for a number of reasons. First, valuation theorists have long recommended present value approaches such as the discounted cash flow (DCF) model and the economic value added (EVA) model for firm valuation (Penman, 1997). Popular textbooks such as Stewart (1991), Rappaport (1998), Copeland et al. (2000), Pratt et al. (2000) and Damodaran (2002) have increased the awareness and understanding of the present value approaches among practitioners. Still, most of the valuation literature ignores firm valuation of privately held firms. For example, neither Rappaport nor Copeland et al. address valuation of privately held businesses.<sup>1</sup>

<sup>2</sup> While a valuation of listed companies is complicated, a valuation of a privately held firm introduces a range of additional issues. For example, stock market data are not available, which

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<sup>1</sup> According to the Free Encyclopedia firms that are not listed on a stock exchange is defined as private firms: 'A private firm is a firm that is not a public company' .... 'the shares are held by a small group of individuals often members of one or a small group of families or otherwise related individuals (or other firms).

<sup>2</sup> A notable exception is Pratt et al. (2000).

make the estimation of cost of capital difficult. Further, the annual report is less informative, which make projections of future cash flows more uncertain. In addition lack of marketability raises the issue of how to estimate a marketability discount.

Second, valuation of privately held firms has become increasingly important in recent years. The number of merger and acquisitions (M&A) involving privately held firms has increased considerably. For example Ang and Kohers (2001) find that the volume of acquisitions involving privately held firms has soared to record heights. Based on US data they find that over the period 1984-1996, more than 22,000 acquisitions involving privately held firms have occurred, while 8,000 mergers and acquisitions have involved publicly traded firms. According to Initiative Europe (2005) privately owned businesses accounted for most of the M&A activities in Europe over the period 2000-2004. This development is further stimulated by generational change. In Denmark approximately 2,000 privately held firms are estimated to change hands within the next few years due to generational change (Vaekstfonden, 2004). This tendency is also found in other parts of Western Europe, the US and in the Asian-Pacific region (e.g. Australia).

Privately held firms are also valued for other reasons than just M&A activities. Employee retirement plans that invest in the common stock of the sponsoring privately held firm have resulted in the need for annual valuations of these firms. The introduction of employee stock ownership plans and employee stock options in privately held firms and litigation issues are other reasons. Further, the recent implementation of IFRS 3 on impairment test of goodwill implies that firms must test the valuation of goodwill on an annual basis. IFRS 3 recommends the DCF model as the basic model for carrying out such impairment tests.

Third, as pointed out by Spremann and Gantenbein (2003) most of the empirical literature including surveys on firm valuation examines data from listed firms. Demirakos et al. (2004) examine valuation practices of financial analysts by studying valuation methodologies contained in 104 analysts report for 26 large UK-listed companies. They find that the DCF model and multiples are widely adopted by UK analysts.<sup>3</sup> Another stream of research examines information collected and processed by financial analysts. Previtts et al. (1994) examine the information needs of sell-side analysts in the US. They find that financial reporting provides an important but incomplete basis for sell-side analyst forecasts of firm performance. A third line of research examines how financial analysts estimate cost of capital. Bierman (1993) surveys 74 Fortune 100 companies and finds that they all use some form of discounting in their capital budgeting, and 93% use weighted average cost of capital (WACC). Also, Bruner et al. (1998) examine best practices in estimating the cost of capital. Based on 27 listed companies and ten leading financial advisers they find a broad acceptance of the weighted average cost of capital as the basis for estimating discount rates. They find large variations, however, for the joint choices of the risk free rate, beta, and the equity market risk premium as well as for the adjustment of costs of capital for specific investment risk.

While Denmark is a small country with only 5.3 million inhabitants there are more than 400.000 privately held firms. Only 185 companies are listed on Copenhagen Stock Exchange. This is consistent with the firm structure of other countries (Pedersen and Thomsen 1996), where the vast majority of firms remain private. According to U.S. Census Bureau more than 99% of US firms are privately held. Spremann and Gantenbein (2003) find that privately held firms account for more than 99% of all German firms, employ 70% of the German workforce and generates over 50% of

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<sup>3</sup> Related studies include Lazaridis (2004) that examines methods to evaluate project investments in Cyprus.

the gross value added in the private sector. Denmark, therefore, offers a useful setting for examining how practitioners apply the present value approaches on privately held firms.

This study proceeds as follows. The next section describes the methodology and sample. The following section presents descriptive statistics. The third section analyses the empirical findings and the final section concludes what might be learned from our study.

### METHODOLOGY AND SAMPLE

We develop our original set of questions based on extensive reviews of books and journal articles.<sup>4</sup>

Our questions can be framed within the following two-stage model present value approach

(Copeland et al. (2000))<sup>5</sup>

$$EV_t = \sum_{\tau=1}^T \frac{FCF_{t+\tau}}{\prod_{j=1}^{\tau} (1+WACC_{t+j})} + \frac{FCF_{T+1}}{WACC_{T+1} - g} \cdot \frac{1}{\prod_{j=1}^T (1+WACC_{t+j})} \quad (1)$$

- EV = Enterprise value
- WACC = Weighted average cost of capital
- FCF = Free cash flow
- g = Growth rate (in the terminal period)

The two-stage present value approach consists of a *forecast period* and a *terminal period*. In the forecast period the free cash flow and cost of capital are allowed to vary over time. In the terminal period the free cash flow is assumed to grow at a constant rate and the cost of capital must remain constant. Based on the present value approach firm valuation boils down to projections of cash flows and cost of capital. Further, Pratt (2000) and Damodaran (2002), among others, suggest adjusting firm value estimates for ownership control and lack of marketability. Accordingly, our

<sup>4</sup> A copy of the questions (10 pages) is available upon request from the authors.

<sup>5</sup> The DCF approach is used as an example.

study focuses on 1) projections of cash flows, 2) estimation of cost of capital and 3) special adjustments of estimated firm value.

We adopt the field study methodology to address each of the three issues listed above. Field studies complement other research methods based on either large samples or case studies. As pointed out by Graham and Harvey (2001) large studies are the most common type of empirical analysis, and offer statistical power and cross-sectional variation. On the other hand, large sample studies have weaknesses related to variable specification and the inability to ask qualitative questions. Case studies offer details and unique information of corporate behaviour. However, the sample is small and the results are often sample-specific. The field study approach offers a balance between large sample studies and case studies. We obtain a larger sample than in case studies which improve the external validity. Further, by interviewing each of the participants we were able to ask qualitative questions, which improve the quality of our data.<sup>6</sup>

### **Sample selection**

Both investors and corporate financial advisers are included in the sample. Different sources of information were gathered to ensure a comprehensive list of participants. We obtained data from the following sources:

- Vaekstfonden is a state backed investment company. They provide funding to fast-growing Danish firms and act as a fund-of-funds investor in the private equity (PE) sector in the Nordic region. Vaekstfonden publish an annual report that gives an overview of the venture – and private equity market. According to Vaekstfonden's report 2004 there are 11 PE

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<sup>6</sup> Given the technical nature of some of the research questions we apply semi-structured interviews.

funds<sup>7</sup> and 62 venture funds located in Denmark with an invested capital of 2.5 and 2.6 billions euro, respectively.<sup>8</sup>

- [www.match-online.dk](http://www.match-online.dk) acts as an intermediary between investors and firms. On their homepage there is a list of corporate financial advisers. Seven auditing firms, including the ‘big-four’, and eight independent corporate financial advisers are listed on their homepage.
- Annual reports and homepages for all major banks and financial institutions were examined. Nine banks and financial institutions are actively involved with corporate finance activities.

In addition, the Institute of State Authorized Public Accountants in Denmark (FSR) and market participants were contacted to ensure that all private equity funds and corporate financial advisers were potentially included in the sample. This yields the following comprehensive list of participants:

Insert table 1 here

The list includes 15 private equity funds and 29 corporate financial advisers. Two corporate financial advisers were not willing to participate in the analysis, which yields an acceptance rate of 96%. Further, as an additional selection criterion the participants must apply a present value approach. 14 private equity funds and 25 corporate financial advisers adopt a present value approach for valuation of privately held firms. Thus, the final sample consists of 39 private equity funds and corporate financial advisers. This is equivalent to 93% of the market for private equity

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<sup>7</sup> Some of the PE funds are a subsidiary of international PE funds.

<sup>8</sup> Interviews with twelve venture funds revealed that they rarely apply valuation approaches such as present value techniques and multiples for investment decisions. This is also in line with Mørch and Wassermann (2002) who find that strategic factors such as management, know how, business plan, market opportunities and exit plans are more important than different valuation approaches. As a consequence venture funds are not included in this study.

and 86% of the market for the corporate financial services. Surveys using questionnaires rather than interviews typically have a much smaller acceptance rate. For example, Graham and Harvey (2001) send out 4.440 questionnaires. 392 completed questionnaires were returned equal to a response rate of 8.8%. Mukherjee et al. (2004) examine merger motives and obtain 75 usable responses, which represents 11.8% of the 636 delivered responses. The high acceptance rate obtained in our study and the extensive use of present value approaches, provide us with valuable insights into valuation of privately held firms in Denmark. Further, the high coverage of the market for valuation of privately held firms ensures high external validity.

It is expected that corporate financial institutions affiliated with major banks, financial institutions and accounting firms are mainly involved in large transactions that involve listed companies. On the other hand, 'independent' corporate financial advisers are expected to focus on smaller transactions that involve privately held firms. Since the two groups of corporate financial advisers are expected to yield different insights into valuation of privately held firms this study divides the answers into data from 'dependent corporate financial adviser', 'independent corporate financial adviser' and private equity funds, respectively. Statistical analysis is performed to examine if there are significant differences in the behaviour across the three groups of participants.

Due to the limited number of private equity funds and corporate financial advisers on the Danish market a concern has been the number of observations. This, among other things, explains the adoption of the field study approach. Further, to ensure a high response rate we limit the scope and address only the most interesting research questions prevalent in the field at present. As the length and complexity of the questions increase, the response rate generally declines, which increases the

potential for non-response bias (Mukherjee et al., 2004). We believe, however, that the tradeoff we make is justified and the high response rate seems to support this view.<sup>9</sup>

## DESCRIPTIVE STATISTICS

The participants experience with firm valuation is on average 9.2 years. However, the average level of experience varies across the three groups of participants. As can be seen from table 2 panel A independent corporate financial advisers have on average a much higher level of experience (12.3 years) than dependent corporate financial advisers (6.7 years) and participants from private equity funds (7.3 years).<sup>10</sup> The  $\chi^2$ -statistic<sup>11</sup> also supports that the level of experience is significantly different between the groups of participants (p-value = 0.099).

Insert table 2 here

Panel B in table 2 reveals that all participants from private equity funds and dependent corporate financial advisers hold a master degree. Furthermore, they seem to have a significantly higher level of education than independent corporate financial advisers (p-value = 0.01). 69% of the independent corporate financial advisers in the sample hold a master degree and 25% hold a bachelor degree. One of the independent corporate financial advisers does not hold an academic degree.

Insert table 3 here

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<sup>9</sup> For various reasons some participants did not answer all questions. Thus, the number of answers does not necessarily match with the number of participants.

<sup>10</sup> Since the three participants that do not apply a present value approach offer some insights regarding control premium and marketability discount we include them in the descriptive statistics (tables 1-3).

<sup>11</sup> The  $\chi^2$ -statistic is based on a Kruskal-Wallis test.

Services offered by corporate financial advisers are listed in table 3 panel A. In most cases corporate financial advisers are involved with M&A activities. Other services include value based management, serving as expert witness and fund raising. There are no significant differences (at the 10%-level) in the type of services offered by dependent and independent corporate financial advisers, respectively (p-value = 0.107).

As reported in table 3 panel B a significantly larger proportion of the businesses in dependent corporate finance units involve listed companies (p-value = 0.001). Five (45%) dependent corporate financial advisers generate 50% or more of their revenue from listed companies. In comparison 15 (94%) independent corporate financial advisers state that all or most of their revenue (75-100%) are generated from privately held firms. Further, the clients of independent corporate finance units are significantly smaller than clients of dependent corporate finance units (p-value = 0.001). 74% of the clients of dependent corporate financial advisers generate revenue that exceeds 500 million DKK (see table 3c). In comparison, only 21% of the clients of the independent corporate financial advisers generate revenue that exceeds 500 million DKK. Thus, a distinction between the two groups of corporate financial advisers seems warranted. Although not reported private equity funds invest in both listed and privately held firms.

Insert table 4 here

Table 4 presents the different present value approaches adopted by the participants. DCF is significantly more used than any other present value approach (p-value = 0.001). 34 (87%) participants apply this technique. Only 8 (20%) participants prefer the EVA model. The interviews revealed that in most cases the EVA model was used to check the results obtained from the DCF

valuation. The participants pointed out that value estimates based on these two approaches should be identical. The DDM is primarily used in valuing banks and utilities. Barker (1999) also provides evidence that UK analysts apply the DDM approach on banks and utilities. ‘Other’ covers the leverage buyout (LBO) model. The LBO model is popular among private equity funds. They use the LBO model to calculate the internal rate of return (IRR). In order to estimate entry and exit values in the LBO model, both the DCF model and multiples were applied. Further, a Fischer Exact test of equal use of different present value approaches across the groups of participants reveal significant differences (p-value = 0.002). For example, the LBO models seem to be more popular among private equity funds.

In summary, the participants are highly experienced and well educated. Thus, they should be able to provide valuable insights into firm valuation of privately held companies.

## **EMPIRICAL RESULTS**

This section presents the empirical findings.

### **Projections of cash flows**

A publicly held company is required by the Danish Accounting Act to provide certain information to its stakeholders. Private firms are not legally required to disclose information about their operations and are generally reluctant to do so.<sup>12</sup> As a result publicly held companies are easier to research than private firms. For example, Boehmer and Ljungquist (2000) and Damodaran (2002) find that information on privately held firms is difficult to obtain in Germany and the US,

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<sup>12</sup> Private limited companies in Denmark must follow the Danish accounting legislation if revenue, total assets and number of employees exceed 7 mio US\$, 3 mio. US\$ and 50, respectively. Furthermore, private limited companies that follow the Danish accounting legislation must make their annual reports available for interested parties.

respectively. The lack of information (accounting data) may therefore complicate the estimation of the free cash flow in the forecast period and in the terminal period.

Figure 1 gives an overview of the information typically requested by private equity funds and corporate financial advisers.

Insert figure 1 here

While both internal and external information were regarded as important, the participants seem to emphasise internal information. One plausible explanation is that both private equity funds and corporate financial advisers have better access to external information than internal information. Several participants were also industry specialists, which imply that they already possess knowledge about the market, competitors and the attractiveness of the industry.

Private equity funds and corporate financial advisers find that the quality of internal information varies. For example, the participants find that firms often make forecasts for one year only and the quality is sometimes questionable. Advisers also have difficulties in obtaining information about profitability by product or market. Data are simply not available in the information systems. These data problems were even more severe in smaller businesses and several of the participants regarded it as one of the main problems in the data collection process. Our results support that useful information on privately held firms is difficult to obtain.

As pointed out by Damodaran (2002) the intermingling of business and private expenses is a particular problem in small businesses, since the owners have absolute access to funds and run the

day to day operations. For example, family members are hired to fill phantom positions in order to distribute income or to reduce taxes. Our findings support the intermingling of business and private expenses. The interviews revealed that primarily independent corporate financial advisers adjust the historical accounting numbers for private expenses. In addition, they also make a salary adjustment in cases where the salary to the owner(s) does not reflect the ‘market salary’. Independent corporate financial advisers consider these problems to be positively correlated with the level of ownership concentration. Thus, with a diverse ownership structure it is less likely that these types of adjustments need to be made. The findings also supports the ones found by Demsetz (1983) and Fama and Jensen (1983) who argue that entrenched manager-shareholders may enjoy benefits other than maximizing shareholder value, such as perk-extracting, thereby effectively receiving tax-free dividends that are not given to potential outside shareholders.

Copeland et al. (2000) recommend using a forecast period of 10 to 15 years for most firms. Firms with high growth rates or cyclical firms may need an even longer time to reach a relatively mature stage. Our results show that all participants operate with a finite forecasting horizon. Most of the corporate financial advisers adjust their forecasts according to the type of business. For example, mature firms require a shorter forecast horizon than firms with high growth.

Insert table 5 here

Further, as shown in table 5 all participants make forecasts ranging from two to 12 years. The average forecasting horizon is six years. The average forecasting horizon by private equity funds is only 4.2 years, which may be expected considering their short investment horizon. In comparison, the average forecast horizon for dependent and independent corporate financial advisers is 6.1 years and 7.2 years, respectively. In general, it is not possible to detect any significant differences in the

length of forecasting horizons across the three groups of participants (p-value = 0.41). Our results also indicate that the length of forecasting period is significantly shorter than the ten years recommended by Copeland et al. (2000) (p-value = 0.001). Copeland et al. argue that the consequences of using a (too) short forecasting period is a significant undervaluation of a firm or it requires heroic long-term growth assumptions. Confronted by this statement several participants argue that it is impossible to prepare sensible forecasts for more than ten years. They add that many industries are mature and experience only modest growth rates which imply a shorter forecast horizon than suggested by Copeland.

According to Copeland et al. (2000) and Damodaran (2002) a fundamental question in the terminal value calculation is whether growth adds value, i.e. does return on invested capital (ROIC) exceeds the weighted average cost of capital (WACC) on new investments. Both Gordon's growth model<sup>13</sup> and the value driver model<sup>14</sup> assume that growth affect value in the terminal period.<sup>15</sup> On the other hand, the convergence model<sup>16</sup> assumes that growth does not contribute to firm value.

Insert table 6 here

Table 6 presents how the participants calculate terminal value. Gordon's growth model is significantly more used than any other terminal value model (p-value = 0.001). 80% of the participants apply Gordon's growth model, while 14% use the value driver model. Thus, most

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<sup>13</sup> Gordon's growth model is equal to  $\frac{FCF}{WACC-g}$ .

<sup>14</sup> The value driver model is equal to  $\frac{NOPAT \cdot (1-g/ROIC)}{WACC-g}$  where NOPAT equals net profit after tax and ROIC equals return on invested capital.

<sup>15</sup> Gordon's growth model and the value driver model are theoretically equivalent.

<sup>16</sup> The convergence model is equal to  $\frac{NOPAT}{WACC}$ .

assume that growth affect firm value. Only six participants assume that ROIC equals WACC, i.e. they apply the convergence model. One (3%) independent corporate financial adviser ignores cash flows after 7 - 9 years. He argues that the value in a small business often depends on the presence of the owner. As soon as (s)he retires, the firm's most valuable asset disappears. Finally, five (14%) participants estimate terminal value using a multiple such as EV/EBIT or EV/EBITDA. The use of multiples as a mean to estimate the terminal value is generally not recommended in the valuation literature. For example, Copeland et al. (2000, 284) argue that traditional terminal value approaches such as Gordon's growth model are easier to apply than multiples. Further, since the terminal value often accounts for 60-80% in a DCF-valuation, applying a multiple should capture the major part of the value creation. Given the inherent problems in multiples, it is at best problematic that multiples are used to estimate value generated by a firm in the terminal period. The participants who apply multiples argue that Gordon's growth model is also based on rather simple assumptions, which, they believe, justify the use of multiples. A Fisher Exact test reveals that the three groups of participants do not apply different types of terminal value models ( $p$ -value = 0.877).

Both Gordon's growth model and the value driver model require an estimate of the long-term growth in a firm. From a theoretical perspective the long-term growth rate in an industry is a function of the expected inflation and expected real growth in the industry. Damodaran (1996, 193) argues that 'in practical terms, the stable growth rate cannot be larger than the nominal growth rate in the economy in which the firm operates.....The growth rate in the economy is determined by the expected inflation and real growth rate in GDP.' The average (median) growth rate is 3.1% across all three categories of participants and test statistics support that there are no significant differences in the growth rate adopted across the different groups of participants ( $p$ -value = 0.887) (results not reported). Currently, the inflation rate is approximately 2% p.a., which means that the

participants on average expect a real growth rate close to 1% p.a. for privately held companies in Denmark. This is close to the average real growth rate in Denmark and EU in the past five years. Thus, there seems to be a correspondence between theory and practice.

### **Estimation of cost of capital**

The following section presents the cost of capital estimation techniques adopted by the participants. Since privately held companies are not traded on a stock exchange, the CAPM and APT model cannot be used in a traditional sense. Further, credit ratings are rarely applied on Danish firms. According to the Danish accounting legislation privately-held firms are neither required to publish interest rates on nor the market value of interest-bearing debt. The lack of market data and credit ratings, therefore, complicates the estimation of the parameters included in WACC. Table 7 represents the results on how the participants estimate cost of equity.

Insert table 7 here

The results show that the capital asset pricing model (CAPM) is adopted by 25 (71%) participants. 16 (46%) participants rely on their experience (from previous transactions) or estimates from their clients. Even though CAPM seems more popular among the participants the difference between the two techniques remain insignificant ( $p$ -value = 0.160). None of the participants seem to use other methodologies such as at the arbitrage pricing theory (APT). A closer examination of the answers reveals interesting differences. In most cases, dependent corporate financial advisers and private equity funds rely on CAPM. On the other hand, independent corporate financial advisers seem to favour their own experience. This difference is significant at the 0.1 level. Several of the independent corporate advisers argue that a ‘common-sense-approach’ is applicable on smaller

businesses where reliable beta estimates are difficult to obtain. As stated by one of the participants: “it does not make sense to apply beta estimates from blue chip stocks on privately held firms – in fact, in most cases it is difficult to find a good match”.

Insert table 8 here

Table 8 shows how the 25 participants adopting the CAPM estimate the systematic risk component. As can be seen from table 8 there is a significant difference in the use of estimation procedures of beta values (p-value = 0.042). 17 (68%) participants apply beta estimates from peers. 8 participants (32%) obtain beta estimates based on fundamental drivers that are expected to affect operational as well as financial risk. 14 (56%) participants obtain beta estimates based on their experience (from previous transactions). The results in table 8 also show that the participants do not purely rely on one method. Often participants combine beta estimates from peers with either beta estimates based on fundamental risk drivers or previous experience. These results remain robust across the three groups of participants. A chi-square test reveal that there are no significant differences in the way participants estimate beta values (p-value = 0.586). The empirical results seem to be in line with the recommendations outline in the valuation literature. For example, Damodaran (2002) suggests using beta estimates from peers and using financial fundamentals, such as earnings variability, dividend payout, and leverage to predict betas for privately held firms.

Out of 17 participants that apply beta estimates from peers only twelve (71%) adjust for differences in financial leverage as recommended in the valuation literature (results not tabulated). They tend to favour the Modigliani and Miller (1963) beta formula for unlevering/levering beta estimates.<sup>17</sup>

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<sup>17</sup> See for example Copeland et al. (2000), Pratt et al. (2000) and Damodaran (2002).

$$\beta_{\text{equity}} = \beta_{\text{asset}} \cdot \left( 1 + (1 - t) \cdot \frac{\text{Debt}}{\text{Equity}} \right)^{18} \quad (2)$$

where

$\beta$  = Systematic risk

$t$  = Corporate tax rate

The M&M beta relation rests on the heroic assumption that debt remains at the same *level* across time. However, empirical evidence supports that *the debt to equity ratio* tends to remain *constant* across time (Jalilvad and Harris, 1984 and Graham and Harvey, 2001). Thus, the M&M beta relation is a relatively poor description of the actual financial leverage in most businesses. If the underlying assumption is a *constant debt to equity ratio* the beta relation should be expressed as in Harris and Pringle (1985):

$$\beta_{\text{equity}} = \beta_{\text{asset}} \cdot \left( 1 + \frac{\text{Debt}}{\text{Equity}} \right) \quad (3)$$

However, only three participants apply the Harris and Pringle beta formula and it is significantly less popular than the M&M beta formula (p-value = 0.083). 5 (29%) participants do not adjust for differences in financial leverage. The effect is a bias in the beta estimates that ultimately will affect firm value estimates. In most cases the participants were not familiar with the methodology used to unlever/lever beta.

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<sup>18</sup> Assuming that debt is risk free ( $\beta_{\text{debt}}=0$ )

Parum (2001) lists previous studies on risk premiums in Denmark and finds that it varies from 1.9% to 3.2%. In our study the average (median) risk premium used for CAPM calculations is 4.5% and the standard deviation is small (results not tabulated). Further, the three groups of participants seem to apply the same level of risk premium (p-value = 0.638). Thus, there seem to be a mutual agreement on the risk premium on the Danish market. The discrepancy between the risk premium measured by academics and the risk premium applied by practitioners in Denmark is puzzling. A possible explanation might be that other studies often find a higher risk premium. For example, Jorion and Goetzmann (2000) find that the average risk premium based on US data is 4.3%.

The CAPM theory assumes that investors are well-diversified and investors should only be compensated for systematic risk. Valuations of privately-held firms, however, often involve owners that are not well-diversified. Both Pratt et al. (2000) and Damodaran (2002) suggests that valuation of privately-held firms should take into account non-systematic risk<sup>19</sup>, if the owners are not well-diversified. Pratt et al recommend a discount that takes into account non-systematic risk.

Damodaran, on the other hand, recommends that beta includes both systematic and non-systematic risk. He dubs it ‘total beta’. The interviews reveal that the majority of participants do not follow these recommendations. 33 participants consider unsystematic risk as irrelevant (results not tabulated). Only two (6%) participants include unsystematic risk. They add an additional 1-3 percentage points to cost of equity to compensate for unsystematic risk. Thus, the participants seem to ignore the recommendations by Pratt et al. (2000) and Damodaran (2002).

Table 9 presents how participants estimate cost of debt.

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<sup>19</sup> Non-systematic risk is also called firm-specific risk and diversifiable risk.

Insert table 9 here

As pointed out by Damodaran (2002) the cost of debt is often known. In cases where information on cost of debt is not available a cost of debt estimate based on a synthetic rating can be used. In our study the participants generally apply the efficient interest rate. The efficient interest rate (from the transaction) is significantly more used than any other methods (p-value = 0.001). However, there are also differences in the way the three groups of participants estimate the cost of debt (p-value = 0.071). For example, dependent corporate financial advisers also estimate cost of debt based on a synthetic rating<sup>20</sup> (55%). Furthermore, they also apply industry averages (27%). Private equity funds rely almost entirely on the efficient interest rate. Since they have superior knowledge of the efficient interest rate from a transaction this finding is expected. Independent corporate financial advisers also rely on the efficient interest rate (91%). For example, only one independent corporate financial adviser applies synthetic rating. The overall impression from the interviews is, however, that the cost of debt can be estimated with a fairly high degree of precision.

Over the past few years private equity funds have been aggressive on the Danish M&A market. They focus on leveraged buy-outs (LBO) and banks that have been willing to provide sufficient funding for the LBOs. Since most LBOs are characterised by an aggressive repayment scheme a variable capital structure (WACC) would mirror the underlying capital structure much better. On the other hand, Levin and Olsson (1995) demonstrate that the error from using a constant capital structure instead of an annually rebalancing is modest and can be ignored for practical purposes.

Our results seem to support the recommendations by Levin and Olsson (1995). 30 (91%) participants apply a constant capital structure. Only one private equity fund and four corporate

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<sup>20</sup> In most cases, the synthetic rating is based on the interest coverage ratio (EBIT/Interest expense) only. Thus, they deviate from credit rating institutions such as Standard & Poors and Moodys who apply a range of financial ratios to classify firms into different risk classes.

financial advisers adjust for changes in the underlying capital structure (results not reported). For example, most private funds argue that the application of a variable capital structure would only have a minor impact on firm value estimates, which implies that it is not worth the effort.

The estimation of a target capital structure is a non-trivial issue since market prices on equity and debt are not observable for privately held firms. Participants must rely on different estimation procedures and the literature has suggested different techniques. For example, Damodaran (2002) suggest using an industry average or an iteration procedure. While both methods are used in practice the capital structure from peers is significantly more used than the iteration procedure (p-value = 0.013). Although not reported our results show that 23 (88%) participants were inspired by the capital structure from listed peers and nine (35%) participants estimate the target capital structure through iteration. There seems to be no variation in the use of estimation procedure of the capital structure across the three groups of participants (p-value = 0.892).

### **Special adjustments of estimated firm value**

The valuation literature on privately-held firms recommends adjustments for <sup>1)</sup> lack of marketability<sup>21</sup> and <sup>2)</sup> ownership control.<sup>22</sup> Pratt et al. (2000) argue that the most common valuation premiums and discounts relate to the degree of ownership and marketability. A controlling interest is considered to have greater value than a minority interest because it gives investors the ability to affect changes in the overall business structure and to influence business politics. A marketable ownership is considered attractive since it gives the ability to convert the ownership to cash quickly,

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<sup>21</sup> A marketable ownership interest is one that can be sold and converted into cash quickly and at a low transaction cost.

<sup>22</sup> A controlling interest is considered to have greater value than a minority interest because it gives the ability to affect changes in the overall business structure and to influence business politics.

with minimum of transaction cost and with a high degree of certainty of realising the expected net proceeds.

Researchers have addressed the illiquidity (marketability) issue in different ways. For example, Amihud and Mendelson (1986) suggest that the value of an asset is reduced by the present value of expected transaction costs, thus creating a discount on value. Kooli et al. (2003) list previous empirical studies on marketability discount and find the following discounts:

1. Restricted Stock Approach (average discount: 35%)
2. IPO-Approach (average discount: 44%)
3. Acquisition Approach (average discount: 24%)

They conclude that based on US studies the average marketability discount is between 24% and 44%. Mayers (1973), on the other hand, argue that the required rate of return on an asset should be adjusted to reflect its illiquidity, with a higher required rate of return for less liquid assets. Acharyan and Pedersen (2003) estimate that illiquid stocks annualised risk premiums are about 1.1% higher than for liquid stocks. Datar et al. (1998) find that illiquid stocks have an annual return that is about 3.25% higher than liquid stocks. Thus, previous studies strongly support that illiquidity is priced. Table 10 panels A and B provide statistics of how the participants adjust firm value for lack of liquidity.

Insert table 10 here

19 participants reduce the estimated market value of equity. The average (median) marketability discount is 31% (30%) which corresponds with the findings based on US data. Furthermore, there

are no significant differences in the level of marketability discount across the three groups of participants (p-value = 0.250). 23 participants add a risk premium to cost of equity to compensate for illiquidity. The average (median) risk premium for illiquidity is 2.8% (2.0%) and there are no significant variations in the risk premium for illiquidity across the three groups of participants (p-value = 0.736). This average risk premium for illiquidity lies between the estimated illiquidity premium of 1.1% and 3.25% based on US data. Thus, the participants seem to incorporate the issue of liquidity in a manner that is consistent with theory and previous empirical studies.

Damodaran (2005) argue that gaining control over a firm adds value as the company might be run more efficiently. Further, the value of control is greater for poorly managed firms than well run ones. Pratt et al. (2000) find support for this view. They find that the average size of the control premium is 26%-45% in the period 1990-1999. Jarrell and Poulsen (1989) find that the average control premium paid for target firms in acquisitions in the US has been between 20% and 30% in the 1980s. Lang and Walking. (1989) find that the premium tend to be slightly higher for hostile acquisitions. Table 10 panel C provides evidence on control premiums on Danish data. As shown in table 10 panel C only a few participants include a control premium. Most participants include the effect from ownership control in the financial forecasts. Thus, to avoid double counting they exclude a control premium. Eight participants argue that if synergies were not included in the forecasts they would apply a control premium. The average (median) control premium is 29% (30%).<sup>23</sup> Thus, the average size of control premium in Denmark seems to correspond with the estimated US control premium.

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<sup>23</sup> Since the number of observations (8) is low a test for differences in control premium across the three groups of participants has not been performed.

## CONCLUSIONS

Our field study of how privately held firms are valued in practice are both reassuring and puzzling. For example, it is reassuring that the majority of practitioners apply present value approaches. The practitioners also apply a target capital structure and cost of debt in line with the recommendations in the valuation literature. On the other hand, many practitioners do not adjust for differences in financial leverage when applying beta estimates obtained from peers. Some practitioners apply multiples as a mean to estimate terminal value. These findings indicate that some practitioners do not apply the present value approaches correctly.

Some participants argue that the valuation literature is inadequate in certain areas. For example, several participants argue that beta estimates from peers contain measurement problems. Thus, to examine the validity of the beta estimates they also examine fundamental risk drivers. The marketability discount and control premium also involve a great deal of guesswork.

We identify interesting differences between independent corporate financial advisers and other participants. For example, it is primarily independent corporate financial advisers that adjust historical accounting numbers for private expenses. Further, while dependent corporate financial advisers and private equity funds emphasise beta values from peers independent corporate financial advisers favour beta values based on fundamental risk drivers. Even when they apply beta estimates from peers they do not adjust for differences in financial leverage.

Our research shows that practitioners apply a shorter forecast horizon than suggested by Copeland et al. (2000). Only one participant applies a forecasting period that exceeds ten years. Copeland et al.'s suggest that this may lead to significant undervaluation of a firm. This raises the question

whether practitioners are using a forecast horizon that is too short or if the academics are exaggerating the forecast horizon? Our research also shows that private equity funds use different versions of the LBO model and that they ignore the effect of changes in the capital structure on cost of capital and firm value. These issues beg questions such as why private equity funds apply different versions of the LBO model and why the interest tax shield is not correctly incorporated in the estimated firm value.

Our research also indicate that valuation of privately held firms often involve investors that are not well-diversified. While the valuation literature suggests that unsystematic risk should be included if investors are not well-diversified, most corporate financial advisers and private equity funds ignores it. This discrepancy between theory and practice deserve further attention. In summary, additional research may shed light on these issues. Finally, our study is based on Danish data. Obviously, further research examining the same issue in different institutional settings help improve our understanding of valuation of privately held firms.

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Participants (Category)	Total number of possible participants	Number of participants included in the sample	Acceptance rate	Adoption of a present value approach
Private Equity Funds	15	15	100,0%	13
Dependent corporate financial advisers	12	11	91,7%	11
Independent corporate financial advisers	17	16	94,1%	15
Total	44	42	95,5%	39

	Average	Median	Std. dev.	Participants
Dependent	6.7	6.0	4.0	11
Independent	12.3	10.0	8.5	16
Private Equity	7.3	7.0	2.8	15
Total	9.2	7.5	6.4	42

Kruskal-Wallis test of equal medians across the three groups of participants (Median<sub>dependent</sub> = Median<sub>independent</sub> = Median<sub>private equity</sub>): p-value = 0.099

	Master	Bachelor	Other	Total
Dependent	11	0	0	11
Independent	11	4	1	16
Private Equity	15	0	0	15
Total	37	4	2	42

Fisher's Exact test of equal level of education across the three groups of participants (dependent = independent = private equity): p-value = 0.035

Fisher's Exact test of equal level of education across independent and dependent+private equity funds (dependent = independent+ private equity): p-value = 0.01

Table 3

## Panel A

Type of services offered by corporate financial advisers

	M&A	Value based Management	Fund Raising	Expert witness	Other
Dependent	11	4	10	6	6
Independent	17	1	3	2	5
Total	28	5	13	8	11

Fisher's Exact test of equal type of services offered by dependent and independent corporate financial advisers, respectively: p-value = 0.107

## Panel B

Proportion of business that involves privately-held firms

	0-25%	25-50%	50-75%	75-100%	Total
Dependent	2	3	5	1	11
Independent	0	1	0	15	16
Total	2	4	5	16	27

Fisher's Exact test of equal distribution of businesses that involves privately-held companies across dependent and independent corporate financial advisers, respectively: p-value = 0.001

## Panel C

Size (based on revenue) on clients

(mio. DKK)	10-100	100-500	500-1,000	1,000+
Dependent	10%	16%	37%	37%
Independent	50%	29%	13%	8%

$\chi^2$  of equal distribution of size on clients across dependent and independent corporate financial advisers = 23.51, p-value= 0.001

Table 4

Which present value approach do you apply on privately-held firms?

	DDM	DCF	EVA	APV	Other (LBO)	Participants
Dependent	45.5%	100.0%	45.5%	9.1%	9.1%	11
Independent	14.3%	85.7%	7.1%	0.0%	21.4%	14
Private equity	0.0%	78.6%	14.3%	0.0%	57.1%	14
Total	17.9%	87.2%	20.5%	2.6%	30.8%	39

a. DCF = Discounted cash flow model

b. DDM = Dividend discount model

c. EVA = Economic value added

d. APV = Adjusted present value

$\chi^2$  of equal use of present value approaches: p-value = 0.001

Fisher's Exact test of equal use of present value approaches across the three groups of participants (dependent = independent = private equity) = 52.03, p-value= 0.002

Figure 1

Information regarded as important by private equity funds and corporate financial advisers

	Quantitative	Qualitative
Internal Information	Budgets/forecasts Internal accounts Tax accounts Historical budgets Financial information at segment level	Interviews with key persons Business plan Dividend policy Investment plans Policy on capital structure Key customers Key suppliers Management letter Memos from board meetings Memos from management meetings
External information	Financial statements Employee stock options Employee stock ownership Ownership structure	Industry information Competitor information Stock reports from listed peers

Table 5

What is the average forecasting horizon?

	1 year	1-5 years	6 - 8 years	9 - 12 years	+ 12 years	Average (years)	Participants
Dependent	0.0%	46.2%	33.3%	23.1%	0.0%	6.1	39
Independent	0.0%	27.3%	36.4%	36.4%	0.0%	7.2	11
Private equity	0.0%	64.3%	21.4%	7.1%	0.0%	4.2	14
Total	0.0%	42.9%	42.9%	14.3%	0.0%	5.8	14

Fisher's Exact test of equal length of forecast horizon across the groups of participants (dependent = independent = private equity): p-value = 0.41

 $\chi^2$  of length of forecast horizon  $\leq 10$ , p-value= 0.001

Table 6  
How do you estimate the terminal value?

	Gordon's growth model	Value driver model	Convergence model (ROIC=WACC)	Multiple	None	Participants
Dependent	90.9%	27.3%	18.2%	18.2%	0.0%	11
Independent	66.7%	0.0%	16.7%	8.3%	8.3%	12
Private equity	83.3%	16.7%	16.7%	16.7%	0.0%	12
Total	80.0%	14.3%	17.1%	14.3%	2.9%	35

$\chi^2$  of equal use of the different terminal value methodologies (Gordon = Value driver model = Convergence = Multiple = None) = 51.8, p-value = 0.001

Fisher's Exact test of equal use of different terminal value methodologies across the three groups of users (dependent = independent = private equity): p-value = 0.877

Table 7  
How do you estimate cost of equity

	CAPM <sup>a</sup>	Experience/required by client	Participants
Dependent	90.9%	27.3%	11
Independent	53.8%	69.3%	13
Private equity	72.7%	36.4%	11
Total	71.4%	45.7%	35

a. CAPM is an abbreviation for Capital Asset Pricing Model

$\chi^2$  of equal use of CAPM and experience/required by client = 1.98, p-value = 0.160

$\chi^2$  of equal use of CAPM and experience/required by client across independent and dependent+private equity (dependent = independent + private equity) = 3.27, p-value = 0.071

Table 8  
How do you estimate Beta values?

	Peers	Fundamental beta	Experience	Other	Participants
Dependent	100.0%	30.0%	70.0%	10.0%	10
Independent	42.9%	57.1%	42.9%	14.3%	7
Private equity	62,5%	25,0%	50,0%	37,5%	8
Total	68,0%	32,0%	56,0%	20,0%	25

$\chi^2$  of equal use of estimation procedures of beta values (peers = fundamental beta = experience) = 8.18, p-value = 0.042

Fisher Exact test of equal use of estimation procedures of beta values across the three groups of participants (dependent = independent = private equity): p-value = 0.586

Table 9  
How do you estimate cost of debt?

	Book value	Efficient interest rate	Industry average	Synthetic rating	Participants
Dependent	9,1%	63,6%	27,3%	54,5%	11
Independent	0,0%	90,9%	18,2%	9,1%	11
Private equity	0,0%	100,0%	0,0%	9,1%	11
Total	3,0%	75,8%	15,2%	24,2%	33

$\chi^2$  of equal use of estimation procedures of interest rates (book value = efficient interest rate = industry average = synthetic rating) = 34.3, p-value = 0.001

Fisher's Exact test of equal estimation procedures of interest rates across the three groups of participants (dependent = independent = private equity): p-value = 0.071

Table 10  
Panel A

What is the average marketability discount?

	Average	Median	Std. dev.	Participants
Dependent	25.0%	22.5%	8.9%	6
Independent	33.8%	30.0%	11.6%	8
Private equity	35.0%	30.0%	14.1%	5
Total	31.3%	30.0%	11.8%	19

Kruskal-Wallis test of equal medians across the three groups of participants (Median<sub>dependent</sub> = Median<sub>independent</sub> = Median<sub>private equity</sub>): p-value = 0.25

Panel B

What is the average risk premium (to cost of equity) due to illiquidity?

	Average	Median	Std. dev.	Participants
Dependent	2.6%	2.0%	1.56%	10
Independent	3.0%	2%	1.64%	10
Private equity	3.3%	4%	1.15%	3
Total	2.8%	2%	1.51%	23

Kruskal-Wallis test of equal medians across the three groups of participants (Median<sub>dependent</sub> = Median<sub>independent</sub> = Median<sub>private equity</sub>): p-value = 0.736

Panel C

What is the average control premium

	Average	Median	Std. dev.	Participants
Dependent	25.0%	25.0%	7.1%	2
Independent	50.0%	50.0%	NA	1
Private equity	27.0%	30.0%	4.5%	5
Total	29.4%	30.0%	9.4%	8